



# QUARK MATTER 2015

Sep. 27 – Oct. 3, 2015 Kobe, Japan 

## Flow

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Student day, September 26th.



# What is the goal of this lecture?

- Explain why we think that nucleus-nucleus collisions at RHIC and LHC colliders produce a **little fluid**.
- Describe several *qualitative* features seen in data which are **naturally** explained by **hydrodynamics**, irrespective of model details.
- Introduce the terminology to be used extensively at this conference regarding **anisotropic flow**, the key observable that characterizes the little fluid.

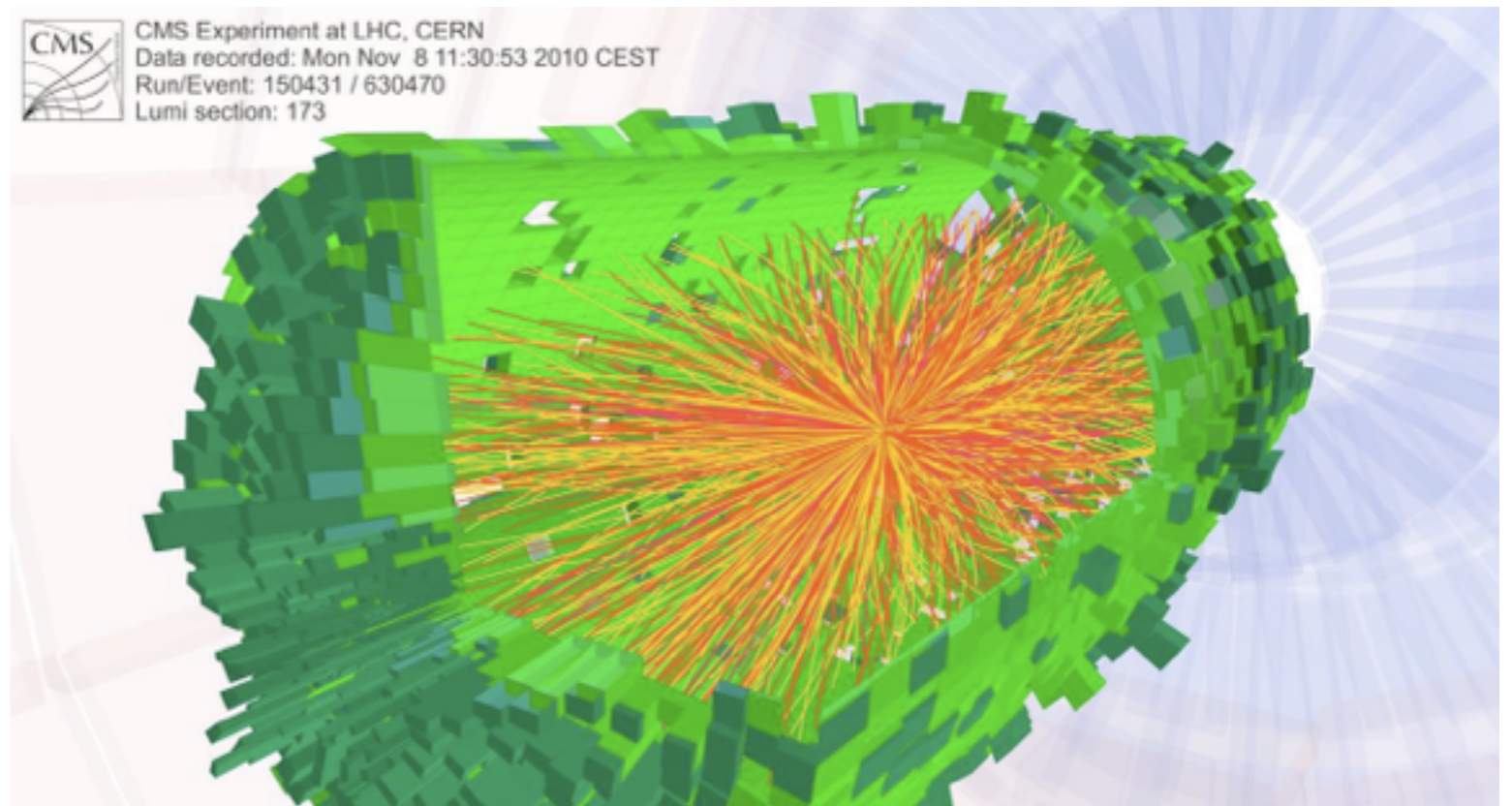
# What is the goal of this lecture?

