I. Introduction

- Violation of CP symmetry or time reversal symmetry in the decay of B mesons was studied in the difference of $\Delta S = 1$ and $\Delta S = 0$ processes. The CP violation observed in $B^0$ decay but not in $B^0\bar{B}^0$ decay.

- Quantitative difference of $\Delta S = 2$ and $\Delta S = 0$ is also observed.

- Cartan’s complex notation under order corrections is $|a + bi| = a^2 + b^2$ and $-a + bi$ transform to $a^2 + b^2$.

- Quark QCD model is based on noncommutative variables. A product of QCD momenta and QCD quark is interpreted as a noncommutative (quantum) space $\mathbb{Q}_a = q_a q_b + i [q_a, q_b]$, where $q_a$ is a quantum number, and $[q_a, q_b]$ is the commutator which depends on $\mathbb{Q}_a$.

- When $G = (\mathbb{C})$ and under order algebra $G^2 = A_0 + A_1\sqrt{\mathbb{Q}}$, there are two different forms $(A_0, A_1) = (\mathbb{Q}_a, 0)$ and $(0, \mathbb{Q}_a)$ which have multiplication tables.

- We can assign vectors $\mathbb{Q}_a = 0$ and $\mathbb{Q}_b = 0$ to $A_0$ and $A_1$, and thus there is no unique solution in addition space, whose basis has characteristics 2 $\mathbb{Q}_a$ (Lumercio2011).

- But combining with the commutation relation with the time component, we can define unique vectors in addition space satisfying Cartan's superstipulated rules.

- The unit vector in $\mathbb{Q}_a$ space can be taken as $\mathbb{Q}_a$ and that in $\mathbb{Q}_b$ space can be taken as $\mathbb{Q}_b$ in the space of $F = (\mathbb{C})$.

- In addition to $\Delta S = 2$, there are quantum diagrams and tree diagrams.

- In the decay of $B^0$, the wave vector $\mathbb{Q}_a = (\mathbb{Q}_a, 0)$ appears.

- The Cabibbo-Kobayashi-Maskawa (CKM) matrices $V_{cb}$ and $V_{cd}$ are given by

$$V_{cb} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \end{pmatrix} \quad \text{and} \quad V_{cd} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \end{pmatrix}$$

- Large $\mathbb{Q}_a$ yields strong $\mathbb{Q}_a = (\mathbb{Q}_a, 0)$ electron momenta.

- Strong $\mathbb{Q}_a = (\mathbb{Q}_a, 0)$ electron momenta comes from relatively large $\mathbb{Q}_a = 0.005$ and small $\mathbb{Q}_a = 0.0005$ and $\mathbb{Q}_a = 0.0004$.

II. CP violation in $B^0$ and $B^0\bar{B}^0$ decay

- In the standard model (SM), violation of the CP symmetry occurs from the interference of the tree diagram and the quark diagram.

$$\text{Fig. 1:} B \to \psi K, \text{Tree diagram and quark diagram.}$$

- Adopting the parameterization of the low mass eigenstate $b_L$ and the high mass eigenstate $b_R$ as

$$|b_L \rangle = \frac{1}{\sqrt{2}} |0 \rangle + \frac{1}{\sqrt{2}} |1 \rangle, \quad \text{and} \quad |b_R \rangle = \frac{1}{\sqrt{2}} |0 \rangle - \frac{1}{\sqrt{2}} |1 \rangle,$$

- Using the CP parity $|c_L \rangle = -|c_R \rangle$ for $F = (\mathbb{C})$.

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III. Cartan’s superstipulated and the CKM matrices

- Hadron weak interactions including $\Delta S = 1$ transitions can be calculated by CKM matrices but do not have good agreement with experiments.

- Cartan’s superstipulated matrices are defined by interactions, and interactions of vector current of fermions and pion decay widths are calculated from the current of fermions, which is straightforward, and gives differences in $\Delta S = 1$ decays and $\Delta S = 0$ decay, which the CKM theory cannot explain.

- The Clifford algebra of $\mathbb{Q}_a$ is equipped with the basis

$$(\mathbb{Q}_a, 0), (\mathbb{Q}_a, 1), (\mathbb{Q}_a, 0), (\mathbb{Q}_a, 1),$$

which expresses Pauli algebras or quaternion algebras.

- Cartan considers fermion fields $\psi = (\psi^1, \psi^2, \psi^3, \psi^4)$ and the operator $\mathbb{Q}_a = (\mathbb{Q}_a, 0)$ and $\mathbb{Q}_a = (\mathbb{Q}_a, 1)$.

- Cartan defines three quadratic forms

$$\mathbb{Q}_a = \mathbb{Q}_a \mathbb{Q}_a, \quad \mathbb{Q}_a = \mathbb{Q}_a \mathbb{Q}_a, \quad \text{and} \quad \mathbb{Q}_a = \mathbb{Q}_a \mathbb{Q}_a.$$