Self-similar inverse cascade of magnetic helicity driven by the chiral anomaly

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Abstract

For systems with charged chiral fermions, the imbalance of chirality in the presence of magnetic field generates an electric current - this is the Chiral Magnetic Effect (CME). We study the dynamical real-time evolution of electromagnetic fields coupled by the anomaly to the chiral charge density and the CME current by solving the Maxwell-Chern-Simons equations. We show that the CME induces the inverse cascade of magnetic helicity, and that at late times the evolution of magnetic helicity is self-similar and is characterized by universal exponents. We devise an experimental signature of this phenomenon in heavy-ion collisions.

Introduction

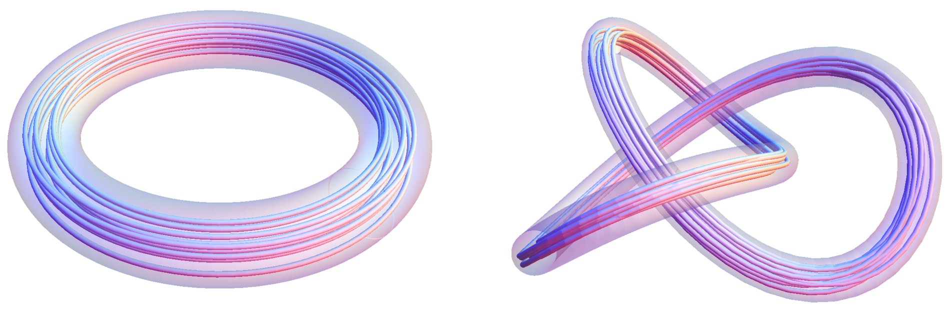
Topology of magnetic fields and chiral anomaly

- Magnetic helicity is a measure of topology

$$h_m \equiv \int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B} = \sum_i \phi_i^2 \mathcal{S}_i + 2\sum_{i,j}\phi_i\phi_j\mathcal{L}_{ij}$$

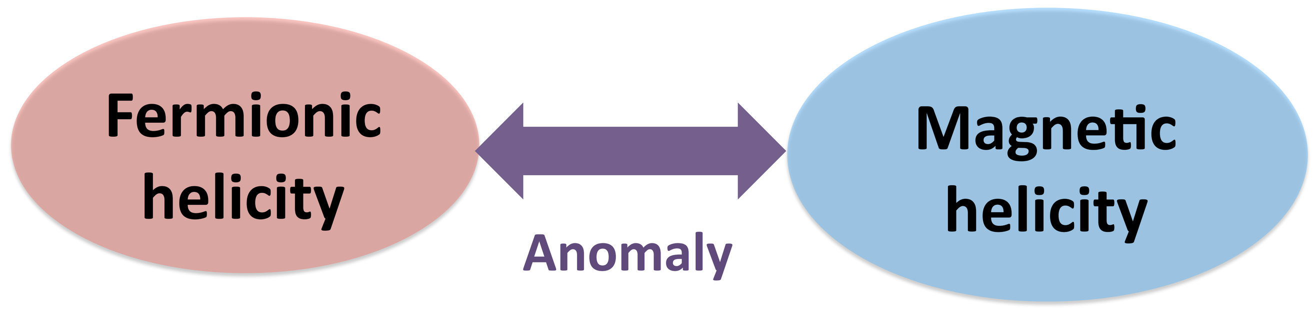
$\mathcal{S}_i$: Self linking number = (writhing #) + (twisting #)

