In-medium quarkonium properties from a lattice QCD based effective field theory

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in collaboration with S.Kim and P. Petreczky

References:
and work in progress
Motivation: Heavy-Ion Collisions
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Our interest: probes susceptible to medium but distinguishable \( Q_{\text{probe}} > > T_{\text{med}} \)

Bound states of c\( \bar{c} \) or b\( \bar{b} \): Heavy quarkonium \( M_Q > > T_{\text{med}} \)
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Bound states of $c\bar{c}$ or $b\bar{b}$: Heavy quarkonium $M_Q \gg T_{\text{med}}$

- In-medium quarkonium properties from a lattice QCD based EFT

**Figure:**
- CMS PbPb $\sqrt{s_{\text{NN}}} = 2.76$ TeV
- ALICE (|y|<0.9, 26% syst.), $\sqrt{s_{\text{NN}}} = 2.76$ TeV
- PHENIX (|y|<0.35, 12% syst.), $\sqrt{s_{\text{NN}}} = 0.2$ TeV

**Graph:**
- Mass($\mu^+\mu^-$) [GeV/c$^2$] vs. Events/(0.1 GeV/c$^2$)
- Data, total PbPb fit, background, pp shape (R$_{AA}$ scaled)

**Graph:**
- $J/\psi$ mid-rapidity $R_{AA}$ vs. $N_{\text{part}}$
- ALICE, PRL 109, 072301 (2012)
- PHENIX (|y|<0.35, 12% syst.), $\sqrt{s_{\text{NN}}} = 0.2$ TeV

**Legend:**
- ALICE (|y|<0.9, 26% syst.), $\sqrt{s_{\text{NN}}} = 2.76$ TeV
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Theory goal: $1^{st}$ principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions
Two limits for in-medium $Q\bar{Q}$

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Static: Kinetically equilibrated heavy quarks

presence of in-medium bound eigenstates?

modern approach: LATTICE QCD meson spectra

compare also G. Aarts et. al.: JHEP 1407 (2014) 097
Two limits for in-medium $Q\bar{Q}$


Quarkonium as Open-Quantum System
see e.g. Y. Akamatsu, A.R. PRD85 (2012) 105011

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Dynamical: real-time approach to equilibrium
redistribution of states over time?

LATTICE QCD based potential description
see poster 0601 by Y. Akamatsu, A.R.
Two limits for in-medium $Q\bar{Q}$

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**LATTICE QCD based potential description**

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Heavy quarks on the lattice

Relativistic treatment of light and heavy d.o.f.

Full Lattice QCD simulation incl. QQ (still too costly)
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Lattice QCD simulation without Q̅Q

\[ \frac{\Lambda_{QCD}}{m_Q} \ll 1 \]

\[ \frac{T}{m_Q} \ll 1 \]
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Kin. eq. non-relativistic Q\bar{Q} in a background of light medium d.o.f.

Q\bar{Q} in NRQCD effective theory

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Q\bar{Q} in NRQCD effective theory

- Lattice Non-Relativistic QCD (NRQCD) well established at $T=0$, applicable at $T>0$
  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes $v\neq 0$ contributions
  - scale setting requires exp. input - -> successful in ab-initio predictions e.g. $m(\eta_b(2S))$
    Dowdall et. al., PRD85, 054509 (2012)
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- Recent progress: realistic simulations of the QCD medium by the HotQCD collab.
  - $m_\pi=161\text{MeV}$, $T= [140-249] \text{MeV}$, $m_b a= [2.759-1.559]$, $m_c a= [0.757-0.427]$
Correlation functions in NRQCD

Non-rel. propagator of a single heavy quark $G$

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Non-rel. propagator of a single heavy quark $G$

QQ propagator projected to a certain channel

„correlator of QQ wavefct.

