Equation of state in two-, three-, and four-color QCD at nonzero temperature and density

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arXiv:1412.6712

Abstract

We calculate the equation of state at nonzero temperature and density from first principles in **two-, three-, and four-color QCD with two fermion flavors in the fundamental and two-index, antisymmetric representation**. By matching low-energy results (from a "hadron resonance gas") to high-energy results from (resummed) perturbative QCD, we obtain results for the pressure and trace anomaly that are in **quantitative agreement with full lattice-QCD studies** for three colors at zero chemical potential. Our results for nonzero chemical potential at zero temperature **constitute predictions for the equation of state in QCD-like theories that can be tested by traditional lattice studies for two-color QCD with two fundamental fermions and four-color QCD with two two-index, antisymmetric fermions**. We find that the speed of sound squared at zero temperature can exceed 1/3, which may be relevant for the phenomenology of high-mass neutron stars.

Methodology

We match resummed perturbative QCD (pQCD) pressure at high temperature T or baryon chemical potential μ to a Hadron Resonance Gas (HRG) at low T or μ.

For the **pQCD pressure**, we use results obtained in Ref. [1, 2, 3, 4] with the resummation scheme described in [5]. In addition, we add a bag constant:

$$P_{\text{pQCD}}(\mu \text{ or } T) = P^0_{\text{pQCD}}(\mu \text{ or } T) + B \quad (1)$$

For the **HRG pressure**, we use

- $\mu = 0$ and $T > 0$, all hadrons $$P_{\text{HRG}}(T) = T^4 \sum_{i\in H} \frac{g_i}{2\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right) \quad (2)$$
- $T = 0$ and $\mu > 0$ for **fermionic baryons** (F) or **bosonic baryons** (B):

$$P^{\text{F}}_{\text{HRG}}(\mu) = \sum_{i\in B} g_i \eta \int_0^{\sqrt{(\mu r_i)^2 - m_i^2}} \frac{\boldsymbol{p}^2 \text{d}\boldsymbol{p}}{2\pi^2}\left(\mu r_i - \sqrt{\boldsymbol{p}^2 + m_i^2}\right)\theta(\mu r_i - m_i) \quad (3)$$

$$P^{\text{B}}_{\text{HRG}}(\mu) = \sum_{i\in B} \frac{g_i}{24\pi^2 N_b^4}((\mu r_i)^2 - m_i^2)^2 \theta(\mu r_i - m_i) \quad (4)$$

- For the hadrons, in the **real-world case, we use the Particle Data Book values** [6] for all hadrons or baryons with a mass up to 2.25GeV; in the **two- and four-color theories, we determine the hadrons using group-theoretic arguments and Fermi statistics**, when necessary. We **ignore the glueballs** in these theories because they tend to be more massive than the lightest hadrons [7].
- Mesons are taken to be the analogs of the flavorless mesons that exist in the real world (up to a mass of about 2 GeV) of which **the masses are written in multiples of the string tension $\sigma_{\text{SU(3)}} = (420\ \text{MeV})^2$**. In the two-color case, we mainly use the analogs of the real-world mesons, substituting the two-color masses calculated by Bali *et al.* [8] when available.
- We **set the scale** using the ratio $\Lambda_{\overline{\text{MS}}}/\sqrt{\sigma}$ given in Ref. [9]. We **scale this pure-glue result** by multiplying by $\Lambda^{N=3}_{\overline{\text{MS}}}(n_f{=}2)/\Lambda^{N=3}_{\overline{\text{MS}}}(n_f{=}0)$ determined from Ref. [10]. In the **antisymmetric-representation case**, no value for this ratio was found in the literature, we thus treat it as a free parameter, and use both the pure-glue and the scaled value for the fundamental-representation case.

Matching scheme:

- We are interested only in **bulk properties**, not the details of the phase transition, so we use simple matching criteria. For $F = T$ or μ:

$$P_{\text{HRG}}(F_0) = P_{\text{pQCD}}(F_0, B_0), \quad (5)$$

$$\left.\frac{\text{d}P_{\text{HRG}}(F)}{\text{d}F}\right|_{F=F_0} = \left.\frac{\partial P_{\text{pQCD}}(F, B_0)}{\partial F}\right|_{F=F_0}. \quad (6)$$

- We vary the renormalization scale $\bar{\Lambda} \in [\pi T, 4\pi T]$ for $\mu = 0$, and $\bar{\Lambda} \in [\mu/2, 2\mu]$ at $T = 0$, and solve for the pair (F_0, B_0).

