Nonlinear hydrodynamic response confronts LHC data

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with Subrata Pal and Jean-Yves Ollitrault

- PLB744 (2015) 82-87 • Work in progress

Harmonic flow in heavy-ion collisions and flow response

- Harmonic flow and azimuthal anisotropy of event-by-event spectrum,

$$\frac{dN}{d\phi_p} \sim \sum_n V_n e^{-in\phi_p} \quad \Longleftrightarrow \quad V_n = v_n e^{in\Psi_n} = \langle e^{in\phi_p} \rangle \quad (\mathit{complex}\,!)$$

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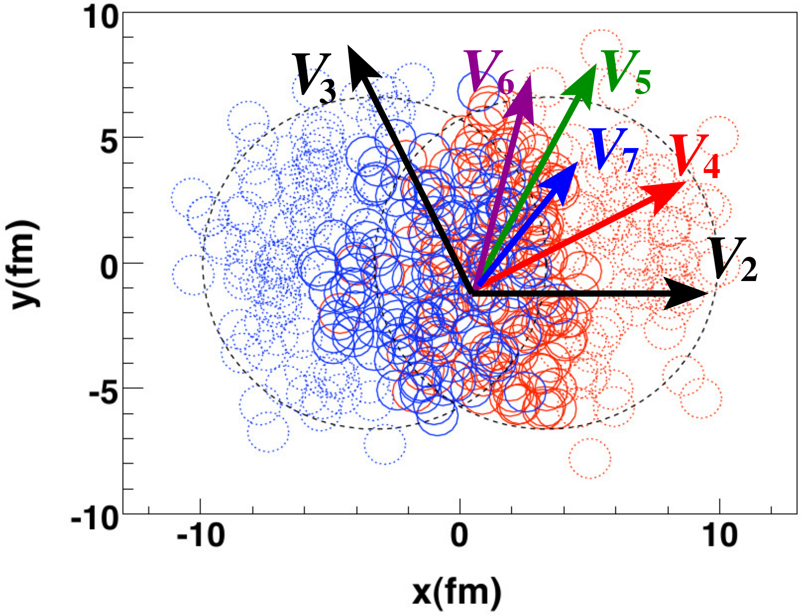

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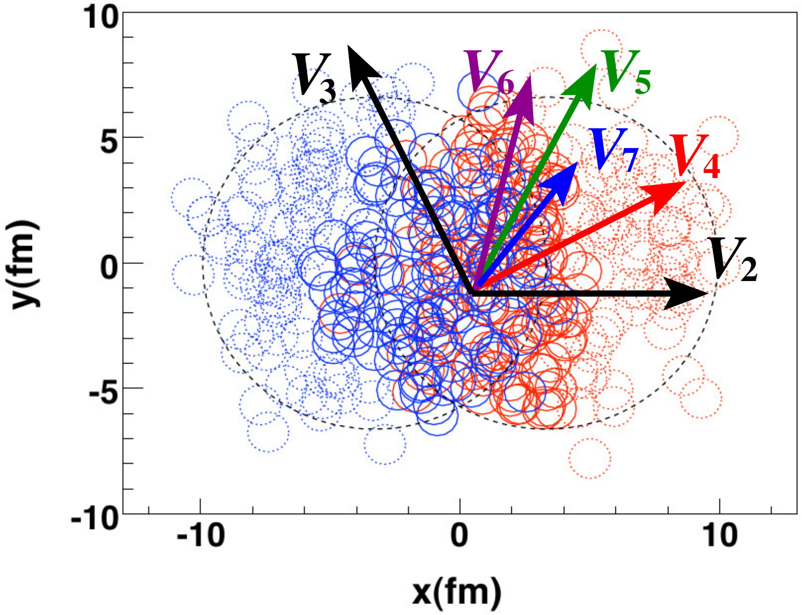

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$$V_n = V_n(\underbrace{\text{initial anisotropy} : \varepsilon_n}_{\text{small and } \frac{2\pi}{n} \text{ symmetric}} ; \eta/s, \dots)$$

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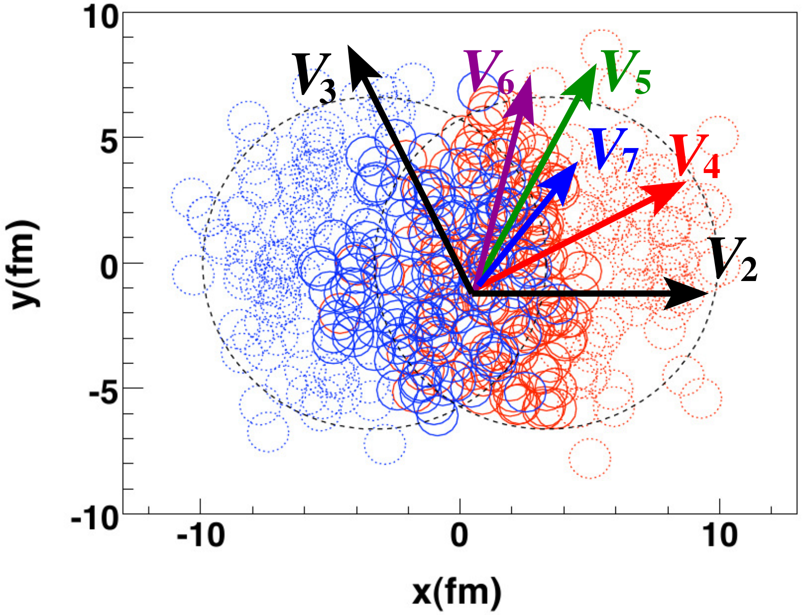

