Anisotropic hydrodynamics for a mixture of quark and gluon fluids

Wojciech Florkowski[1,2], **Ewa Maksymiuk**[1], **Radosław Ryblewski**[2], **Leonardo Tinti**[1]

[1] *Institute of Physics, Jan Kochanowski University, Kielce, Poland*
[2] *The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland*

1 Introduction

Relativistic hydrodynamics is a fundamental theory used for the description of the matter produced in the ultra relativistic heavy-ion collisions at RHIC (Brookhaven National Laboratory) and LHC. The energies used there are enough to produce quark-gluon plasma. Despite the success of second order viscous hydrodynamics in reproducing collective behavior and particle spectra, there are still theoretical shortcomings that may question the validity of the approach in heavy-ion experiments conditions. Large gradients and fast longitudinal expansion produce very large pressure corrections, in contrast to the founding hypothesis of small deviation from local equilibrium and the perturbative treatment viscous corrections.

One way to address this problem is anisotropic hydrodynamics. In this theory the large pressure corrections are taken into account in a non-perturbative manner already at the leading order of the expansion.

As a lot of work has been already done in this context, for simple (i.e., one-component) fluids, the analysis of mixtures has been so far quite limited [1,2]. One of the problems of the previous approaches using the concept of anisotropic hydrodynamics in this context [1-2] was that they were based only on the zeroth and first moments of the kinetic equations. Assuming that the distribution functions used in anisotropic hydrodynamics are described by the original Romatschke-Strickland form [3], one finds underdetermined set of equations, where the number of unknown parameters is larger than the number of equations. Consequently, to close the system of equations, in Refs. [1-2] the transverse-momentum scale parameters for quarks and gluons were taken to be equal.

In this work we use the zeroth, first, as well as the second moments of the kinetic equations for quarks, antiquarks and gluons. This follows us to the five independent equations for five unknown parameters. The new approach eliminates solutions with exponentially damped anisotropy parameters, found in [1], which do not agree with the kinetic-theory solutions [2].

The complete analysis of the problem is presented in [4].

2 Anisotropic hydrodynamics

In following description we have a five independent variables we would like to find as a functions of the proper time: effective temperature $T(\tau)$, transverse momentum scales of quarks and gluons, respectively Λ_q, Λ_g and anisotropic parameters for quarks and gluons ξ_q and ξ_g. To do this we use five equations found from the zeroth, the first and the second moments of the Boltzmann equation.

- The zeroth moment is used as a linear combination of the non-equilibrium and equilibrium particle densities.

$$\frac{d}{d\tau}\left(\alpha\frac{\sqrt{1+D^2}\Lambda_q^3}{\sqrt{1+\xi_q}}+(1-\alpha)\frac{\tilde{r}\Lambda_g^3}{\sqrt{1+\xi_g}}\right)+\left(\frac{1}{\tau}+\frac{1}{\tau_{\rm eq}}\right)\left(\alpha\frac{\sqrt{1+D^2}\Lambda_q^3}{\sqrt{1+\xi_q}}+(1-\alpha)\frac{\tilde{r}\Lambda_g^3}{\sqrt{1+\xi_g}}\right) = \frac{T^3}{\tau_{\rm eq}}\left(\alpha\sqrt{1+D^2/\kappa_q^2}+(1-\alpha)\tilde{r}\right) \quad (1)$$

α parameter is equal 1 for quarks, 0 for gluons, and 1/2 for quarks and gluons.

- Landau matching condition for the energy-momentum conservation requires that the energy determined from the non-equilibrium distribution functions is the same as the energy obtained with the equilibrium distribution functions $\varepsilon = \varepsilon_q + \varepsilon_g = \varepsilon_{\rm eq} = \varepsilon_{q,{\rm eq}} + \varepsilon_{g,{\rm eq}}$. This follows to the equation

$$T^4 = \frac{\Lambda_q^4\sqrt{1+D^2}\,\mathcal{R}(\xi_q)+\Lambda_g^4\tilde{r}\,\mathcal{R}(\xi_g)}{\sqrt{1+D^2/\kappa_q^2}+\tilde{r}} \quad (2)$$

- On the other hand on the (0+1)D case considered here the energy and momentum conservation takes the form $d\varepsilon/d\tau = -(\varepsilon+P_L)/\tau$. This leads directly to the formula

$$\frac{d}{d\tau}\left[\Lambda_q^4\sqrt{1+D^2}\,\mathcal{R}(\xi_q)+\tilde{r}\Lambda_g^4\mathcal{R}(\xi_g)\right]=\frac{2}{\tau}\left[\Lambda_q^4\sqrt{1+D^2}\,(1+\xi_q)\mathcal{R}'(\xi_q)+\tilde{r}\Lambda_g^4(1+\xi_g)\mathcal{R}'(\xi_g)\right] \quad (3)$$

- From the second moment of the kinetic equation we have a following form for quarks

$$\frac{d}{d\tau}\ln\left(\frac{\Lambda_q^5}{(1+\xi_q)^{1/2}}\sqrt{1+D^2}\right)-\frac{d}{d\tau}\ln\left(\frac{\Lambda_q^5}{(1+\xi_q)^{3/2}}\sqrt{1+D^2}\right)-\frac{2}{\tau}=\frac{T^5}{\tau_{\rm eq}\Lambda_q^5}\xi_q(1+\xi_q)^{1/2}\frac{\sqrt{1+D^2/\kappa_q^2}}{\sqrt{1+D^2}} \quad (4)$$

- Similarly, using the formalism of the second moment of the kinetic equation we get a formula for gluons