Results

- In the **real-world case, lattice data agree reasonably well with the band resulting from our HRG+pQCD calculation**, both for the pressure as well as for the trace anomaly
- Defining the critical temperature T_c as the average of the upper and lower matching values of T and μ_c for the deconfinement transition similarly, we find that when we plot T or μ in multiples of T_c or μ_c respectively, **all the theories show similar behavior both for the pressure and trace anomaly, a phenomenon well-known (in the $\mu = 0$ case) from pure-gauge theories [11]**.
- Differences at low T or μ are due to the different numbers of Nambu–Goldstone bosons.
- Results suggest that the T_c values for the different theories are within 20 percent of each other, while the extracted μ_c values span a much broader range.
- We found that at $T = 0$, $c_s^2 > 1/3$ for some values of μ in all fundamental QCD-like theories, where c_s is the speed of sound. This finding could be of interest because restricting $c_s^2 < 1/3$ has previously been noted to be in tension with astrophysical observations [12].
- Our work provides **predictions for lattice gauge theory**:
  - The values of T_c and μ_c themselves, along with the pressure and trace anomaly plots.
  - Our assertion that in the four-color, antisymmetric case, $\Lambda_{\overline{\text{MS}}}/\sqrt{\sigma}$ is closer to the pure-glue value than it is in the real-world, three-color case; for our matching was not possible for the scaled value of $\Lambda_{\overline{\text{MS}}}/\sqrt{\sigma}$ at $\mu = 0$.
  - Our finding that c_s^2 can exceed 1/3 at $T = 0$ in theories with fundamental fermions.

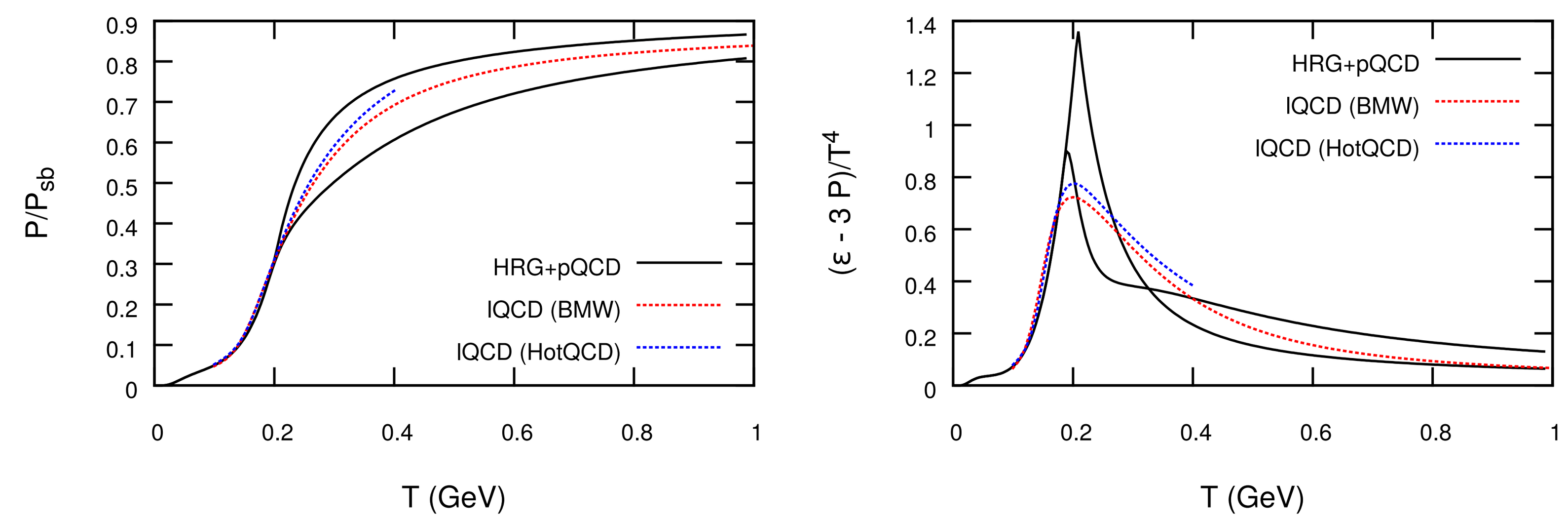

Figure 1 : **Normalized pressure** (left) and **trace anomaly** (right) at $\boldsymbol{\mu = 0}$ for the **three-color, three-massless-quark case** from HRG+pQCD in comparison to lattice-QCD data from the Budapest–Marseille–Wuppertal Collaboration [13] and the HotQCD Collaboration [14].

