CP violation in B^0 and $\bar{B}^0$ decay and the flavor-tagged Δt distributions

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I. Introduction

- Violation of CP symmetry or time reversal symmetry in the decay of B mesons was studied in the difference of

$$B^0 \to \ell^- \nu_\ell + X, J/\psi \bar{K}^0_L, \quad \text{v.s.}\ \bar{B}^0 \to \ell^+ \nu_\ell + X, J/\psi K^0_L$$
$$B_s \to \ell^- \nu_\ell + X, J/\psi \bar{K}^0_L, \quad \text{v.s.}\ \bar{B}_s \to \ell^+ \nu_\ell + X, J/\psi K^0_L$$

- CP violation is observed in B^0 decay but not in B_s decay.

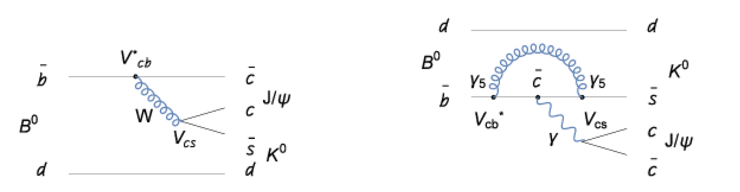
- Qualitative difference of $B^0 \to \ell\bar{\ell}$ and $B_s \to \ell\bar{\ell}$ is also observed.

- Since complete non leading order corrections in $b \to s\gamma$ and $b \to sg$ transitions are not yet available[Buchalla et al, 1996], we apply Cartan's octonion theory to solve these problems.

II. $\mathcal{CP}$ violation in B^0 and $\bar{B}^0$ decay

- In the standard model (SM), violation of the CP symmetry occurs from the interference of the tree diagram and the penguin diagram.

Fig.1 $B^0 \to K^0_L J/\psi$ decay. Tree diagram and penguin diagram.



- Adopting the parametrization of the low mass eigenstate B_L and the high mass eigenstate B_H as

$$|B_L\rangle \propto p\sqrt{1-z}|B^0\rangle - q\sqrt{1+z}|\bar{B}^0\rangle$$
$$|B_H\rangle \propto p\sqrt{1+z}|B^0\rangle + q\sqrt{1-z}|\bar{B}^0\rangle$$

the amplitude S_f and C_f are parametrized as

$$S_f = 2Im\lambda_f/(1+|\lambda_f|^2), \quad C_f = (1-|\lambda_f|^2)/(1+|\lambda_f|^2)$$

- Using the CP parity $\eta_f = -1(+1)$ for $f = J/\psi K^0_S(J/\psi K^0_L)$

$$\lambda_f = \eta_f \frac{q\bar{A}p_K}{pAq_k} = \frac{qp_K\langle J/\Psi \bar{K}^0|\mathcal{D}|\bar{B}^0\rangle}{pq_k\langle J/\Psi K^0|\mathcal{D}|B^0\rangle}$$

where

$$|K^0_S\rangle \propto p_K|K^0\rangle - q_k|\bar{K}^0\rangle$$
$$|K^0_L\rangle \propto p_K|K^0\rangle + q_k|\bar{K}^0\rangle$$

- In the decay of $B^0(\bar{B}^0)$, the vertex $V_{cb}{}^*V_{cs}(V_{cb}V_{cs}{}^*)$ appears.

- The Cabibbo-Kobayashi-Maskawa CKM matrices V_{cs}, V_{cb}, V_{cd} are given by

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1+4A^2)$$
$$V_{cb} = A\lambda^2$$
$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)]$$

- Large $V_{cs} = 0.973$ yields strong $B^0(\bar{B}^0) \to K^0 + X(\bar{K}^0) + X$ decay modes.

- Strong $B^0(\bar{B}^0) \to D^{*-}\ell^+\nu_\ell(D^{*+}\ell^-\bar{\nu}_\ell)$ comes from relatively large $V_{cd} = 0.225$ and small $V_{ub} = 0.00355$ and $V_{cb} = 0.0414$.

