# The Curvature of the QCD Phase Transition Line

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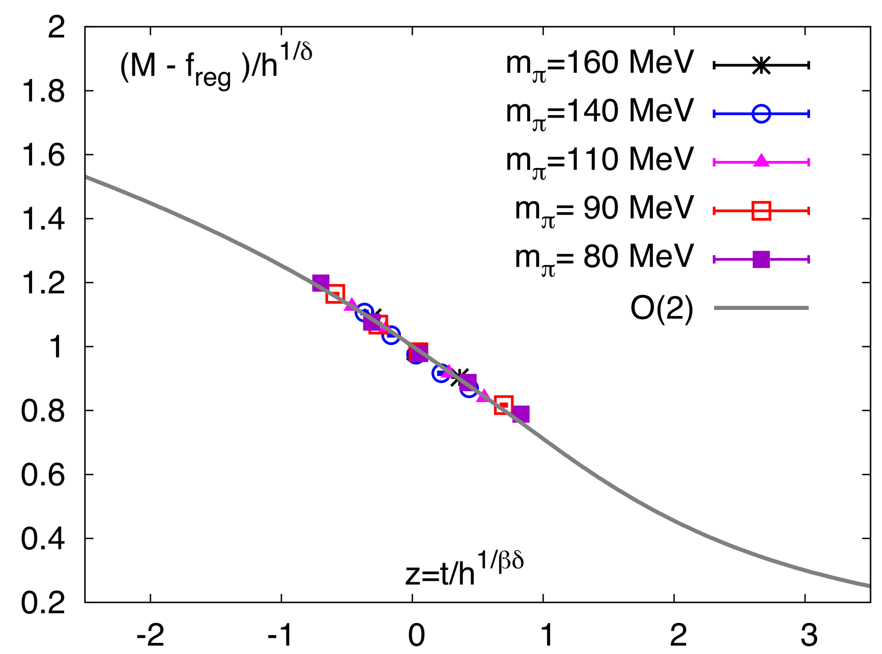
## Synopsis

The QCD chiral phase transition is known to be second-order for the case of massless light quarks, $m_l = 0$. The introduction of non-zero quark chemical potentials $\hat{\mu} = (\mu_l/T, \mu_s/T)$ does not change the order; however the transition temperature $T_c(\hat{\mu})$ decreases as the potentials are increased. In this work, we shall calculate this rate of decrease to lowest order in $\hat{\mu}$ at $\mu_l = \mu_s = 0$. This rate is determined by the $2 \times 2$ *curvature matrix*, given by

$$T_c(\hat{\mu}) = T_c(0) - \hat{\mu}^T \mathbf{K} \hat{\mu} + \mathcal{O}\left(\left(\hat{\mu}^T \hat{\mu}\right)^2\right), \qquad \mathbf{K} = \begin{pmatrix} \kappa_{ll} & \kappa_{ls} \\ \kappa_{ls} & \kappa_{ss} \end{pmatrix}.$$

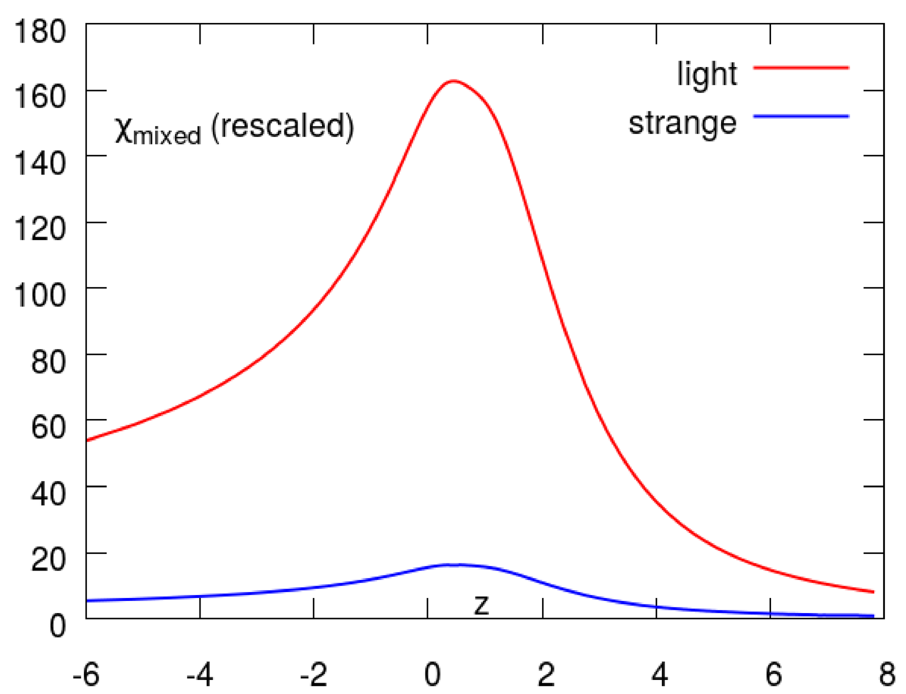
The light quark curvature has been calculated for physical quark masses by locating the peak of the chiral susceptibility as a function of imaginary $\mu_l$. Note however that for $m_l > 0$, the transition is not a genuine phase transition but simply a crossover. Therefore, here we will take an alternative approach and calculate $\mathbf{K}$ in the chiral limit from universality arguments.