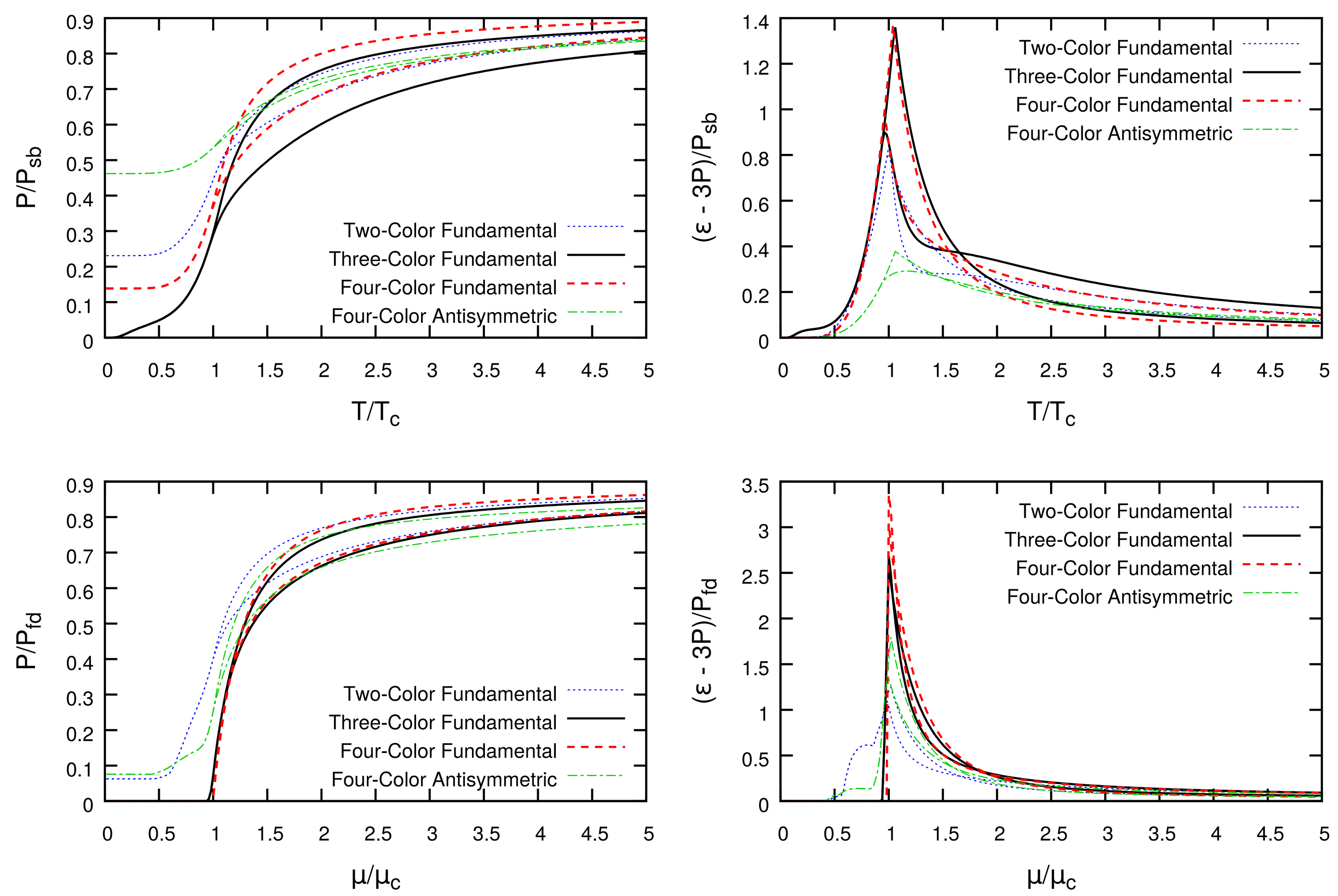

Figure 2 : Top: **Normalized pressure** (left) and **trace anomaly** (right) at $\boldsymbol{\mu = 0}$ for the two-color, three-color, four-color fundamental, and four-color antisymmetric theories in HRG+pQCD. Note that the T axis has been scaled by the critical temperature. Bottom: **Normalized pressure** (left) and **trace anomaly** (right) at $\boldsymbol{T = 0}$ for the two-color, three-color, four-color fundamental, and the four-color antisymmetric theories in HRG+pQCD. Note that the μ axis has been scaled by the critical chemical potential.