$\mathcal{L}_{ij}$: Gauss linking number



Magnetic & fermionic helicities

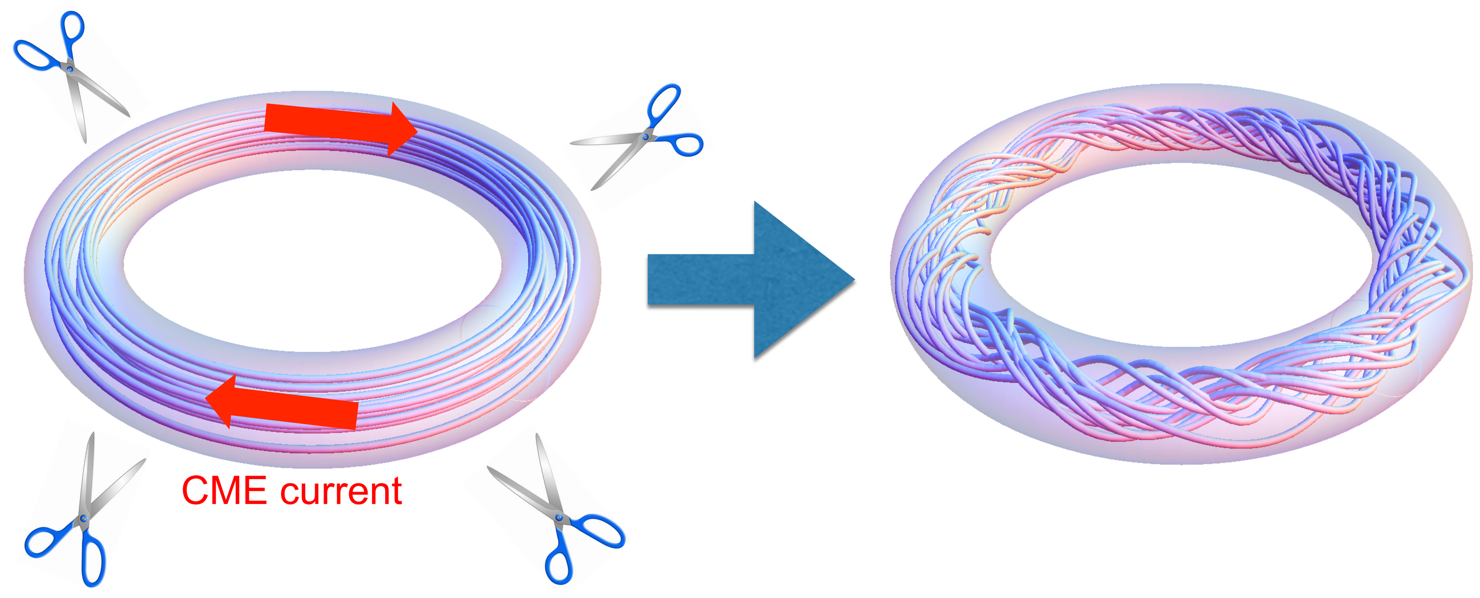
$$\partial_\mu j_A^\mu = C_A \boldsymbol{E}\cdot\boldsymbol{B} \;\Longrightarrow\; \frac{d}{dt}\left[h_F + h_m\right] = 0$$



$$h_F \equiv \frac{2}{C_A}\int d^3x\, n_A \qquad h_m \equiv \int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B}$$

- Anomaly converts one helicity to the other
- Total helicity is conserved

Fermions change the topology of magnetic fields



Time evolution of magnetic fields coupled with massless fermions

Maxwell-Chern-Simons equations

$$\nabla\times\boldsymbol{B} = \frac{\partial\boldsymbol{E}}{\partial t} + \boldsymbol{j}_{\mathrm{EM}} \quad \nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \quad \nabla\cdot\boldsymbol{B} = \nabla\cdot\boldsymbol{E} = 0$$

$$\boldsymbol{j}_{\mathrm{EM}} = \sigma\boldsymbol{E} + \boxed{\sigma_A\boldsymbol{B}}$$ **Chiral magnetic effect**

Equation of motion for magnetic fields and axial charge

$$\sigma\partial_t\boldsymbol{B}(t,\boldsymbol{x}) = \nabla^2\boldsymbol{B} + \sigma_A\left(\nabla\times\boldsymbol{B}\right)$$

$$\sigma_A(t) = \frac{C_A n_A(t)}{\chi} \approx \frac{C_A}{\chi V}\int d^3x\, n_A(\boldsymbol{x},t)$$

$$\frac{d}{dt}n_A(t) = \frac{C_A}{V}\int d^3x\, \boldsymbol{E}\cdot\boldsymbol{B}$$

