Crystalline chiral condensate in an external magnetic field

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Motivation

Elucidate QCD phase structure by taking into account both of inhomogeneity and magnetic field

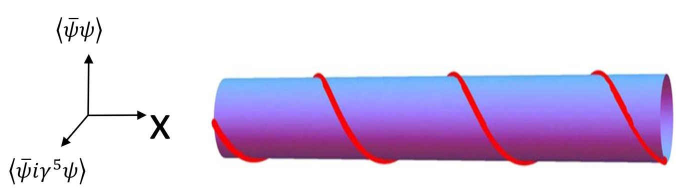
Preceding Study and problems

Inhomogeneous Chiral Condensate

$$\Delta(x) = -2G[\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau^3\psi\rangle]$$

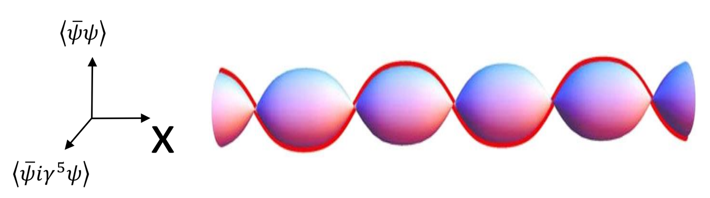
・Dual Chiral Density Wave(DCDW)
Plane wave type condensate

$$\Delta_{\mathrm{DCDW}}(x) = \lambda\, e^{iqx}$$



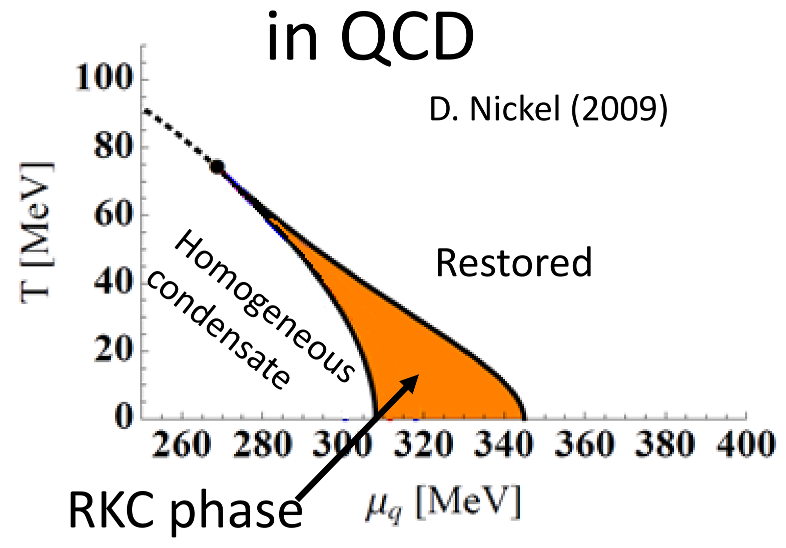
・Real kink Crystal(RKC)
Multi soliton type condensate

$$\Delta_{\mathrm{RKC}}(x) = \lambda\left(\frac{2\sqrt{\nu}}{1+\sqrt{\nu}}\right)\mathrm{sn}\left(\frac{2\lambda x}{1+\sqrt{\nu}};\nu\right)$$



G.Basar, et. al.(2009)

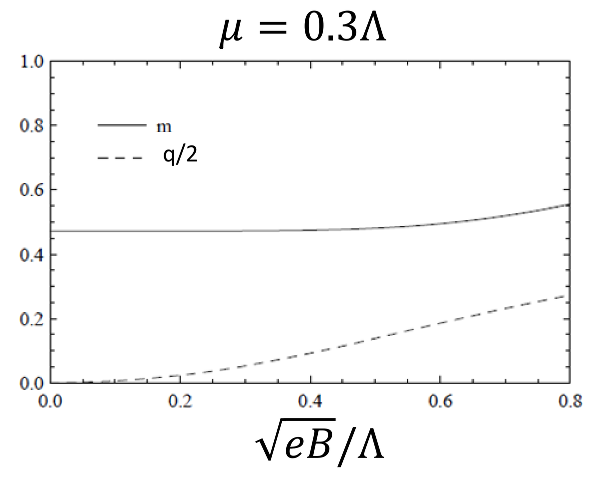
Inhomogeneous chiral Phase in QCD



Without magnetic field, RKC is favored

DCDW in Magnetic Field

I.E. Frolov, et. al. (2010)



Inhomogeneity grows with the increase of B

・Which phase is favored in external magnetic field?
・Why does magnetic field drive DCDW?