$D_{J/\psi}(\tau) \triangleq \langle \psi_{J/\psi}(\tau)\psi_{J/\psi}^\dagger(0) \rangle$“

Brambilla et al. Rev. Mod. Phys. 77 (2005) 1423
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Jpsi correlator at $T\approx 0$ for different lattice spacings
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$QQ$ propagator projected to a certain channel

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Ratio of $T>0$ and $T\approx 0$ correlators: estimate of overall in-medium effects

Jpsi correlator at $T\approx 0$ for different lattice spacings
Bayesian spectra in lattice NRQCD

Inversion of Laplace transform required to obtain spectra: Inherently Ill-defined
Bayesian spectra in lattice NRQCD

\[ D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_1 \tau_i] \rho_1 \Delta \omega_1 \]

1. \( N_\omega \) parameters \( \rho_1 >> N_\tau \) datapoints
2. Simulation data \( D_i \) has finite precision

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2. Simulation data \( D_i \) has finite precision

- Inversion of Laplace transform required to obtain spectra: Inherently Ill-defined
- Give meaning to problem by incorporating prior knowledge: Bayesian approach
  - Bayes theorem: Regularize the naïve \( \chi^2 \) functional \( P[D|\rho] \) through a prior \( P[\rho|I] \)

\[ P[\rho|D, I] \propto P[D|\rho] P[\rho|I] \]

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459
### Bayesian spectra in lattice NRQCD

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\beta=6.64$</th>
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**Equation:**

$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i] \rho_l \Delta \omega_l$$

**Notes:**

1. $N_\omega$ parameters $\rho_l$ $\gg$ $N_\tau$ datapoints
2. Simulation data $D_i$ has finite precision

**Inversion of Laplace transform required to obtain spectra:** Inherently Ill-defined

**Give meaning to problem by incorporating prior knowledge:** Bayesian approach

- **Bayes theorem:** Regularize the naïve $\chi^2$ functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D, I] \propto P[D|\rho] P[\rho|I]$$

**Recent progress:** Regulator that remedies flat directions issue in Maximum Entropy Method

$$P[\rho|I] \propto e^S \quad S = \alpha \sum_{l=1}^{N_\omega} \Delta \omega_l \left(1 - \frac{\rho_l}{m_l} + \log \left[\frac{\rho_l}{m_l}\right]\right)$$

**References:**

- Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459
- Y.Burnier, A.R. PRL 111 (2013) 18, 182003

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The XXVth Int. Conference on Ultrarelativistic Nucleus-Nucleus Collisions QM2015 - September 29th 2015
Bayesian spectra at $T=0$

$m_{J/\psi}$ from PDG calibrates freq. scale

$\beta=6.664$ – $\beta=7.280$
Bayesian spectra at $T=0$

- $m_{J/\Psi}$ from PDG calibrates freq. scale

- Check systematic error of lattice computation by postdiction of P-wave ground state mass
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$$M_{\chi_{c1}} = 3.546(4)\text{GeV} \quad M_{\chi_{c1}}^{\text{exp}} = 3.51066(7)\text{GeV}$$
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Check systematic error of lattice computation by postdiction of P-wave ground state mass

$M_{\chi_{c1}} = 3.546(4)\text{GeV}$  $M_{\chi_{c1}}^{\text{exp}} = 3.51066(7)\text{GeV}$

$M_{\chi_{b1}} = 9.917(3)\text{GeV}$  $M_{\chi_{b1}}^{\text{exp}} = 9.89278(3)\text{GeV}$
High precision of the improved Bayesian reconstruction (narrow width resolved)

How does accuracy suffer from limited available information at $T>0$ ($N_\tau=12$) ?

One of the tests we ran: truncate $T=0$ dataset ($N_\tau=32/64$) to $N_\tau=12$

A benchmark for $T>0$ spectra
A benchmark for T>0 spectra

- High precision of the improved Bayesian reconstruction (narrow width resolved)
- How does accuracy suffer from limited available information at T>0 ($N_\tau=12$)?
- One of the tests we ran: truncate T=0 dataset ($N_\tau=32/64$) to $N_\tau=12$

**Example: limits for Upsilon**

- $\beta = 6.664$ : $\Delta m_T < 2\text{MeV}$, $\Delta \Gamma_T < 5\text{MeV}$
- $\beta = 7.280$ : $\Delta m_T < 40\text{MeV}$, $\Delta \Gamma_T < 21\text{MeV}$
Finite temperature results
Sequential in-medium modification

$E_{\text{bind}}^{T=0} \approx 1.1 \text{ GeV}$
Sequential in-medium modification

\[ E_{\text{bind}}^{T=0} \approx 1.1 \text{Gev} \]

max 1% for $\Upsilon$

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Characteristic non-monotonicity
Sequential in-medium modification

\[ E_{\text{bind}}^{T=0} \approx 1.1 \text{GeV} \]

\[ E_{\text{bind}}^{T=0} \approx 640 \text{MeV} \]

