Using Unfolding Techniques to Separate Charm and Bottom Contributions to Single Electron Yields from Semi-leptonic Decays of Heavy Flavor Hadrons Measured by PHENIX

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In [1], we apply Bayesian inference techniques to the problem of separating electrons from charm and bottom hadron decays using measured distributions of electron DCAₜ and pﻙ. 

Given a vector of measured data, x and a vector of model parameters, y, we use Bayes’ theorem

\[ p(y|x) \propto p(x|y) \pi(y) \]

(1)
to compute the posterior probability density \( p(y|x) \) from the likelihood \( P(x|y) \) and prior information \( \pi(y) \). The denominator \( P(x|\pi(y)=0) \) serves as an overall normalization of the combined likelihood \( P(x|\pi(y)) \) such that \( p(y|x) \) can be interpreted as a probability density.

Here \( y \) consists of 17 bins in \( p_{cke} \) of both charm and bottom hadron yields. We employ an affine-invariant ensemble sampler described in Ref. [2] to draw samples of \( y \) in proportion to \( p(y|x) \).

**Modeling the likelihood function**

This analysis is based on 21 data points of total heavy flavor electron invariant yield, \( Y_{e-h} \), in the range 1.0-9.0 GeV/c from the 2004 data set [3], and five electron DCAₜ distributions \( D_{lns} \), where \( l \) indexes each electron \( p_{cke} (p_{cb}) \) interval within the range 1.5-5.0 GeV/c from the 2013 data set. Therefore, 

\[ x = \left[ \begin{array}{c} \ln\left( D_{lns} \right) \end{array} \right] \]

(2)

For each trial set of hadron yields, \( Y_{e} \), and electron DCAₜ, \( D_{lns} \), is calculated by

\[ Y(\theta) = M^{\lns} \theta_0 + M^{\lns} \theta_1 \]

(3)

\[ D_{lns}(\theta) = M^{D_{lns}} \theta_0 + M^{D_{lns}} \theta_1 \]

(4)

where \( M^{\lns} \) and \( M^{D_{lns}} \) are matrix decays (discussed later). The (log) likelihood between the prediction and each measurement in the datasets \( \text{Y}_{\text{data}} \) and \( \text{D}_{\text{lns}} \) is then evaluated using

\[ \ln P_{\text{Y}}(\theta) = \ln P_{\text{Ydata}}(Y(\theta)) + \sum_{l} \ln P_{\text{Ylns}}(D_{lns}(\theta)) \]

(5)

The likelihood \( P_{\text{Ydata}}(Y(\theta)) \) is modeled as a multivariate Gaussian distribution with diagonal covariance. The likelihood \( P_{\text{Ylns}}(D_{lns}(\theta)) \) is described by a multivariate Poisson distribution, as the DCAₜ data is in counts.

**Re-folded comparisons to data**

In [3], we apply the method \( M^{\lns} \) encoding the probability of charm and bottom hadron decays to electrons with DCAₜ > 1 cm, using the measured DCAₜ distributions of electrons from charm and bottom hadron decays. The distributions are then re-folded back to the parent hadron. This is done using the method described in Ref. [3].

The re-folded method serves as an overall normalization of the combined likelihood \( P(x|\pi(y)) \) such that \( p(y|x) \) can be interpreted as a probability density.

Here \( y \) consists of 17 bins in \( p_{cke} \) of both charm and bottom hadron yields. We employ an affine-invariant ensemble sampler described in Ref. [2] to draw samples of \( y \) in proportion to \( p(y|x) \).

**Results**

A large positive correlation is seen for adjacent bins in Fig. 2 for high-\( p_{cke} \) charm and bottom hadron yields. This is a consequence of the regularization, which requires a smooth \( p_{cke} \) distribution, and is stronger where there is less constraint from the data. There is also a region of anti-correlation between the mid to high \( p_{cke} \) charm hadrons and the low to mid \( p_{cke} \) bottom hadrons. Charm and bottom hadrons in these regions contribute decay electrons in the same \( p_{cke} \) region, and appear to compensate for each other to some extent.

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**REFERENCES**


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**Figure 1**: A scatter plot of the DCAₜ distribution of electrons from charm and bottom hadron decays. The green shades represent the re-folded charm and bottom hadron yields.

**Figure 2**: The joint probability distributions for the vector of hadron yields, \( \theta \), showing the 2-dimensional correlations between parameters. The diagonal plots show the marginalized probability distributions for each hadron \( p_{cke} \) bin (i.e., the 1-dimensional projection over all other parameters). Along the x-axis the plots are organized from top to bottom as the charm hadron \( p_{cke} \) bins from low to high \( p_{cke} \) followed by the bottom hadron \( p_{cke} \) bins from low to high \( p_{cke} \). The x-axis is organized similarly from left to right. The \( p_{cke} \) and \( p_{cb} \) binning follows that shown in Fig. 3. The region of green (blue) plots shows the correlations between charm (bottom) hadron yields. The region of orange plots shows the correlations between charm and bottom hadron yields. A circular contour in the 2-dimensional panels represents no correlation between the corresponding \( p_{cke} \) bins. An oval shape with a positive slope indicates a positive correlation between corresponding bins, and an oval shape with a negative slope represents an anti-correlation between corresponding bins. Sub-panels (b)-(d) show a set of example distributions.

**Figure 3**: The heavy flavor electron invariant yield as a function of \( p_{cke} \) compared to electrons from the re-folded charm and bottom hadron yields.

**Figure 4**: The DCAₜ distribution for measured electrons compared to the decomposed DCAₜ distributions for background components, electrons from charm decays, and electrons from bottom decays. The gray band indicates the region in DCAₜ considered in the unfolding procedure. See Ref. [3] for all DCAₜ comparisons.

**Figure 5**: Unfolded charm and bottom hadron invariant yield as a function of \( p_{cke} \), integrated over all rapidities, as constrained by electron yield vs DCAₜ at 3 GeV, in the range 1.0-9.0 GeV/c from the 2004 data set [3]. The bottom electron fraction as a function of \( p_{cke} \) compared to measurements in p+p collisions at \( \sqrt{s} = 200 \text{ GeV} \) from PHENIX [3] and STAR [6], as well as the central values for PHENIX [5] for p+p collisions at \( \sqrt{s} = 200 \text{ GeV} \).