III Cartan's supersymmetry and the CKM matrices

- Hadron weak interactions including $b \to s$ transitions calculated by CKM matrices do not have good agreement with experiments.

- In Cartan's supersymmetry, fermions are defined by octonions, and interactions of vector current of fermions and gauge fields are studied. Its extension to interaction of axial current of fermions is straightforward, and gives differences in B_s decay and B_d decay, which the CKM theory cannot explain.

- The Clifford algebra of $\boldsymbol{R}_{3,0}$ is equipped with the basis

$$\{\boldsymbol{1}, \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_1\boldsymbol{e}_2, \boldsymbol{e}_2\boldsymbol{e}_3, \boldsymbol{e}_1\boldsymbol{e}_3, \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\},$$

which expresses Pauli algebra or quaternion algebra.

- Cartan considers fermion fields ψ

$$\psi = \xi_1\boldsymbol{i} + \xi_2\boldsymbol{j} + \xi_3\boldsymbol{k} + \xi_4 = \begin{pmatrix} \xi_4 + i\xi_3 & i\xi_1 - \xi_2 \\ i\xi_1 + \xi_2 & \xi_4 - i\xi_3 \end{pmatrix}$$
$$C\psi = -\xi_{234}\boldsymbol{i} - \xi_{314}\boldsymbol{j} - \xi_{124}\boldsymbol{k} + \xi_{123} = \begin{pmatrix} \xi_{123} - i\xi_{124} & -i\xi_{234} + \xi_{314} \\ -i\xi_{234} - \xi_{314} & \xi_{123} + i\xi_{124} \end{pmatrix}$$

and the spinor operator

$$\phi = \xi_{14}\boldsymbol{i} + \xi_{24}\boldsymbol{j} + \xi_{34}\boldsymbol{k} + \xi_0 = \begin{pmatrix} \xi_0 + i\xi_{34} & i\xi_{14} - \xi_{24} \\ i\xi_{14} + \xi_{24} & \xi_0 - i\xi_{34} \end{pmatrix}$$
$$C\phi = -\xi_{23}\boldsymbol{i} - \xi_{31}\boldsymbol{j} - \xi_{12}\boldsymbol{k} + \xi_{1234} = \begin{pmatrix} \xi_{1234} - i\xi_{12} & -i\xi_{23} + \xi_{31} \\ -i\xi_{23} - \xi_{31} & \xi_{1234} + i\xi_{12} \end{pmatrix},$$

and vector field

$$\{\boldsymbol{x}, \boldsymbol{x}'\} = \{x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3, x'_4\}.$$

- Cartan defines three quadratic forms

$$F = x_1x'_1 + x_2x'_2 + x_3x'_3 + x_4x'_4$$
$$\Psi = -\xi_1\xi_{234} - \xi_2\xi_{314} - \xi_3\xi_{124} + \xi_4\xi_{123}$$
$$\Phi = \xi_0\xi_{1234} - \xi_{23}\xi_{14} - \xi_{31}\xi_{24} - \xi_{12}\xi_{34}$$

and obtained 5 Triality transformations

$$\{G_{23}, G_{12}, G_{13}, G_{132}, G_{123}\}$$

that leave F, Ψ and Φ invariant. The transformation table of the 5 transformations are given in [Furui,2012].

- Cartan studied interaction of even dimensional field $\boldsymbol{x}, \boldsymbol{x}'$ and spinor fields $\psi, C\psi$ and $\phi, C\phi$ which are expanded by quaternions. Quadratic algebra of quaternions induces octonions, and Cartan showed that there is a group G which contains octonion transformation and leave ${}^t\phi CX\psi$, $F = \boldsymbol{x}\cdot\boldsymbol{x}'$, $\Phi = {}^t\phi C\phi$ and $\Psi = {}^t\psi C\psi$ invariant.

