Determining η/s in Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ GeV and below via a statistical analysis of a hybrid model

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ABSTRACT

We perform a systematic comparison of a hybrid UrQMD + (3+1)D viscous hydrodynamics model [1] to data from the RHIC BES program:

- Utilize state-of-the-art analysis, based on Bayesian statistics [2, 3]
- Probe multiple model parameters, including fundamental quark-gluon plasma properties such as the specific shear viscosity η/s
- Calibrate the model to optimally reproduce experimental data
- Extract quantitative constraints for all parameters simultaneously

Bayes' theorem

Given a set $X = \{\vec{x}_k\}_{k=1}^N$ of points in parameter space and a corresponding set $Y = \{\vec{y}_k\}_{k=1}^N$ of points in observable space,

$$P(\vec{x}^*|X, Y, \vec{y}^{\exp}) \propto P(X, Y, \vec{y}^{\exp}|\vec{x}^*) P(\vec{x}^*)$$

- $P(\vec{x}^*|X, Y, \vec{y}^{\exp})$ is the *posterior* probability distribution of $\vec{x}^*$ for given $(X, Y, \vec{y}^{\exp})$
- $P(\vec{x}^*)$ is the *prior* probability distribution (simplest case: ranges of parameter values)
- $P(X, Y, \vec{y}^{\exp}|\vec{x}^*)$ is the *likelihood* of $(X, Y, \vec{y}^{\exp})$ for given $\vec{x}^*$ (to be determined with statistical analysis)

$$P(X, Y, \vec{y}^{\exp}|\vec{x}^*) = \exp\left(-\tfrac{1}{2}(\vec{y}^* - \vec{y}^{\exp})^T \Sigma^{-1} (\vec{y}^* - \vec{y}^{\exp})\right),$$

where

- Σ is the covariance matrix. In this study $\Sigma = \mathrm{diag}(\sigma^2 \vec{y}^{\exp})$, with $\sigma \approx 0.05$ as a global estimate of relative uncertainty
- $\vec{y}^*$ is model output for the input parameter point $\vec{x}^*$

$\Rightarrow$ Need a way to predict model output for arbitrary input parameter point

Gaussian process

Model emulation using Gaussian processes: Set Y_a of values of observable y_a, corresponding to set X of points in parameter space, has a multivariate normal distribution:

$$\mathcal{G} : X \to Y_a \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} = \mu(X) = \{\mu(\vec{x}_1), ..., \mu(\vec{x}_N)\}$ is the mean and $\boldsymbol{\Sigma}$ is the covariance matrix with covariance function $\sigma(\vec{x}, \vec{x}')$ (model-dependent choice; constant, linear, exponential, periodic, ...).

Choice: Squared-exponential covariance function with a noise term

$$\sigma(\vec{x}, \vec{x}') = \theta_0 \exp\left(-\sum_{i=1}^{n} \frac{(x_i - x_i')^2}{2\theta_i^2}\right) + \theta_{\text{noise}} \delta_{\vec{x}\vec{x}'}$$

The *hyperparameters* $\vec{\theta} = (\theta_0, \theta_1, ...\theta_n, \theta_{\text{noise}})$ are not known a priori and must be estimated from the simulation output (emulator training)

Markov Chain Monte Carlo

The posterior distribution is sampled with Markov Chain Monte Carlo (MCMC) method

- Random walk in parameter space, where each step is accepted or rejected based on a relative likelihood
- Converges to posterior distribution as number of steps $N \to \infty$

References

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This work has been supported by NSF grant no. PHY-0941373 and by DOE grant no. DE-FG02-05ER41367. CPU time was provided by the Open Science Grid, supported by DOE and NSF.

