Entropy production in the early stage of relativistic heavy ion collisions

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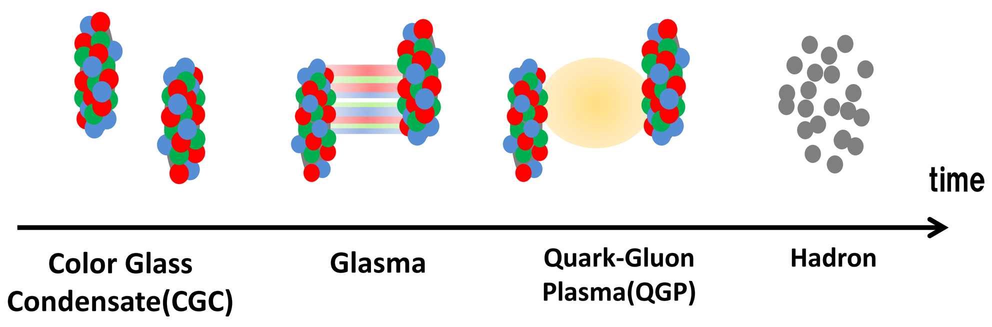
Abstract

- We extend the numerical method to calculate Husimi-Wehrl(HW) entropy in quantum mechanics[1] to Yang-Mills field theory.
- We first show that the production of HW entropy occurs in Yang-Mills theory though with using product ansatz.
- We find that the entropy production rate agrees with Lyapunov exponent which characterizes classical chaotic behaviors of systems.

1. Early thermalization in relativistic heavy ion collisions

Understanding early themalization or entropy production is a theoretical challenge in the area of relativistic heavy ion collision. There are many proposals for pinning down the underlying mechanics for it.

After a collision strong (almost classical) gluon field exists and quantum fluctuations on the top of the classical configurations (glasma) induce instabilities, which in turn may trigger the chaotic behavior of the gauge field and eventually give rise to entropy production.



Our analysis is based on semi-classical approximation. We assume that an initial state is coherent, which is caused by CGC. We calculate the semi-classical time evolution for the initial quantum state.

At first, we demonstrate our formalism in a quantum mechanical model. Next ,we extend the formalism to Yang-Mills field theory.

2. Classical chaos and Kolmogorov-Sinai(KS) entropy

Chaotic systems have a sensitivity to initial value. This property is characterized by Lyapunov exponents λ_i , which is given from eigenvalue of a time evolution operator about distance $\delta\vec{X}$;

$$U(t,t+\tau)=\mathcal{T}[\exp(\int_t^{t+\tau}\mathcal{H}(t')dt')] \qquad \delta X_i(t+\tau)=e^{\lambda_i\tau}\delta X_i(t)$$

$\delta\vec{X}$: distance between classical trajectories

$\mathcal{H}$: Hessian

$\delta X_i(t)$

KS entropy is related with Lyapunov exponents

$$h_{KS}=\sum_{\lambda>0}\lambda_i$$

It was calculated in YM field case [2].

3. Husimi-Wehrl(HW) entropy

Wigner function [Wigner(1932)]

$$f_W(p,q;t)=\int d\eta\exp(-ip\eta/\hbar)\langle q+\eta/2|\hat{\rho}|q-\eta/2\rangle$$

Wigner function has a problem in serving as a quantum distribution function. It is not positive definite.

Husimi function [Husimi(1940)] Gaussian smeared Wigner function

$$f_H(p,q;t)=\int\frac{dp'dq'}{\pi\hbar}\exp(-\frac{1}{\Delta\hbar}(p-p')^2-\frac{\Delta}{\hbar}(q-q')^2)f_W(p',q';t)$$

Husimi function is semi-positive definite and is considered as a quantum distribution function.

Husimi-Wehrl entropy [Wehrl(1978)]

$$\boxed{S_{HW}(t)=-\int\frac{dpdq}{2\pi\hbar}f_H(p,q;t)\log f_H(p,q;t)}$$

4. Semi-classical approximation of the time evolution of Wigner function

We can derive the time evolution of Wigner function from Liouville equation.