- The vertex of vector particles and spinors $\gamma_0 x^\mu \gamma_\mu$ can be extended to include weak interaction by replacing the vertex to $\gamma_0 x^\mu \gamma_\mu(1-\gamma_5)$ and choosing -1 or γ_5 such that the couplings are unified in the form

$$\sum_{i=1}^{4}(x_i\mathcal{C}\phi\mathcal{C}\psi + x'_i\mathcal{C}\phi\,\psi)$$

- The operator $\gamma_5 x_4$ and x'_4 couples between $\phi\mathcal{C}\psi$ or $\psi\mathcal{C}\phi$, and the coupling between $\phi\psi$ is stronger than that between $\phi\mathcal{C}\psi$, which makes difference in the strength of $1-\gamma_5$ coupling from that of the CKM model.

- Quantum mechanics is based on noncommutative variables. A product of Clifford numbers $\boldsymbol{x}$ and $\boldsymbol{y}$ in n dimensional linear space V over a field $\boldsymbol{F}$ defined by Chevalley is

$$\boldsymbol{xy} = \boldsymbol{x} \wedge \boldsymbol{y} + \boldsymbol{x} \lrcorner \boldsymbol{y} = \boldsymbol{x} \wedge \boldsymbol{y} + B(\boldsymbol{x}, \boldsymbol{y})$$

where $\boldsymbol{x} \wedge \boldsymbol{y}$ is the antisymmetric product, and $\boldsymbol{x} \lrcorner \boldsymbol{y}$ is the contraction which depends on $\boldsymbol{F}$[Lounesto, 2001]

- When $\boldsymbol{F} = \{0,1\}$ and an exterior algebra $\wedge V$ has the basis $\{1, \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_1 \wedge \boldsymbol{e}_2\}$, there are two bilinear forms $B_1(\boldsymbol{x}, \boldsymbol{y}) = x_1y_2$ and $B_2(\boldsymbol{x}, \boldsymbol{y}) = x_2y_1$ which have multiplication tables.

- We can assign vectors of $\psi, \mathcal{C}\psi$ to B_1 and $\phi, \mathcal{C}\phi$ to B_2. There is no preference of $\psi, \mathcal{C}\psi$ and $\phi, \mathcal{C}\phi$, and thus there are no unique bivectors in octonion space, whose field has characteristic 2 [Lounesto,2001].

- But combining with the commutation relation with the time component, we can define unique bivectors in octonion space satisfying Cartan's supersymmetry.

  The unit vector in $\psi, \mathcal{C}\psi$ space $\boldsymbol{e}_0$ can be taken as $\boldsymbol{e}_1$ and that in $\phi, \mathcal{C}\phi$ space can be taken as $\boldsymbol{e}_2$ in the space of $\boldsymbol{F} = \{0,1\}$.

- In $B^0 \to J/\Psi K^0$, there are penguin diagrams and tree diagrams.

Fig.2 Typical penguin diagrams of $B^0 \to K^0_L J/\psi$ and $\bar{B}^0 \to \bar{K}^0_L J/\psi$ decay.