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max 1% for $\gamma$

max 5% for $\chi_{b1}$
Sequential in-medium modification

\[ E_{\text{bind}}^{T=0} \approx 1.1 \text{ GeV} \]

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- \( \Upsilon \) Correlator ratio
- \( \chi_{b1} \) Correlator ratio
- \( J/\psi \) Correlator ratio

- Characteristic non-monotonicity

max 1% for \( \Upsilon \)

max 5% for \( \chi_{b1} \)
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\[ E^T=0_{\text{bind}} \approx 1.1\text{Gev} \]

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Sequential in-medium modification

\( E_{\text{bind}}^{T=0} \approx 1.1 \text{ Gev} \)

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\( E_{\text{bind}}^{T=0} \approx 200 \text{ MeV} \)

Characteristic non-monotonicity

max 1% for \( \Upsilon \)

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I\( N \) - MEDIUM QUARKONIUM PROPERTIES FROM A LATTICE QCD BASED EFT
Sequential in-medium modification

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max 12% for \( \chi_{c1} \)

characteristic non-monotonicity
Interpreting lattice correlator ratios
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- Use a phenomenological spectrum to compute correlator ratios
Interpreting lattice correlator ratios

Use a phenomenological spectrum to compute correlator ratios

from poster 0021 by Y. Burnier, A.R.

Y. Burnier, O. Kaczmarek, A.R.

arXiv:1509.07366
Interpreting lattice correlator ratios

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Potential based spectra qualitatively reproduce the T-dependence of the correlator ratio.

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Bayesian lattice spectra at $T>0$

- NRQCD Bottomonium S-wave and P-wave spectra between $T=140\text{ - }249\text{MeV}$
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- S-wave ground state peak present up to $T=249\text{MeV}$
Bayesian lattice spectra at $T>0$

- NRQCD Bottomonium S-wave and P-wave spectra between $T=140$ - $249$ MeV

- S-wave ground state peak present up to $T=249$ MeV

- Naïve inspection by eye fails for P-wave: first vs. second peaked structure
A systematic definition of survival

Our strategy: systematic comparison to non-interacting spectra
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Analytically known, no peaked features

\[
a_T E_p = -\log \left( 1 - \frac{p_{\text{lat}}^2}{8 M_b \alpha_s} \right)
\]

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\rho_S(\omega) = \frac{4\pi N_c}{N_s^2} \sum_p \delta(\omega - 2E_p)
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G. Aarts et. al., JHEP 1111 (2011) 103
A systematic definition of survival

- **Our strategy:** systematic comparison to non-interacting spectra

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  - Expectation: Presence of wiggly features due to numerical **Gibbs ringing**

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\( \Upsilon(1S) \) signal survives at \( T=249 \text{MeV} \)
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![Upsilon (1S) signal survives at $T=249\text{MeV}$](image)

![Charm-anticharm (1P) signal survives at $T=249\text{MeV}$](image)
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**Graphs:**

- **$\Upsilon(1S)$ signal survives at $T=249$ MeV**
  - Ground state ringing $\approx 10$

- **$\chi_b(1P)$ signal survives at $T=249$ MeV**
  - Ground state ringing $\approx 3$

- **$J/\psi$ signal survives at $T=249$ MeV**
  - Ground state ringing $\approx 3$
Conclusions

- Heavy quarkonium represents a precision probe of QCD at T>0
- Combining established EFT methods (NRQCD) and lattice QCD at T>0
  - **Progress I:** Realistic simulations of the QCD medium close to physical point (HotQCD)
  - **Progress II:** Improved Bayesian spectral reconstruction method available
- In-medium results for Quarkonium in lattice NRQCD:
  - **Sequential** in-medium **modification** of correlators according to vacuum $E_{\text{bind}}$
  - In-medium correlator behavior compatible with sequential peak melting
  - Comparison of free and interacting spectra disentangles ringing from bound state
  - **Survival** of the **bound state signal** at $T=249\text{MeV}$ for $\Upsilon(1S)$, $\chi_b(1P)$ and $J/\psi(1S)$
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Thank you for your attention
ご清聴ありがとうございました。