Spectral function analysis of the hydrodynamical mode around the QCD critical point with use of functional renormalization group

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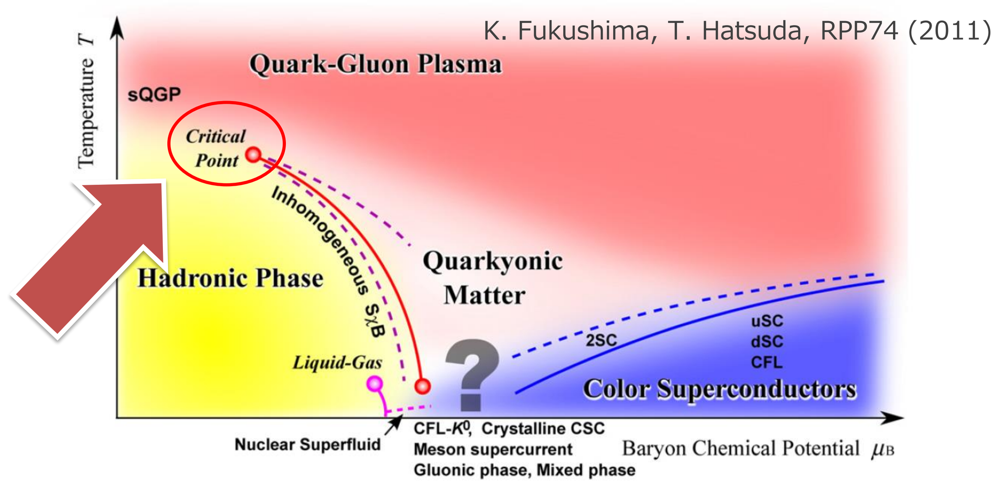
Introduction

QCD Phase diagram and QCD CP (critical point)

K. Fukushima, T. Hatsuda, RPP74 (2011)



- The soft mode at QCD CP is tricky.
- The understanding of the fluctuations is important for specifying detectable signals of QCD CP with heavy ion collision experiment.

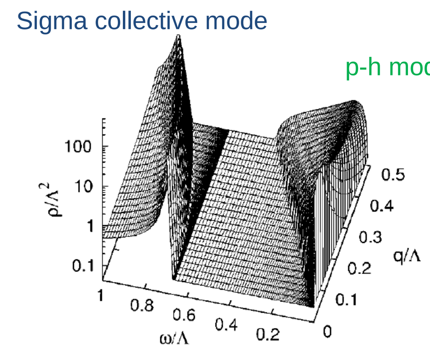
Soft Mode at QCD CP

- At CP, phase transition is 2nd order and some collective mode enhances at low frequency.
  - ⇒ Soft Mode e.g.) phonon mode at crystal structure phase transition point
- What is the soft mode at QCD CP?
  - Chiral limit: Sigma mode (and partially particle-hole mode)
  - Off chiral limit: p-h mode (particle-hole mode)
    - ⇒ Space-like dispersion relation
    - Large quark number density fluctuation
    - Same universality class as liquid gas CP (Model H)

H. Fujii, M. Ohtani, PRD70 (2004)
D. T. Son, M. A. Stephanov, PRD70 (2004)
Y. Hatta, T. Ikeda, PRD67 (2003)

Purpose of this research

- Investigate p-h mode enhancement near QCD CP
  - Using a non-perturbative method, FRG (functional renormalization group)
  - 2-flavor quark-meson model
  - Calculate spectral function of σ and see mode enhancement directly



RPA with mean field theory result

H. Fujii, M. Ohtani, PRD70 (2004)

Method

Functional renormalization group

- Include appropriate quadratic term (Regulator term)

C. Wetterich, PLB 301 (1993)

- Regulate IR modes and vanish with $k \to 0$

$$S_\Lambda[\phi] \to S_\Lambda[\phi] + \Delta S_k[\phi] = S_\Lambda[\phi] + \frac{1}{2}\int_q \phi(-q) R_k(q) \phi(q)$$

$$W_k[J] = \ln \int \mathscr{D}\varphi\, e^{-S_\Lambda[\varphi] - \Delta S_k[\varphi] + J\cdot\varphi}$$

$$\Gamma_k[\phi] = \sup_J \left(J\cdot\phi - W_k[J]\right) - \Delta S_k[\phi]$$

Flow eq. for effective action (Wetterich equation)