## Procedure

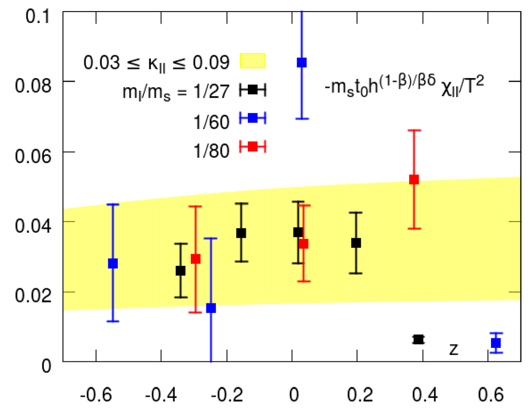


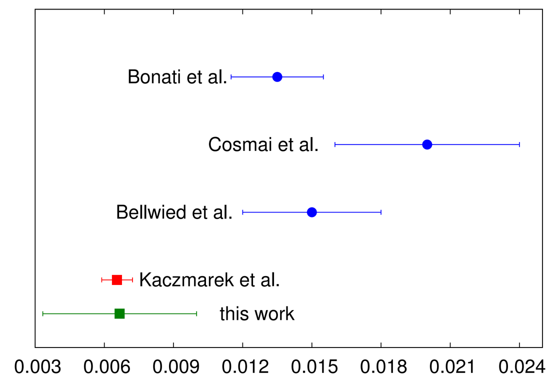

The QCD chiral phase transition belongs to the universality class of the $3d$-$O(4)$ model. On the lattice, this is broken to $O(2)$ when one uses staggered fermions. The analog of the magnetization $M$ is the chiral condensate, while the ratio of quark masses $m_l/m_s$ plays the role of the symmetry-breaking field $h/h_0$. The chemical potentials do not break chiral symmetry; therefore they enter via the reduced temperature $t_0 t = (T - T_c)/T_c + \hat{\mu}^T \mathbf{K} \hat{\mu}$. For a suitable combination of these variables, the chiral condensate for different condensates should fall on a universal curve [1]. We use this fact to determine the normalization constants $t_0$, $h_0$ and $T_c$. Once these are known, the curvature is determined from a calculation of the *mixed susceptibilities* whose definition and scaling function are shown below viz.

$$\frac{1}{T}\frac{\partial \langle \bar{\psi}\psi \rangle_l}{\partial \mu_i \partial \mu_j} = \kappa_{ij} \, \frac{2T}{t_0 m_s} \, h^{-(1-\beta)/\beta\delta} f'_G(z)$$

