Pre-equilibrium evolution effects on heavy-ion collision observables

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In collaboration with Jia Liu and Chun Shen

Reference:
J. Liu, C. Shen and U. Heinz, PRC 91 (2015) 064906

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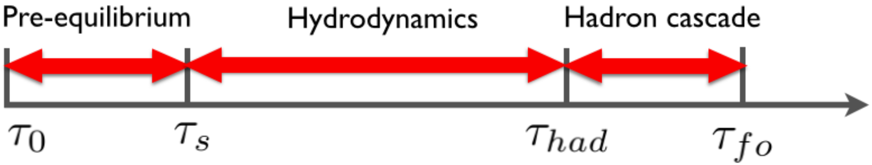
Overview

Motivation

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions
- Hydrodynamics does not become valid until the medium has reached a certain degree of local momentum isotropization
- In an inhomogeneous system, collective flow (i.e. space-momentum correlations) begin, however, to develop already before hydrodynamics becomes valid.
- The hydrodynamic stage thus starts with a non-vanishing pre-equilibrium flow.
- **Goal: To perform a systematic study of pre-equilibrium flow effects on heavy-ion collision observables.**

Weak vs. strong coupling



- Here: model pre-equilibrium stage by kinetic theory (no mean fields, no plasma instabilities)
- **Weak coupling:** very few collisions, long thermalization time: $\tau_s \gg \tau_0$
- **Strong coupling:** frequent collisions, very rapid thermalization: $\tau_s \approx \tau_0$
- $\Longrightarrow$ Use τ_s to parametrize the rate of approach to hydrodynamic behavior;
  model the period $\tau_0 < \tau < \tau_s$ by free-streaming massless degrees of freedom (extreme weak-coupling limit)

Overview

Free-streaming

Collisionless BE: $p^\mu \partial_\mu f(x,p) = 0.$

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To match to VISH2+1, impose longitudinal boost invariance:

$$f(\boldsymbol{x}_\perp, \eta_s, \tau;\, \boldsymbol{p}_\perp, y) = \frac{\delta(y-\eta_s)}{\tau m_\perp \cosh(y-\eta_s)} \tilde{f}(\boldsymbol{x}_\perp, \tau;\, \boldsymbol{p}_\perp, y).$$

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Assume massless degrees of freedom. Analytic soln. of collisionless BE:

$$f(\boldsymbol{x}_\perp, \eta_s, \tau_s; \boldsymbol{p}_\perp, y) = f(\boldsymbol{x}_\perp - (\tau_s - \tau_0)\hat{\boldsymbol{p}}_\perp, \eta_s, \tau_0; \boldsymbol{p}_\perp, y).$$

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Energy-momentum tensor $T^{\mu\nu}(x) = \frac{g}{(2\pi)^3} \int \frac{d^3p}{E} p^\mu p^\nu f(x,p)$

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Energy-momentum tensor $T^{\mu\nu}(x) = \frac{g}{(2\pi)^3} \int \frac{d^3p}{E} p^\mu p^\nu f(x,p)$

Need it only at midrapidity $\eta_s = 0$ (boost invariance):

$$T^{\mu\nu}(\boldsymbol{x}_\perp, \eta_s{=}0, \tau) = \frac{1}{\tau} \int_{-\pi}^{\pi} d\phi_p\, \hat{p}^\mu \hat{p}^\nu F(\boldsymbol{x}_\perp, \tau; \phi_p),$$

where

$$F(\boldsymbol{x}_\perp, \tau; \phi_p) = F_0(\boldsymbol{x}_\perp - (\tau - \tau_0)\hat{\boldsymbol{p}}_\perp) = \frac{g}{(2\pi)^3} \int_0^\infty p_\perp^2 dp_\perp \tilde{f}(\boldsymbol{x}_\perp - (\tau - \tau_0)\hat{\boldsymbol{p}}_\perp, \tau_0; p_\perp, 0)$$

is independent of how $\tilde{f}$ depends on the magnitude of $p_\perp$, and $F_0(\boldsymbol{x}_\perp)$ is the spatial distribution function at $\tau = \tau_0$.