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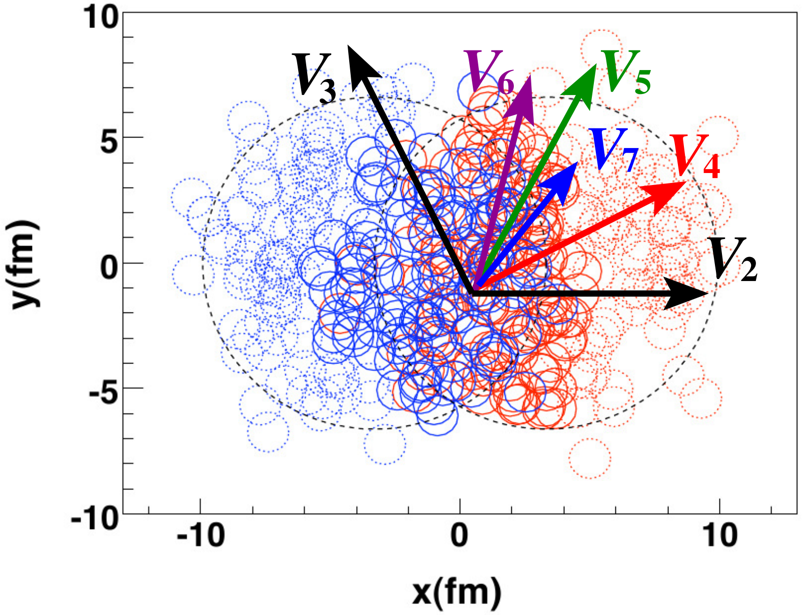

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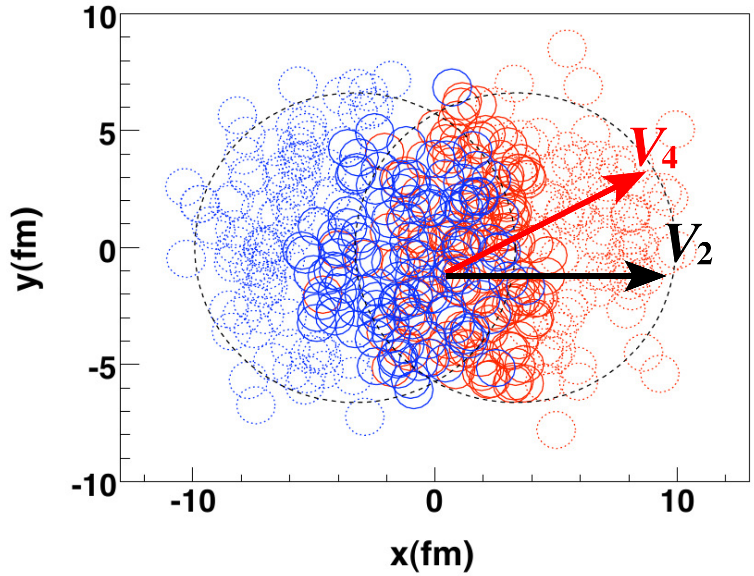
Measurements of higher order harmonic flow: V_4, (also for V_5, V_6 etc.)



- V_4 measurements in experiment:

$V_4\{\Psi_2\}$ w.r.t. lower order harmonics

[STAR nucl-ex/0310029 / PHENIX arXiv:1003.5586]

$V_4\{\Psi_4\}$ w.r.t. its own event plane

[ALICE 1105.2865 / PHENIX 1105.3928]

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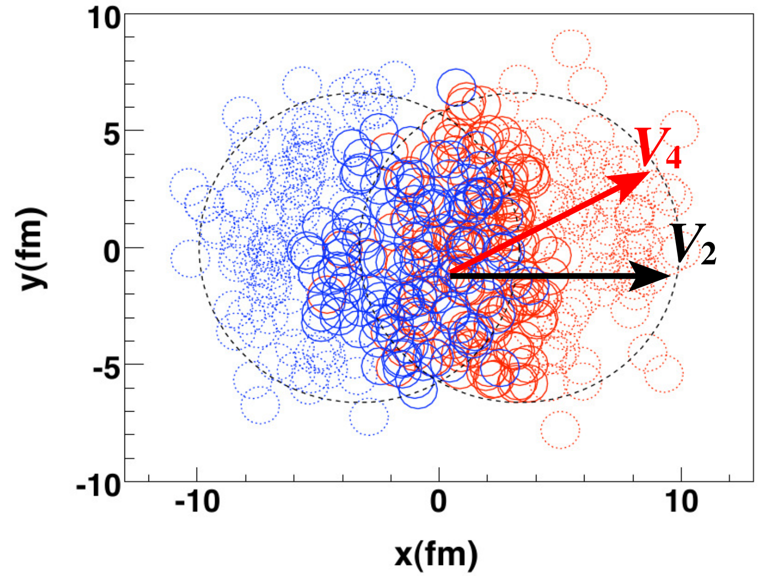

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$$V_4\{\Psi_4\}(\approx V_4\{2\} = \langle |V_4|^2 \rangle^{1/2}) \quad \text{w.r.t. its own event plane}$$

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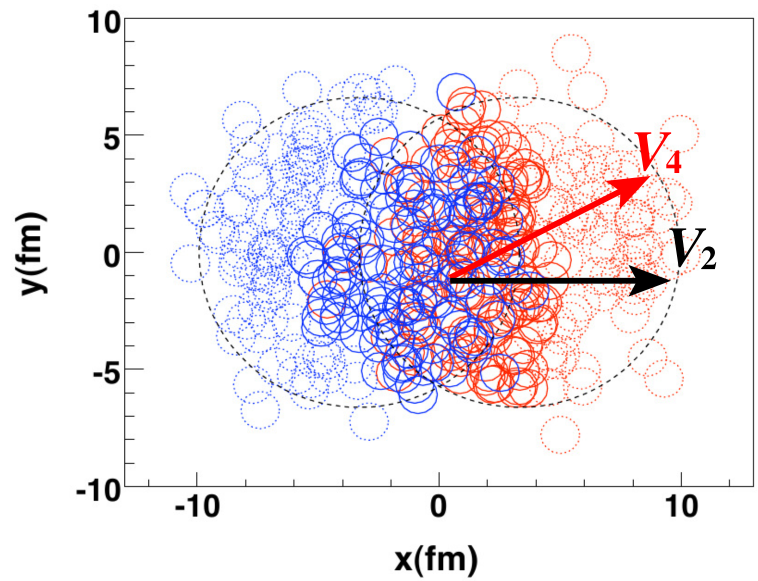

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Similarly, V_6 has been measured with respect to Ψ_2 and Ψ_6.