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)}[\phi] + R_k}\right]$$

- Solve flow equation ↔ Coarse graining from microscopic theory

Calculation spectral function with FRG

R. Tripolt, N. Strodthoff, L. Smekal, J. Wambach, PRD89 (2014)

- Spectral function

$$\rho(\omega, \vec{p}) = \frac{1}{\pi}\mathrm{Im} G_R(\omega, \vec{p}) = \frac{1}{\pi}\mathrm{Im}\frac{1}{\Gamma^{(2)}(\omega, \vec{p})} \Leftarrow \frac{\delta^2 \Gamma[\phi]}{\delta\phi(p)\delta\phi(p')} = (2\pi)^4 \delta^4(p+p')\Gamma^{(2)}(p)$$

- Update $\Gamma^{(2)}[\phi]$ with $\Gamma^{(2)}[\phi]$ flow equation

$$\partial_k \Gamma^{(2)}[\phi] = \mathrm{STr}\left[\frac{1}{\Gamma^{(2)}[\phi] + R_k}\Gamma^{(3)}[\phi]\frac{1}{\Gamma^{(2)}[\phi] + R_k}\Gamma^{(3)}[\phi]\frac{1}{\Gamma^{(2)}[\phi] + R_k}\partial_k R_k\right] - \frac{1}{2}\mathrm{STr}\left[\frac{1}{\Gamma^{(2)}[\phi] + R_k}\Gamma^{(4)}[\phi]\frac{1}{\Gamma^{(2)}[\phi] + R_k}\partial_k R_k\right]$$

Regulator $\Gamma^{(3)}[\phi]$ $-\frac{1}{2}$ $\Gamma^{(4)}[\phi]$

- Finite temperature system ⇒ Analytical continuation
  - Do analytical continuation after Matsubara sum at the level of flow equation.
  - Take care of ω upper half plane analyticity.
    - $i\omega_n \to \omega + i\varepsilon$ after $n_{\mathrm{B,F}}(E + i\omega_n) \to n_{\mathrm{B,F}}(E)$ $(\omega_n = 2\pi n T)$

Model and Regulator

- 2-flavor Quark-Meson model.
  - Truncation: Local potential approximation and ignoring Yukawa coupling flow.

$$\Gamma_k\left[\bar{\psi}, \psi, \phi = (\sigma, \vec{\pi})\right] = \int d^4x \left\{\bar{\psi}\left(\partial\!\!\!/ + g_s(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5) - \mu\gamma_0\right)\psi + \frac{1}{2}(\partial_\mu\phi)^2 + U_k(\phi^2) - c\sigma\right\}$$

- Initial condition

$$U_\Lambda = \frac{1}{2}m_\Lambda^2\phi^2 + \frac{1}{4}\lambda_\Lambda(\phi^2)^2$$

- Litim's optimized regulator

$$R_k^B(q) = (k^2 - \vec{q}^2)\theta(k^2 - \vec{q}^2) \quad R_k^F(q) = i\,\vec{q}\!\!\!/\left(\sqrt{k^2/\vec{q}^2} - 1\right)\theta(k^2 - \vec{q}^2)$$

Flow equations

- Flow equation for U_k

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[-2N_f N_c\left[\frac{1}{E_q}\tanh\frac{E_q + \mu}{2T} + \frac{1}{E_q}\tanh\frac{E_q - \mu}{2T}\right] + \frac{1}{E_\sigma}\coth\frac{E_\sigma}{2T} + \frac{3}{E_\pi}\coth\frac{E_\pi}{2T}\right]$$

$$E_\alpha^2 = k^2 + M_\alpha^2 \quad M_q^2 = g_s^2\sigma^2 \quad M_\sigma^2 = \partial_\sigma^2 U_k \quad M_\pi^2 = \partial_\sigma U_k/\sigma$$

⇓ Input U_k and physical σ (meson masses, vertex)

- Flow equation for $\Gamma_{k,\alpha}^{(2)}$

$$\partial_k \Gamma_{k,\sigma}^{(2)}(p) = \sigma\,\sigma\,\sigma + \sigma\,\pi\,\sigma - \frac{1}{2}\,\sigma\,\sigma - \frac{1}{2}\,\pi\,\sigma - 2\,\sigma\,\psi\,\bar{\psi}\,\sigma$$