- Spatial dependence of axial charge is neglected in this study
- Solved in the so-called "Chardrasekhar-Kendall" basis

$$\nabla\times\boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) = \pm k\boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) \qquad \nabla\cdot\boldsymbol{W}^{\pm}_{lm}(\boldsymbol{x};k) = 0$$

$$\int d^3x\, \boldsymbol{W}^a_{lm}(\boldsymbol{x};k)\cdot\boldsymbol{W}^b_{l'm'}(\boldsymbol{x};k') = \frac{\pi}{k^2}\delta(k-k')\delta_{ll'}\delta_{mm'}\delta_{ab}$$

Time evolution

$h_m(t)$: magnetic helicity

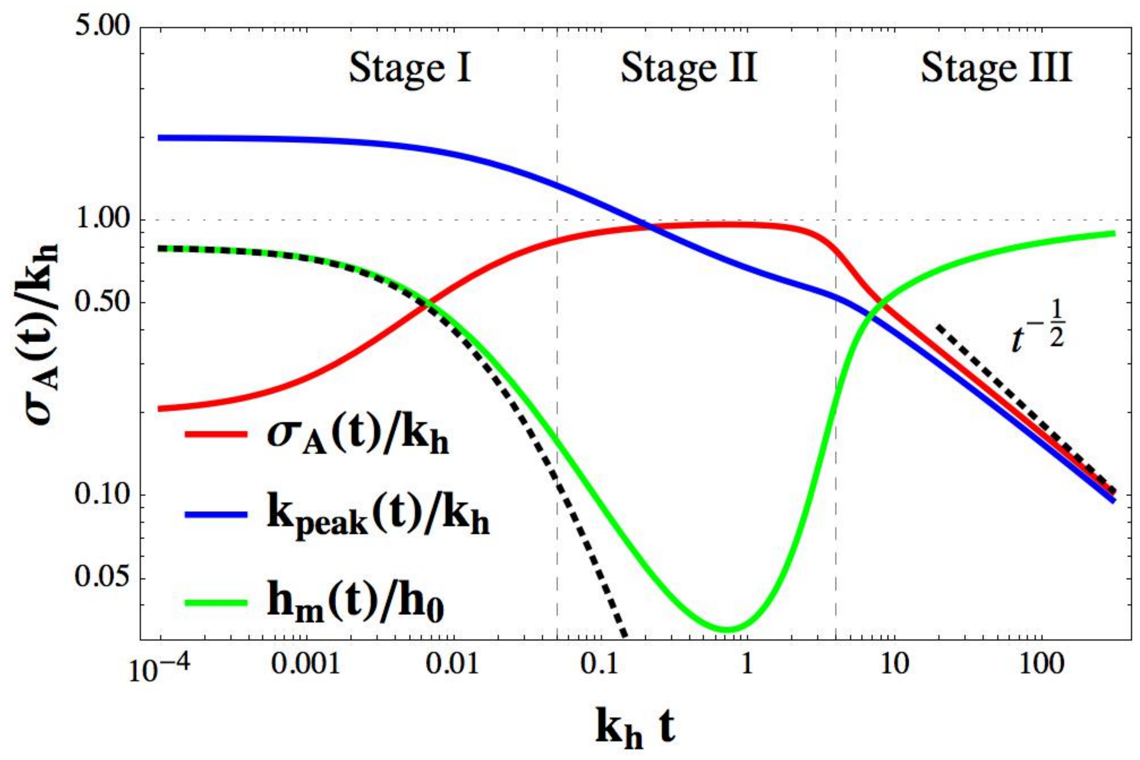
$\sigma_A(t)$: chiral magnetic conductivity (energy cost per unit fermionic helicity)

$k_{\mathrm{peak}}(t)$: peak energy of magnetic helicity spectrum

$$h_m(t) = \int_0^\infty \frac{dk}{\pi}\, k\,\boxed{g_+(k,t) - g_-(k,t)}$$

Quantities are expressed in the unit of $k_h \equiv \dfrac{C_A^2 h_0}{\chi V}$,

which is the energy cost per unit fermionic helicity, when all the helicity is stored in fermions



