Calculation of high-order cumulant with canonical ensemble method in lattice QCD

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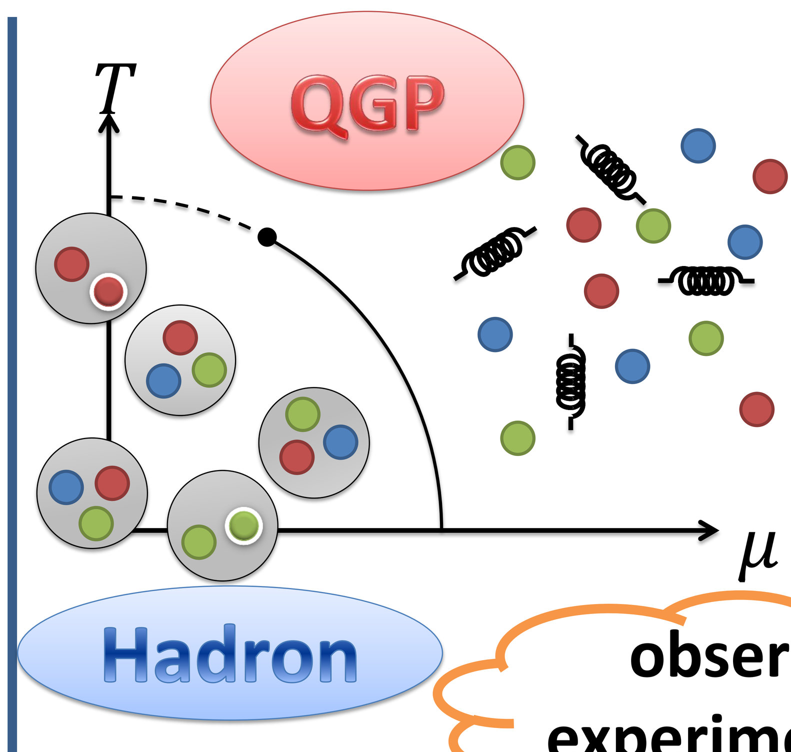
Background



We are interested in the **QCD phase diagram!**

I would like to see H.R. gas to Q.G. gas tr.

Quark number cumulants are convenient !

observed experimentally

computable In the models

Conclusion

- We saw H.R. gas to Q.G. gas transition.
- At low temperature and density, lattice results consistent with H.R. gas.
- At high temperature or density, lattice results approach to Q.G. gas.
- We measured a "singular behavior" of high order cumulant for $\beta = 1.6$.

Method

1.Canonical ensemble method

Fugacity expansion

$$Z_{G.C.}(\mu) = \sum_n Z_{can.}(n) e^{n\frac{\mu}{T}}$$

Canonical ⟷ Grand Canonical

Compatible!

Fourier transformation

$$Z_{can.}(n) = \frac{1}{2\pi}\int d\frac{\mu}{T}\, e^{-in\frac{\mu}{T}} Z_{G.C.}(i\mu)$$

2.Winding number expansion

$$Z_{can.}(n) = \left\langle \frac{1}{2\pi}\int_{-\pi}^{\pi} d\frac{\mu}{T}\, e^{-i\frac{\mu}{T}n} \frac{Det\{D(i\mu)\}}{Det\{D(0)\}} \right\rangle_g$$

? Instability of Fourier transf.

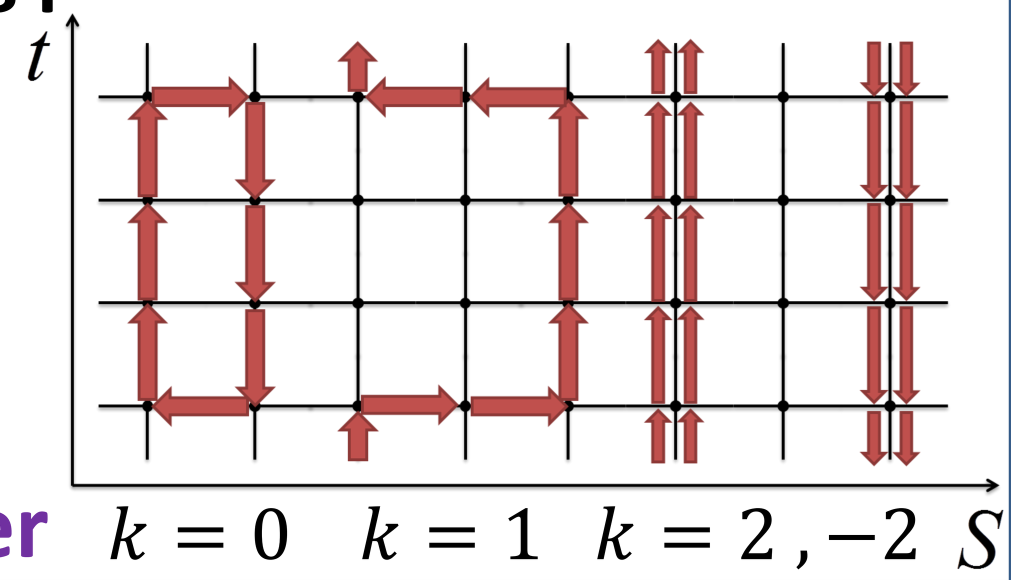
real and positive (for 2-flavors)

calculate $\det D(i\mu)$ at low numerical cost !

