The phase diagram of QCD

The detailed structure of the QCD phase diagram at high baryon densities is still under investigation. **Low-energy models** provide us with some guidance but:

- parameters of the models are fine-tuned (scale $\Lambda_{NJL}$ and couplings at $\Lambda_{NJL}$)
- parameters do not depend on $T$ and $\mu$
- typically not all possible interaction channels are included (not Fierz complete)

$\Rightarrow$ Models require improvement from the point of view of the fundamental theory

From QCD to the low-energy model

**QCD in the chiral limit:**

$$\mathcal{L} = \bar{\psi}(i\partial - \vec{g}A + i\gamma_\mu A_\mu)\psi + \frac{1}{4}F_{\mu}^a F_{\mu}^a + \frac{g^2}{32} F_{\mu}^a F_{\mu}^a$$

- Quark self-interactions $\lambda_\psi$ are induced by quark-gluon interactions
- Large $\lambda_\psi$ triggers chiral symmetry breaking

Basic idea:

- **Momentum scale**
- **QCD**
- **Low-energy model**
- **Low-energy observables**

Functional Renormalization Group (FRG) and Model

**Wetterich’s flow equation** [1]:

$$\frac{\partial}{\partial k} \Gamma_k[\phi_k] = \frac{1}{2} Tr k \partial R_k \left( \Gamma_k[\phi_k] + R_k \right)^{-1} = \frac{1}{2} \bigcirc$$

$\Gamma_k$: scale-dependent effective action

$$\lim_{k \to \infty} \Gamma_k[\phi_k] = S[\phi]$$

$$\lim_{k \to 0} \Gamma_k[\phi_k] = \Gamma_0[\phi]$$

**Fierz complete ansatz** [2]:

$$\Gamma_k = \int_0^{\Lambda_{NJL}} \left[ \bar{\psi}(\partial \tau - \bar{A} + i\gamma_\mu A_\mu)\psi + \frac{1}{4} F_{\mu}^a F_{\mu}^a + \frac{(\partial \bar{A})^2}{2} \right]$$

$$+ \frac{1}{2} \left[ \lambda_\psi (V - A) + \lambda_\sigma (V + A) + \lambda_\lambda (S - P) \right] + \lambda_{NJL} [2(V - A)^4k + 1/N_c(V - A)]$$

with

- $V - A = (\bar{\psi}\gamma_\tau \psi)^2 + (\bar{\psi}\gamma_{\tau b} \gamma_\tau \psi)^2$
- $V + A = (\bar{\psi}\gamma_{\tau b} \gamma_\tau \psi)^2 - (\bar{\psi}\gamma_\tau \psi)^2$
- $S - P = (\bar{\psi}\gamma_\tau \psi)^2 - (\bar{\psi}\gamma_{\tau b} \gamma_\tau \psi)^2 - (\bar{\psi}\gamma_{\tau b} \gamma_\tau \psi)^2$, with $a, b, \ldots$ flavor indices
- $V - A)^4k = (\bar{\psi}\gamma_\tau T^a \psi)^2 + (\bar{\psi}\gamma_\tau \gamma_{\tau b} T^a \psi)^2$, with $T^a$ SU(Nc) generators
- Landau gauge
- Running coupling $g(k)$ is taken as input from [3]
- 4-fermion couplings are set to zero at initial scale $\Lambda_W = 20$ GeV

$\Rightarrow$ Coupled equations for $\lambda_\psi$, where all of them include contributions proportional to:

-Fierz-complete ansatz for interactions in the NJL-model

Results for $\mu = 0$

**Mean field**

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<tr>
<th>$\lambda_{NJL}/(\Lambda_{NJL})$</th>
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**With fluctuations**

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Conclusion and outlook

**Conclusion:**

- Estimate of low-energy QCD model parameters from QCD RG-flows
- $T$-dependent model parameters alter the $T$-dependence of low-energy observables as well as the critical temperature
- Present results still show regulator dependence

**Outlook:**

- Compute $T$- and $\mu$-dependence of the model parameters and employ them to study the phase diagram in the $(T, \mu)$ plane
- Fierz-complete ansatz for interactions in the NJL-model

References