Critical Behavior of QCD Phase Transition at Finite Isospin Density

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Abstract

We investigate QCD phase transition at finite isospin density with functional renormalization group (FRG). There exists a dimension crossover at finite temperature, its effects on critical exponents and universality class are analyzed.

Model and FRG

- $SU(2)$ linear σ model:

$$\mathcal{L}^E = \frac{1}{2}(\partial_\mu \hat{\vec{\pi}})^2 + \frac{1}{2}(\partial_\mu \hat{\sigma})^2 + i\mu_I(\hat{\pi}_1\partial_0\hat{\pi}_2 - \hat{\pi}_2\partial_0\hat{\pi}_1) - \frac{\mu_I^2}{2}(\hat{\pi}_1^2 + \hat{\pi}_2^2) + U(\hat{\vec{\pi}}, \hat{\sigma})$$

$$U(\hat{\vec{\pi}}, \hat{\sigma}) = \frac{m^2}{2}(\hat{\vec{\pi}}^2 + \hat{\sigma}^2) + \frac{\lambda}{4}(\hat{\vec{\pi}}^2 + \hat{\sigma}^2)^2 - c\hat{\sigma}$$

- Chiral condensate $\langle\sigma\rangle$, pion condensate $\langle\pi\rangle$
- Normal phase $\langle\pi\rangle = 0$, superfluid phase $\langle\pi\rangle \neq 0$
- Flow equation

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]$$

- IR regulator $R_k = (k^2 - \vec{p}^2)\Theta(k^2 - \vec{p}^2)$
- Under uniform field configuration and local potential approximation
- Effective action

$$\Gamma_k = V_4 U_k = \int_x \left[\frac{m_k^2}{2}(\pi_k^2 + \sigma_k^2) + \frac{\lambda_k}{4}(\pi_k^2 + \sigma_k^2)^2 - \frac{\mu_I^2}{2}\pi_k^2 - c\sigma_k\right]$$

- Flow equation in normal phase

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\sum_{\pm}\frac{\coth\left(\frac{E_{\pi,k}\pm\mu_I}{2T}\right)}{E_{\pi,k}} + \frac{\coth\left(\frac{E_{\pi,k}}{2T}\right)}{E_{\pi,k}} + \frac{\coth\left(\frac{E_{\sigma,k}}{2T}\right)}{E_{\sigma,k}}\right]$$

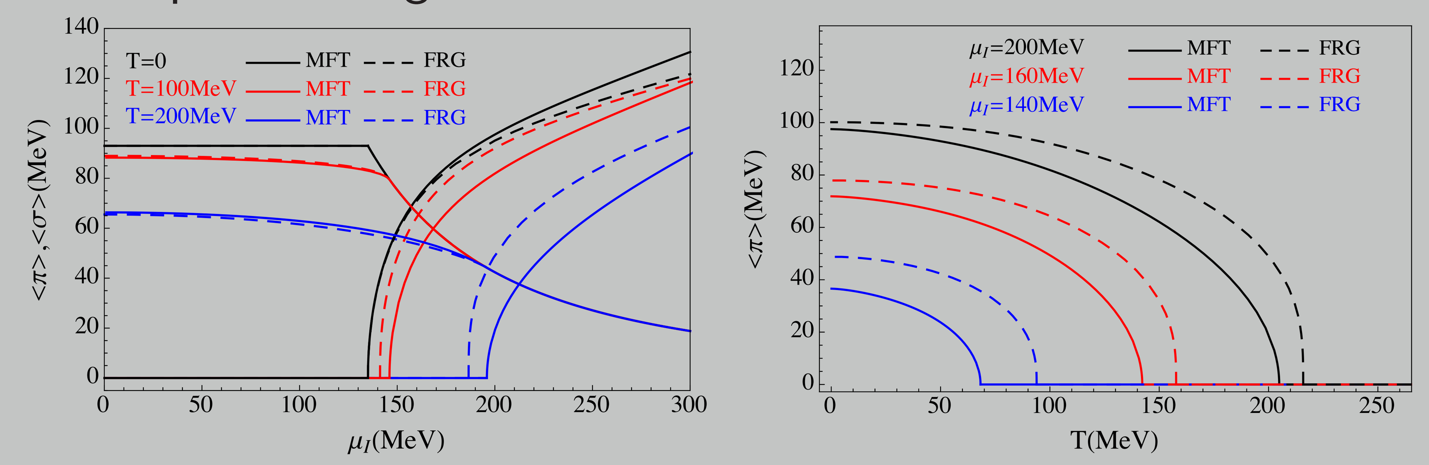
- Flow equation in superfluid phase

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{\coth\left(\frac{E_{\pi,k}}{2T}\right)}{E_{\pi,k}} + \sum_{i=1}^{3}\frac{R_i \coth\left(\frac{E_{i,k}}{2T}\right)}{E_{i,k}}\right]$$

- Derivative expansion around the condensates
- Initial parameters fixed by vacuum properties

