Simulating chiral magnetic and separation effects with cold atoms

Xu-Guang Huang
huangxuguang@fudan.edu.cn
Physics Department and Center for Particle Physics and Field Theory, Fudan University, Shanghai

Poster for Quark Matter 2015, Kobe, Japan

September 25, 2015

Chiral magnetic and separation effects

- Chirality imbalance + magnetic field = chiral magnetic effect (CME) **(Kharzeev 2004, Kharzeev, Mclerran, Warringa, Fukushima 2007-2008)**:

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B}$$

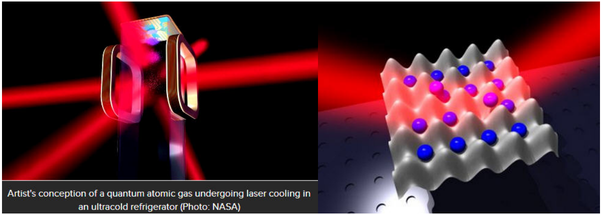
- A dual effect to chiral magnetic effect: chiral separation effect (CSE) **(Son and Zhitnitsky 2004, Metlitski and Zhitnitsky 2005)**

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B} \;\; \Rightarrow \;\; \mathbf{J}_A = \frac{N_c e}{2\pi^2}\mu_V \mathbf{B}$$

- The coupled evolution of CME and CSE induces massless collective modes, Chiral magnetic wave (CMW) **(Kharzeev and Yee 2011)**
- CMW $\Rightarrow$ electric quadrupole in QGP$\Rightarrow$ $v_2(\pi^-) - v_2(\pi^+) \propto A_\pm$ with $A_\pm = (N_+ - N_-)/(N_+ + N_-)$**(Burnier, Kharzeev, Liao, and Yee, 2011)**
- Phenomenology in heavy-ion collisions: charge-charge azimuthal correlation and charged pions elliptic flow splitting.

Motivation

- The CME/CSE/CESE are masked by various backgrounds in HICs, it is very hard to pin down and to explore their properties in HICs.
- Question: Is there any system that exhibits anomalous transport in a controllable way?
- Answer: Yes! One example is the Dirac or Weyl semimetal **(Li, et al, 1412.6543) and many other recent experimental progresses.**
- Here we propose another possibility: The cold atomic gases.
- Atomic gases experiments. $10^5 - 10^6$ atoms put in magnetic trap or optical trap, and cooled down to nano Kelvin by using laser cooling or evaporating cooling


Artist's conception of a quantum atomic gas undergoing laser cooling in an ultracold refrigerator (Photo: NASA)

- A lot of exciting low-temperature phenomena have been observed: superfluidity, Bose-Einstein condensation, BCS-BEC crossover, novel superfluid, polaron gases, ferromagnetisim,......

Spin-orbit coupled atomic gases

- In 2011, a new type of cold Bose gases generated in which the spin is coupled to the orbital motion of the atoms **(Spielman et al 2011).**
  The single-particle Hamiltonian(Rashba-Dresselhaus SOC):

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \sigma_x p_y$$

- In 2012, same type spin-orbit coupling (SOC) for Fermi gases produced in MIT **(Zwierlein group 2012)** and in Shanxi**(Zhang group 2012).**
- Other types of SOC also possible, e.g., the Weyl SOC: **(Spielman et al 2012)**

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda \sigma \cdot \mathbf{p}$$

- Now we show: there are CME and CSE in Weyl spin-orbit coupled Fermi gases.

Semiclassical equations of motion

- Consider the Weyl SOC, $\lambda\sigma \cdot \mathbf{p}$, in single atom Hamiltonian

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m} - \lambda\mathbf{p} \cdot \sigma$$

- Along $\mathbf{p}$, the spin has two projection which defines two chiralities, right-hand (project along $\mathbf{p}$) and left-hand (project along $-\mathbf{p}$).
- Consider atoms in a harmonic trap and let them rotate.

$$\begin{aligned} \mathcal{H} &= \frac{[\mathbf{p} - \mathbf{A}(\mathbf{x})]^2}{2m} - \lambda[\mathbf{p} - \mathbf{A}(\mathbf{x})] \cdot \sigma + A_0(\mathbf{x}) \\ A_0(\mathbf{x}) &= V(\mathbf{x}) - (m/2)(\omega \times \mathbf{x})^2 - \mu \\ \mathbf{A}(\mathbf{x}) &= m\omega \times \mathbf{x} \end{aligned}$$

- Integrate out the spin degree of freedom and at $O(\hbar)$ level: the semiclassical EOM **(Niu 1998-)**

$$\begin{aligned} \sqrt{G_c}\dot{\mathbf{x}} &= \nabla_{\mathbf{k}}\varepsilon_c + c\hbar\mathbf{E} \times \Omega + c\hbar(\Omega \cdot \nabla_{\mathbf{k}}\varepsilon_c)\mathbf{B}, \\ \sqrt{G_c}\dot{\mathbf{k}} &= \mathbf{E} + \nabla_{\mathbf{k}}\varepsilon_c \times \mathbf{B} + c\hbar(\mathbf{E} \cdot \mathbf{B})\Omega \end{aligned}$$

where $\mathbf{k} = \mathbf{p} - \mathbf{A}$ is the kinetic momentum, $\sqrt{G_c} = 1 + c\hbar\mathbf{B} \cdot \Omega$, $\mathbf{E} = -\nabla V(\mathbf{x})$— effective E-field, $\mathbf{B} = 2m\omega$—effective B-field, Ω—Berry curvature. $c = \pm$ for right- or left-hand.