You have asked very good questions.  
They have helped me a lot.

I will do my best to answer them

# Where should we look?

- This is not about rare events (e.g. jets) or rare particles.
- It is about how the system behaves as a whole.
- We need to look at the **bulk** of particle production.  
**All** the particles you see are useful here



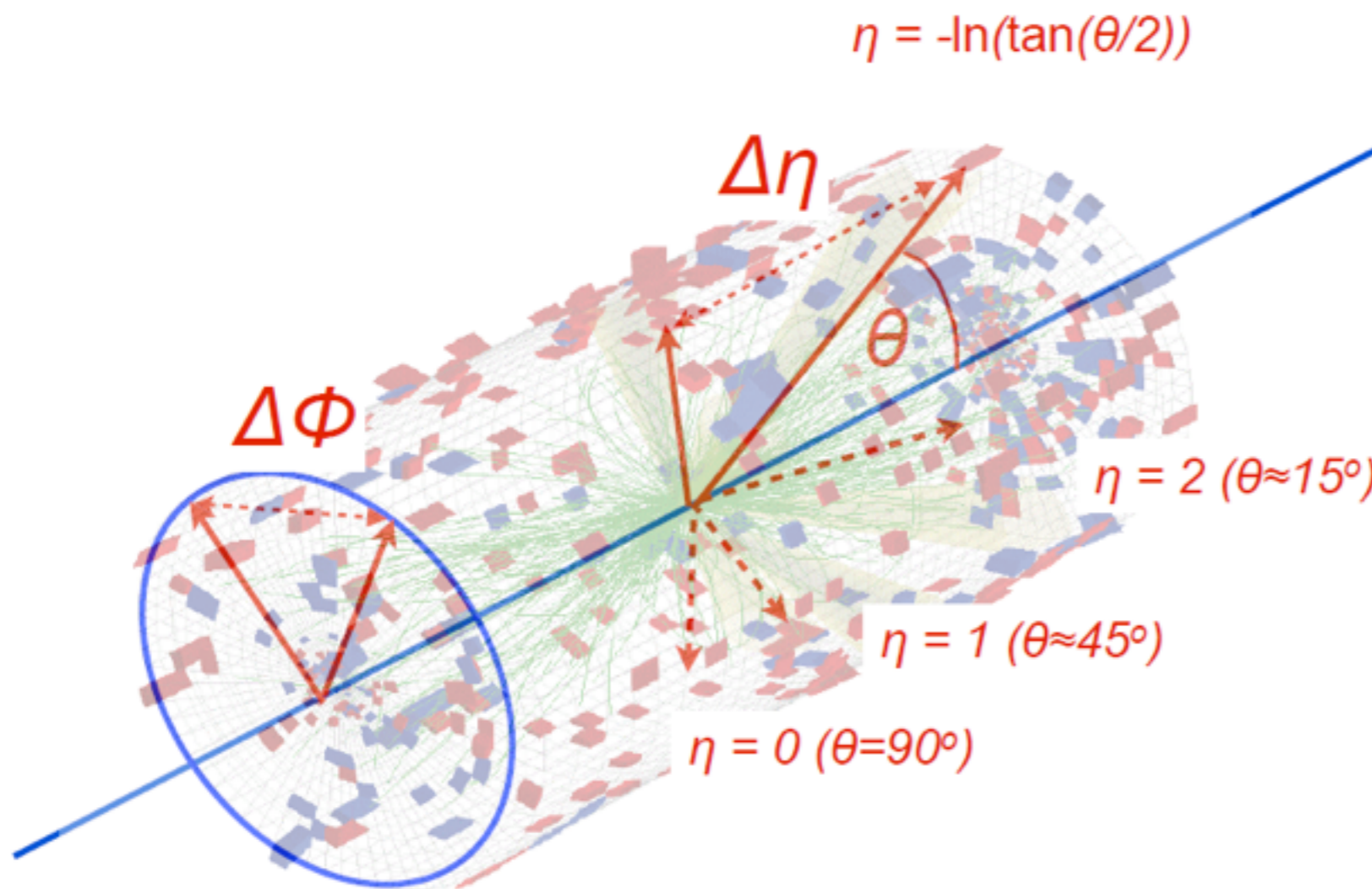
How the energy of the collision is spent? How much of the initial energy is used to produce particles in the collisions, and what are the proportions (how much is used for particle masses, how much for their momentum)?

