The QCD phase diagram from analytic continuation of lattice data [1507.07510]

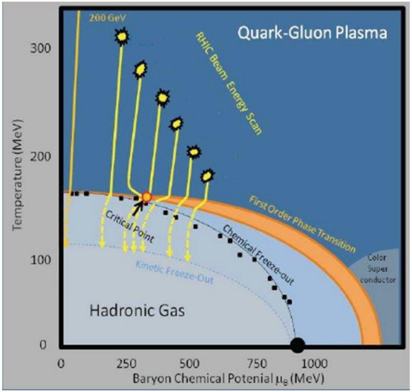
Szabolcs Borsanyi
R. Bellwied, Z. Fodor, J. Günther, S. D. Katz, A. Pásztor, C, Ratti and K. K. Szabó

(Wuppertal-Budapest)

Quark Matter 2015, Kobe

Sep 28, 2015

The (T, μ)-phase diagram of QCD



We determine T_c, the equation of state and freeze-out conditions at finite μ_B.

Taylor expansion:

WB: [1102.1356,1204.6710]

Bielefeld: [hep-ph/0303042], [hep-lat/0512040,1011.3130,1408.6305]

MILC [1003.5682]

Analytical continuation:

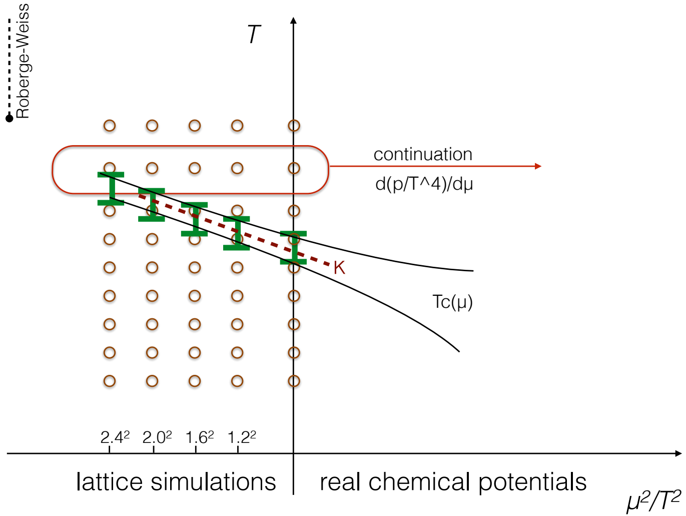
WB: [1507.07510]

Bonati et al: [1410.5758]

Cea et al: [hep-lat/0612018, 1403.0821]

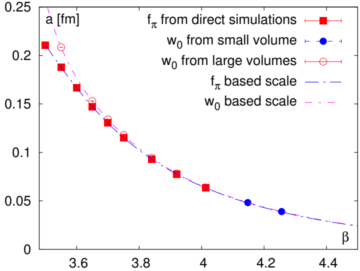
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \mathcal{O}(\mu_B^4)$$

$$\frac{p(\mu_B)}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T} \right)^2 + c_4 \left(\frac{\mu_B}{T} \right)^4 + c_6 \left(\frac{\mu_B}{T} \right)^6 + \mathcal{O}(\mu_B^8)$$



Many exploratory studies: [de Forcrand & Philipsen hep-lat/0205016]

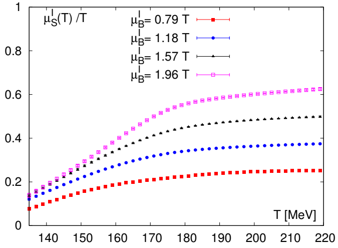
[Philipsen 0708.1293] [Philipsen 1402.0838] [Cea et al hep-lat/0612018,0905.1292,1202.5700]

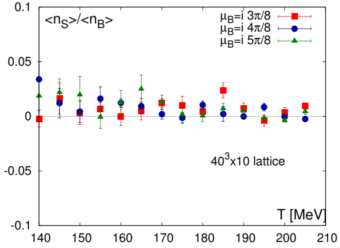


- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, until now there are simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = 0,\ 1.18i,\ 1.57i$ and $1.96i$
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Tuning to strangeness neutrality: $\langle n_S \rangle = 0$

We simulate each ensemble with an imaginary μ_B, μ_S pair such that $\langle n_S \rangle = 0$. This requires a non-trivial fine-tuning.