Chiral anomaly

- The kinetic equation reads **(Son and Yamamoto 2012, Stephanov and Yin 2012, Gao,Wang,Pu,Chen,Wang 2012)**

$$\partial_t f_c + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} f_c + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f_c = I[f_c]$$

- Direct calculation gives the $U(1)$ chiral anomaly in current of chirality c:

$$\partial_t n_c + \nabla_{\mathbf{x}} \cdot \mathbf{j}_c = c(\mathbf{E} \cdot \mathbf{B}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_c \nabla_{\mathbf{k}} \cdot \Omega = c f_c(\mathbf{k}_0) \frac{W}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

where W is the winding number of the Berry curvature.

- Write down the current $\mathbf{j}_c$ explicitly:

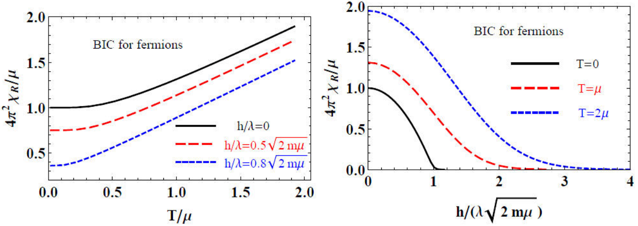
$$\begin{aligned} \mathbf{j}_c &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_c \nabla_{\mathbf{k}} \varepsilon_c + c\mathbf{E} \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Omega f_c \\ &\quad + c\mathbf{B} \int \frac{d^3\mathbf{k}}{(2\pi)^3} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_c) f_c. \end{aligned} \tag{1}$$

- The third term is $\mathbf{B}$-induced currents:

$$\mathbf{j}_c^{\mathbf{B}-\mathrm{ind}} = \chi_c \mathbf{B}, \quad \chi_c = c \int \frac{d^3\mathbf{k}}{(2\pi)^3} (\Omega \cdot \nabla_{\mathbf{k}} \varepsilon_c) f_c$$

Chiral magnetic/separation effects

- The $\mathbf{B}$-induced conductivity χ_c for Fermi gas **(XGH, arXiv:1506.03590)**



- *If there is parity-odd domains* in the Fermi gases $\Rightarrow$ $\mu_R = \mu + \mu_A, \mu_L = \mu - \mu_A \Rightarrow$

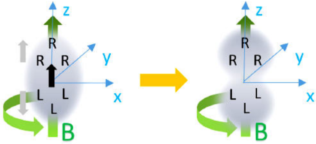
$$\mathbf{j}_V^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_R^{\mathbf{B}-\mathrm{ind}} + \mathbf{j}_L^{\mathbf{B}-\mathrm{ind}} = \frac{\mu_A}{2\pi^2}\mathbf{B},$$

$$\mathbf{j}_A^{\mathbf{B}-\mathrm{ind}} \equiv \mathbf{j}_R^{\mathbf{B}-\mathrm{ind}} - \mathbf{j}_L^{\mathbf{B}-\mathrm{ind}} = \frac{\mu}{2\pi^2}\mathbf{B}$$

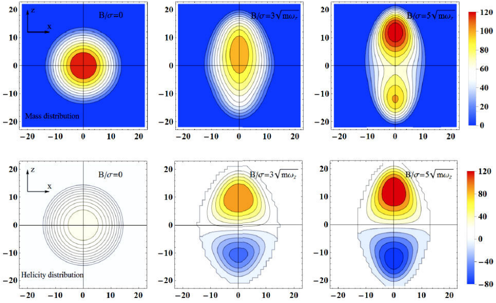
- These are exactly the chiral magnetic/speration effects!
- Question: how can produce parity-odd domins in Fermi gases?

Chiral dipole and mass quadrupole

- Very like what happen in QGP, the CMW exists in SOC atomic gases, which transport chirality and mass **(XGH, arXiv:1506.03590)**

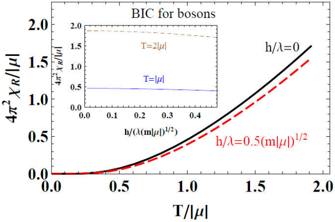


- Unlike in QGP, the presence of trap will finally stop these transport currents and system reaches a equilibrium configuration where appear a mass quadrupole and chiral dipole

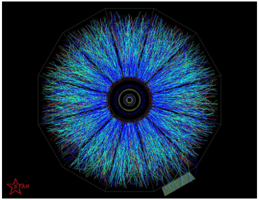


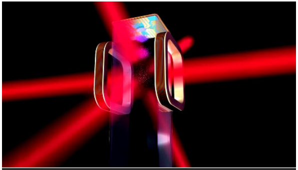
Link the hottest to the coldest

- The similar thing happens also in Bose gases, e.g., the BIC



- The CME/CSE initiated in the study of the hottest matter, the QGP, can possibly be realized in the coldest matter, the cold atoms.





Artist's conception of a quantum atomic gas undergoing laser cooling in an ultracold refrigerator (Photo: NASA)