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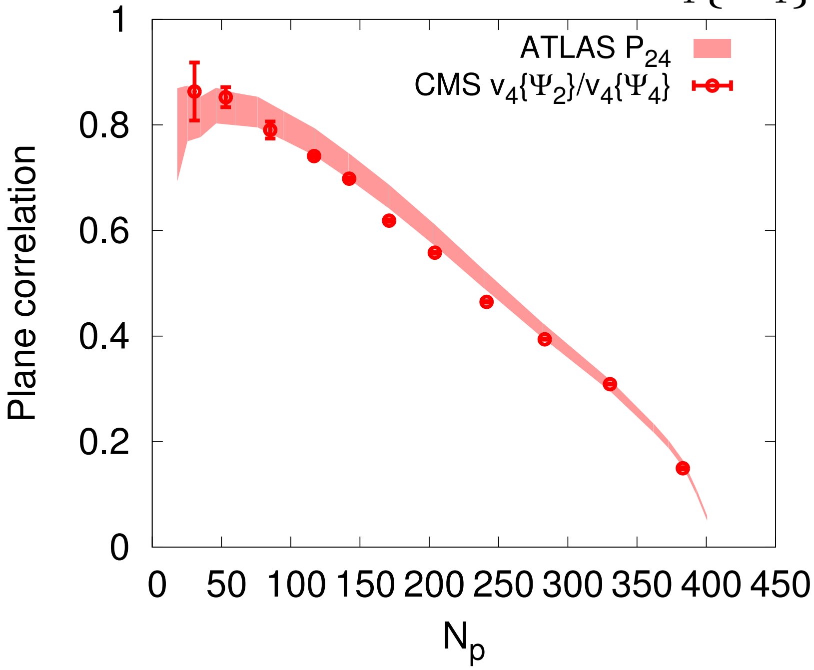
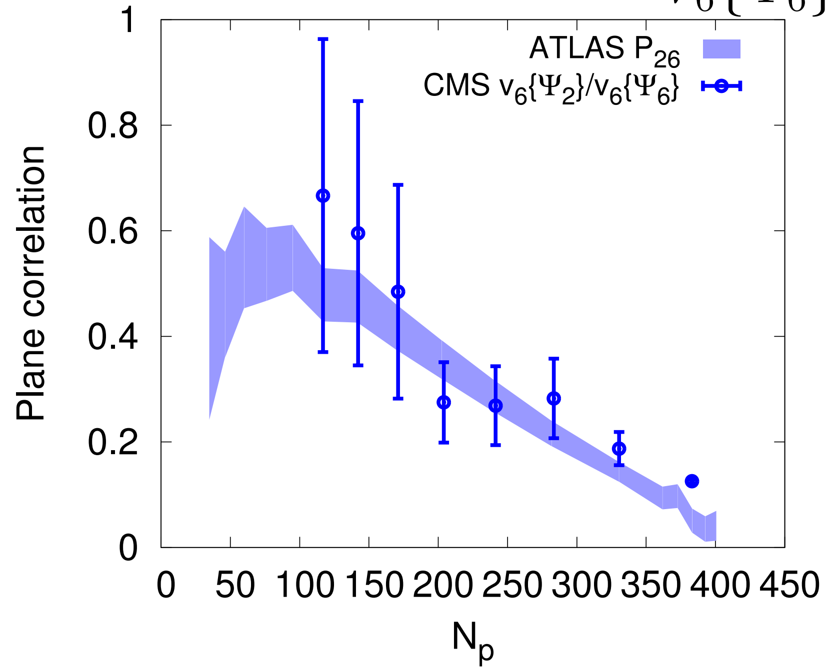
- Event-plane correlations from ATLAS are related to these quantities :

$$\langle \cos 4(\Psi_4 - \Psi_2) \rangle = \frac{V_4\{\Psi_2\}}{V_4\{\Psi_4\}} \qquad \langle \cos 6(\Psi_6 - \Psi_2) \rangle = \frac{V_6\{\Psi_2\}}{V_6\{\Psi_6\}}$$





The event-plane correlations are compatible with CMS V_4 and V_6 data.

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- Similary, V_5 and V_7 could be measured w.r.t. a plane constructed with Ψ_2 and Ψ_3 (Ψ_{23}); V_6 could also be measured w.r.t. Ψ_3. These *projected* measurements are smaller, but can be measured with better accuracy !
  - Better resolution in experiments for lower harmonics.

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	Ψ_2	Ψ_3	Ψ_5	Ψ_6	Ψ_7
V_5	✗		✓	N/A	N/A
V_6	✓	✗	N/A	✓	N/A
V_7	✗		N/A	N/A	✗

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V_7	✗		N/A	N/A	✗

We still need from experiments: $V_5\{\Psi_{23}\}$, $V_6\{\Psi_3\}$, $V_7\{\Psi_{23}\}$.

New measurables $\chi_n \Leftrightarrow$ nonlinear hydro. resp. of higher harmonics

- Definition of χ_4 – decomposition of V_4 in terms of V_2 :

$$\text{hydro. resp. :} \qquad V_4 = \underbrace{\kappa_4^L \varepsilon_4}_{V_4^L} + \underbrace{\kappa_4^{NL} \varepsilon_2^2}_{\sim V_2^2} + \ldots, \quad V_2 = \kappa_2 \varepsilon_2$$

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Ratio between hydro. flow resp.: $\chi_4 = \kappa_4^{NL} / \kappa_2^2$

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- Why is χ_4 interesting?