In the case of $H=\frac{p^2}{2m}+V(q)$, the time evolution of Wigner function is given by;

$$\frac{\partial}{\partial t}f_W=\frac{\partial V}{\partial q}\frac{\partial f_W}{\partial p}-\frac{p}{m}\frac{\partial f_W}{\partial q}+\mathcal{O}(\hbar^2)$$

The semi-classical solution leads to

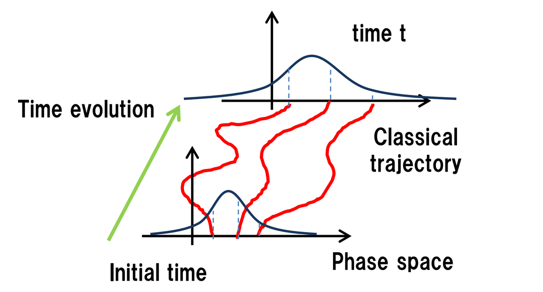
$$\frac{d}{dt}f_W(p,q;t)=0$$

With classical equation of motion $\dot{q}=\frac{p}{m},\dot{p}=-\frac{\partial V}{\partial q}$

This means that Wigner function is constant along the classical trajectory;

$$f_W(p(t),q(t);t)=f_W(p(0),q(0),t=0)$$

We can obtain the semi-classical time evolution of Wigner function by solving classical EOM.



5. Two numerical methods for HW entropy

Ref. [1]

Explicit form of HW entropy in terms ofWigner function is

$$S_{HW}(t)=-\int\frac{dpdq}{2\pi\hbar}\exp(-\frac{1}{\Delta\hbar}p^2-\frac{\Delta}{\hbar}q^2)\int\frac{dp'dq'}{\pi\hbar}f_W(p',q';t)$$
$$\times\log\int\frac{dp''dq''}{\pi\hbar}\exp[-\frac{1}{\Delta\hbar}(p+p'-p'')^2-\frac{\Delta}{\hbar}(q+q'-q'')^2]\,f_W(p'',q'';t)$$

We introduce two numerical methods to calculate the integral with the semi-classical time evolution of Wigner function.

Two step Monte-Carlo method : direct Monte-Carlo evaluation

Test particle method : Wigner function is a sum of delta functions

$$f_W(p,q;t)=\frac{2\pi\hbar}{N}\sum_i^N\delta(p-\bar{p}^i(t))\delta(q-\bar{q}^i(t))$$

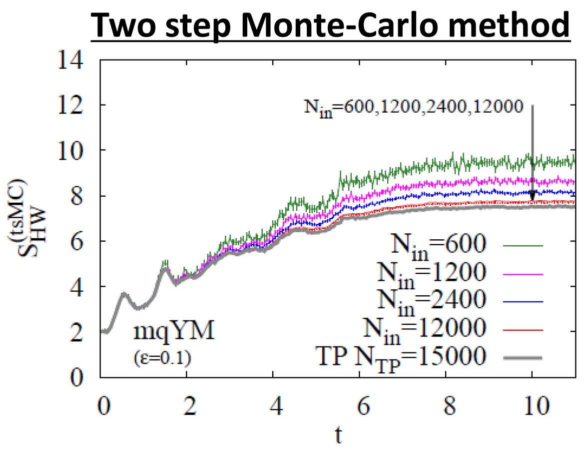
6. Results in quantum mechanical model

Ref. [1]

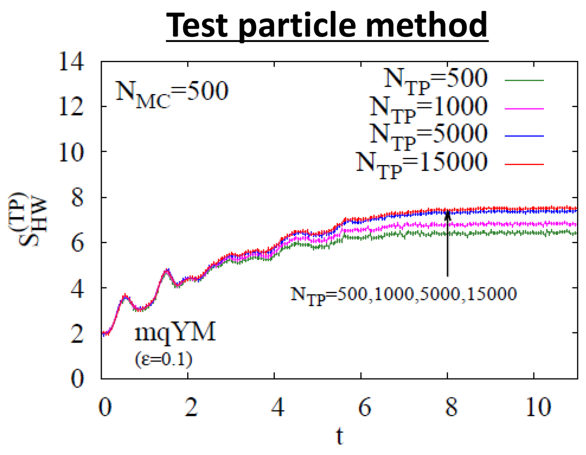
Hamiltonian : $$H=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+\frac{1}{2}g^2q_1^2q_2^2+\frac{0.1}{4}q_1^4+\frac{0.1}{4}q_2^4 \qquad (1)$$

with $m=1,g=1,\epsilon=0.1$

Two step Monte-Carlo method



Test particle method



Two numerical methods describe the entropy production. Both results are consistent within error bars.