To compansated the remaining slight inaccuracies of the tuning and to achieve $\langle n_Q \rangle = 0.4 \langle n_B \rangle$ we correct all generalized quark number susceptibilities using higher order μ_Q and μ_S deviratives for each simulation point.

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial\left(m_q\right)^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right)\frac{m_l^2}{m_\pi^4}$$

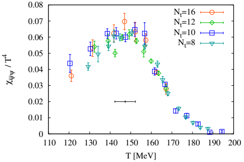
Chiral condensate:

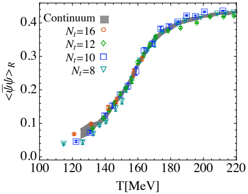
$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

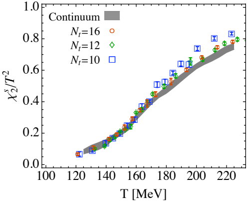
$$\langle\bar{\psi}\psi\rangle^r = -\left(\langle\bar{\psi}\psi\rangle(T,\beta) - \langle\bar{\psi}\psi\rangle(0,\beta)\right)\frac{m_l}{m_\pi^4}$$

Strangeness susceptibility:

$$\chi_{SS} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial\left(\mu_S\right)^2}$$





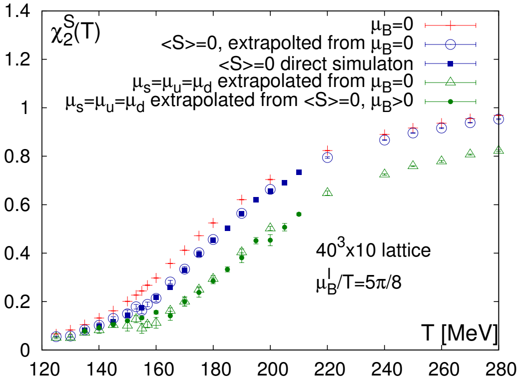


[WB arXiv:1005.3508]

Strange quark number susceptibility χ_{SS}

For this observable we can caluclate the Taylor extrapolation, too:

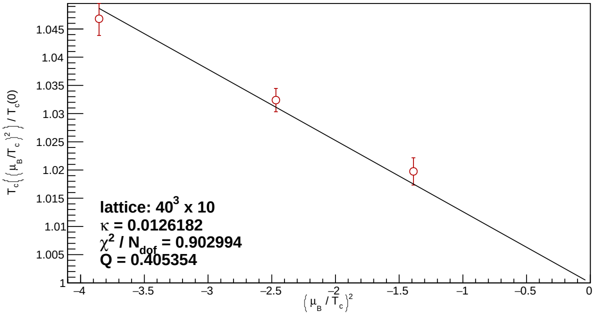
→ our μ_B range is in the leading order extrapolation regime



Fit function:

$$\chi_{SS}(\mu, T) = A(\mu)\left(1 + B\tanh\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$$

(or $\chi_{SS}(\mu, T) = A(\mu)\left(1 + B\arctan\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$)



Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

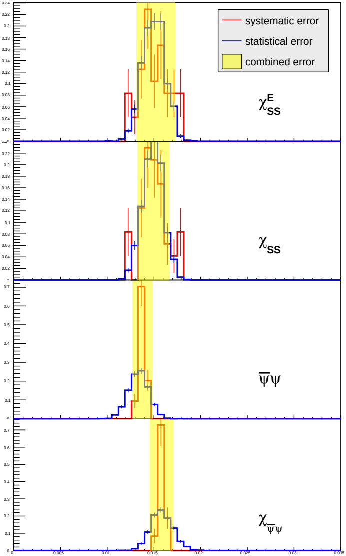
For error analysis we also fit:

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

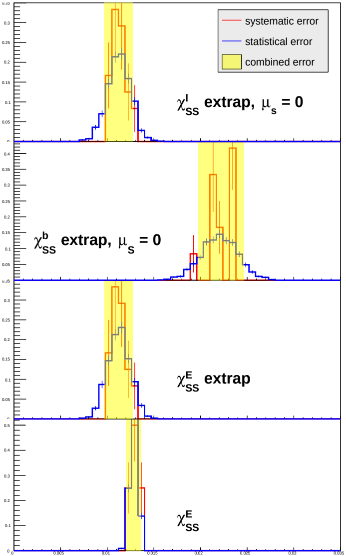
Comparison for different observables



χ_{SS}^{E}: $\langle n_S \rangle = 0$ and $0.5\langle B \rangle = \langle Q \rangle$

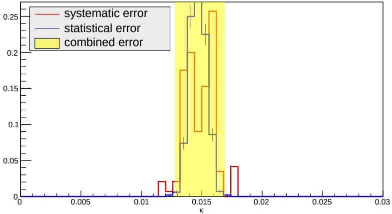
χ_{SS}: $\langle n_S \rangle = 0$ and $0.4\langle B \rangle = \langle Q \rangle$

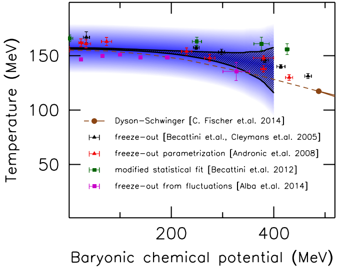
Comparison of Taylor expansion and analytic continuation



Comparison for results at $N_t = 10$, since here the precision is higher than in the continuum.

Combined result from χ_{SS}, $\bar{\psi}\psi$ and $\chi_{\bar{\psi}\psi}$:

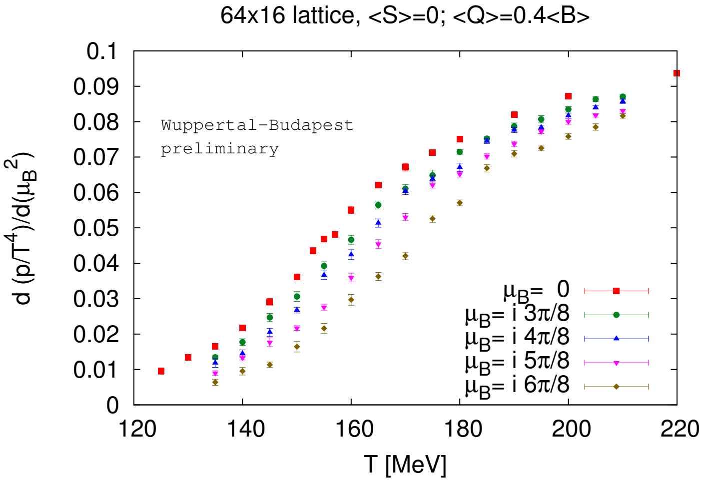




$$\kappa = 0.015 \pm 0.002^{+0.001}_{-0.001}$$

The derivative of the pressure

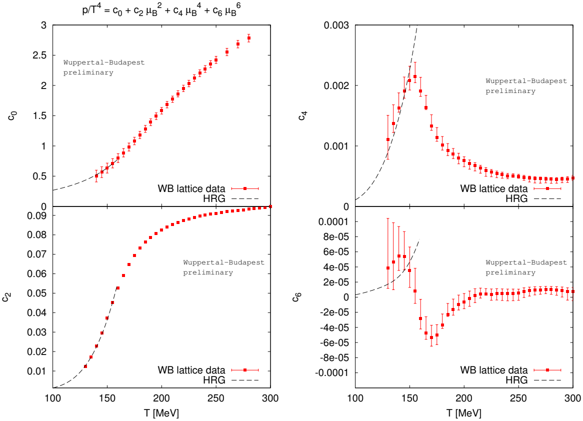
The $\mu_B = 0$ curve gives the LO μ_B dependenc, already determined in [WB 1204.6710]. The $\mu_B = 0$ equation of state was published in [WB 1309.5258].