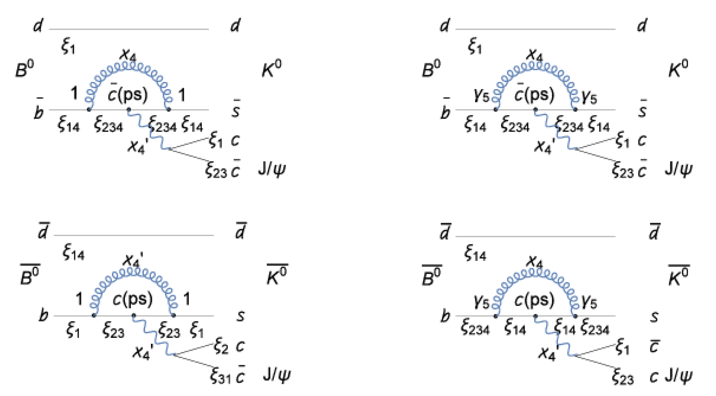
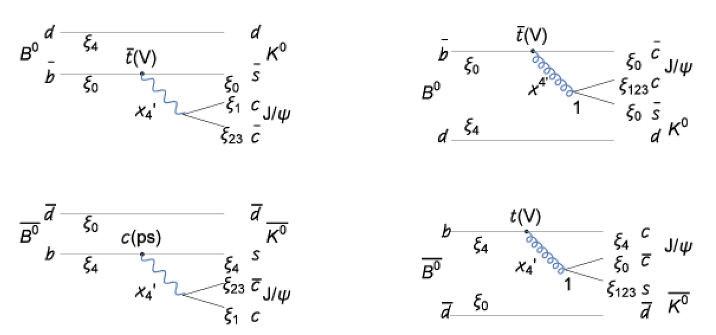
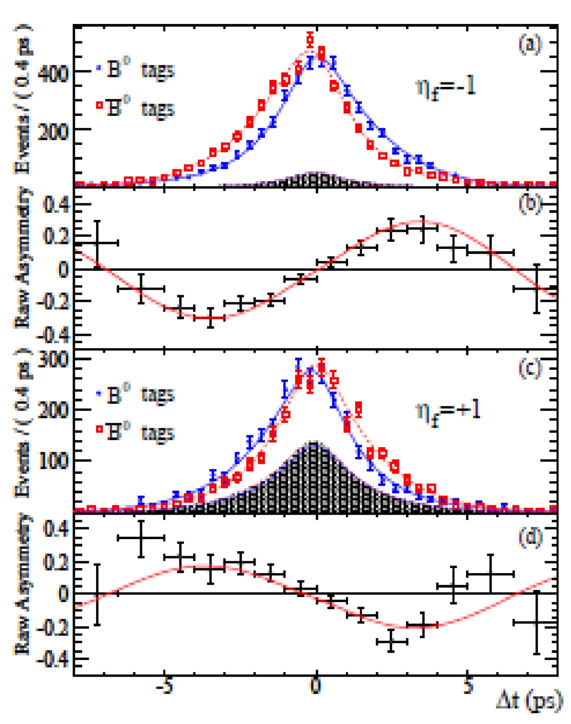


Fig.3 Typical tree diagrams of $B^0(\bar{B}^0) \to K^0_L(\bar{K}^0_L) J/\psi$ decay.

In tree diagrams, there are type I in which $c\bar{c}$ quark pair is created, and type II in which $c\bar{s}$ quark pair is created. In type II, the quark wave function of J/ψ from B^0 decay is suppresed as compared to that from $\bar{B}^0$ decay. The type II occurs at large Δt region.





Experiment and simulation of $B^0 \to J/\Psi K^0$ by BABAR Collab. Phys. Rev. D79, 072009 (2009).
When CP parity of the final state $\eta_f = -1$, theory is consistent with experiment.
When $\eta_f = +1$, and Δt is small, penguin diagram gives consistent experimental data, but when Δt is large, there appears discrepancy which is expected to be due to the tree diagram contribution.

- Experimental branching ratios of $B \to \mu\bar{\mu}$:

$$\bar{B}(B_s \to \mu^-\mu^+) = (2.9 \pm 0.7) \times 10^{-9}$$

is consistent with theoretical values but

$$\bar{B}(B_d \to \mu^-\mu^+) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}$$

is about 3 times larger than theoretical values[Bobeth et al.,2014]

- In SM, the dominant combination of V and A is fixed by $\bar{\psi}(1-\gamma_5)\gamma_\mu\psi$. Cartan's supersymmetry defines $\bar{\phi}\gamma_\mu\psi$ and $\bar{\psi}\gamma_5\gamma_\mu\psi$ indivisually.

- There are three sectors of quarks (t, b), (c, s) and (u, d), and in the decay of a B meson, incorporation of a quark triangular loop is important. In usual decay process t quark loop plays an essential role, but when there are s quark in the initial meson state, c quark loop plays also an important role and enhances the decay process.

