Calculation of high-order cumulant with canonical ensemble method in lattice QCD

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Background

We are interested in the QCD phase diagram!
I would like to see H.R. gas to Q.G. gas tr. Quark number cumulants are convenient!

Method

1. Canonical ensemble method

Fugacity expansion

\[ Z_{G.C.}(\mu) = \sum_n Z_{\text{can.}}(n) e^{n \mu T} \]

Canonical

\[ Z_{\text{can.}}(n) = \frac{1}{2\pi} \int d\mu T e^{-in\mu T} Z_{G.C.}(i\mu) \]

Grand Canonical

Fourier transformation

2. Winding number expansion

\[ Z_{\text{can.}}(n) = \left\{ \frac{1}{2\pi} \int \frac{d\mu}{T} e^{-i\mu n} \frac{\text{Det}(D(\mu))}{\text{Det}(D(0))} \right\} g \]

\[ \text{Instability of Fourier transf.} \]

\[ \text{real and positive} \]

(for 2-flavors)

calculate \( \text{det} D(\mu) \) at low numerical cost!

\[ \text{det} D(\mu) = \text{det} (1 - \kappa Q(\mu)) = e^{\text{Tr} \{\log(1 - \kappa Q)\}} \]

\( \kappa \) : hopping parameter

\[ \text{Tr} \{\log(1 - \kappa Q)\} = -\sum_n \frac{k_n}{n} \text{Tr} \{Q^n\} = \sum_k W_k e^{i\mu k} \]

\( k \); winding number

\( k = 0 \quad k = 1 \quad k = 2, -2 \quad S \)

Numerical Results

1. Canonical partition function

Lattice size: \( 8^3 \times 4 \)

\[ Z_{\text{can.}}(N_B) \]

\[ T_C = 222.5(11) \text{ MeV} \]

2-flavors Wilson-Clover

2. Partition function to cumulants

\( Z_{\text{can.}} \)

Quark number moments \( \langle \hat{N}^k \rangle \)

Quark number cumulants \( \langle \hat{N}^k \rangle_c \)

\[ \langle \hat{N}^k \rangle(\mu) = \sum_n n^k Z_{\text{can.}}(n) e^{n \mu T} \]

\[ \langle \hat{N}^1 \rangle = \langle \hat{N} \rangle = \langle \hat{N}^1 \rangle_c = \langle \hat{N}^1 \rangle_c = \langle \hat{N}^2 \rangle_c = \langle \hat{N}^2 \rangle_c - \langle \hat{N}^1 \rangle_c^2 \text{ etc.} \]

3. \( \langle \hat{N}^2 \rangle_c / \langle \hat{N}^1 \rangle_c \text{ vs. } \mu_B / T \)

Low temp. low dens. \( \rightarrow \) consistent with H.R. gas;
Low temp. high dens. \( \rightarrow \) approach to Q.G. gas;
High temp. \( \rightarrow \) approach to Q.G. gas

4. High order cumulants for \( \beta = 1.6 \)

H.R. \( \rightarrow \) “singular behavior” \( \rightarrow \) Q.G. 5th order is inconsistent with H.R. gas.

Conclusion

- We saw H.R. gas to Q.G. gas transition.
- At low temperature and density, lattice results consistent with H.R. gas.
- At high temperature or density, lattice results approach to Q.G. gas.
- We measured a “singular behavior” of high order cumulant for \( \beta = 1.6 \).