Nonlinear hydrodynamic response confronts LHC data

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with Subrata Pal and Jean-Yves Ollitrault

• PLB744 (2015) 82-87 • Work in progress
Harmonic flow in heavy-ion collisions and flow response

- Harmonic flow and azimuthal anisotropy of event-by-event spectrum,

\[ \frac{dN}{d\phi_p} \sim \sum_n V_n e^{-in\phi_p} \quad \iff \quad V_n = v_n e^{in\Psi_n} = \langle e^{in\phi_p} \rangle \quad (\text{complex!}) \]
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Azimuthal symmetry of \( V_n \): \( \phi_p \to \phi_p + 2\pi/n \)
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V_n = V_n(\text{ initial anisotropy } : \varepsilon_n ; \eta/s, \ldots) \]

small and \( \frac{2\pi}{n} \) symmetric
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  \( \kappa_n^L \times \varepsilon_n \) **linear**
  \( \kappa_n^{NL} \times O(\varepsilon_n^2) \) **nonlinear**
Measurements of higher order harmonic flow: $V_4$, (also for $V_5$, $V_6$ etc.)

- $V_4$ measurements in experiment:

  $$V_4\{\Psi_2\} \quad \text{w.r.t. lower order harmonics}$$

  \[\text{[STAR nucl-ex/0310029 / PHENIX arXiv:1003.5586]}\]

  $$V_4\{\Psi_4\} \quad \text{w.r.t. its own event plane}$$

  \[\text{[ALICE 1105.2865 / PHENIX 1105.3928]}\]
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$$V_4\{\Psi_2}\approx\frac{\langle V_4 (V_2^*)^2 \rangle}{\langle |V_2|^4 \rangle^{1/2}}$$

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$$V_4\{\Psi_4\} (\approx V_4\{2\} = \langle |V_4|^2 \rangle^{1/2})$$

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[ALICE 1105.2865 / PHENIX 1105.3928]

Similarly, $V_6$ has been measured with respect to $\Psi_2$ and $\Psi_6$. 
Event-plane correlations from ATLAS are related to these quantities:

\[ \langle \cos 4(\Psi_4 - \Psi_2) \rangle = \frac{V_4\{\Psi_2\}}{V_4\{\Psi_4\}} \]

\[ \langle \cos 6(\Psi_6 - \Psi_2) \rangle = \frac{V_6\{\Psi_2\}}{V_6\{\Psi_6\}} \]

The event-plane correlations are compatible with CMS $V_4$ and $V_6$ data.
• Similarly, $V_5$ and $V_7$ could be measured w.r.t. a plane constructed with $\Psi_2$ and $\Psi_3$ ($\Psi_{23}$); $V_6$ could also be measured w.r.t. $\Psi_3$. These projected measurements are smaller, but can be measured with better accuracy!

  – Better resolution in experiments for lower harmonics.
• Similary, $V_5$ and $V_7$ could be measured w.r.t. a plane constructed with $\Psi_2$ and $\Psi_3$ ($\Psi_{23}$); $V_6$ could also be measured w.r.t. $\Psi_3$. These projected measurements are smaller, but can be measured with better accuracy!

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<tr>
<td>$V_5$</td>
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We still need from experiments: $V_5\{\Psi_{23}\}$, $V_6\{\Psi_3\}$, $V_7\{\Psi_{23}\}$.
New measurables $\chi_n \Leftrightarrow$ nonlinear hydro. resp. of higher harmonics

- Definition of $\chi_4$ – decomposition of $V_4$ in terms of $V_2$:

  \[
  \text{hydro. resp.: } V_4 = \kappa_4^L \varepsilon_4 + \kappa_4^{NL} \varepsilon_2^2 + \ldots, \quad V_2 = \kappa_2 \varepsilon_2
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  \[\sim V_2^2\]

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Similarly, $\chi_5 = \kappa_5^{NL} / (\kappa_2 \kappa_3)$, etc.
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  Ratio between hydro. flow resp.: $\chi_4 = \frac{\kappa_4^{NL}}{\kappa_2^2}$

  Similarly, $\chi_5 = \frac{\kappa_5^{NL}}{(\kappa_2 \kappa_3)}$, etc.

- Why is $\chi_4$ interesting?

  \[\text{Indep. of initial state (condition) by construction}!\]

  equivalently: \[V_4(\varepsilon_2, \varepsilon_4, \ldots) = V_4^L(\varepsilon_4, \ldots) + \chi_4(V_2(\varepsilon_2, \ldots))^2\]
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  - $\chi_4$ is determined by collective properties of medium ($\eta/s$) and by **freeze-out**

  $\Rightarrow$ a scaling relation from $V_4^{NL} = \frac{1}{2} V_2^2$. 
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  **Approachable in experiments!**
χ₄ can be measured experimentally

- Considering the definition of χ₄: \( V₄ = V₄^L + χ₄(V₂)^2 \)

We assume that the 'linear' term \( V₄^L \) is uncorrelated with \( V₂^2 \).