- ● : Boson regulator $R_k^B(q)$ ----- : Boson propagator $\left(\Gamma_{k,\alpha}^{(2)}(q) + R_k^B(q)\right)^{-1}$
- ● : Fermion regulator $R_k^F(q)$ —— : Fermion propagator $\left(\Gamma_{k,\bar{\psi}\psi}^{(2)}(q) + R_k^F(q)\right)^{-1}$

Numerical method details

- Grid method: calculate $U_k(\sigma)$ on discretized σ
  - It reveals global structure of $U_k(\sigma)$.
  - Condition to solve non-linear partial differential equation for U_k without error enhancement

$$\left|\frac{\Delta k}{k}\right| \le \min\left(\frac{2|G|}{F^2}, \frac{\Delta\sigma^2}{2|G|}\right)$$

$$F = -\frac{k^5}{8\pi^2\sigma E_\pi^3}\left(\coth\frac{E_\pi}{2T} + \frac{E_\pi}{2T}\frac{1}{\sinh^2\frac{E_\pi}{2T}}\right)$$

$$G = -\frac{k^5}{24\pi^2 E_\sigma^3}\left(\coth\frac{E_\sigma}{2T} + \frac{E_\sigma}{2T}\frac{1}{\sinh^2\frac{E_\sigma}{2T}}\right)$$