Indep. of initial state (condition) by construction !

$$\text{equivalently :} \qquad V_4(\varepsilon_2, \varepsilon_4, \ldots) = V_4^L(\varepsilon_4, \ldots) + \chi_4 (V_2(\varepsilon_2, \ldots))^2$$

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$\Rightarrow$ a scaling relation from $V_4^{NL} = \frac{1}{2} V_2^2$.

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Approachable in experiments !

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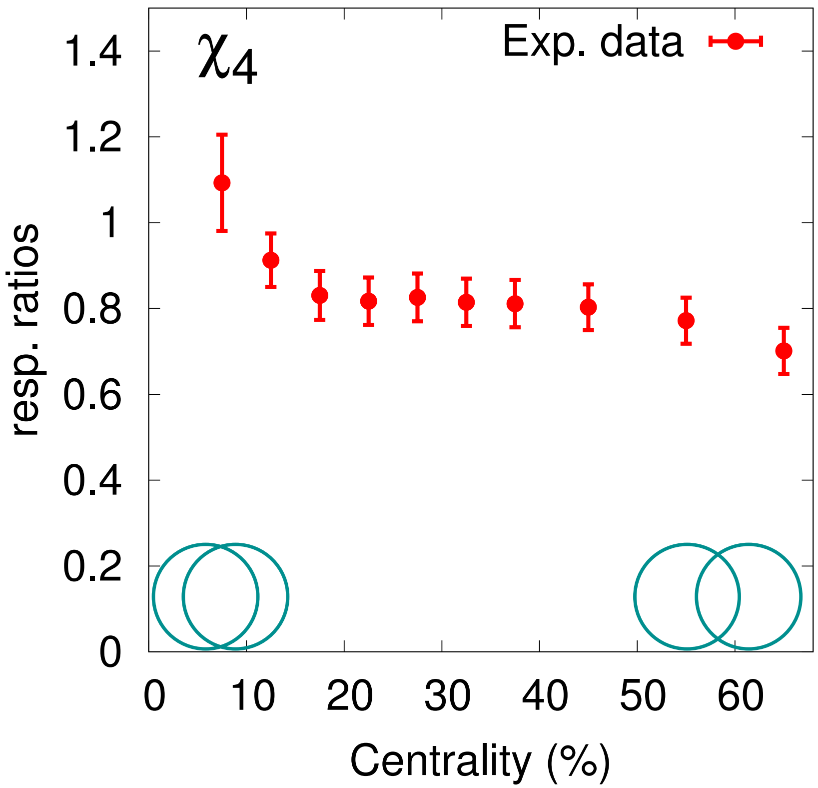

(See also ATLAS event-shape selection.)

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χ_4 : hydro. vs. experiments

- Hydro calculation with a smooth Gaussian density profile + deformations :

$$\chi_4 = \frac{v_4}{v_2^2} \quad \leftrightarrow \quad \text{deforming Gaussian profile by an ellipticity } \varepsilon_2.$$

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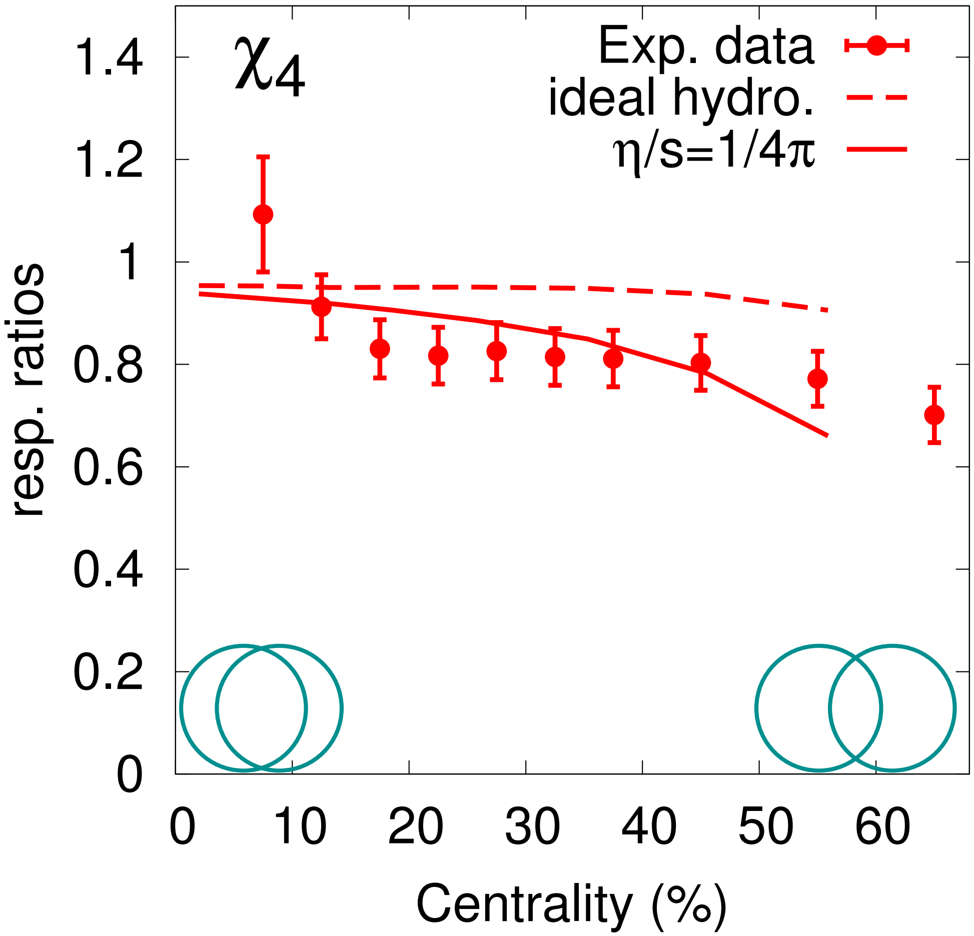
$T_{\rm fo} = 150$ MeV, Lattice EOS, direct pions :