- Pb+Pb collision at LHC: total energy = 500 TeV
- 25000 particles produced in a central collision, mostly pions: mass energy = 5 TeV = 1%
- Most of the remaining 99% is probably kinetic energy of these particles. Detectors typically miss the most energetic, almost parallel to the collision axis (large rapidity)

What are differences between collisions of elementary particles and heavy ions in that regard?

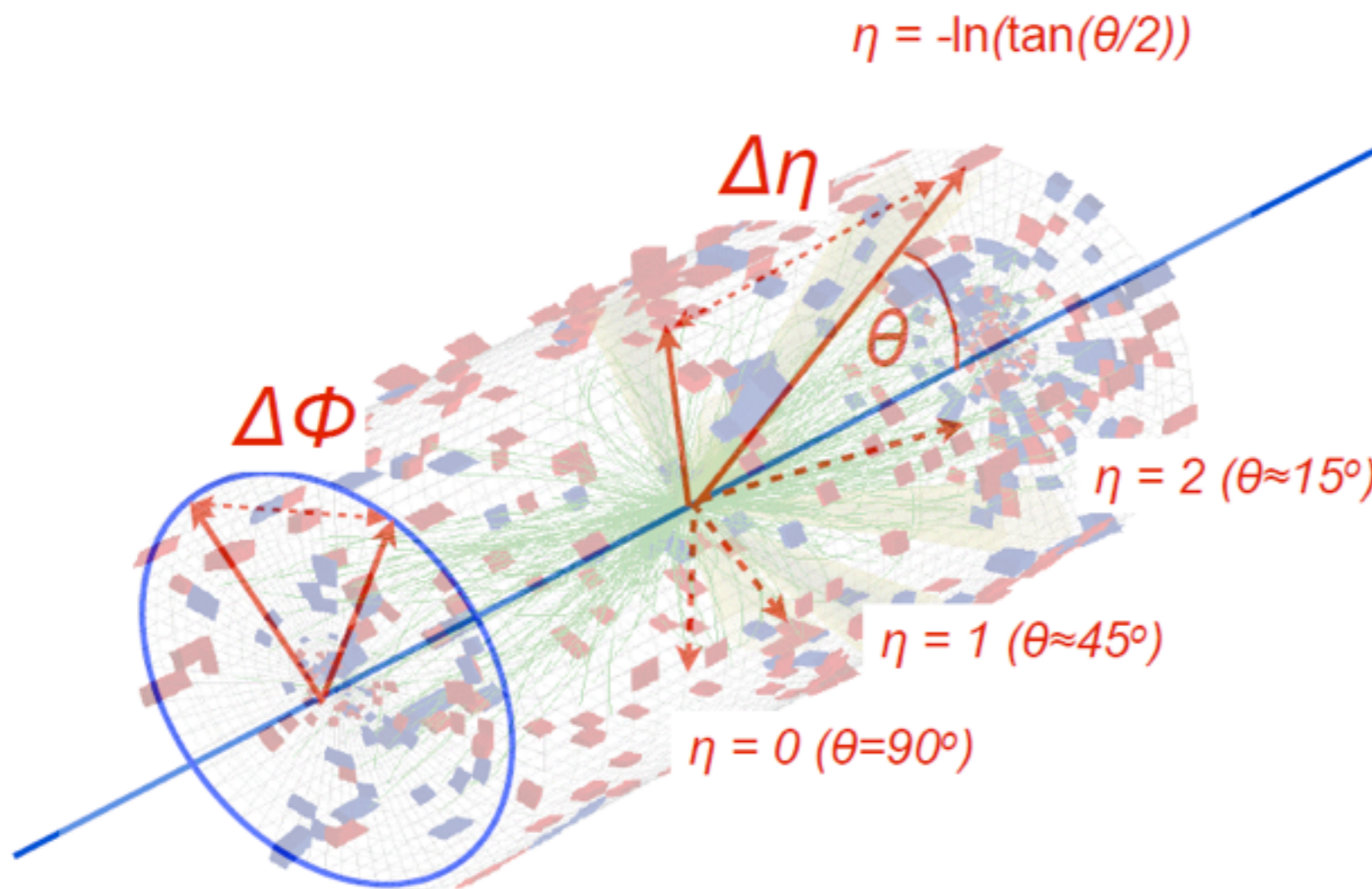
Nothing spectacular if you just count particles