Critical Behaviors of Isospin System

- Evolution of pion and sigma condensate



- Critical exponents

$$\langle\pi\rangle = c_1\left(\frac{T_c - T}{T_c}\right)^\beta, \qquad \frac{1}{m_\pi} = c_2\left(\frac{T - T_c}{T_c}\right)^{-\nu}$$

- Scaling law $\Rightarrow$ Effective dimension

$$\beta = \frac{d-2}{2}\nu \quad \Rightarrow \quad d_{\text{eff}} = 2 + 2\beta/\nu$$

- Compare results of **FRG** and **MFT**

T_c(MeV)	$\beta(FRG)$	$\beta(MFT)$	$\nu(FRG)$	$\beta(MFT)$	$d_{\text{eff}}(FRG)$	$d_{\text{eff}}(MFT)$
0	0.5	0.5	0.5	0.5	4	4
10	0.442	0.5	0.517	0.520	3.71	3.92
100	0.352	0.5	0.619	0.750	3.14	3.33
250	0.313	0.5	0.628	0.948	3	3.05

Dimension Crossover

- Dimension reduction by temperature
- Momentum integral in $S^1 \times R^d$ space

$$\int_0^\infty d^D p \longrightarrow \int_0^\beta dp_0 \int_0^\infty d^d \vec{p}$$

- two length scale: $\beta = 1/T$ the circumference of S^1
  $L = 1/k$ the inverse of RG-scale
- Zero temperature $D = d + 1$
- Intermediate temperature $d < D < d + 1$
- High temperature limit $D = d$

Comparison with O(N) Model in Continuous Dimension

- Euclidean Lagrangian involving a set of N real scalar field $\Phi = (\phi_1, \cdots, \phi_N)^T$

$$\mathcal{L}_N = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i + \frac{1}{2}a_k^{(2)}\phi_i\phi_i + \frac{1}{4}a_k^{(4)}(\phi_i\phi_i)^2$$

- Rescaling method

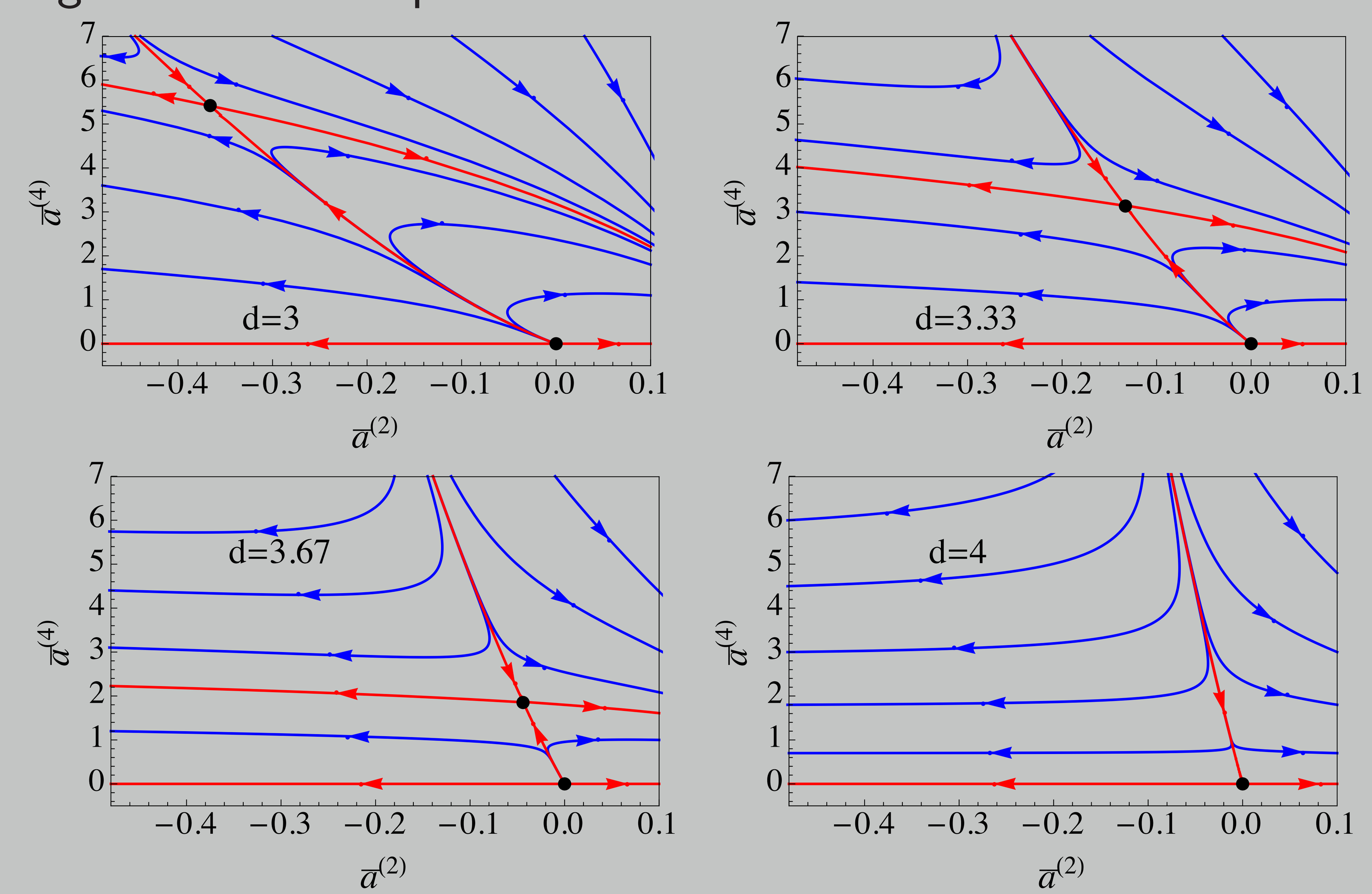
$$\overline{\phi}_k = k^{-(d-2+\eta)/2}\phi_k$$
$$\overline{a}_k^{(2)} = k^{-2}a_k^{(2)}$$
$$\overline{a}_k^{(4)} = k^{d-4+2\eta}a_k^{(4)}$$