Hybrid condensate

$$\Delta(x) \coloneqq \mathrm{M}(x)e^{iqx} = \frac{2m\sqrt{\nu}}{1+\sqrt{\nu}}\mathrm{sn}\left(\frac{2mx}{1+\sqrt{\nu}};\nu\right)\times e^{iqx}$$

$$\Delta_{\mathrm{DCDW}}(x) \xleftarrow{\nu\to 1} \Delta(x) \xrightarrow{q\to 0} \Delta_{\mathrm{RKC}}(x)$$

Free energy calculation

・1 Particle Energy Spectrum

$$\mathrm{E}_{n,\zeta,\alpha} = \left(F_\alpha + \zeta\frac{q}{2}\right)\sqrt{1+\frac{2eBn}{\left(F_\alpha+\zeta\frac{q}{2}\right)^2}} \quad \text{n=1,2,.....}$$

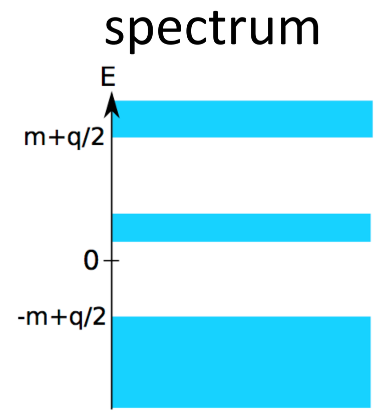
$\zeta = \pm 1$
n : Landau level
F_α : 1+1dim RKC spectrum
(symmetric with respect to F=0)

$$\mathrm{E}_{n=0,\alpha} = F_\alpha + \frac{q}{2} \quad \text{n=0}$$

$\mathrm{E}_{n\geq 1,\zeta,\alpha}$:symmetric
$\mathrm{E}_{n=0,\alpha}$:asymmetric(for q≠0)

B≠0 and q≠0, Energy spectrum is asymmetric.

spectrum



$E_{n=0}$

・Anomalous Quark Number Density

$$\left.\frac{N}{V}\right|_{T=0} = -\left.\frac{\partial\Omega}{\partial\mu}\right|_{T=0} = N_c\sum_f\frac{|q_fB|}{2\pi}\sum_\zeta\sum_{n=0}^{\infty}\int dF\,\rho(F)\theta(\mathrm{E}_{n\zeta F})\theta(\mu-\mathrm{E}_{n\zeta F})$$

$$+N_c\sum_f\frac{1}{2}\frac{|q_fB|}{2\pi}\sum_F\mathrm{sgn}(\mathrm{E}_{n=0,F})$$