Table 1 : The ratios $\boldsymbol{T_c/\sqrt{\sigma}}$ and $\boldsymbol{\mu_c/\sqrt{\sigma}}$ for the theories analyzed in our work. Errors are given by the number of significant figures.

Group, Representation, $\boldsymbol{n_f}$	$\boldsymbol{T_c/\sqrt{\sigma}}$	$\boldsymbol{\mu_c/\sqrt{\sigma}}$
SU(2), fundamental, 2	0.400	3.24
SU(3), fundamental, 3	0.47	2.382
SU(4), fundamental, 2	0.44	2.853
SU(4), antisymmetric 2 ($\Lambda_{\overline{\text{MS}}}/\sqrt{\sigma} = 0.527$)	0.29	5.09
SU(4), antisymmetric 2 ($\Lambda_{\overline{\text{MS}}}/\sqrt{\sigma} = 0.723$)	no matching	5.0

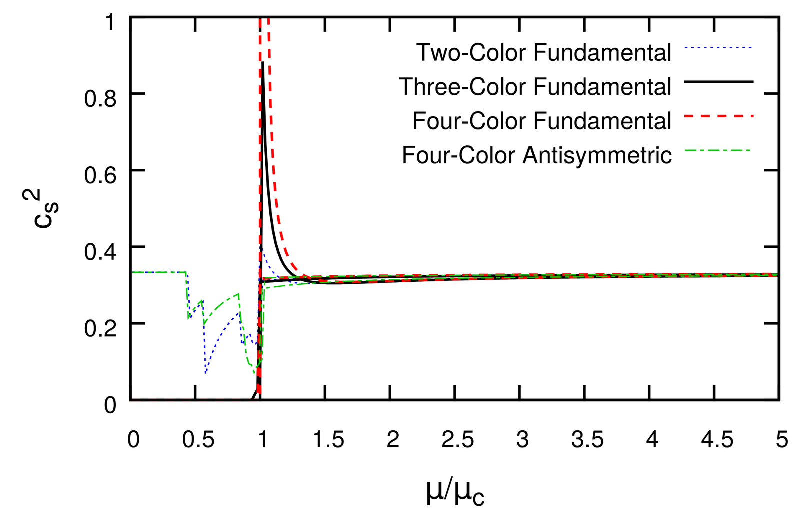

Figure 3 : The **speed of sound squared at $\boldsymbol{T = 0}$** for the two-color, three-color, four-color fundamental, and the four-color antisymmetric theories in HRG+pQCD. Note that the μ axis has been scaled by the critical chemical potential.

Conclusion

We have calculated the equation of state at nonzero temperatures and densities in a first-principles approach, by matching physics from the hadron resonance gas at low energies to perturbative QCD at high energies for two-, three-, and four-color QCD. In particular, **our work provides predictions for results in future lattice studies at zero temperature and nonzero chemical potential for two-color QCD with two fundamental fermions and four-color QCD with two flavors of fermions in the two-index, antisymmetric representation**. While some aspects of our study are systematically improvable, we **expect the current HRG+pQCD results to be sufficiently robust that a direct comparison with future lattice-QCD studies in the two- and four-color cases could validate or rule out the HRG+pQCD method, depending on the quantitative agreement. In the case of agreement, one could thus also reasonably expect HRG+pQCD results to be quantitatively accurate in the physically relevant, three-color-QCD case**. We stress that our analysis is intended to investigate bulk thermodynamic properties of these theories from first principles and that our approach cannot be used to investigate the details of the phase transition region. Nevertheless, we were able to obtain bounds on thermodynamic quantities that quantitatively agree with data in the real-world case without free parameters. To make our results accessible, we have made them electronically available [15].

Acknowledgments & References

We thank Gert Aarts, Tom DeGrand, Simon Hands, Yuzhi Liu, Marco Panero, Andreas Schmitt, and Aleksi Vuorinen for helpful discussions and suggestions. This work was funded by the Department of Energy, DOE Award No. DE-SC0008132

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