Landau matching I:

Hydrodynamic form of energy-momentum tensor:

$$T^{\mu\nu}_{\rm hyd} = e u^\mu u^\nu - (\mathcal{P} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Flow vector = timelike eigenvector of $T^{\mu\nu}$: $\quad T^{\mu\nu} u_\nu = e u^\mu$

LRF energy density e is the associated eigenvalue; pressure $\mathcal{P} = \mathcal{P}(e)$ from EOS.

Project with LRF spatial projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ to get bulk viscous pressure Π:

$$\Pi = -\frac{1}{3}\mathrm{Tr}(\Delta_{\mu\nu} T^{\mu\nu}) - \mathcal{P}$$

Use double projector $\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$ to get $\pi^{\mu\nu}$:

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} T^{\alpha\beta}$$

Alternatively $\pi^{\mu\nu} = T^{\mu\nu} - e u^\mu u^\nu + (\mathcal{P} + \Pi)\Delta^{\mu\nu}$.

Landau matching II: Entropy generation and bulk pressure

Free-streaming preserves entropy (collisionless!)
After matching to hydro, entropy density is given by EOS from $s = \partial\mathcal{P}/\partial T$

$\Longrightarrow$ s jumps (increases) after Landau matching.

For successful phenomenology, normalize entropy density profile after Landau matching such that, upon completion of the dynamical evolution, it correctly reproduces the observed final multiplicity dN_{ch}/dy.

In spite of the entropy jump at τ_s, the normalization of the entropy density profile after Landau matching has a one-to-one relation with the normalization of the initial distribution function. Since the entropy jump depends on τ_s, so does the initial normalization. As τ_s is varied, the normalization is adjusted to preserve dN_{ch}/dy.

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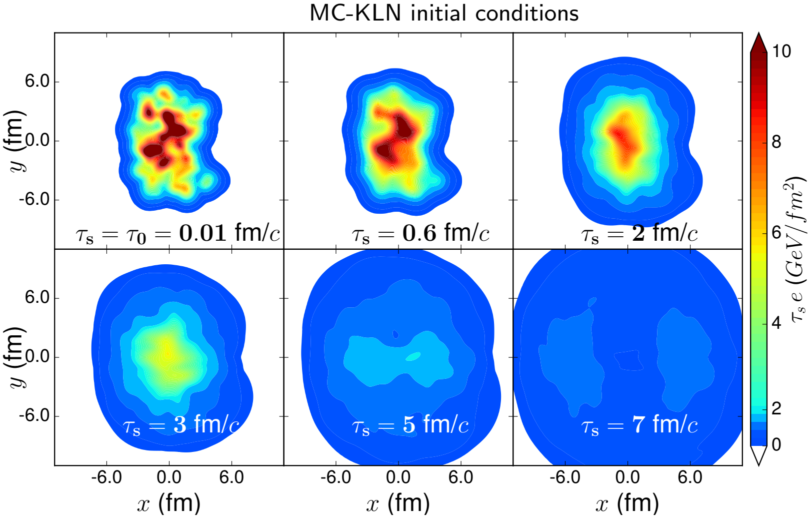
Since $m = 0$, $\Pi = 0$ in the free-streaming stage. Since EOS from latttice QCD breaks conformal symmetry, $\Pi \neq 0$ after Landau matching. We here set $\zeta = 0$ and let Π evolve back to zero with IS EOM over a short relaxation time τ_Π

Overview

Initial LRF energy density profile after free-streaming:



Initial radial flow after free-streaming:

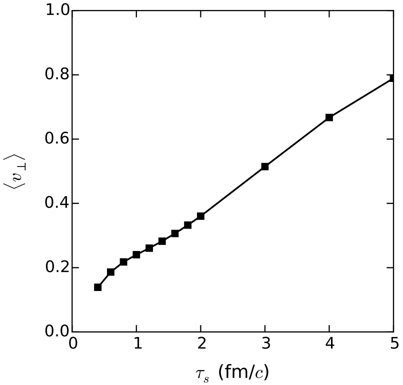
$$\{v_\perp\} = \frac{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, v_\perp(\boldsymbol{r}_\perp)\, e(\boldsymbol{r})_\perp}{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)},$$

$$\langle v_\perp \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} \{v_\perp\}^{(i)}.$$

Rises initially very quickly, reaching 25% of speed of light after 1 fm/c

Continues to grow over the next 5 fm/c at an approximate rate $\langle a_\perp \rangle \approx \frac{d\langle v_\perp \rangle}{d\tau_s} = 0.13\, c^2/\text{fm}$.

Free-streaming should yield an upper limit for this radial pre-equilibrium flow.



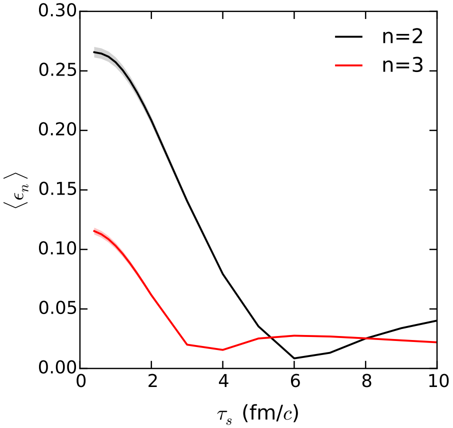
Initial eccentricities after free-streaming:

Eccentricities drive anisotropic flow:

$$\mathcal{E}_n(\tau_s) = \epsilon_n(\tau_s) e^{in\Phi_n(\tau_s)}$$
$$= -\frac{\int_{\tau_s} d^3\sigma_\mu(x)\, T^{\mu\nu}_{\rm hyd}(x)\, u_\nu(x)\, r_\perp^n\, e^{in\phi}}{\int_{\tau_s} d^3\sigma_\mu(x)\, T^{\mu\nu}_{\rm hyd}(x)\, u_\nu(x)\, r_\perp^n}$$
$$= -\frac{\int d^2r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)\, r_\perp^n\, e^{in\phi}}{\int d^2r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)\, r_\perp^n},$$

$$(n > 1)$$

Note, this counts only contributions from fluid elements, not from cells that are already frozen out after Landau matching!



Initial shear stress I:

Inverse Reynold number measures importance of first-order viscous stress relative to ideal hydro pressure:

$$R^{-1} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{-\Delta^{\mu\nu}T_{\mu\nu}/3} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathcal{P}+\Pi}$$

[For conformal systems, $\mathcal{P}=e/3$ and $\Pi=0$.]
Initial value can be calculated from free-streamed distribution function:

$$\pi^{\mu\nu}\pi_{\mu\nu} = \int \frac{g\,d^3p}{(2\pi)^3 p^0}\int \frac{g\,d^3p}{(2\pi)^3 p'^0}\left[(\boldsymbol{p}\cdot\boldsymbol{p}')^2 - \frac{1}{3}\boldsymbol{p}^2\boldsymbol{p}'^2\right] f(p)f(p')$$