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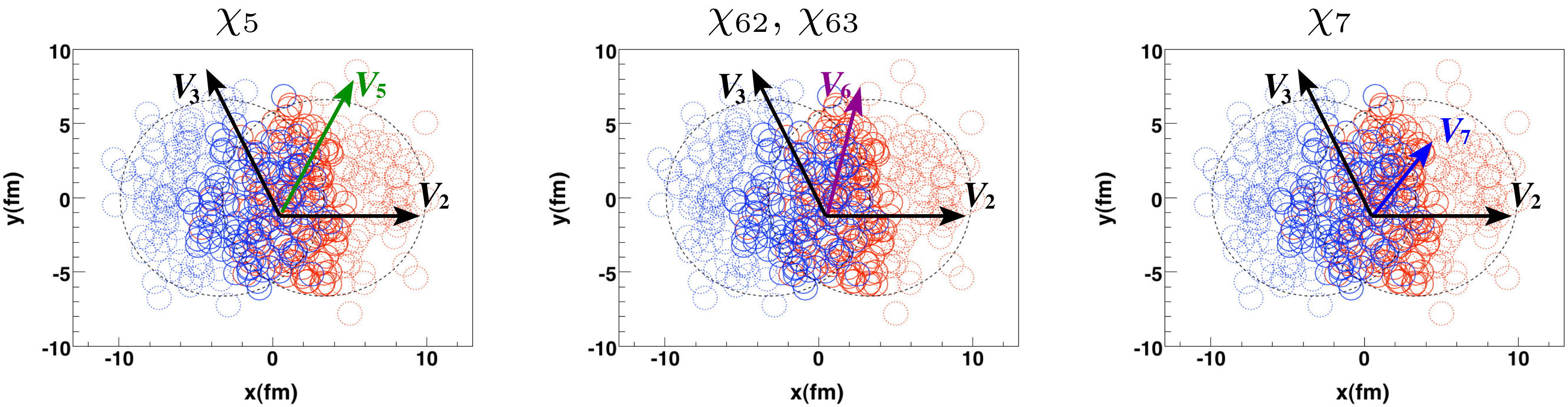
χ_n of higher order harmonics: V_5, V_6 and V_7



- From nonlinear resp. allowed by rotational symmetry :

$$\frac{2\pi}{5} \Rightarrow (V_5, V_2 V_3) \quad \Leftrightarrow \quad \chi_5 = \frac{\langle V_5 (V_2^* V_3^*) \rangle}{\langle |V_2|^2 |V_3|^2 \rangle}$$

$$\frac{2\pi}{6} \Rightarrow (V_6, V_2^3, V_3^2) \quad \Leftrightarrow \quad \chi_{62} = \frac{\langle V_6 V_2^{*3} \rangle}{\langle |V_2|^6 \rangle}, \quad \chi_{63} = \frac{\langle V_6 V_3^{*2} \rangle}{\langle |V_3|^4 \rangle}$$

$$\frac{2\pi}{7} \Rightarrow (V_7, V_2^2 V_3) \quad \Leftrightarrow \quad \chi_7 = \frac{\langle V_7 (V_2^{*2} V_3^*) \rangle}{\langle |V_2|^4 |V_3|^2 \rangle}$$

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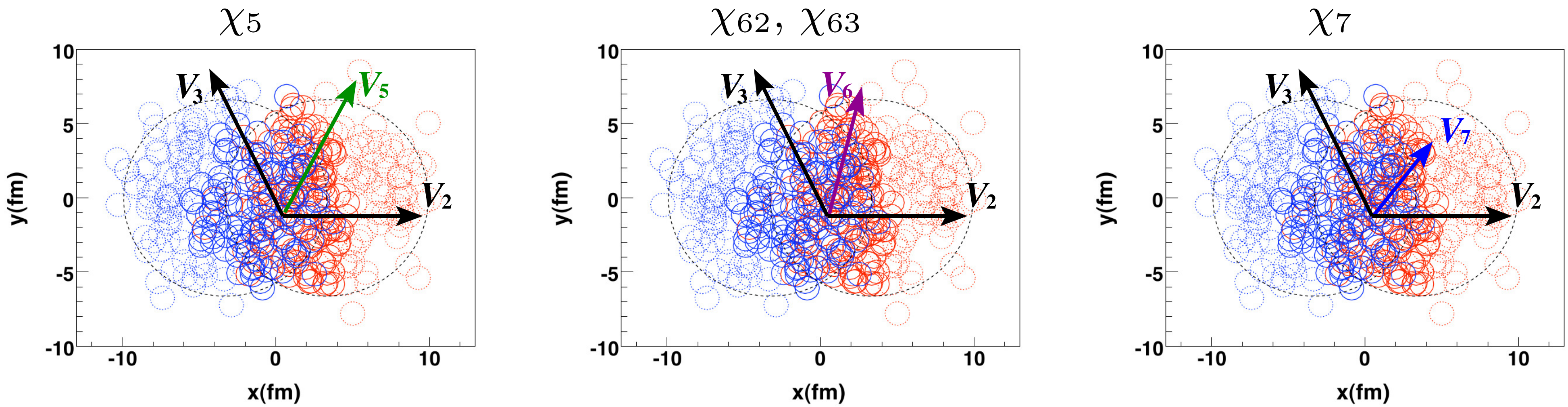

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- Scaling relations from **freeze-out**: (recall $V_4^{NL} = \frac{1}{2}V_2^2$)

$$\Rightarrow \quad V_5^{NL} = V_2V_3, \quad V_{6|3}^{NL} = \frac{1}{2}V_3^2, \quad V_{6|2}^{NL} = \frac{1}{6}V_2^3, \quad V_7^{NL} = \frac{1}{2}V_2^2V_3$$

- χ_n from experiments:

V_n w.r.t. lower harmonics / moments of lower harmonics

* Extract V_n w.r.t. lower harmonics by, e.g., ATLAS event-plane correlations.
* Extract moments from cumulants [or Bhalerao et. al., PLB742 94-98].