7. Extension to the Yang-Mills field theory

We will work in temporal gauge $A_0^a=0$

Then Hamiltonian in a non-compact formalism is given by

$$H=\frac{1}{2}\sum_{x,a,i}E_i^a(x)^2+\frac{1}{4}\sum_{x,a,i,j}F_{ij}^a(x)^2$$
$$F_{ij}^a=\partial_iA_j^a(x)-\partial_jA_i^a(x)+\sum_{b,c}f^{abc}A_i^b(x)A_j^c(x)$$

Canonical variables are $(A_i^a(x),E_i^a(x))$ EOM
$$\dot{A}_i^a(x)=E_i^a(x)$$
$$\dot{E}_i^a(x)=\sum_j\partial_jF_{ij}^a(x)+\sum_{b,c,j}f^{abc}A_j^b(x)F_{ji}^c(x)$$

For the extension, we consider

$$(q,p)\rightarrow(A_i^a(x),E_i^a(x))$$

c.f. S. Mrowczynski, B. Muller(1994) (in a scalar field case)

8. Product ansatz

In higher dimension, we need a larger number of samples and test particles. We consider product ansatz to converge numerical results.

We assume that Husimi function is decomposed into the product of that of 1-dim degree of freedom.

$$f_H[A,E;t]=\prod^{N_D}h(A_i,E_i;t)$$

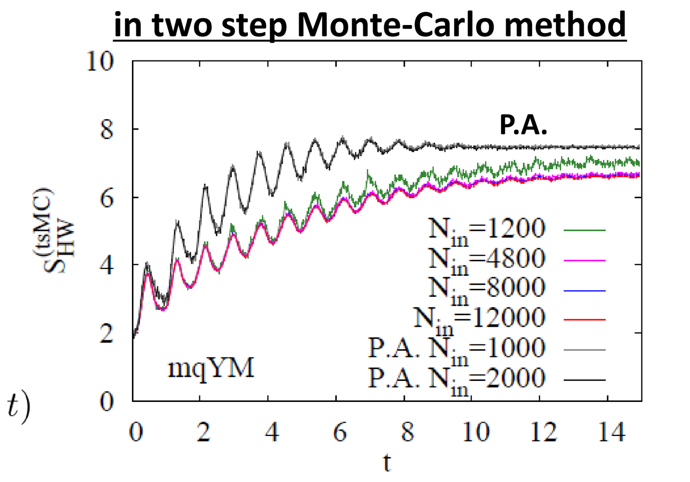
But we solve a equation of motion of full degrees of freedom unlike Hartree approximation.

Then, HW entropy is written by

$$S_{HW}(t)\simeq-\sum_i^{N_D}\int\frac{dA_idE_i}{2\pi\hbar}h(A_i,E_i;t)\log h(A_i,E_i;t)$$

quantum mechanical model ($\epsilon=1$ in (1))