# Counting **pairs**: correlations



- Trajectories of charged particles:
- polar angle  $\theta$  (or pseudorapidity  $\eta = -\ln \tan \theta/2$ )
- azimuthal angle  $\phi$

# Counting **pairs**: correlations



In each collision, construct all pairs of particles.

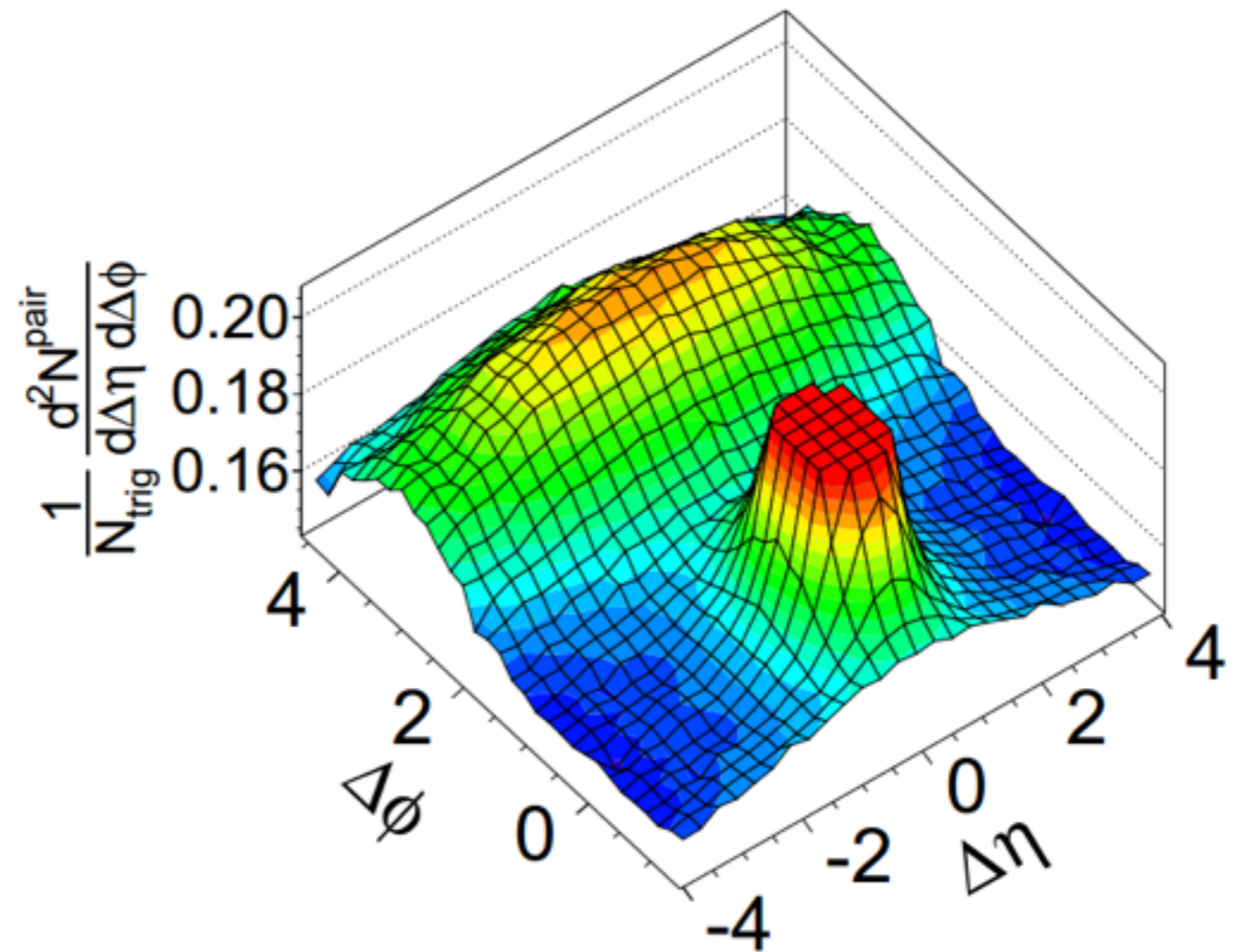
Count as a function of *relative* angles  $\Delta\phi$  and  $\Delta\eta$