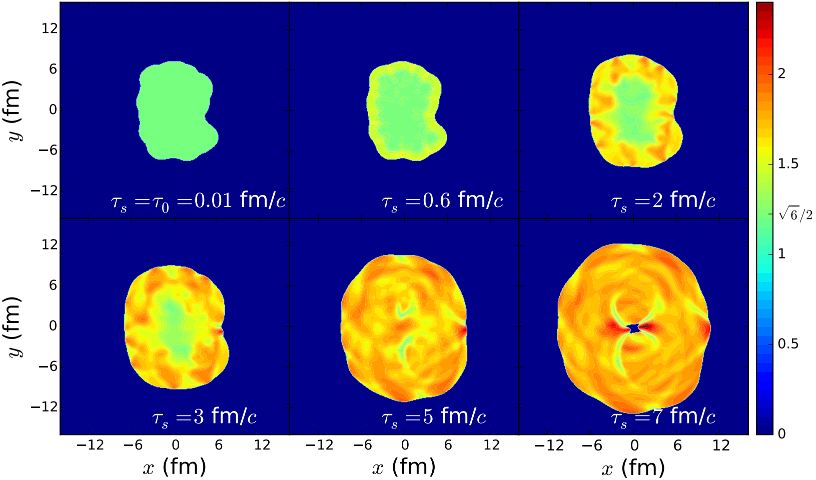
The value at τ_0 (before onset of free-streaming) can be worked out exactly:

$$\pi^{\mu\nu}\pi_{\mu\nu}\Big|_{\tau_0} = \frac{2\pi^2}{3}C^2, \qquad \mathcal{P}+\Pi = \frac{2\pi}{3}C,$$

where $C \equiv \frac{g}{\tau_0}\int \frac{1}{(2\pi)^3} p_\perp^2\, dp_\perp\, \tilde{f}(p_\perp)$.

Hence, $\boldsymbol{R^{-1}}\Big|_{\boldsymbol{\tau_0,\eta_s=0}} = \boldsymbol{\sqrt{3/2} \approx 1.225}$.

Initial shear stress II: Inverse shear Reynolds number after Landau matching



Effects of pre-flow on final radial flow

Initial conditions rescaled such that

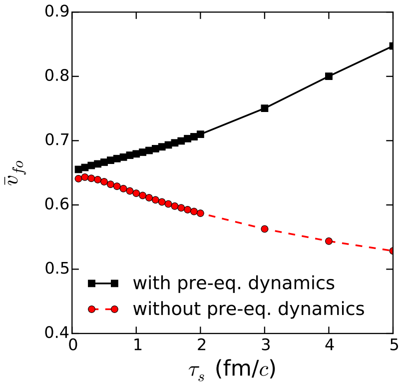
$$\frac{dE}{dy}\Big|_{\Sigma_{\rm fo}} = \frac{dE}{d\eta_s}\Big|_{\Sigma_{\rm fo}} = \int_{\Sigma_{\rm fo}} T^{0\mu}(x)\, \frac{d^3\sigma_\mu(x)}{d\eta_s}$$

is held fixed when varying τ_s (see discussion below).

For $T_{\rm dec} = {\rm const.} = 120\,{\rm MeV}$

$$v_{\rm fo} \equiv \frac{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, v_\perp\, e}{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, e} = \frac{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, v_\perp}{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu}$$

$v_{\rm fo}$ controls slope if final hadron p_T spectra.



Strong effect from pre-flow on final radial flow for all values of τ_s!

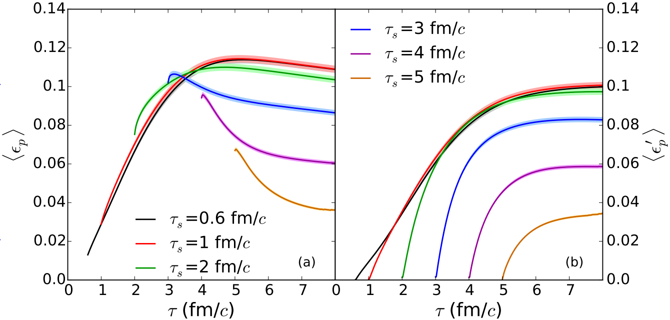
Effects of pre-flow on final flow anisotropy

Total momentum anisotropy

$$\epsilon'_p = \frac{\int d^2r_\perp (T^{xx} - T^{yy})}{\int d^2r_\perp (T^{xx} + T^{yy})}$$