Transport+hydrodynamics hybrid model

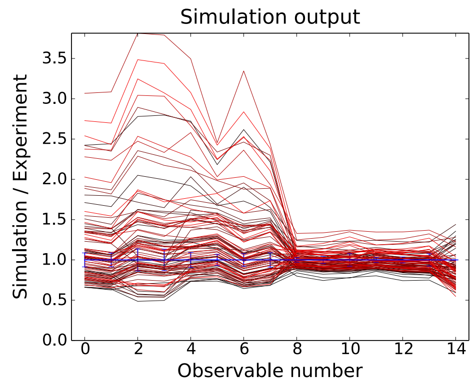
- Initial State from UrQMD [4, 5] hadron+strings cascade
- Start the hydrodynamical evolution when nuclei have passed through each other: $\tau_0 \geq \frac{2R_{\text{nucleus}}}{\sqrt{\gamma_{CM}^2 - 1}}$
- Particle properties (energy, baryon number) to densities: 3D Gaussians with "smearing" parameters R_{trans}, R_{long} ($\equiv \sqrt{2}\sigma$)
- 3+1D viscous hydrodynamics [6] with viscosity parameter η/s
- Transition from hydro back to UrQMD ("particlization") when energy density $\epsilon \leq \epsilon_C$

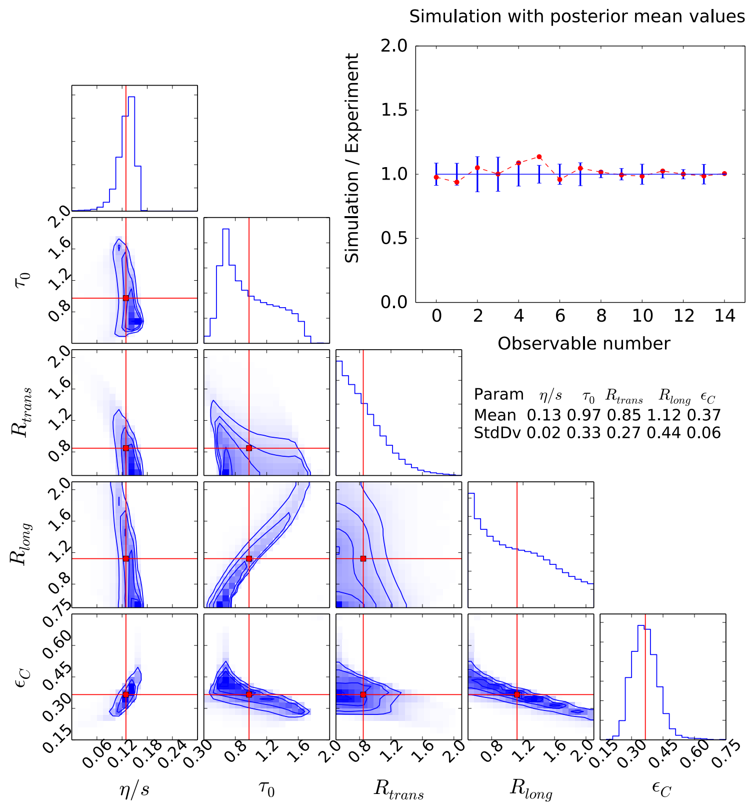
Investigated input parameters: τ_0, R_{trans}, R_{long}, η/s, ϵ_C

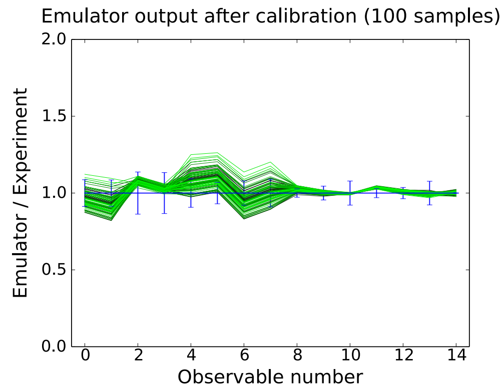
Results at $\sqrt{s_{NN}} = 62.4$ GeV

The following observables were used in the analysis (Observable ID in parenthesis):

- N_{ch} in $|\eta| < 0.5$ in (0-5)%, (10-20)% centrality [STAR] (0, 1)
- $N_{\bar{p}}$ at $y = 0$ in (0-15)%, (15-35)% centrality [PHOBOS] (2 ,3)
- $dN_{\text{ch}}/d\eta$ at $\eta = 0.1$ and 1.1 in (0-3)%, (20-25)% centrality [PHOBOS] (4 - 7)
- $\langle p_T \rangle$ for π^-, K^+, p in (0-5)%, (20-30)% centrality [STAR] (8 - 13)
- $v_2\{\text{EP}\}$ in $|\eta| < 0.3$ in (10-40)% centrality [STAR] (14)