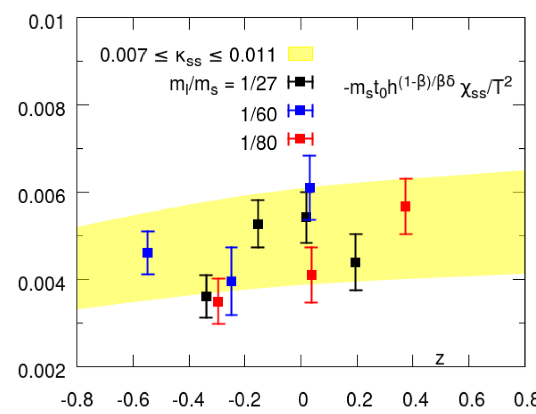


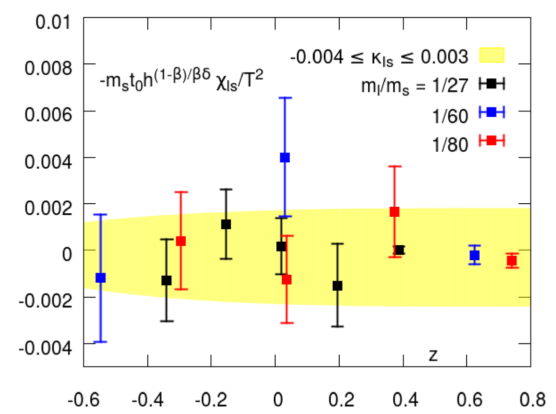

## Results




To measure these $\hat{\mu}$-derivatives, we generated around 1,000 gauge configurations each for quark masses $m_l = m_s/27$, $m_s/60$ and $m_s/80$. The smallest of these corresponded to a pion mass of around 80 MeV. The strange quark mass was fixed to its physical value. On each configuration, we calculated these derivatives stochastically, using around 500-1000 random vectors per configuration. While the curvatures $\kappa_{ij}$ will be eventually determined from a one-parameter ($\kappa_{ij}$) fit to the rescaled mixed susceptibilities, here we merely obtain bounds by varying $\kappa_{ij}$ by hand. The light quark curvature $\kappa_{ll}$ has been determined by other groups too. While the work of Kaczmarek et al. [2] is our previous result, obtained by the same method but on a coarser lattice and with a different action, the rest [3, 4, 5] were all obtained by the method of analytic continuation at physical quark masses. The other two coefficients are shown below. We see that $\kappa_{ss}$ is an order of magnitude smaller than $\kappa_{ll}$. This tells us that the curvature along the baryochemical direction $\hat{\mu}_B$ equals $\kappa_{ll}/9$ to a very good approximation; thus, $\kappa_2^B \approx 0.0067(33)$. Lastly, the off-diagonal coefficient $\kappa_{ls}$ is zero within errors, and in any case not much larger than $\kappa_{ss}$.



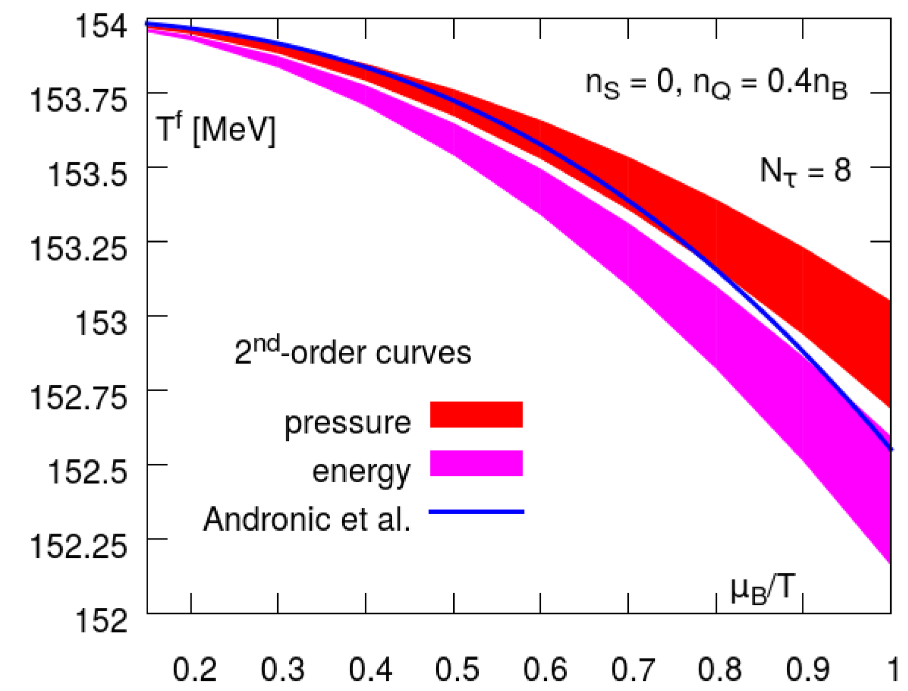




## Lines of Constant Physics

Our value for $\kappa_2^B$ may be compared to the curvature of the experimental freeze-out curve. Currently, there exist both phenomenological parametrizations of the curve (J. Cleymans et al., PRC **73**, 034905 (2006)) as well as *ab initio* determinations of the curvature via lattice QCD (A. Bazavov et al., arXiv:1509.05786 [hep-lat]). Empirically, it is found that the pressure $p$ and energy density $\varepsilon$ stay approximately constant along this curve. We made use of this fact by calculating contours of constant $p$ and $\varepsilon$ to fourth-order from a Taylor series expansion in $\hat{\mu}$. Our second-order results are shown below. While our fourth-order results were zero within the errors, the errors themselves served as a sort of upper bound on the magnitude of $\kappa_4^f$. We state our numbers below.

$$\kappa_2^f = 0.0073(12) \text{ (pressure)}, \qquad |\kappa_4^f| \le 0.004 \text{ (pressure)},$$
$$= 0.0105(14) \text{ (energy)}. \qquad \le 0.006 \text{ (energy)}.$$

Thus, the second-order curvatures are comparable to $\kappa_2^B$, while the fourth-order curvatures are probably not more than $\sim 60\%$ of the second-order ones, and possibly much smaller.




## References

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