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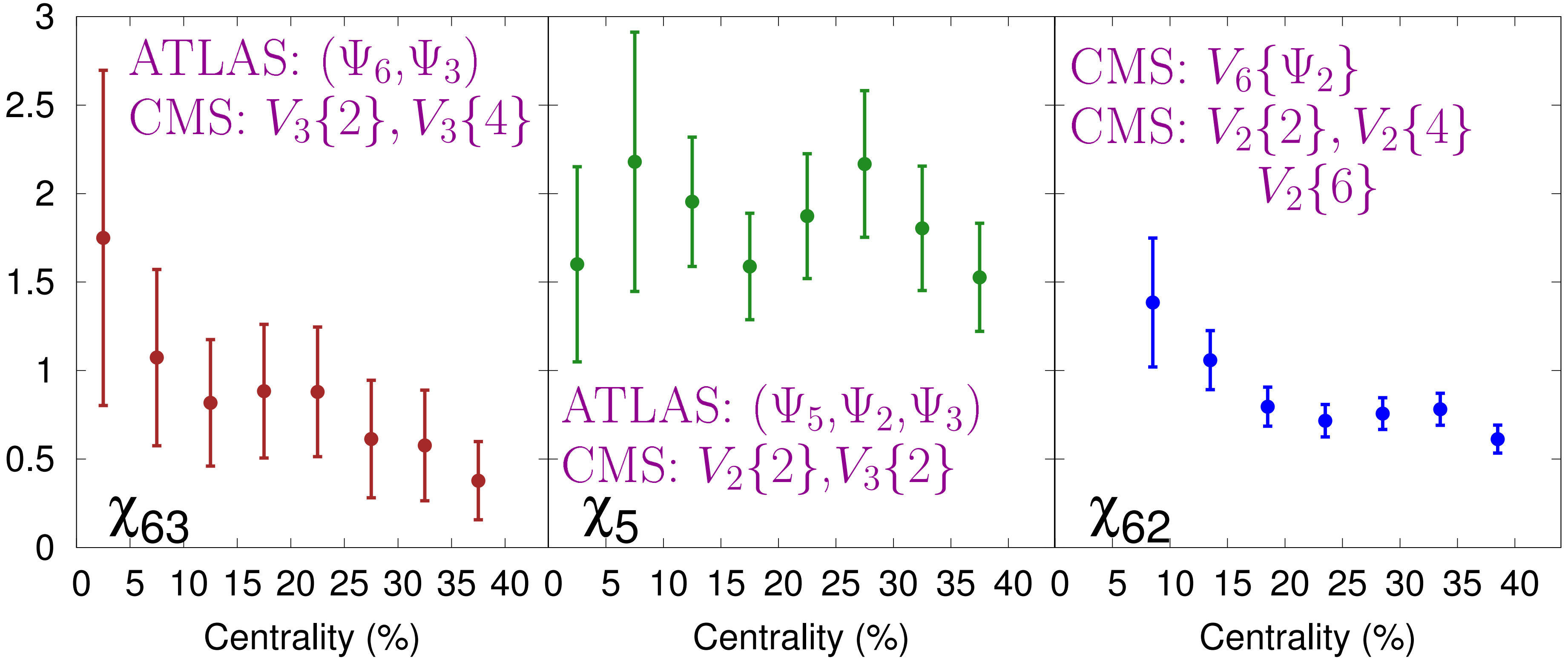

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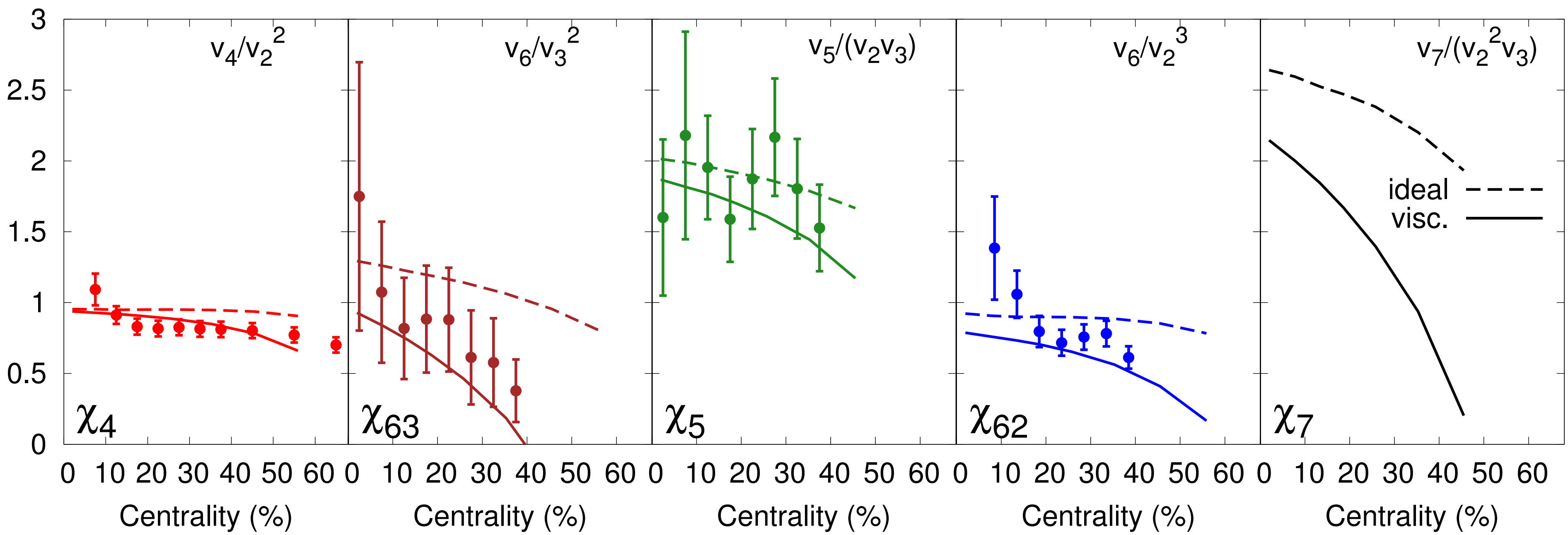
- Smooth hydro. vs. experiments :



- Hydro. captures right trend and magnitude.