- Flow equation of rescaled coupling parameter

$$\partial_t\overline{a}_t^{(2)} = -2\overline{a}_t^{(2)} + 2S_d\overline{a}_t^{(4)}\left[-\left(\frac{3}{\overline{E}_1^4} + \frac{N-1}{\overline{E}_2^4}\right) + 2\overline{a}_k^{(2)}\left(\frac{9}{\overline{E}_1^6} + \frac{N-1}{\overline{E}_2^6}\right)\right],$$

$$\partial_t\overline{a}_t^{(4)} = (d-4)\overline{a}_t^{(4)} + 4S_d(\overline{a}_t^{(4)})^2\left(\frac{9}{\overline{E}_1^6} + \frac{N-1}{\overline{E}_2^6}\right),$$

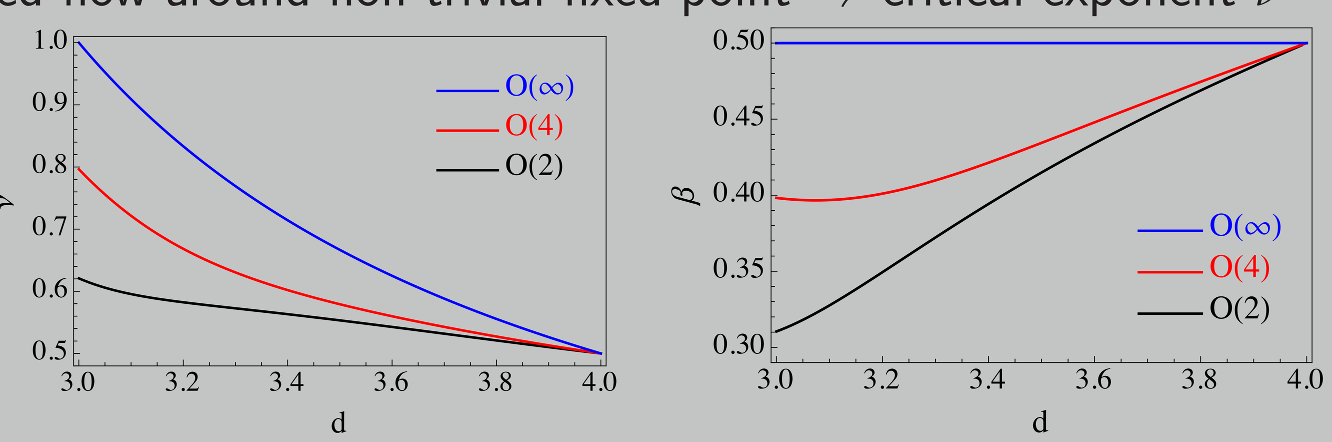
- Flow diagrams and fixed points in continuous dimension



- $3D \rightarrow 4D \Rightarrow$ Wilson-Fisher fixed point $\rightarrow$ Gaussian fixed point
- $\overline{a}_t^{(2)}$: relevant parameter; $\overline{a}_t^{(4)}$: irrelevant parameter

Critical Exponents in Continuous Dimension

- Linearized flow around non-trivial fixed point $\Rightarrow$ critical exponent ν



- $d = 4$ Gaussian fixed point, free boson gas, fluctuation negligible, mean field approach is self-consistent (Ginzburg criteria), $\beta = \nu = 0.5$
- $d = 4 - \epsilon$ Wilson Fisher fixed point is close to Gaussian fixed point, $4 - \epsilon$ expansion, $\nu^{-1} = 2 - \frac{N+2}{N+8}\epsilon$
- $d < 4$ Wilson Fisher fixed point is away from Gaussian fixed point, strong coupling near phase transition, fluctuation is important, non-perturbative method required

Conclusion

- Fluctuation induced difference in critical behavior of QCD phase transition.
- Dimension reduction at finite temperature.
- Effects of dimension on critical exponents and universality class.

References

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K. Fukushima *et al*, Phys. Rev. D **83**, 116005 (2011)