The coefficients in the continuum, $< Q >= 0.4 < B >, < S >= 0$

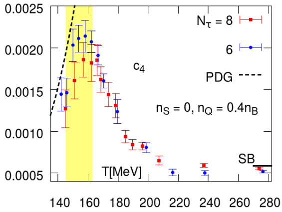
$$\frac{p(\mu_B)}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 + c_4 \left(\frac{\mu_B}{T}\right)^4 + c_6 \left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

Continuum result ($N_t = 8, 10, 12, 16$) with physical quark masses at the first time: *(systematic error estimate is incomplete)*

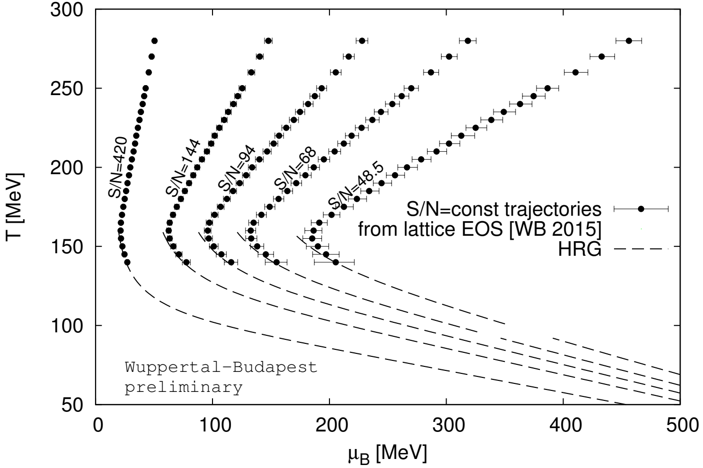


For comparion:
BNL-Bielefeld-CCNU:
$N_t = 6, 8$ HISQ,
next-to-leading order

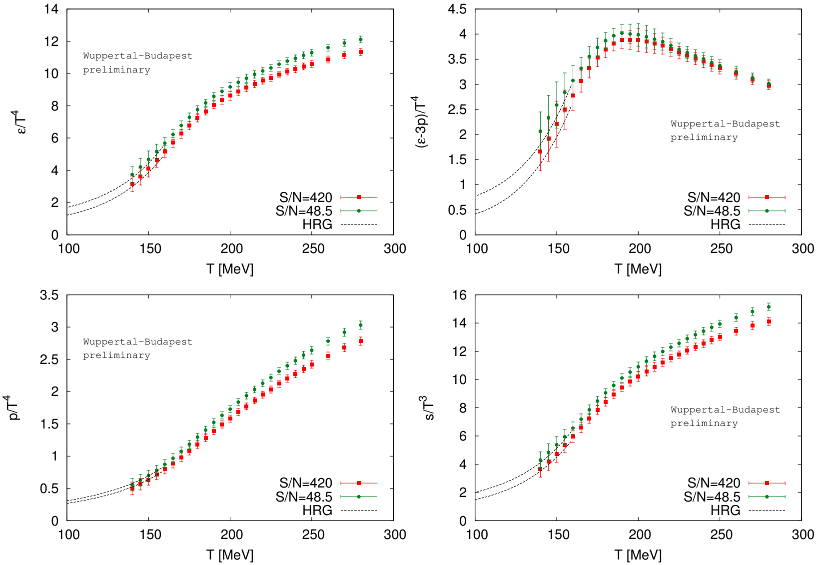
[QM'14: 1412.6727]



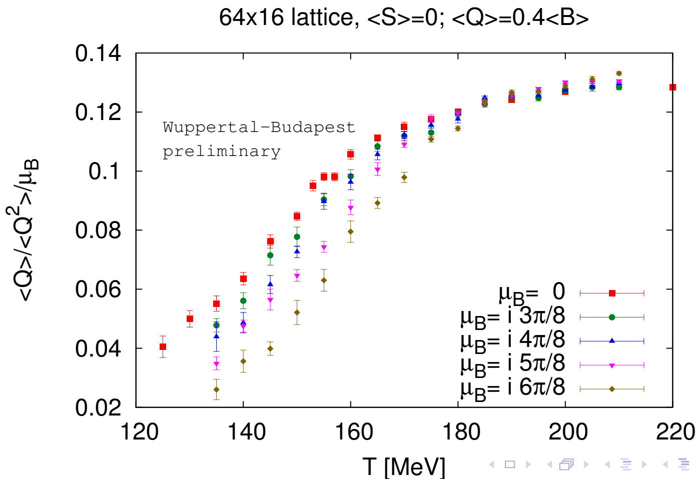
Adiabatic trajectories in the phase diagram