$$Det\{D(\mu)\} = Det\{1 - \kappa Q(\mu)\} = e^{Tr\{log(1-\kappa Q)\}}$$

κ : hopping parameter

$$Tr\{log(1-\kappa Q)\} = -\sum_n \frac{\kappa^n}{n} Tr\{Q^n\} = \sum_k W_k e^{\frac{\mu}{T}k}$$



k ; winding number

Numerical Results

1.Canonical partition function

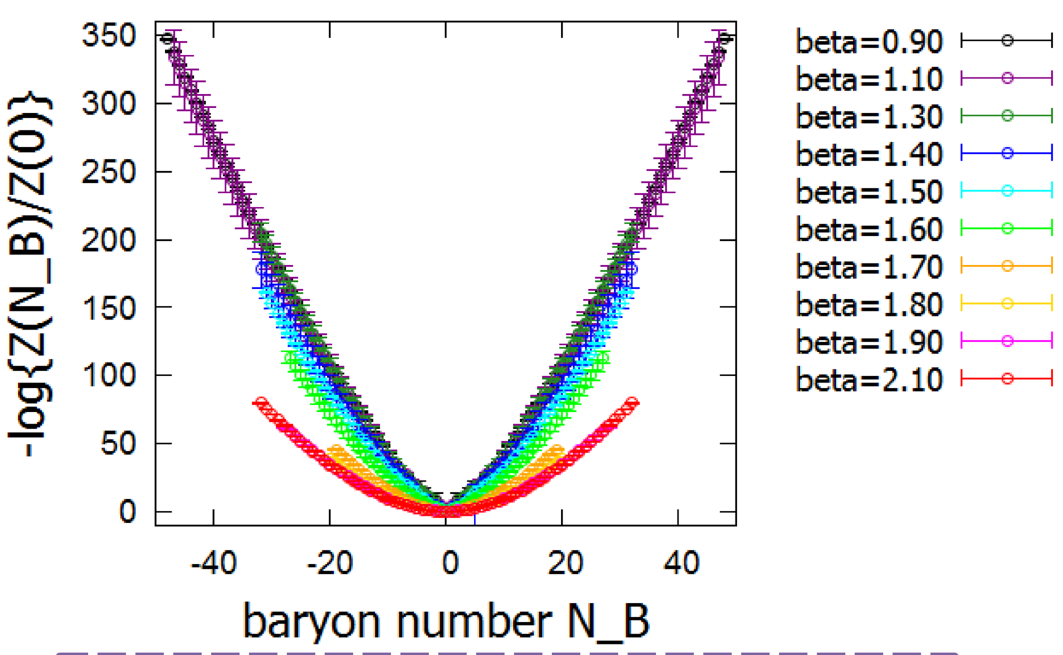
Lattice size: $8^3 \times 4$

β	κ	T/Tc
0.90	0.1370	0.644
1.10	0.1330	0.673
1.30	0.1330	0.706
1.40	0.1320	
1.50	0.1310	0.813
1.60	0.1300	
1.70	0.1290	1.00
1.80	0.1260	
1.90	0.1250	1.68
2.10	0.1220	3.45

$T_C = 222.5(11)\ MeV$

2-flavors Wilson-Clover

$Z_{can.}(N_B)$



Low temp. → flat
High temp. → pointed

2.Partition function to cumulants

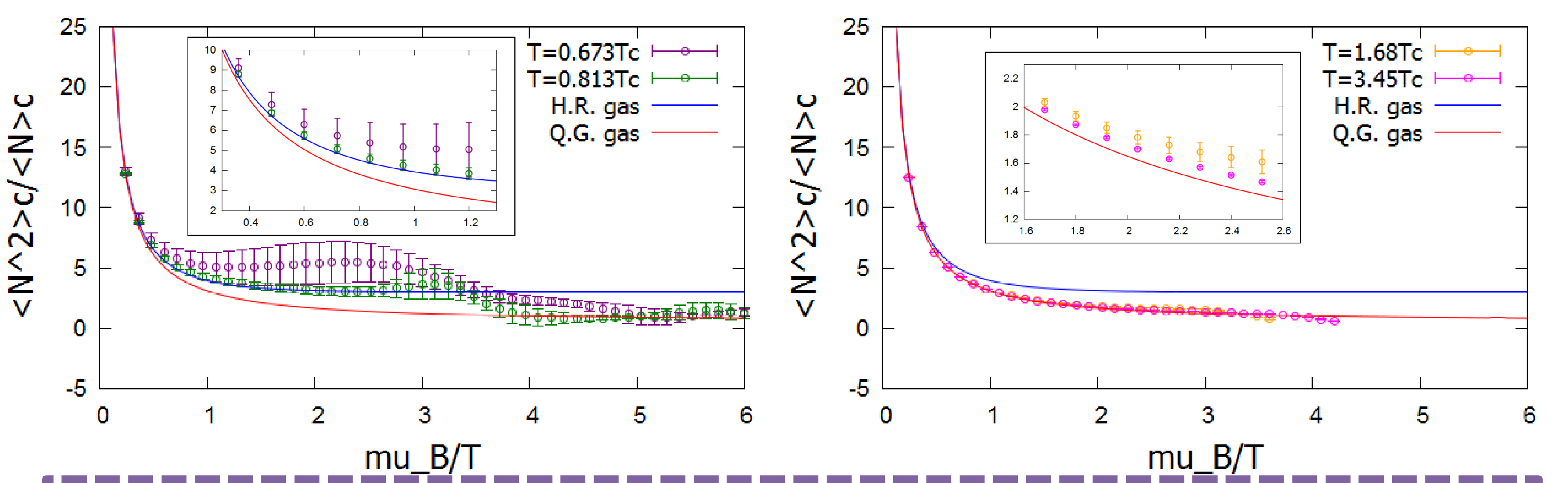
Can. partition function $Z_{can.}$ → Quark number moments $\langle \hat{N}^k \rangle$ → Quark number cumulants $\langle \hat{N}^k \rangle_c$