- In the B_d decay, the contribution of $t\bar{t}$ pair transition to $\ell\bar{\ell}$ is expected to enhance the gluon transition to $\ell\bar{\ell}$.

- In the B_s decay, the effect is cancelled by the $c\bar{c}$ pair transition to $\ell\bar{\ell}$.

Fig.4a Typical diagrams of $B_s(B_d) \to \ell\bar{\ell}$ decay in SM.
They contain two $1-\gamma_5$ operators.

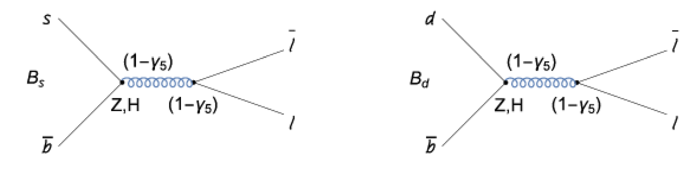
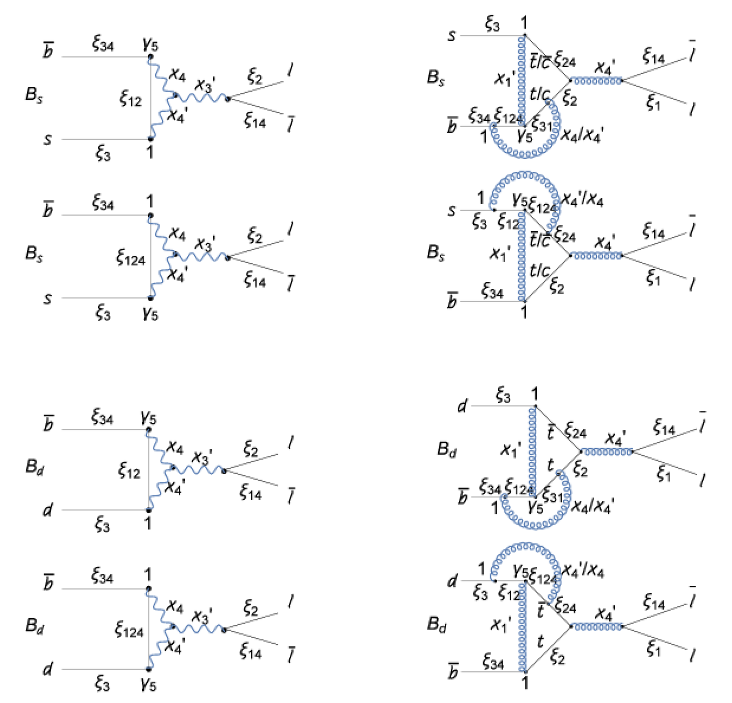


Fig.4b Typical diagrams of $B \to \ell\bar{\ell}$ decay which contain one γ_5.
$B \to \ell\bar{\ell}$ (left) contains one loop, and $B \to \ell\bar{\ell}$ (right) contains two loops.

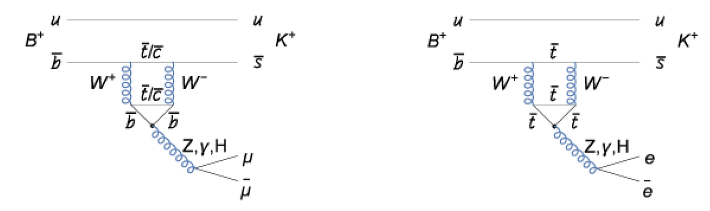


- Similar problem exists in the deficit of $B^+ \to K^+\mu\bar{\mu}$ as compared to $B^+ \to K^+e\bar{e}$

$$\frac{Br(B^+ \to K^+\mu\bar{\mu})}{Br(B^+ \to K^+e\bar{e})} = 0.745^{+0.090}_{-0.074}(stat) \pm 0.036(syst)$$

- In the case of μ in final state, c quark loops make correction to t quark loops, which is dominant in the CKM model.

Fig.5 Typical diagrams of $B^+ \to K^+\ell\bar{\ell}$ decay.