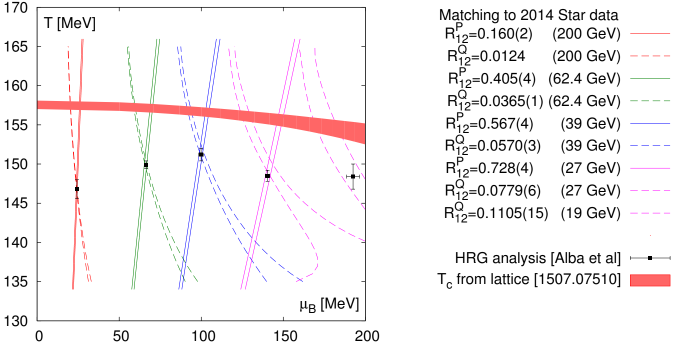
Equation of state along these trajectories



The fluctuations of the net electric charge are available from experiment. Here we are using M/σ^2.
Lattice can calculate this ratio at finite densities using analytic continuation.

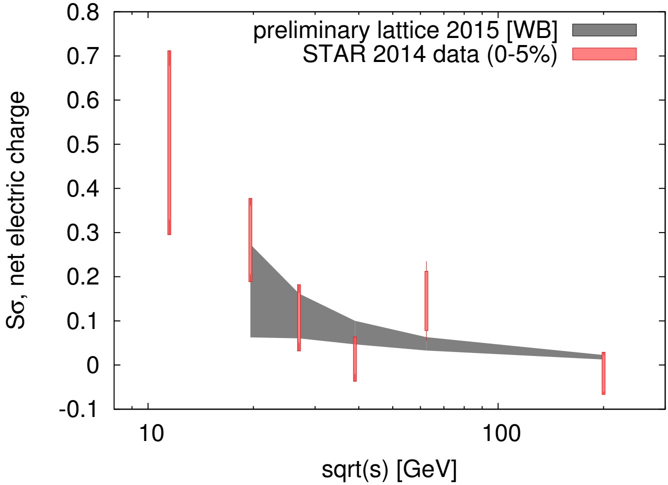


Matching the M/σ^2 to the electric charge and proton fluctuation data of STAR

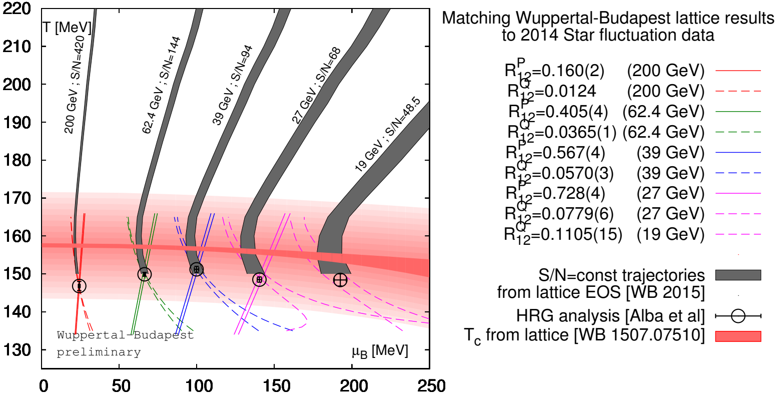


There are caveats in comparing fluctuation data to lattice directly.
For the proton fluctuations a HRG-based assumtion was used.

For the chemical freeze-out points of the previous slide we can extrapolate χ_{QQQ}/χ_{QQ} to finite chemical potential.