\[
χ₄ = \frac{\langle V₄(V₂^*)^2 \rangle}{\langle |V₂|^4 \rangle} = \frac{V₄\{Ψ₂\}}{\langle |V₂|^4 \rangle^{1/2}} = \frac{V₄ \text{ w.r.t. } Ψ₂}{\text{moments of } V₂}
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- Moments \( \langle |V_2|^4 \rangle^{1/2} \) are extracted from CMS cumulants \( v_2\{2\} \) and \( v_2\{4\} \),

(See also ATLAS event-shape selection.)
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\( \chi_4 : \) hydro. vs. experiments

- Hydro calculation with a smooth Gaussian density profile + deformations:

\[
\chi_4 = \frac{v_4}{v_2^2} \iff \text{deforming Gaussian profile by an ellipticity } \varepsilon_2.
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  \]

\( T_{f_0} = 150 \text{ MeV}, \text{Lattice EOS, direct pions:} \)

![Graph showing \( \chi_4 \) vs. Centrality (%)](image)

Hydro. captures right trend and magnitude.
$\chi_n$ of higher order harmonics: $V_5$, $V_6$ and $V_7$

- From nonlinear resp. allowed by rotational symmetry:

\[
\frac{2\pi}{5} \Rightarrow (V_5, V_2V_3) \quad \Leftrightarrow \quad \chi_5 = \frac{\langle V_5(V_2^*V_3^*) \rangle}{\langle |V_2|^2|V_3|^2 \rangle}
\]

\[
\frac{2\pi}{6} \Rightarrow (V_6, V_2^3, V_3^2) \quad \Leftrightarrow \quad \chi_{62} = \frac{\langle V_6V_2^3 \rangle}{\langle |V_2|^6 \rangle}, \quad \chi_{63} = \frac{\langle V_6V_3^2 \rangle}{\langle |V_3|^4 \rangle}
\]

\[
\frac{2\pi}{7} \Rightarrow (V_7, V_2^2V_3) \quad \Leftrightarrow \quad \chi_7 = \frac{\langle V_7(V_2^2V_3^*) \rangle}{\langle |V_2|^4|V_3|^2 \rangle}
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  \]

- Scaling relations from freeze-out: (recall \( V_4^{NL} = \frac{1}{2} V_2^2 \))
  
  \[
  \Rightarrow V_5^{NL} = V_2 V_3, \quad V_6^{NL} = \frac{1}{2} V_3^2, \quad V_6^{NL} = \frac{1}{6} V_2^3, \quad V_7^{NL} = \frac{1}{2} V_2^2 V_3
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• $\chi_n$ from experiments:

$V_n$ w.r.t. lower harmonics / moments of lower harmonics

* Extract $V_n$ w.r.t. lower harmonics by, e.g., ATLAS event-plane correlations.
* Extract moments from cumulants [or Bhalerao et. al., PLB742 94-98].
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- Smooth hydro. vs. experiments:

- Hydro. captures right trend and magnitude.

- A simple, but non-trivial scaling relation – nonlinearities from freeze-out

\[ \chi_4 \sim \chi_{63} \sim \frac{1}{2} \chi_5 \]

quadratic: \( \langle p_t^2 \rangle / \langle p_t \rangle^2 \)

\[ \chi_{62} \sim \frac{1}{3} \chi_7 \]

cubic: \( \langle p_t^3 \rangle / \langle p_t \rangle^3 \)
Smooth hydro. vs. experiments:

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- A simple, but non-trivial scaling relation – nonlinearities from freeze-out

\[ \chi_4 \sim \chi_{63} \sim \frac{1}{2} \chi_5 \] (exp. confirmed !)

\[ \chi_{62} \sim \frac{1}{3} \chi_7 \] (need \( \chi_7 \) from exp.)

- Quadratic: \( \frac{\langle p_t^2 \rangle}{\langle p_t \rangle^2} \)

- Cubic: \( \frac{\langle p_t^3 \rangle}{\langle p_t \rangle^3} \)
Also from event-by-event calculations – AMPT with $\sigma = 1.5$ mb:

- Good test of independency of $\chi_n$ on initial state fluctuations.
- Scaling relation also seen in AMPT results, in particular $\chi_7 = 3\chi_{62}$.
Hydro. prediction of $V_7\{\Psi_{23}\} = \chi_7 \times \sqrt{\langle |V_2|^4 |V_3|^2 \rangle}$
Summary and outlook

- New set of measurables for higher harmonics:

\[ \chi_n = \frac{\text{Nonlinear hydro. resp.}}{(\text{powers of}) \, \text{linear hydro. resp.}} \text{theory} = \frac{V_n \text{ w.r.t. lower harmonics}}{(\text{moments of}) \, \text{lower harmonics}} \text{experiment} \]

- Indep. of initial state geometry and fluctuations. \( \Leftrightarrow \) Naïve hydro. and AMPT.
- Measurables directly related to medium collective properties – \( \eta/s \).
- Scaling relations suggest nonlinearities dominated by freeze-out.

- Event-by-event hydro. calculations.
- Direct measurements from experiments.