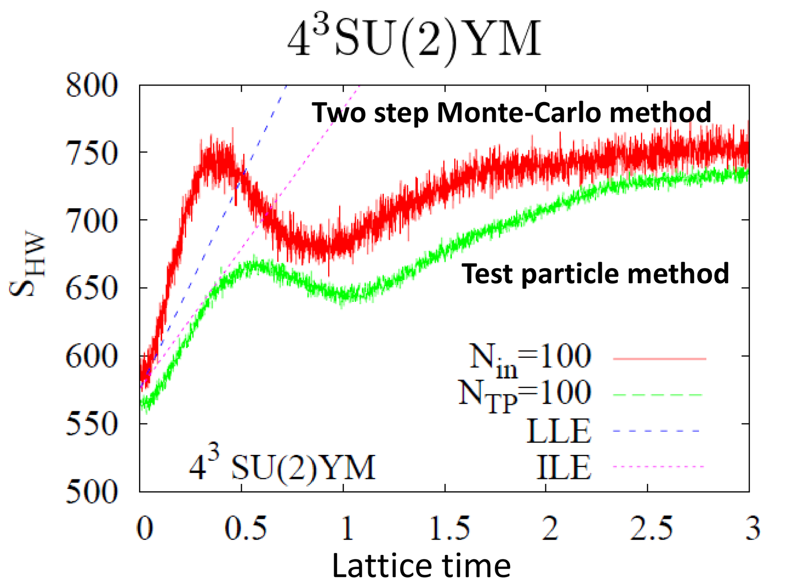
in two step Monte-Carlo method



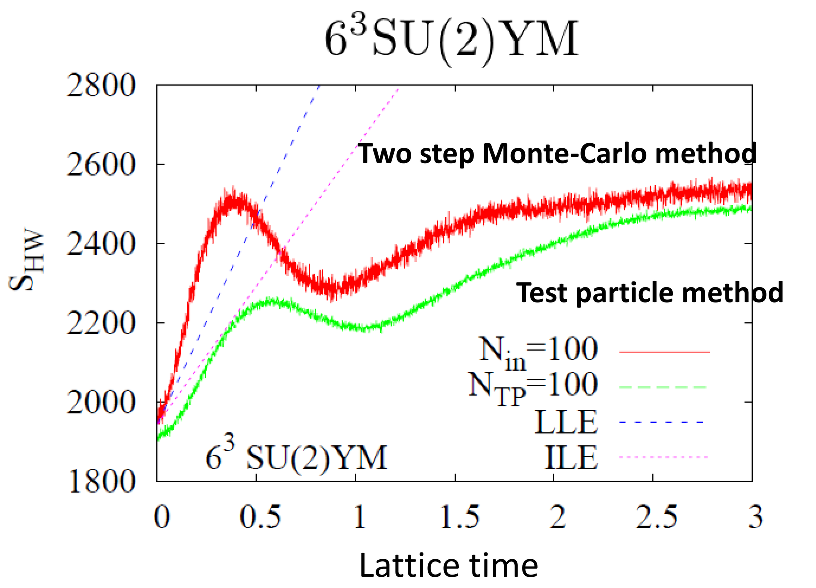
Product ansatz only gives 10% overestimate compared to “exact” results for quantum mechanical model(1).

9. Results in Yang-Mills field theory

$4^3\mathrm{SU(2)YM}$



$6^3\mathrm{SU(2)YM}$



The red(green) line is the time evolution of HW entropy calculated in two step Monte-Carlo method(test particle method). The solid lines are $\lambda_{\mathrm{sum}}^{\mathrm{LLE(ILE)}}t+S_{\mathrm{HW}}(t=0)$.

These figures show the growth rate of HW entropy agrees with Lyapunov exponent.

The right figure suggests that HW entropy is produced in a larger lattice size. The behaviors in 6^3 lattice are the same as that in 4^3 lattice qualitatively.

10. Conclusions

- We have calculated H-W entropy in Yang-Mills field theory on lattice and showed that H-W entropy has been produced. This result suggests that thermal entropy has been created in Yang-Mills theory.
- We have proposed product ansatz and found that it gives H-W entropy within 10% accuracy in quantum mechanical systems.
- We have showed that Lyapunov exponents give the growth rate of HW entropy in short time.

11. Future works

- Consider the physical meaning of product ansatz.
- Calculate H-W entropy on a larger lattice and check the dependence of the initial condition.
- Estimate physical time scale of the entropy growth.
- Check the entropy production in expanding geometry and discuss a relation to early thermalization.

Reference

[1] H.T., H.Iida, T.Kunihiro, A.Ohnishi, and T.T.Takahashi, Prog. Theor. Exp. Phys. (2015) 083A01.

[2] T.Kunihiro, B.Muller, A.Ohnishi, A.Schafer, T.T.Takahashi, and A.Yamamoto, Phys. Rev. D **82** (2010) 114015.