Momentum anisotropy from collective flow

$$\epsilon_p = \frac{\int d^2r_\perp (T^{xx}_{\mathrm{id}} - T^{yy}_{\mathrm{id}})}{\int d^2r_\perp (T^{xx}_{\mathrm{id}} + T^{yy}_{\mathrm{id}})}$$

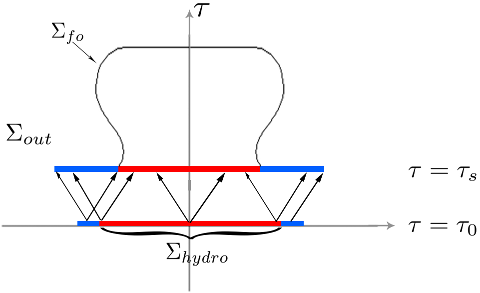


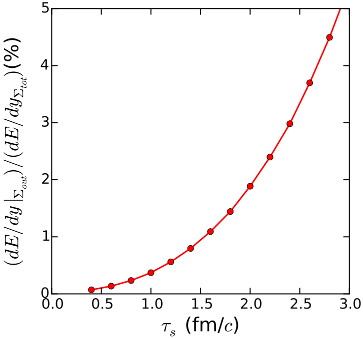
Calculated by rotating for each event $T^{\mu\nu}$ in transverse plane to maximize ϵ'_p or ϵ_p, respectively.

Not much effect from preflow on final momentum anisotropy unless $\tau_s > 2\,\mathrm{fm}/c$.

Overview

Avoiding particle loss from the non-thermalized halo





For large τ_s we have a large halo of partons that never become part of the fluid since the density is too low $\Longrightarrow$ big problem, since we don't know how to convert them correctly to final hadrons, and we need hadron spectra to compute v_n

But we know how to account for their energy!

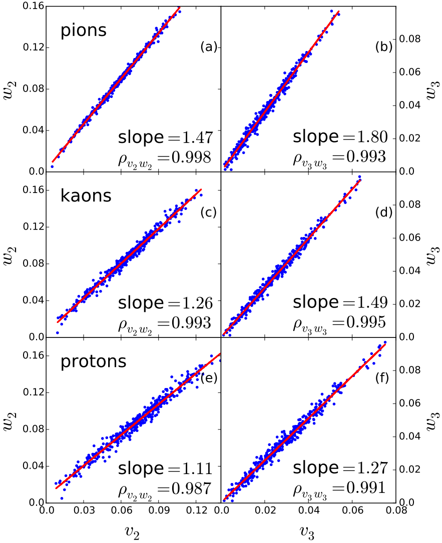
Way out: use energy flow rather than particle flow to define anisotropic flow coefficients!

Anisotropic energy flow coefficients w_n as proxy for v_n

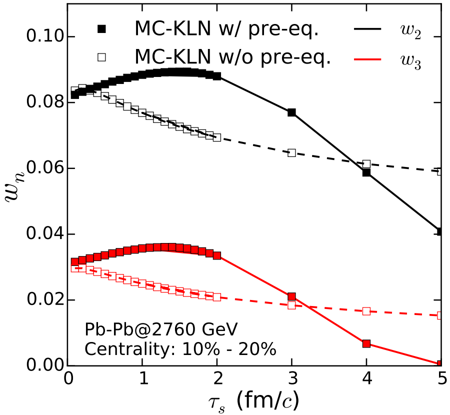
- $dE/dyd\phi$ receives contributions from $\Sigma_{\rm out}$ (non-thermalized parton halo) and $\Sigma_{\rm fo}$ (thermal emission from the liquid at freeze-out)
- **Anisotropic energy flow coefficients:**

$$w_n e^{in\bar{\Psi}_n} = \frac{\int \frac{dE}{dyd\phi_p} e^{in\phi_p}\, d\phi_p}{\int \frac{dE}{dyd\phi_p}\, d\phi_p}$$