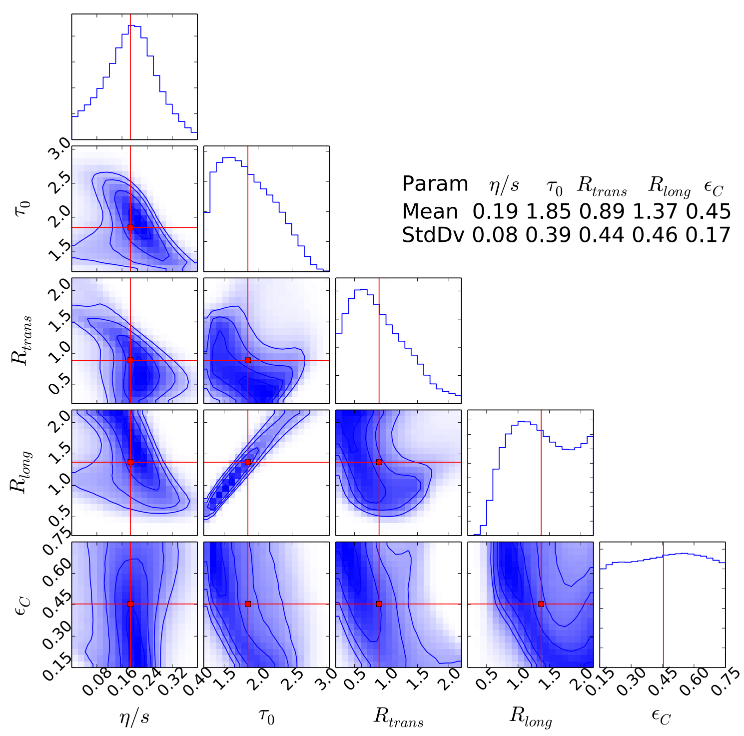


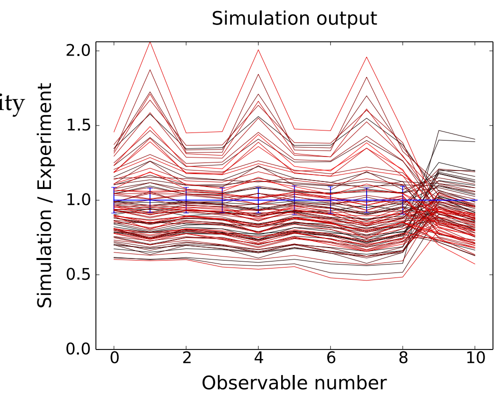
- Smaller values of τ_0 preferred by $dN_{\text{ch}}/d\eta$
- ϵ_C constrained mainly by $\langle p_T \rangle$
- η/s is limited below by $\langle p_T \rangle$ and above by $v_2\{\text{EP}\}$
- Good agreement with experimental data achieved using average input parameter values

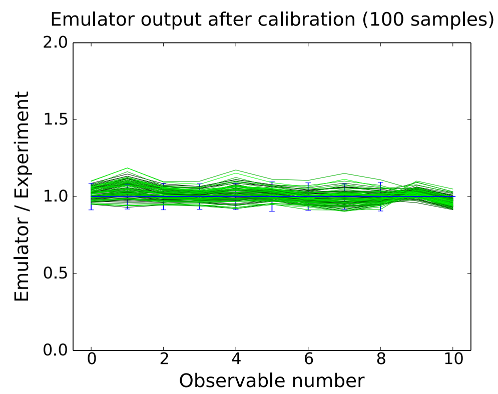
Results at $\sqrt{s_{NN}} = 19.6$ GeV

The following observables were used in the analysis (Observable ID in parenthesis):

- $dN_{\text{ch}}/d\eta$ at $\eta = -1.1, 0.1$ and 1.1 in (0-6)%, (6-15)%, (15-25)% centrality [PHOBOS] (0-8)
- $v_2\{\text{EP}\}$ in $|\eta| < 1.0$ in (10-20)%, (20-30)% centrality [STAR] (9-10)







- Variance remains large for all parameters; information about p_T spectra of particles is necessary to achieve more decisive analysis results