anomalous quark number density : ρ_{anom}

For DCDW

$$\rho_{anom} = N_c\sum_f\frac{1}{2}\frac{|q_fB|}{2\pi}\frac{q}{\pi}$$

$$\Omega_{anom} = -\mu\rho_{anom} \propto \mu Bq$$

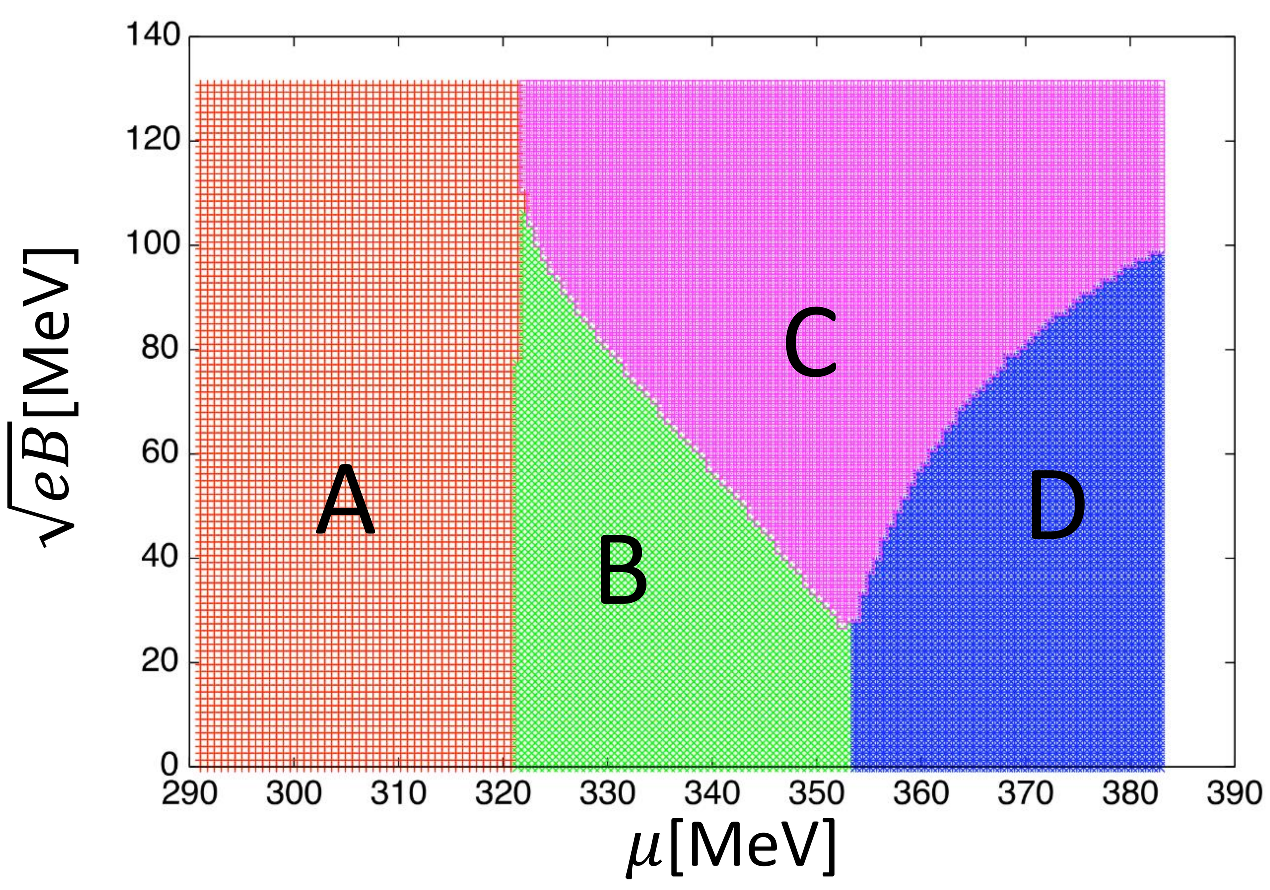
B≠0 , homogeneous phase is unstable

・Free Energy

$$\Omega = \frac{\langle|\Delta(z)|^2\rangle}{4G} - \frac{1}{\beta}N_c\sum_f\frac{|q_fB|}{4\pi^2}\sum_\zeta\sum_{n=0}^{\infty}\int dF\left[\frac{2F^2-2\left(m\frac{1-\sqrt{\nu}}{1+\sqrt{\nu}}\right)^2-m^2+\langle|\Delta(z)|^2\rangle}{\sqrt{(F^2-m^2)\left(F^2-\left(m\frac{1-\sqrt{\nu}}{1+\sqrt{\nu}}\right)^2\right)}}\right]\ln\left(1+e^{-\beta(\mathrm{E}_{n\epsilon\zeta}(\alpha)-\mu)}\right)$$

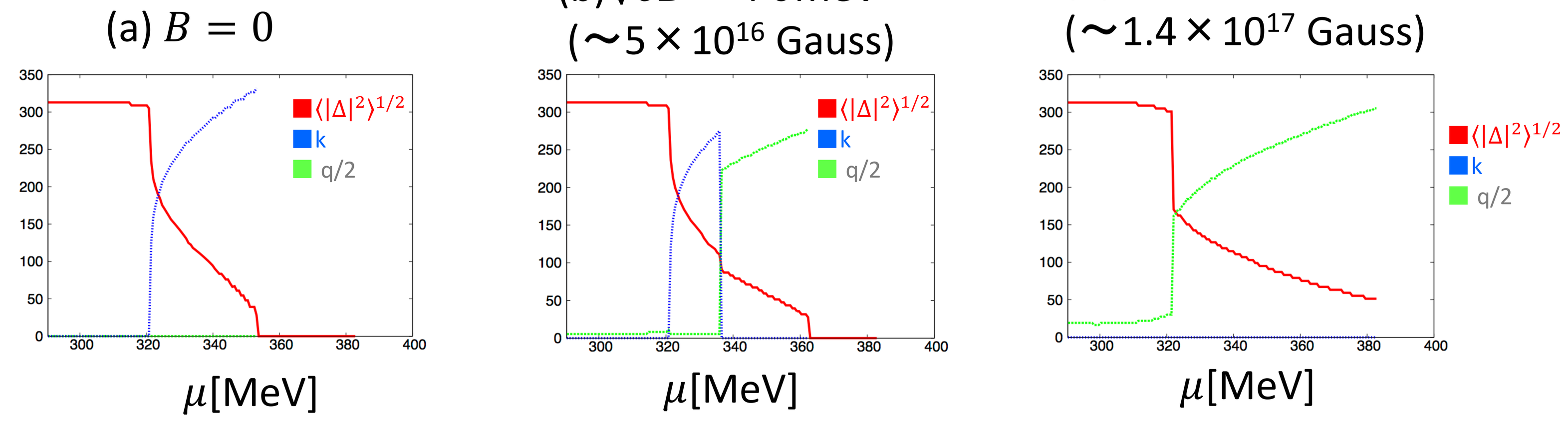
Phase structure is determined by $\frac{\partial\Omega}{\partial m} = \frac{\partial\Omega}{\partial\nu} = \frac{\partial\Omega}{\partial q} = 0$

・Phase Diagram (μ-B plane, T=0)



A: Weak DCDW
B: Hybrid
C: Strong DCDW
D Restored

(a) $B = 0$
(b)$\sqrt{eB} = 70$MeV (∼5 × 10^{16} Gauss)
(b)$\sqrt{eB} = 120$MeV (∼1.4 × 10^{17} Gauss)



k is wavenumber of amplitude modulation
$k = m/(2(1+\sqrt{\nu})\mathrm{K}(\nu))$

Summary and Outlook

・DCDW is favored in strong B due to spectral asymmetry
・Twisted kink crystal appears in weak B