- Plot shows that, for small τ_s when surface loss through $\Sigma_{\rm out}$ can be neglected, $v_n^{\pi,K,p}$ are all tightly linearly correlated with w_n for $n = 2, 3$.
- $\Rightarrow$ **use $w_{2,3}$ as proxy for $v_{2,3}$ for all τ_s**

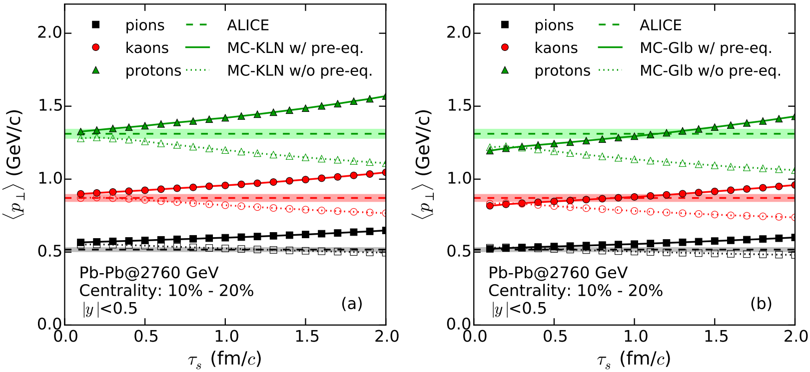


Constraints from elliptic and triangular flow



- Without pre-flow, $w_{2,3}$ drop quickly with increasing τ_s, due to decreasing initial eccentricity $\Longrightarrow$
  old lore: *"Large anisotropic flow requires fast thermalization"*
- With pre-flow, $w_{2,3}$ do not begin to decrease appreciably until $\tau_s > 2\,\mathrm{fm}/c \Longrightarrow$
  new lore: *"Large anisotropic flow almost independent of τ_s unless $\tau_s > 2$ fm/c"*
- For very weakly interacting pre-equilibrium stage, pre-flow more than compensates for the decrease in eccentricity during the first $2\,\mathrm{fm}/c$

Constraints from $p_\perp$-spectra



- Without pre-flow, $\bar{v}_{\rm fo}$ and $\langle p_\perp \rangle$ *decrease* with increasing $\tau_s \Longrightarrow$ old lore: *"Without fast thermalization not enough radial flow"*
- With pre-flow, $\bar{v}_{\rm fo}$ and $\langle p_\perp \rangle$ *increase* with increasing $\tau_s \Longrightarrow$ new lore: *"Without fast thermalization too much radial flow"*
- $\Longrightarrow$ **Once pre-flow is properly accounted for, $\langle p_\perp \rangle$ yields tighter constraint on τ_s than anisotropic flow** (especially for MC-KLN initial conditions)

Overview

1. Motivation
2. Free-streaming and Landau matching
   - Free-streaming
   - Landau matching
3. Hydrodynamic evolution
   - Hydrodynamic initial conditions after free-streaming
   - Effects of free-streaming on hydro evolution
4. Constraining τ_s from data
   - A technical issue: how to treat the non-thermalized halo
   - Constraining τ_s
5. Parameter optimization
6. Conclusions

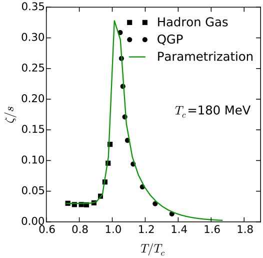
Parameter optimization

In PRC 91 (2015) 064906 we published a parameter search analysis with pre-flow + hydro only.

Here we use pre-flow + VISHNU. We switch from VISH2+1 to UrQMD 3.4 at $T_{\rm sw} = 155$ MeV.

Vary three parameters simultaneously that all have strong influence on spectra and anisotropic flow: τ_s, η/s, and the normalization factor for $(\zeta/s)(T)$.