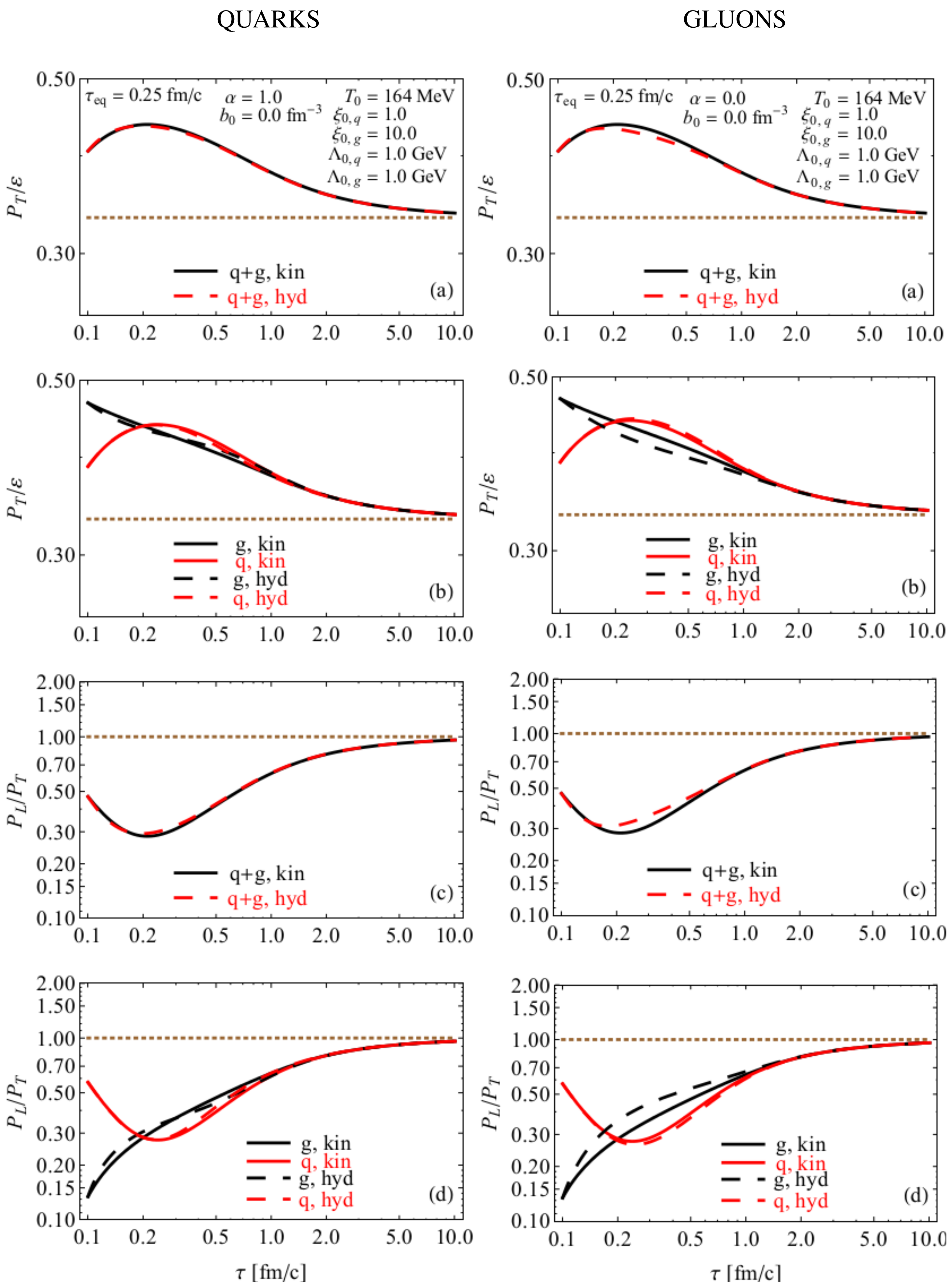
$$\frac{d}{d\tau}\ln\left(\frac{\Lambda_g^5}{(1+\xi_g)^{1/2}}\right)-\frac{d}{d\tau}\ln\left(\frac{\Lambda_g^5}{(1+\xi_g)^{3/2}}\right)-\frac{2}{\tau}=\frac{T^5}{\tau_{\rm eq}\Lambda_g^5}\xi_g(1+\xi_g)^{1/2} \quad (5)$$

- It is impotiant to notice, that we introduced D and $\tilde{r}$ functions, defined as

$$D(\tau,\Lambda_q,\xi_q)=\left(\frac{3\pi^2 b_0\tau_0\sqrt{1+\xi_q}}{2g_q\tau\Lambda_q^3}\right), \qquad \tilde{r}=\frac{g_g}{2g_q}.$$

For numerical calculations presented here we use the value $\tilde{r}=2/3$.

3 Results for Oblate-Oblate System



4 Conclusions

- We have constructed a new set of equations for anisotropic hydrodynamics describing a mixture of anisotropic quark and gluon fluids.
- The consistent treatment of the zeroth, first and the second moments of the kinetic equations allows us to take a different values of the transverse-momentum scale parameters for quarks and gluons.
- New approach gives a very good agreement with the kinetic theory.
- Two ways of treatment the linear equation from the zeroth moment show better agreement for quarks (α=1) than for gluons (α=0).

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*Research supported in part by Polish National Science Center grants No. DEC-2012/05/B/ST2/02528 (WF), DEC-2012/06/A/ST2/00390 (WF, EM, LT), and DEC-2012/07/D/ST2/02125 (RR).