Stage I $\quad \sigma_A < k_{\mathrm{peak}}$: Helicity is transferred from the magnetic field to fermions

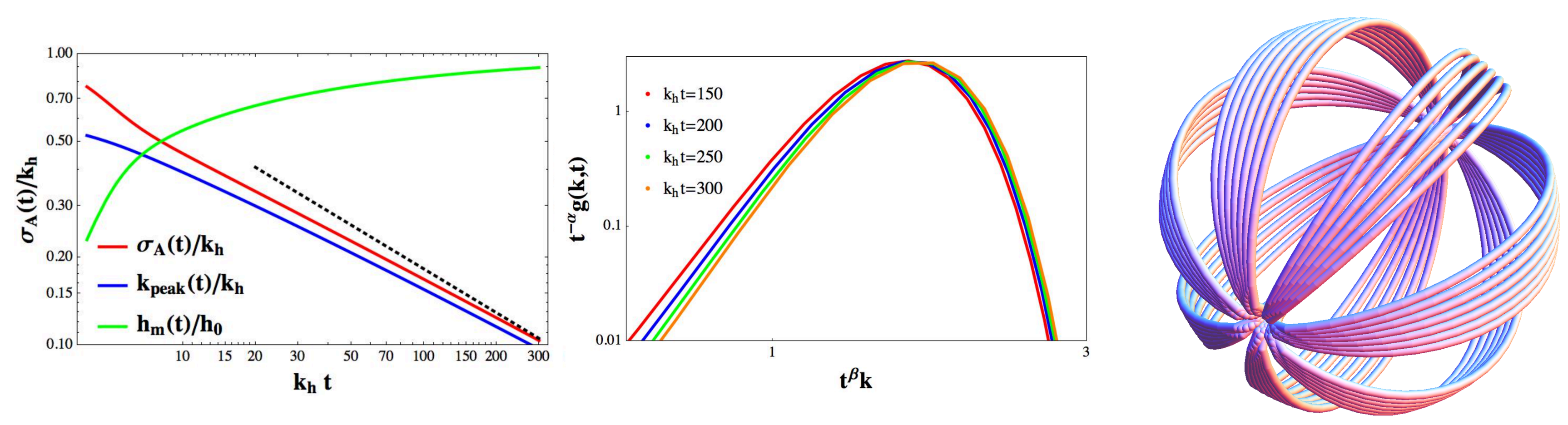
Stage II $\quad \sigma_A \approx k_{\mathrm{peak}}$: Helicity transfer stops

Stage III $\quad \sigma_A > k_{\mathrm{peak}}$: Helicity comes back to the magnetic field

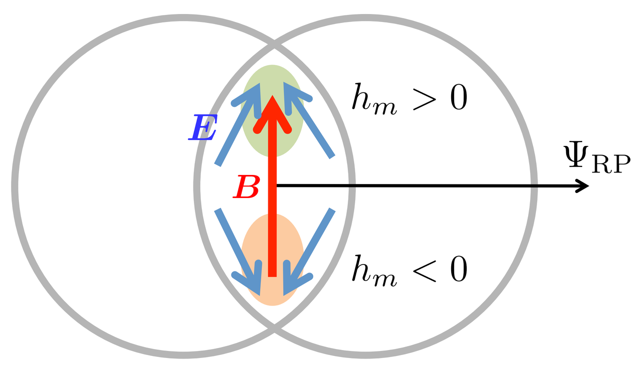
- Long-time behavior is **self-similar**

$$g(k,t) \sim t^\alpha \tilde{g}(t^\beta k) \to \exp\left[-2\sigma^{-1}(k-k_{\mathrm{peak}})^2 t\right] \to \delta(k - k_{\mathrm{peak}}(t))$$

- Exponents: $\alpha = 1,\ \beta = 1/2$



Implications for heavy-ion collisions



Helical magnetic field *should* be created, which will emit polarized photons

- Condition for inverse cascade

$k_h < k_m \approx 1/L \qquad k_m$: Lowest magnetic energy

$L \approx 10$ [fm]

$eB = c_B m_\pi^2$

$c_B > 100$ RHIC

$c_B > 26$ LHC