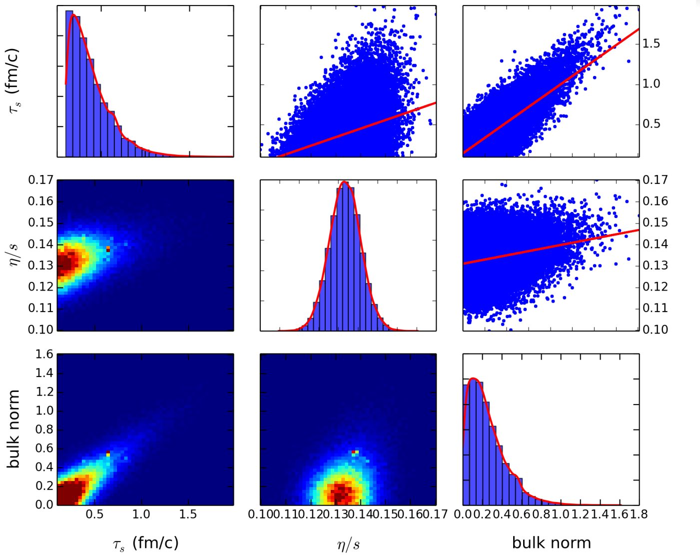
Fit five observables measured in 2.76 A TeV Pb+Pb collisions at 10-20% centrality: $\langle v_2^{\rm ch}\rangle$, $\langle v_3^{\rm ch}\rangle$, $\langle p_\perp\rangle_{\pi^+}$, $\langle p_\perp\rangle_{K^+}$, $\langle p_\perp\rangle_p$.



$\langle v_2^{\rm ch}\rangle$	0.0782 ± 0.0019
$\langle v_3^{\rm ch}\rangle$	0.0316 ± 0.0008
$\langle p_\perp\rangle_{\pi^+}$ (GeV/c)	0.517 ± 0.017
$\langle p_\perp\rangle_{K^+}$ (GeV/c)	0.871 ± 0.027
$\langle p_\perp\rangle_p$ (GeV/c)	1.311 ± 0.034

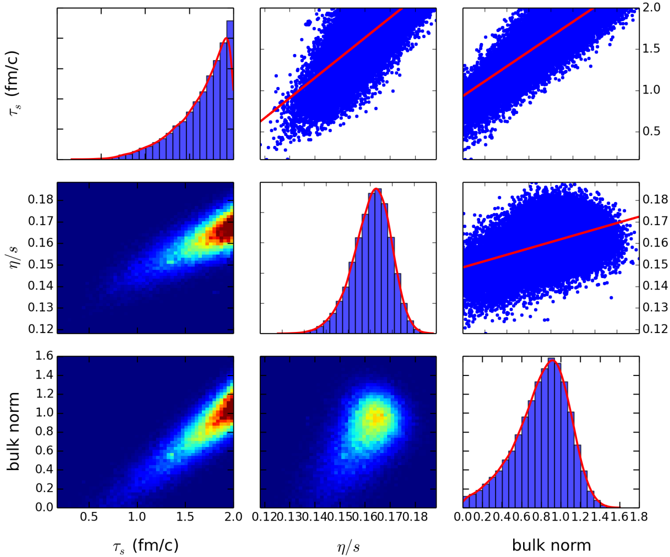
Parameter optimization: MC-KLN ICs with pre-flow

Using MADAI tools (see **poster by J. Bernhard, S. Moreland and S.A. Bass** for a more elaborate analysis fitting 8 model parameters simultaneously to LHC Pb+Pb data):



