QCD inspired determination of NJL–parameters

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The phase diagram of QCD

The detailed structure of the **QCD phase diagram** at high baryon densities is still under investigation. **Low-energy models** provide us with some guidance **but**:

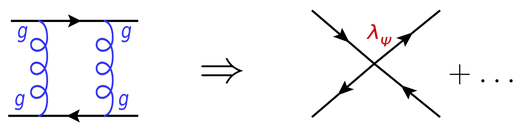
- parameters of the models are fine-tuned (scale Λ_{NJL} and couplings at Λ_{NJL})
- parameters do not depend on T and μ
- typically not all possible interaction channels are included (not Fierz complete)

$\Rightarrow$ **Models require improvement from the point of view of the fundamental theory**

From QCD to the low-energy model

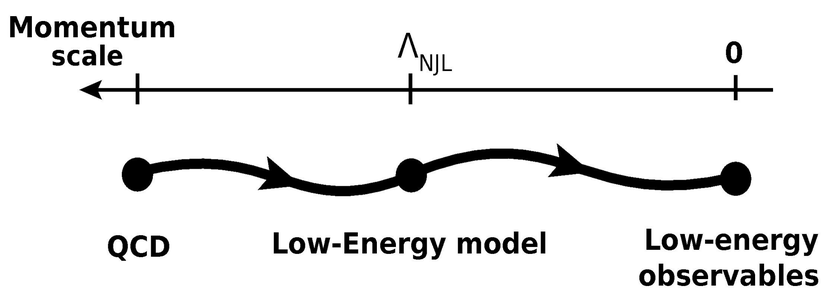
QCD in the chiral limit:

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g}A\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \tfrac{1}{4}F_z^{\mu\nu}F_{\mu\nu}^z$$



- Quark self-interactions λ_ψ are induced by quark-gluon interactions
- **Large** λ_ψ triggers **chiral symmetry breaking**

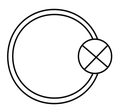
Basic idea:



Functional Renormalization Group (FRG) and Model

Wetterich's flow equation [1]:

$$k\partial_k\Gamma_k[\Phi_k] = \frac{1}{2}\mathrm{Tr}k\partial_k R_k\left(\Gamma_k^{(2)}[\Phi_k] + R_k\right)^{-1} = \frac{1}{2}$$



Γ_k: scale-dependent effective action $\qquad$ R_k: IR cut-off function

$$\lim_{k\to\Lambda}\Gamma_k[\phi_k] = S[\phi] \qquad \lim_{k\to 0}\Gamma_k[\phi_k] = \Gamma[\phi]$$

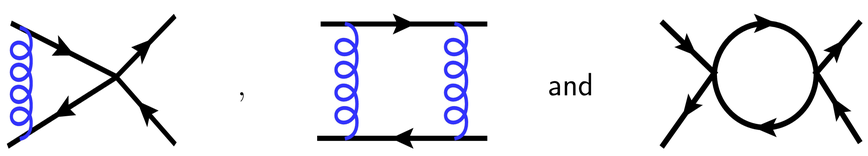
Fierz complete ansatz [2]:

$$\Gamma_k = \int \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g}A\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{1}{4}F_z^{\mu\nu}F_{\mu\nu}^z + \frac{(\partial_\mu A^\mu)^2}{2\xi} + \frac{1}{2}\Big[\bar{\lambda}_-(\mathsf{V}-\mathsf{A}) + \bar{\lambda}_+(\mathsf{V}+\mathsf{A}) + \bar{\lambda}_\sigma(\mathsf{S}-\mathsf{P}) + \bar{\lambda}_{\mathsf{VA}}[2(\mathsf{V}-\mathsf{A})^{\mathsf{adj}} + 1/\mathsf{N_C}(\mathsf{V}-\mathsf{A})]\Big] ,$$

with

- $(\mathsf{V}-\mathsf{A}) = (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2$
- $(\mathsf{V}+\mathsf{A}) = (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2$
- $(\mathsf{S}-\mathsf{P}) = (\bar{\psi}^a\psi^b)(\bar{\psi}^b\psi^a) - (\bar{\psi}^a\gamma_5\psi^b)(\bar{\psi}^b\gamma_5\psi^a)$, with $(a, b, \ldots)$ flavor indices
- $(\mathsf{V}-\mathsf{A})^{\mathsf{adj}} = (\bar{\psi}\gamma_\mu T^z\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5 T^z\psi)^2$, with T^z SU($\mathsf{N_C}$) generators
- Landau gauge
- Running coupling $g(k)$ is taken as input from [3]
- 4-fermion couplings are set to zero at initial scale $\Lambda_{\text{UV}} = 20$ GeV

$\Rightarrow$ Coupled equations for $\bar{\lambda}_i$, where all of them include contributions proportional to:



Mapping effective couplings in QCD to those in NJL

- **Idea**: We project our Fierz-complete ansatz for the 4-fermion interactions from QCD at a particular scale $k = \Lambda_{\text{NJL}}$ onto the 4-quark interaction channels used in conventional NJL/QM-type model studies
- Here: Consider only conventional scalar-pseudoscalar channel in the NJL model:

$$\Gamma_k = \int \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{\bar{\lambda}_{\sigma,\text{NJL}}}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2] .$$

- We work with the partially bosonized version of the action of the NJL model:

$$\Gamma_k = \int \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0\mu)\psi + \frac{1}{2}(\partial_\mu\bar{\Phi})^2 + \mathrm{i}\bar{h}\bar{\psi}(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi + \frac{1}{2}\bar{m}^2\bar{\Phi}^2 + \frac{1}{8}\bar{\lambda}_\Phi\bar{\Phi}^4 ,$$

where $\bar{\Phi}^T = (\sigma, \vec{\pi})$, $\lim_{k\to\Lambda_{\text{NJL}}}\bar{\lambda}_\Phi = 0$ and $\lim_{k\to\Lambda_{\text{NJL}}}\frac{\bar{h}^2}{\bar{m}^2} = \bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$

- **Fix** Λ_{NJL} such that the low-energy model yields the correct results for the constituent quark mass, pion decay constant, and pion mass
- Keep Λ_{NJL} fixed for finite T-studies
- $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$ is now a **function of temperature** (*see figure to the right*)

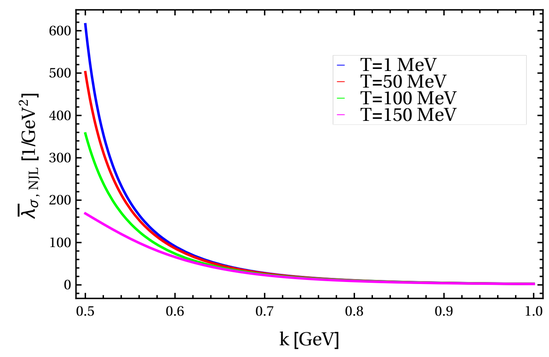


Figure : 4-quark coupling $\bar{\lambda}_{\sigma,\text{NJL}}$ as a function of k for four different values of the temperature in the regime $k > \Lambda_{\text{NJL}}$.

Investigate the low-energy model

- on the **mean field** level
- **with fluctuations** included

Results for $\mu = 0$

Mean field

	4d regulator [4]		3d regulator [5]	
	Λ_{NJL} in MeV	T_C in MeV	Λ_{NJL} in MeV	T_C in MeV
constant $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$	518	113	561	145
T-dependent $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$		100		112

With fluctuations

	4d regulator		3d regulator	
	Λ_{NJL} in MeV	T_C in MeV	Λ_{NJL} in MeV	T_C in MeV
constant $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$	540	104	611	155
T-dependent $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$		95		128

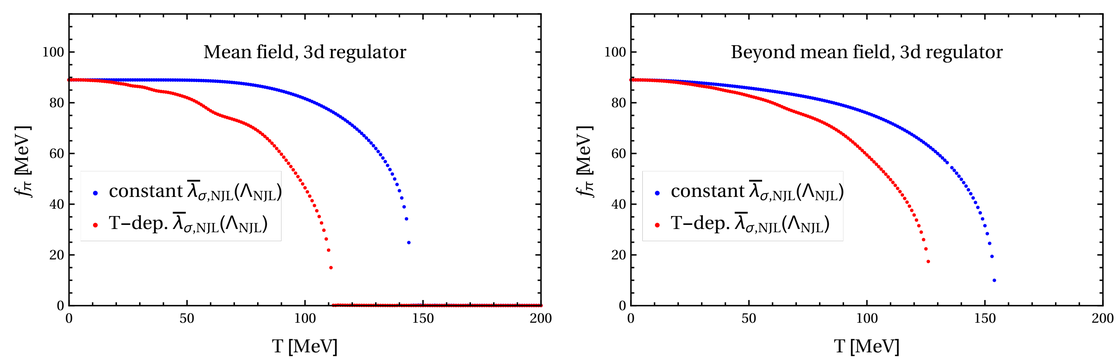


Figure : Pion decay constant (order parameter) as function of temperature. We consider constant and temperature dependent starting parameter $\bar{\lambda}_{\sigma,\text{NJL}}(\Lambda_{\text{NJL}})$. *Left:* **Mean field**, *right:* **with fluctuation effects**.

Conclusion and outlook

Conclusion:

- Estimate of low-energy QCD model parameters from QCD RG-flows
- T-dependent model parameters alter the T-dependence of low-energy observables as well as the critical temperature
- Present results still show regulator dependence

Outlook:

- Compute T- and μ-dependence of the model parameters and employ them to study the phase diagram in the (T, μ) plane
- Fierz-complete ansatz for interactions in the NJL-model

References

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