T_c-line, freeze-out and equation of state from imaginary-μ_B runs:

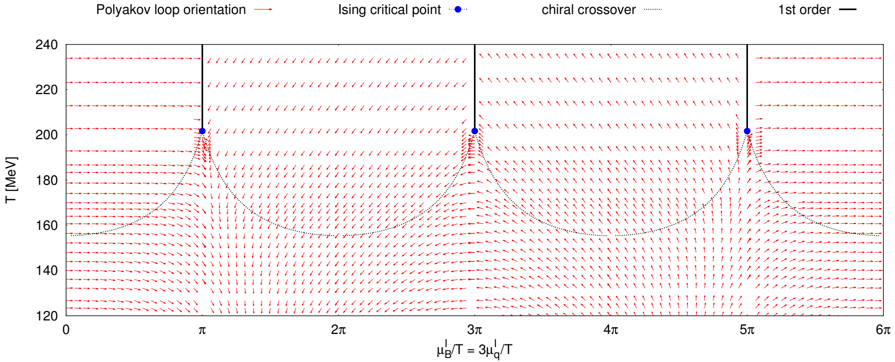


Backup slides

Many exploratory studies: [de Forcrand & Philipsen hep-lat/0205016]

[Philipsen 0708.1293] [Philipsen 1402.0838] [Cea et al hep-lat/0612018,0905.1292,1202.5700]

At imaginary μ_B there is no sign problem. The observables, including the crossover line, are analytical functions of μ_B^2.

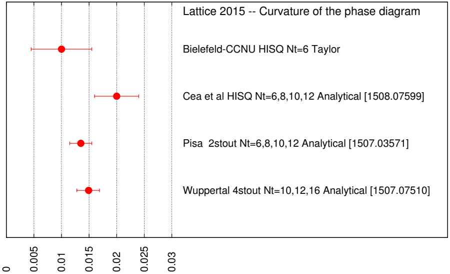


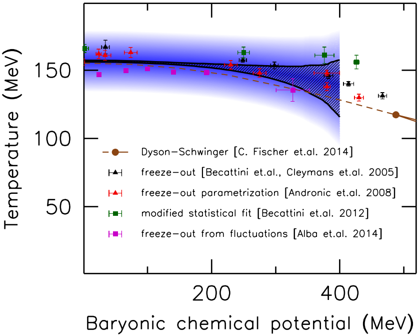
The phase diagram is perodic $\mu_B \to \mu_B + i2\pi T$, with simultaneous rotation between the Z(3) sectors.

The $T_c(\mu_B)$ can be expandedd around $\mu_B = 0$ (Taylor method) or found through analytical continuation with $\mathrm{Im}\ \mu_B > 0$.

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu)}\right)^2 + \ldots$$

Lattice 2015, Kobe:





Aim: For a given μ_B determine μ_S so that $\langle n_S \rangle = 0$. This means solving the differential equation

$$\langle n_S \rangle = 0 \Leftrightarrow \frac{\partial \log Z}{\partial \mu_S} = 0$$

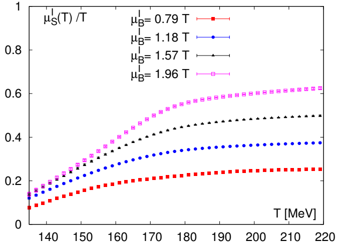
Notation: $\chi_{udsc} = -\frac{1}{T^4} \frac{\partial^4}{\partial(\mu_u/T)\partial(\mu_d/T)\partial(\mu_s/T)\partial(\mu_c/T)} \frac{T}{V} \log Z$

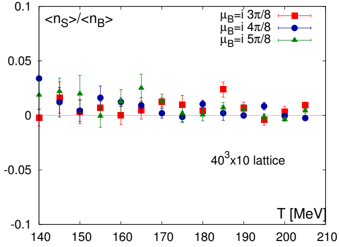
Assuming we know the value for $\mu_S(\mu_B)$ so that $\langle n_s \rangle = 0$ for $\mu_S(\mu_B^0)$ and $\mu_S(\mu_B^0 - \Delta\mu_B)$ with all the derivatives. Then:

$$\mu_S(\mu_B^0 + \Delta\mu_B) = \mu_S(\mu_B^0 - \Delta\mu_B) + 2\Delta\mu_B \frac{\mathrm{d}\mu_S}{\mathrm{d}\mu_B}(\mu_B^0).$$