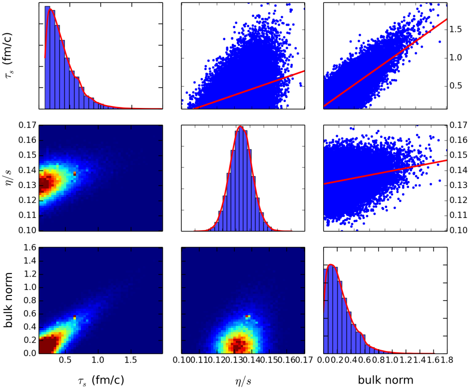
Parameter optimization: MC-Glauber ICs with pre-flow

Using MADAI tools (see **poster by J. Bernhard, S. Moreland and S.A. Bass** for a more elaborate analysis fitting 8 model parameters simultaneously to LHC Pb+Pb data):

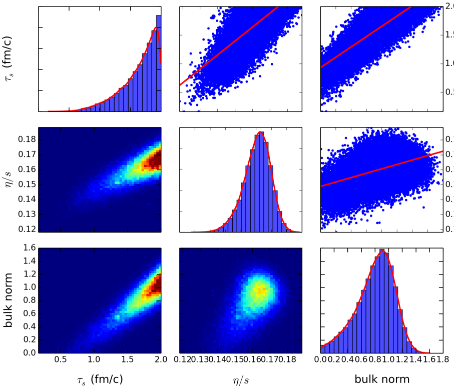


MC-KLN vs. MC-Glauber ICs with pre-equilibrium flow

MC-KLN with pre-equilibrium flow



MC-Glauber with pre-equilibrium flow



Optimal parameter sets

Model	Pre-eq.	τ_s (fm/c)	η/s	bulk norm.
MC-KLN	Yes	0.644	0.138	0.554
MC-KLN	No	—	—	—
MC-Glb	Yes	1.340	0.151	0.554
MC-Glb	No	—	—	—

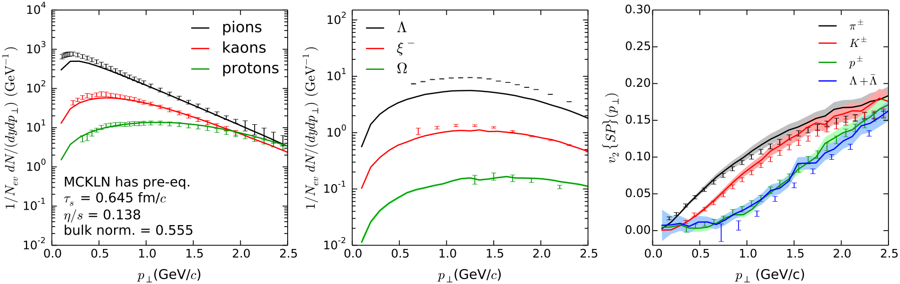
Best-fit parameters

Model	Pre-eq.	τ_s (fm/c)	η/s	bulk norm.
MC-KLN	Yes	0.381 (0–0.843)	0.133 (0.118–0.149)	0.248 (0–0.625)
MC-KLN	No	—	—	—
MC-Glb	Yes	1.51 (0.875–2.15)	0.151 (0.136–0.165)	0.714 (0.130–1.30)
MC-Glb	No	—	—	—

Mean values and 95% confidence intervals for the posterior parameter distributions

Spectra and $v_2(p_T)$ for best-fit parameter set

(Decay products from weak decays not included)



Pre-equil. flow may help to restore $p-\Lambda$ mass ordering of $v_2(p_T)$!

Overview

Conclusions

- Inclusion of pre-equilibrium evolution in hydrodynamical modeling of heavy-ion collisions **is important**.
- **Qualitatively different conclusions** as to how long the pre-equilibrium stage can last are obtained from comparisons with data when its effects on flow development are included.
- **Mean transverse momenta** of hadrons (radial flow) **provide tighter upper limits on** the duration τ_s of the pre-equilibrium stage **than anisotropic flow coefficients**.
- **Pre-equilibrium radial flow shortens** URQMD **stage and may help to restore p—Λ mass ordering of $v_2(p_T)$.**