Thermodynamics of an exactly solvable confining quark model

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Introduction and motivation

One of the fundamental problems related to confinement is to understand how perturbative degrees of freedom are absent from the physical spectrum and how this is reflected in quantum effective actions written in terms of them. This question concerns all low energy QCD models constructed with quark degrees of freedom and may in principle affect their predictions. We present a model of dressed quarks that are confined in the sense that their excitations cannot be associated with physically propagating asymptotic states due to positivity violation. The model is inspired by QCD with soft BRST breaking and possesses a quark mass function in agreement with results from lattice and Dyson-Schwinger equations. Even though fundamental excitations of the model are not physical, the produced thermodynamic behaviour is non-trivial and stable for all temperatures and chemical potentials. Results for pressure, entropy and trace anomaly are qualitatively compatible with thermal lattice data and the finite density thermodynamics is also non-trivial [1].

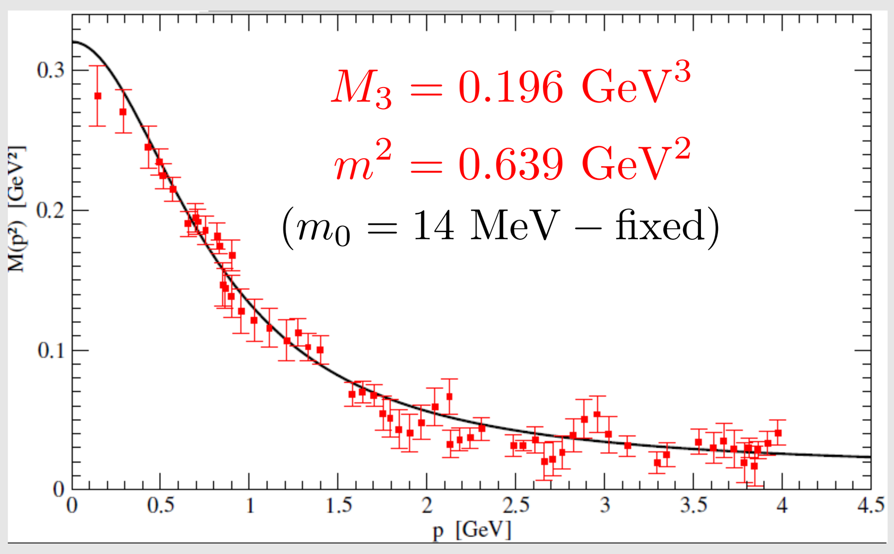
A model of confined quarks

The model is based on the introduction of a soft breaking of the BRST symmetry in the infrared quark sector. A breaking of this type has been investigated in pure gauge systems and has provided promising results, also concerning the thermodynamics [2,3]. Moreover, a profound modification of the BRST symmetry has been related to the quantization of Yang-Mills theories in the non-perturbative regime and the problem of Gribov copies [4,5]. Here we adopt the simplest form of the quark infrared effective action in the Landau gauge,

$$\mathcal{L}_{IRq} = \bar{\psi}\left[i\partial\!\!\!/ - \left(\frac{M_3}{-\partial^2 + m^2} + m_0\right)\right]\psi \qquad (1)$$

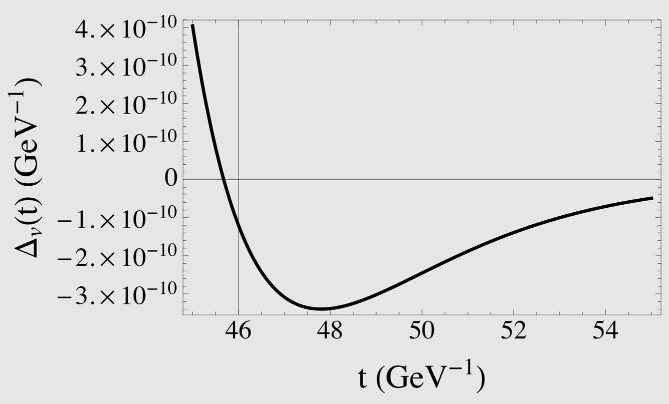
that *reduces to perturbative QCD in the ultraviolet regime* [6] and further displays the following features:

- quark mass function that fits lattice data [7]:



- positivity violation: absence from spectrum

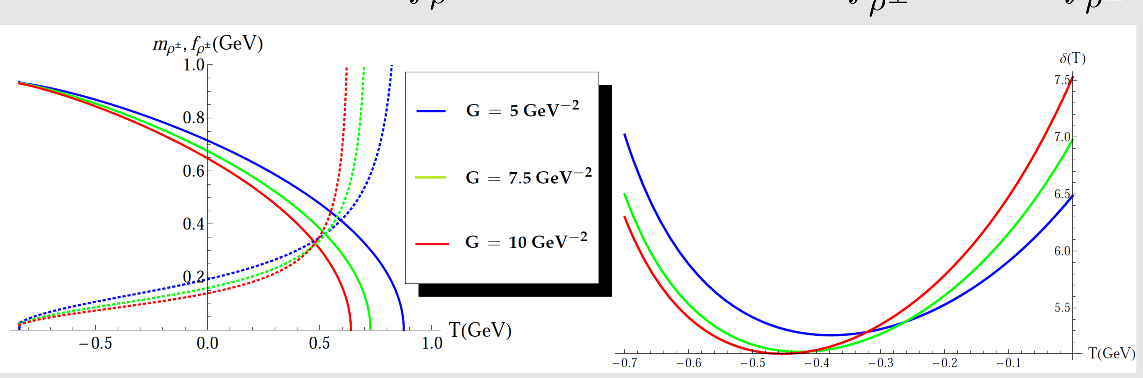
The vectorial Schwinger function defines a probability of propagation of a given physical excitation only if it is positive, which is clearly violated for our confined quarks [8].



- from confined quarks to mesons [8]:

Including interactions via an effective four-fermion coupling, mesonic composite operators develop physical poles, despite the unphysical nature of their constituent quarks. For the charged rho meson, mass and decay constant estimates yield: $m_{\rho^\pm} \sim 0.84\ \text{GeV} = 1.08 m^{\text{exp}}_{\rho^\pm}$

$f_{\rho^\pm} \sim 0.12\ \text{GeV} = 0.58 f^{\text{exp}}_{\rho^\pm} = 0.52 f^{\text{latt}}_{\rho^\pm}$



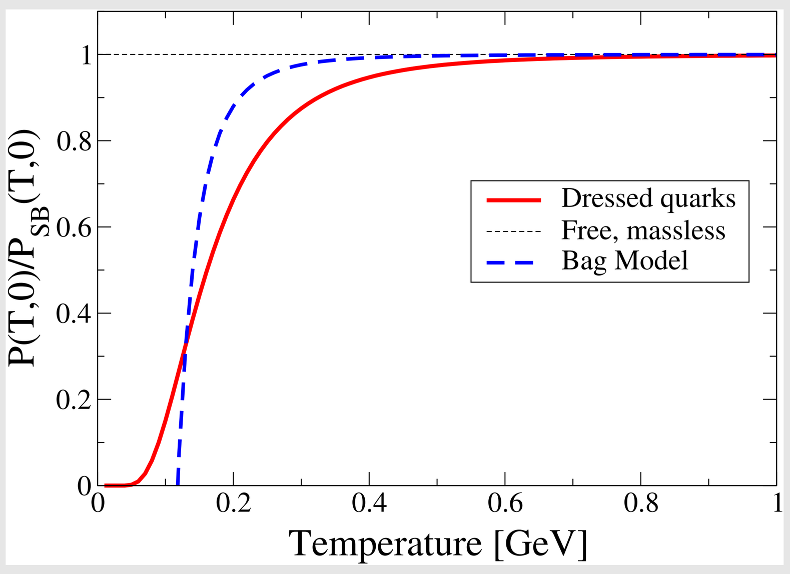
Thermal results

The lowest-order confining quark model defined by the Lagrangian (1) is quadratic and its partition function can be computed exactly.

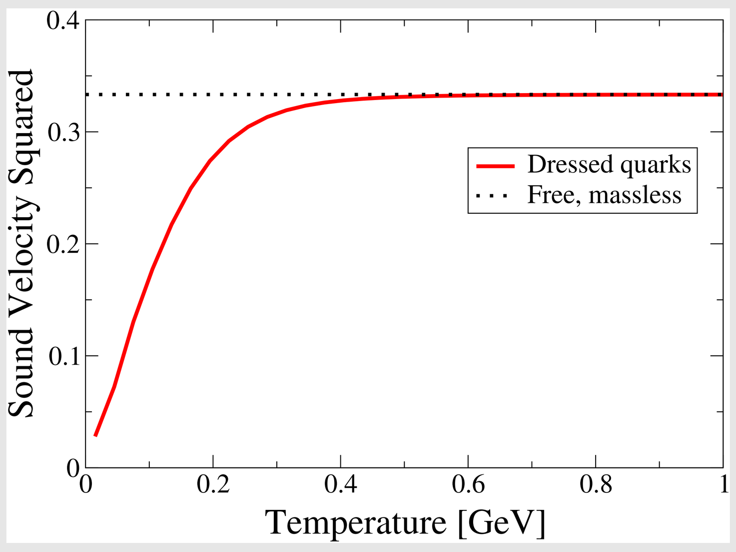
Using standard finite-temperature field theory techniques, the leading-order contribution to the thermodynamics of the confining quark model is shown below.