$$\langle \hat{N}^k \rangle(\mu) = \sum n^k \frac{Z_{can.}(n)}{Z_{G.C.}(\mu)} e^{n\frac{\mu}{T}}$$

$$\langle \hat{N}^1 \rangle_c = \langle \hat{N}^1 \rangle$$

$$\langle \hat{N}^2 \rangle_c = \langle \hat{N}^2 \rangle - \langle \hat{N}^1 \rangle^2 \text{ etc.}$$

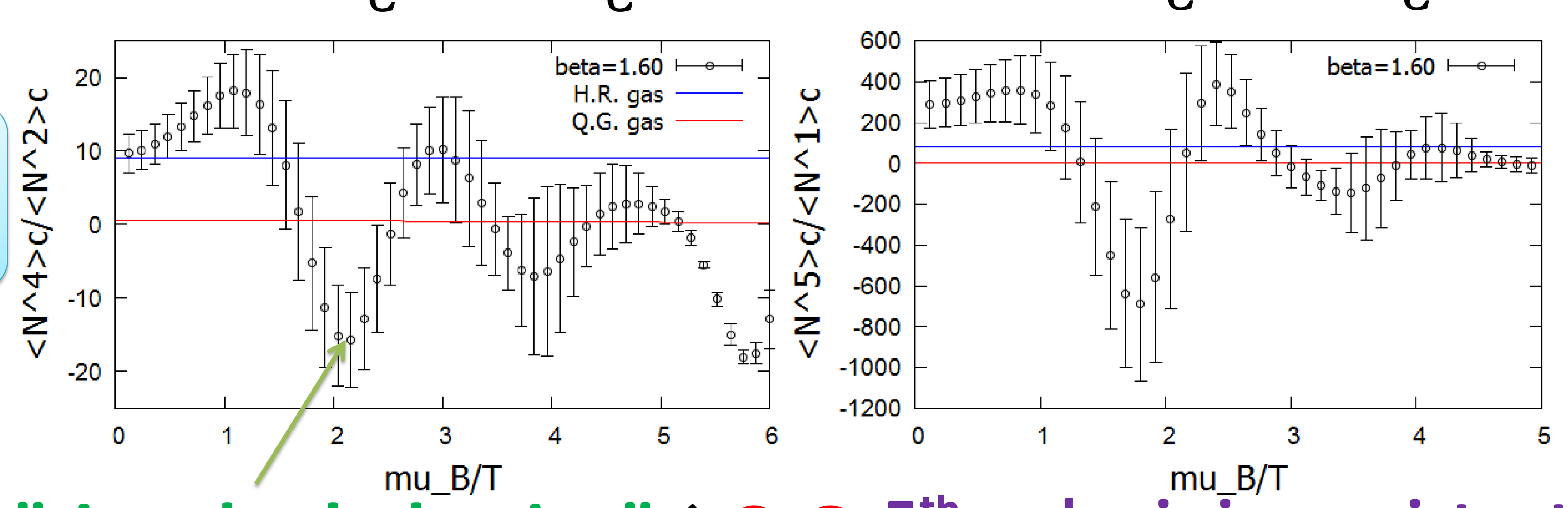
3.$\langle \hat{N}^2 \rangle_C / \langle \hat{N}^1 \rangle_C$ vs. μ_B/T



low temp. low dens.→ consistent with H.R. gas
low temp. high dens.→ approach to Q.G. gas
high temp. → approach to Q.G. gas

4.High order cumulants for $\beta = 1.6$

$\langle \hat{N}^4 \rangle_C / \langle \hat{N}^2 \rangle_C$ $\langle \hat{N}^5 \rangle_C / \langle \hat{N}^1 \rangle_C$



H.R.→"singular behavior"→Q.G.
evidence of P.T.?

5th order is inconsistent with H.R. gas.