In the simulations with μ_B^0 and $\mu_B^0 - \Delta\mu_B$, μ_S might not precisely tuned. There we want to extrapolate to a better value. We assume that correct value of μ_S is $\tilde{\mu}_S = \mu'_S + \Delta\mu'_S$. Then:

$$\langle n_S \rangle = \frac{\partial \log Z}{\partial \tilde{\mu}_S} = \frac{\partial \log Z}{\partial \mu'_S} + \frac{\partial^2 \log Z}{\partial \mu'^2_S} \Delta\mu'_S = 0$$





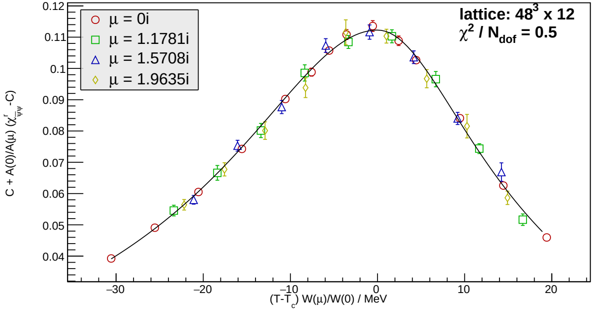
This yields

$$\Delta \mu'_S = -\frac{\chi_S}{\chi_{SS}}$$

Similar for the derivative we get:

$$\frac{\mathrm{d}\tilde{\mu_S}}{\mathrm{d}\mu_B} = -\frac{\tilde{\chi}_{SB}}{\tilde{Z}_{SS}}\Big|_{\langle n_S \rangle = 0} = -\frac{\chi_{SB}}{\chi_{SS}} - \frac{\chi_{SSB}\chi_{SS} - \chi_{SSS}\chi_{SB}}{(\chi_{SS})^2}\Delta\mu'_S + \mathcal{O}(\Delta\mu'^2_S)$$

Chiral susceptibility $\chi_{\bar{\psi}\psi}$

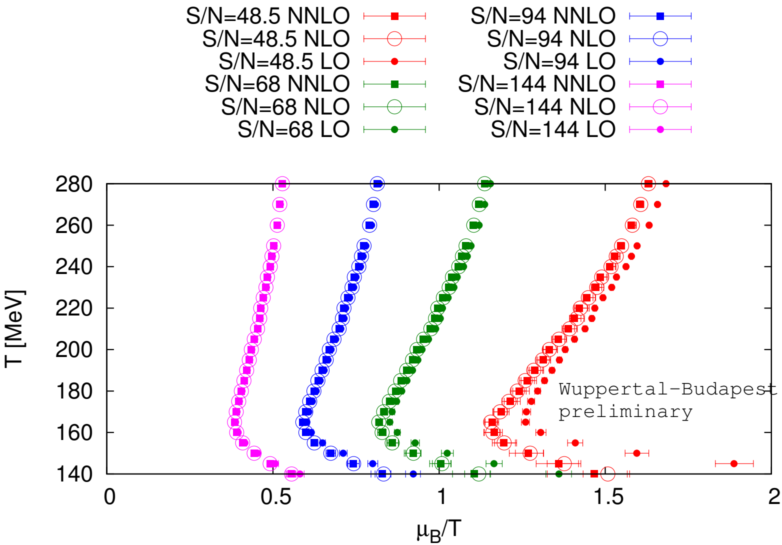


Fit function: $\chi^r_{\bar{\psi}\psi}(T) =$

$$\begin{cases} C + A^2(\mu)\left(1 + W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T \le T_c \\ C + A^2(\mu)\left(1 + b^2 W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T > T_c \end{cases}$$

(or $\chi^r_{\bar{\psi}\psi}(T) = C + \frac{A(\mu)}{1+W^2(\mu)(T-T_c(\mu))^2+a_3 W^3(\mu)(T-T_c(\mu))^3}$)

Orders of the μ_B-dependence



Continuum extrapolation:

$\kappa = \kappa^{c} + A\left(\frac{1}{N_t}\right)^2$

Combined curvature fit and continuum extrapolation with:

$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{c} + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$

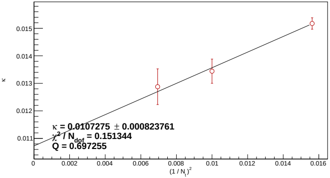
Continuum extrapolation:

$$\kappa = \kappa^c + A\left(\frac{1}{N_t}\right)^2$$

Combined curvature fit and continuum extrapolation with:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^c + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$$

Extrap. with $Nt = 8,\ 10,\ 12$



Continuum extrapolation

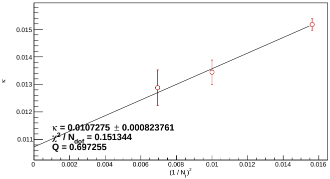
Continuum extrapolation:

$$\kappa = \kappa^c + A\left(\frac{1}{N_t}\right)^2$$

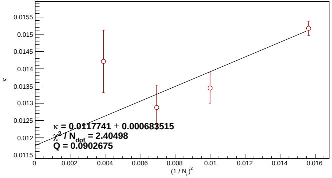
Combined curvature fit and continuum extrapolation with:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^c + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$$

Extrap. with $Nt = 8,\ 10,\ 12$



Extrap. with $Nt = 8,\ 10,\ 12,\ 16$



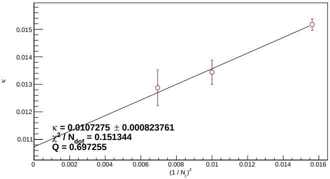
Continuum extrapolation:

$$\kappa = \kappa^c + A\left(\frac{1}{N_t}\right)^2$$

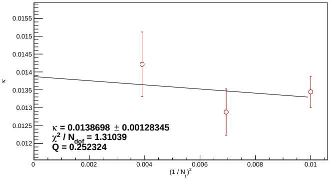
Combined curvature fit and continuum extrapolation with:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^c + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$$

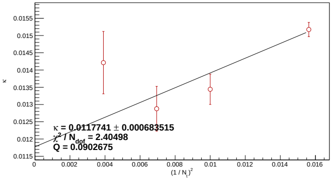
Extrap. with $Nt = 8, 10, 12$



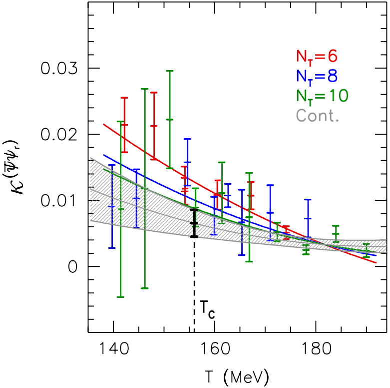
Extrap. with $Nt = 10, 12, 16$



Extrap. with $Nt = 8, 10, 12, 16$



Analysis of systematic error sources



Sources for systematic error:

- There is a strong dependence on the crossover temperature
- Continuum extrapolation only from three relatively coarse lattices
- Systematics of fit functions

Analysis was done at $\mu_s = 0$ not $\langle n_S \rangle = 0$.

Caveats

- *Effects due to volume variation because of finite centrality bin width*
  Experimentally corrected by centrality-bin-width correction method
- *Finite reconstruction efficiency*
  Experimentally corrected based on binomial distribution
  [A. Bzdak, V. Koch, PRC (2012)]
- *Spallation protons*
  Experimentally removed with proper cuts in p_T
- *Canonical vs Grand Canonical ensemble*
  Experimental cuts in the kinematics and acceptance
  [V. Koch, S. Jeon, PRL (2000)]
- *Proton multiplicity distributions vs baryon number fluctuations*
  Numerically very similar once protons are properly treated
  [M. Asakawa and M. Kitazawa], [PRC (2012), M. Nahrgang et al., 1402.1238]
- *Final-state interactions in the hadronic phase* [J.Steinheimer et al., PRL (2013)]
  Consistency between different charges = fundamental test