- A simple, but non-trivial scaling relation – nonlinearities from freeze-out

$$\underbrace{\chi_4 \sim \chi_{63} \sim \frac{1}{2}\chi_5}_{\text{quadratic}: \langle p_t^2\rangle/\langle p_t\rangle^2} \qquad \underbrace{\chi_{62} \sim \frac{1}{3}\chi_7}_{\text{cubic}: \langle p_t^3\rangle/\langle p_t\rangle^3}$$

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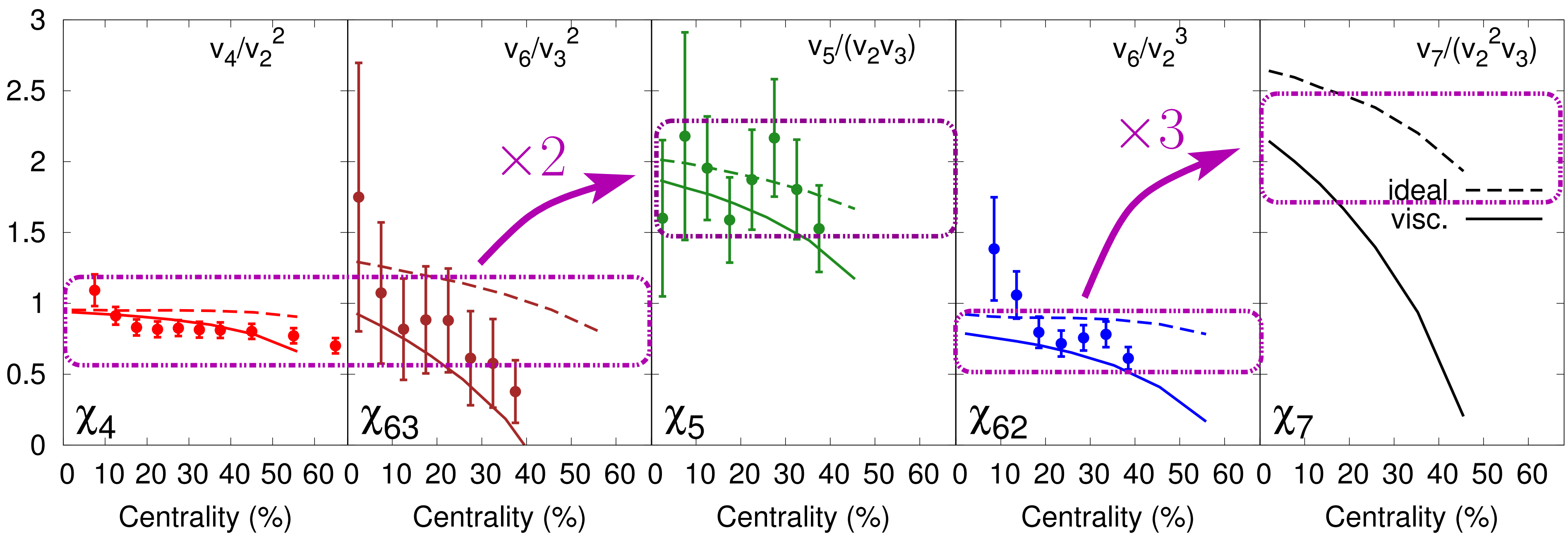

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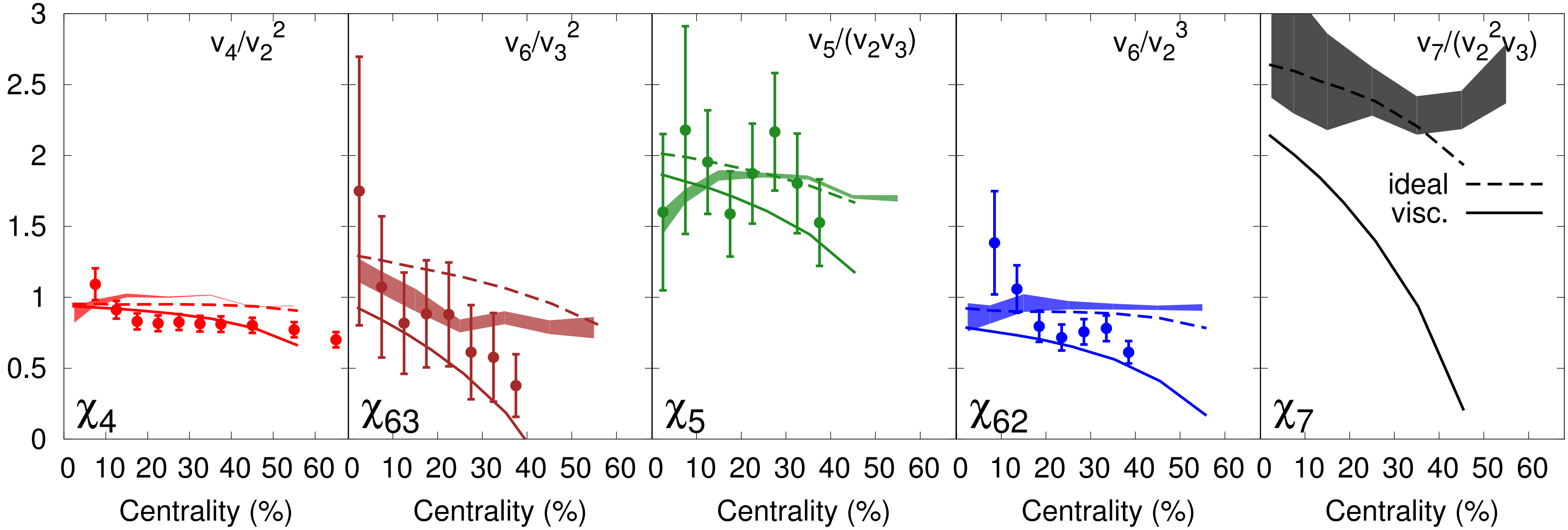
- Also from event-by-event calculations – AMPT with $\sigma = 1.5$ mb:



  - Good test of independency of χ_n on initial state fluctuations.
  - Scaling relation also seen in AMPT results, in particular $\chi_7 = 3\chi_{62}$.

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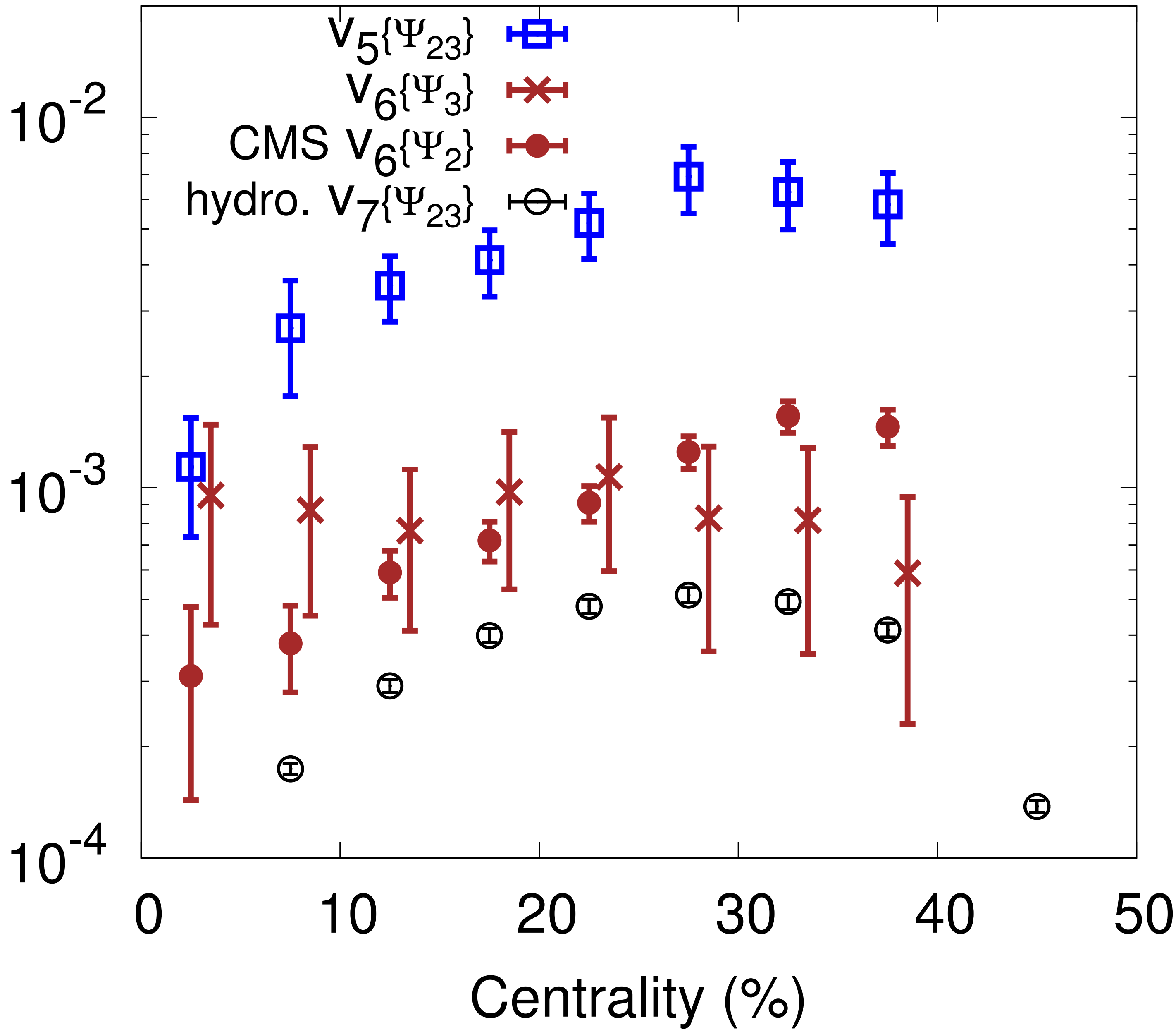
Hydro. prediction of $V_7\{\Psi_{23}\} = \chi_7 \times \sqrt{\langle |V_2|^4 |V_3|^2 \rangle}$



Summary and outlook

- New set of measurables for higher harmonics :

$$\chi_n = \underbrace{\frac{\text{Nonlinear hydro. resp.}}{\text{(powers of) linear hydro. resp.}}}_{\text{theory}} = \underbrace{\frac{V_n \text{ w.r.t. lower harmonics}}{\text{(moments of) lower harmonics}}}_{\text{experiment}}$$

  - Indep. of initial state geometry and fluctuations. ⇔ Naïve hydro. and AMPT.
  - Measurables directly related to medium collective properties – η/s.
  - Scaling relations suggest nonlinearities dominated by **freeze-out**.
- Event-by-event hydro. calculations.
- Direct measurements from experiments.