IV. Discussion and Conclusion

- Description of Dirac fermions is not unique. The sub algebra $\boldsymbol{R}^+_{3,1}$ with a basis

$$\{\boldsymbol{1}, \boldsymbol{e}_1\boldsymbol{e}_2, \boldsymbol{e}_1\boldsymbol{e}_3, \boldsymbol{e}_2\boldsymbol{e}_3, \boldsymbol{e}_1\boldsymbol{e}_4, \boldsymbol{e}_2\boldsymbol{e}_4, \boldsymbol{e}_3\boldsymbol{e}_4, \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\boldsymbol{e}_4\}$$

where $\boldsymbol{e}_i$ satisfy $\boldsymbol{e}_1^2 = \boldsymbol{e}_2^2 = \boldsymbol{e}_3^2 = \boldsymbol{1}, \boldsymbol{e}_4^2 = -\boldsymbol{1}$ is equivalent to Pauli algebra, defined by $\boldsymbol{e}_1\boldsymbol{e}_4 = \sigma_1, \boldsymbol{e}_2\boldsymbol{e}_4 = \sigma_2, \boldsymbol{e}_3\boldsymbol{e}_4 = \sigma_3$ and

$$\boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\boldsymbol{e}_4 = \sigma_1\sigma_2\sigma_3 = \boldsymbol{i}.$$

This non-commutative algebra can be calculated on PC using Mathematica [Aragon et al.,2008].

- In the n dimensional linear space V over a field $\boldsymbol{F}$ and exterior algebra $\wedge V$, octonion appears by defining $\boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3 = \boldsymbol{\ell}$, and choosing

$$\{\boldsymbol{1}, \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, -\boldsymbol{e}_3\boldsymbol{\ell}, -\boldsymbol{e}_2\boldsymbol{\ell}, -\boldsymbol{e}_1\boldsymbol{\ell}, \boldsymbol{\ell}\}$$

as the basis of the field[Dray and Manogue, 1998]. The commutation relations of $\boldsymbol{e}_i$ are not same as those of Cartan's ξ_i.

- The CKM model was successful since Higgs meson separated t quark from other quarks, and $m_u \simeq m_d$. The problem of B meson decay branching ratios indicates importance of the difference of $m_s < m_c < m_H < m_t$ and $m_K < m_c < m_B$, and supersymetric interactions.

- Presence of c meson in the same sector as that of s meson is not crucial in the decay of K mesons since $m_c > m_K$, but crucial in that of B_s since $m_c < m_B$. In the case of Dirac spinors, our model contains the triality symmetry, which can be interpreted as the color degrees of freedom (r, g, b). The neutrinos in a definite flavor sector are

$$\nu_e, [\nu_u, \nu_d]_{r,g,b}, \quad \nu_\mu, [\nu_c, \nu_s]_{r,g,b}, \quad \nu_\tau, [\nu_t, \nu_b]_{r,g,b}$$

all left-handed, and there are seven right-handed neutrinos.

- We expect that our electromagnetic detector can detect electromagnetic fields transformed by G_{23}, but cannot detect electromagnetic fields transformed by G_{12}, G_{13}, G_{123} and G_{132}. There are six lepton sectors

$$|e, \nu_e)^*, |\mu, \nu_\mu)^*, |\tau, \nu_\tau)^*, |e, \nu_e)^{**}, |\mu, \nu_\mu)^{**}, |\tau, \nu_\tau)^{**}$$

whose massive neutrino components cannot be detected by our detectors, and two additional right-handed neutrinos.

- The number of massive neutrinos becomes $21 + 6 = 27$, and the number of right-handed neutrinos becomes $7 + 2 = 9$ in this model. It is possible to construct a model satisfying the Z_3 symmetry using Dirac lepton neutrinos and Majorana quark neutrinos.

- If in the univese there are world which are transformed by G_{ij} and G_{ijk}, and the uncertainty principle applys not only in our world but also in whole universe, we can understand the presence of dark matter.

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