Results

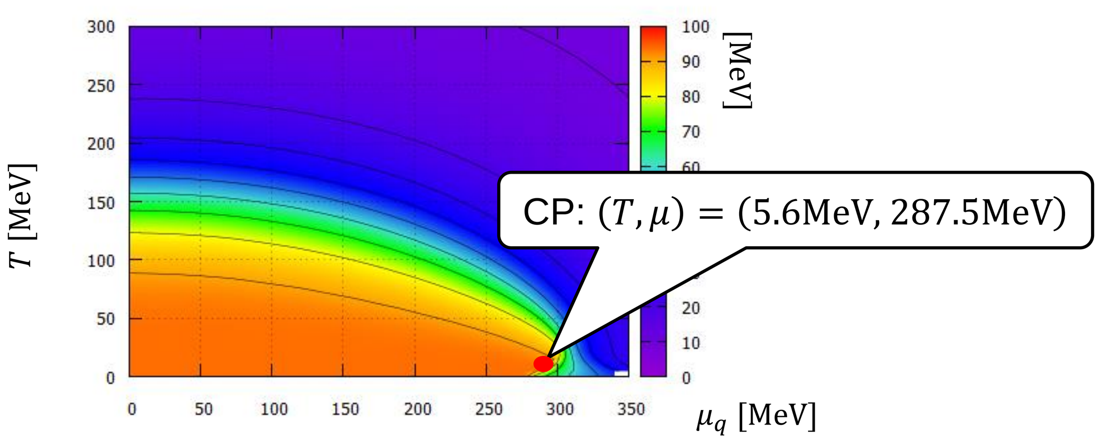
Parameters

Λ	m_Λ/Λ	λ_Λ	c/Λ^3	g_s
1000MeV	0.794	2.00	0.00175	3.2

- Realize vacuum meson masses, quark mass and chiral condensate

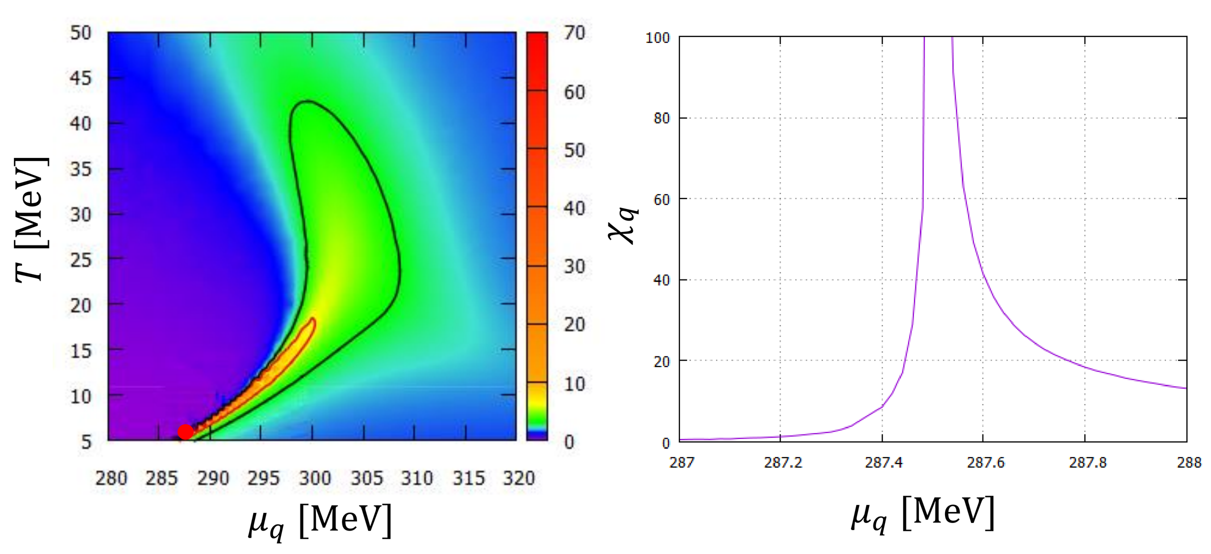
Phase diagram

- Chiral condensate ⋯ determined by $U_k(\sigma^2) - c\sigma$ minimum



Critical region and CP

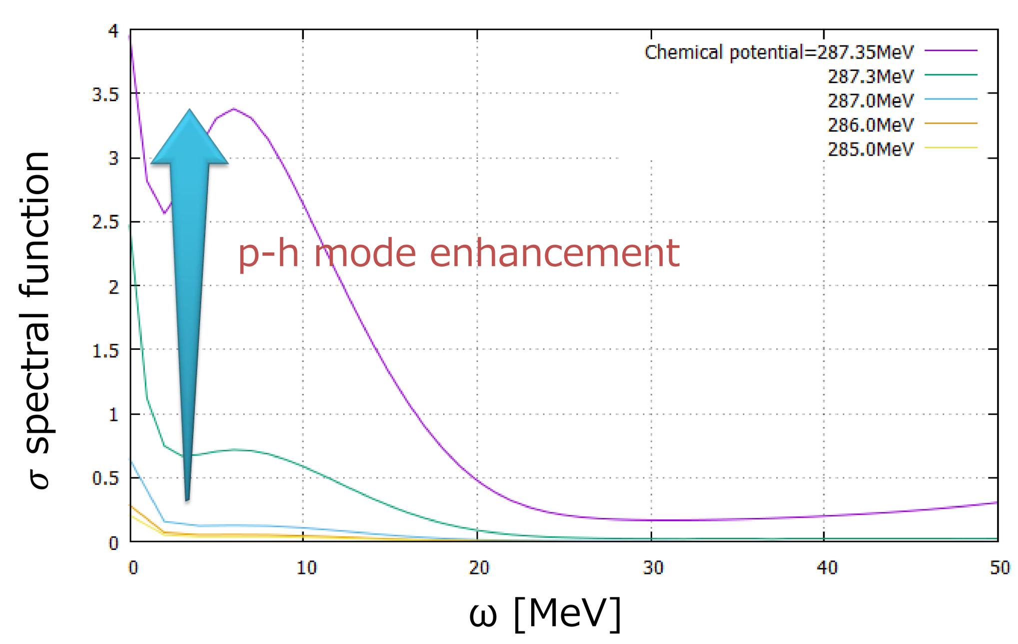
- Quark number susceptibility: $\chi_q = -\frac{\partial^2}{\partial\mu^2}\frac{\Omega}{V} = -\frac{\partial^2}{\partial\mu^2}\min_\sigma(U_k(\sigma^2) - c\sigma)$



- Normalized by the values of free quark gas
- Black line ⋯ contour of value 3
  Red line ⋯ contour of value 6
- $T = 5.6$MeV fixed.

Spectral near Critical point

- Space-like region spectral function



- $T = 5.6$MeV and $|\vec{p}| = 50$MeV fixed.
- Approach to CP from small chemical potential

Summary

- We have calculated σ spectral function with FRG in Quark-meson model.
- Particle-hole mode enhancement at low frequency toward QCD CP has been showed by the result of space-like region calculation. ⇒ consistent to particle-hole mode softening
- Future works
  - Calculate chiral susceptibility from spectral function and compare particle-hole mode and sigma mode contribution to susceptibility.
  - Derive quark number density spectral function and investigate particle-hole mode contribution to quark number susceptibility.
  - Calculate transport coefficients from spectral function and investigate their behavior near CP.