Results for thermodynamic quantities are non-trivial as well as stable for all temperatures. *The qualitative behavior is compatible with lattice data for the thermal crossover.*

- Pressure:

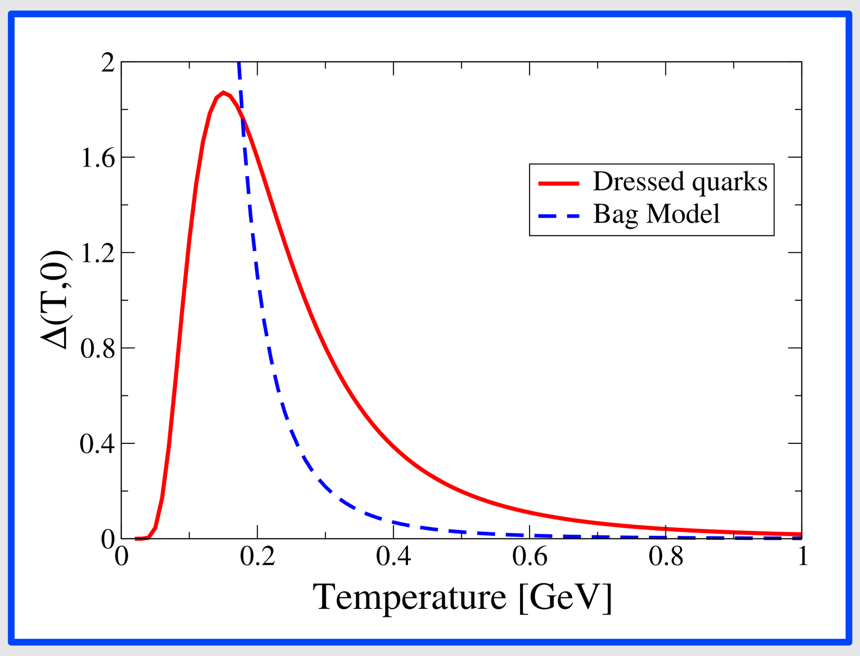


- Sound velocity:

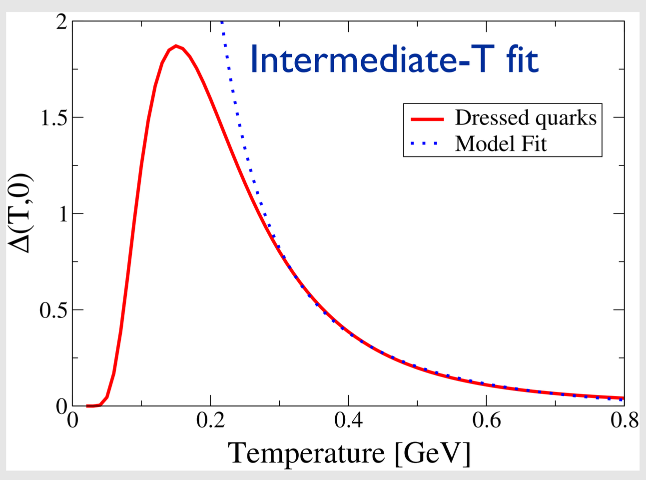


- Trace anomaly:

The interaction measure displays non-monotonic behavior, with a peak around T~150 MeV, which is the typical energy of the QCD thermal crossover.



A bag model fit for intermediate temperatures above the peak (T = [300,800] MeV) reveals an effective negative pressure (bag constant) naturally present in the model:

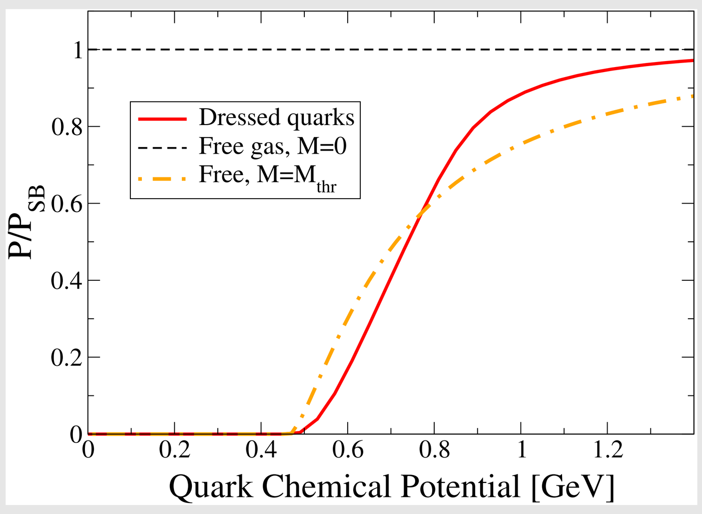


$$\Delta_{\text{fit}}(T) = a + \frac{b}{T^2} + \frac{4B}{T^4}$$

$$a = -0.069$$
$$b = 0.062\ \text{GeV}^2$$
$$B = (141\ \text{MeV})^4$$

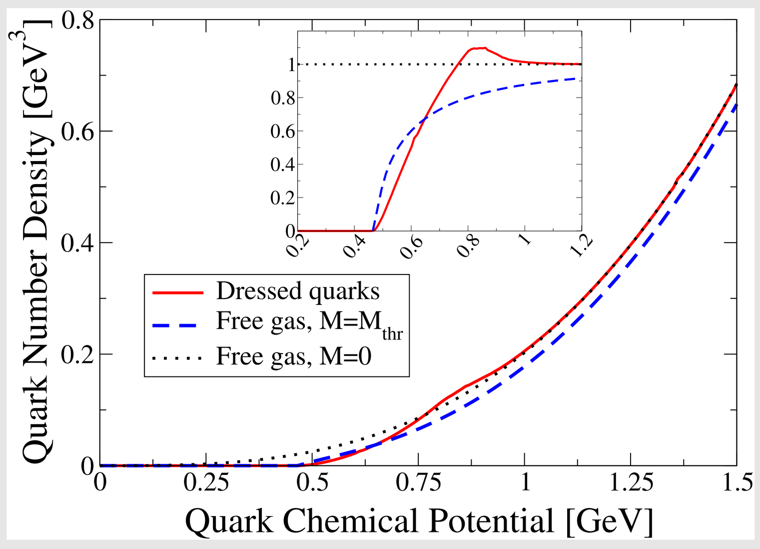
Cold & dense thermodynamics

- Pressure @ T = 0:

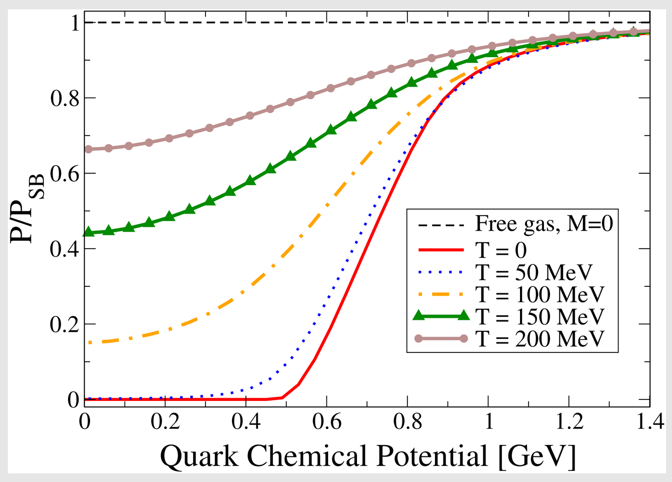


- gluons suppressed: good approximation, but no comparison with lattice available (Sign problem)
- dynamical threshold mass
- Silver Blaze problem: ok!
- No phase transition in this approximation (T- and μ-independent parameters)

- Quark number density:



- Pressure for nonzero T: thermal excitation



- smoothening of the transition,
- shift of inflection point to lower μ's.

Final remarks

A soft breaking of the BRST symmetry could in principle be present in the infrared regime of QCD without affecting any ultraviolet (perturbative) predictions. In this case, it may be intimately related to other nonperturbative phenomena, such as confinement and chiral symmetry breaking, representing a complementary way to develop low energy QCD models. Indeed, several examples of theories with soft BRST breaking display violation of reflection positivity in 2-pt correlation functions of fundamental fields, being consistent with the absence of the corresponding asymptotic states and confinement.

For the case of confined quarks, we have shown that a well-defined model may be proposed and worked out at lowest order to give already highly nontrivial results, like dynamical mass generation and propagators that are qualitatively in line with lattice findings, as well as a fully consistent thermodynamic behavior at non-zero temperatures and quark chemical potential.

The full QCD case, including gluons explicitly and the Polyakov loop via the background field method, is currently under investigation.

Acknowledgments

This work was partially supported by CNPq, FAPERJ and L'Oréal Foundation (For Women in Science research grant). LFP is a BJT fellow from the brazilian program « Ciências sem Fronteiras » (grant n. 301111/2014-6).

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