Average over many collisions in a centrality class

Why nuclear collisions are special:  
*first occurrence of emergent phenomena*  
*in high-energy physics experiments.*

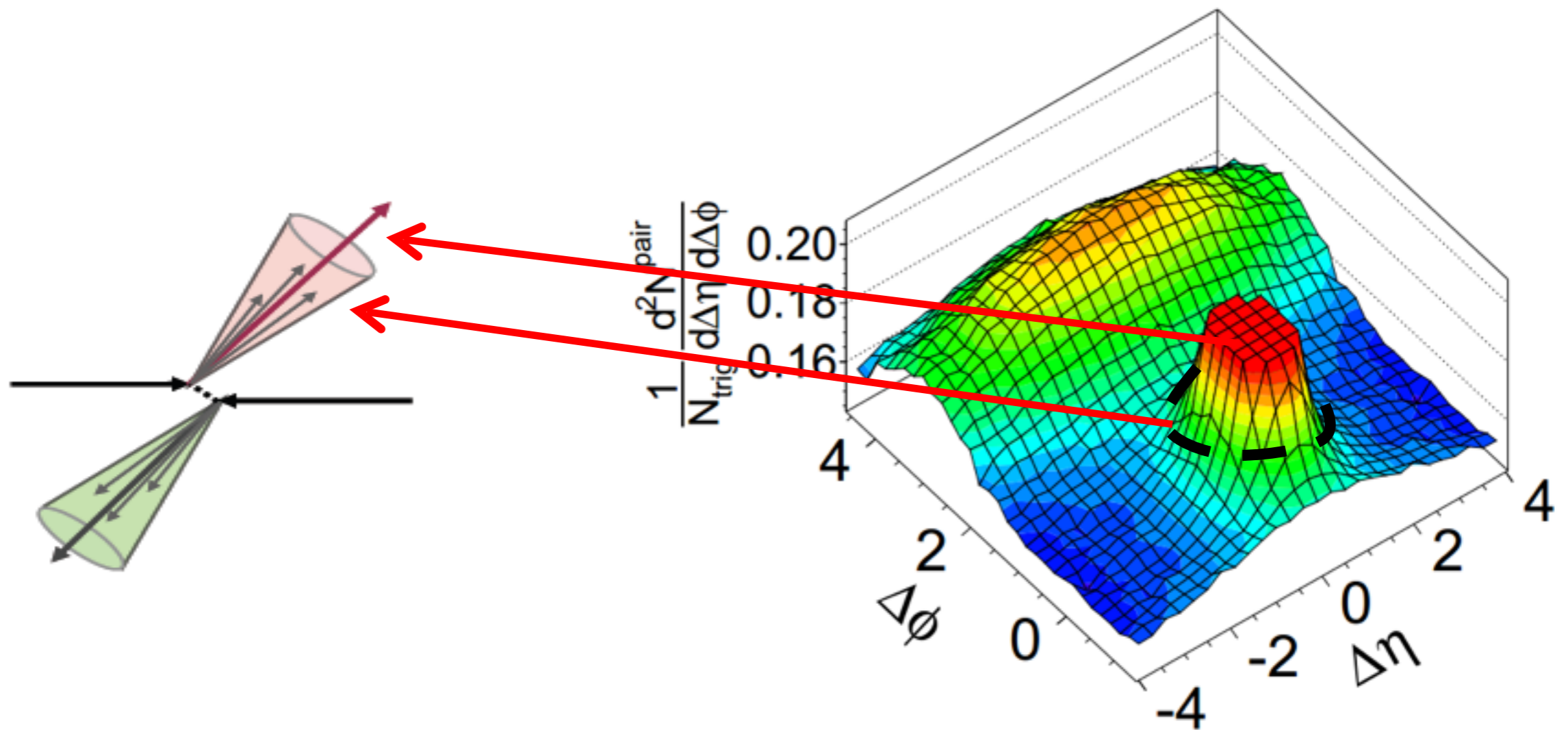


# Correlations in proton-proton collision



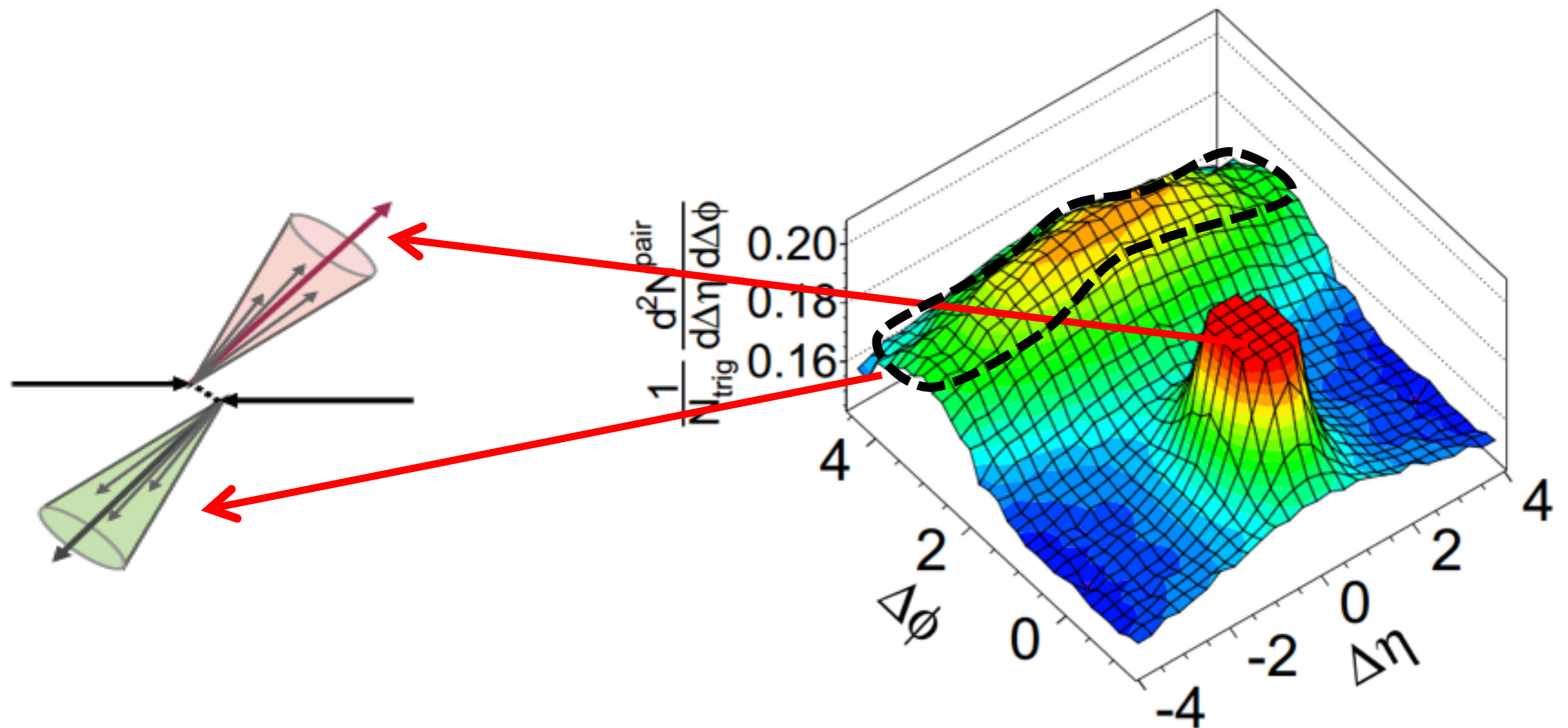
A non-trivial correlation pattern  
driven by elementary processes

# Correlations in proton-proton collision



*Near-side* ( $\Delta\phi, \Delta\eta \sim 0$ ) correlations from single jets

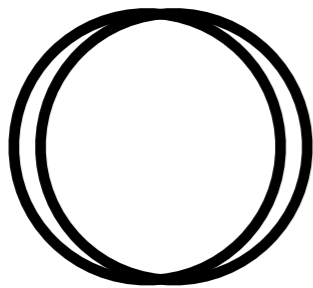
# Correlations in proton-proton collision



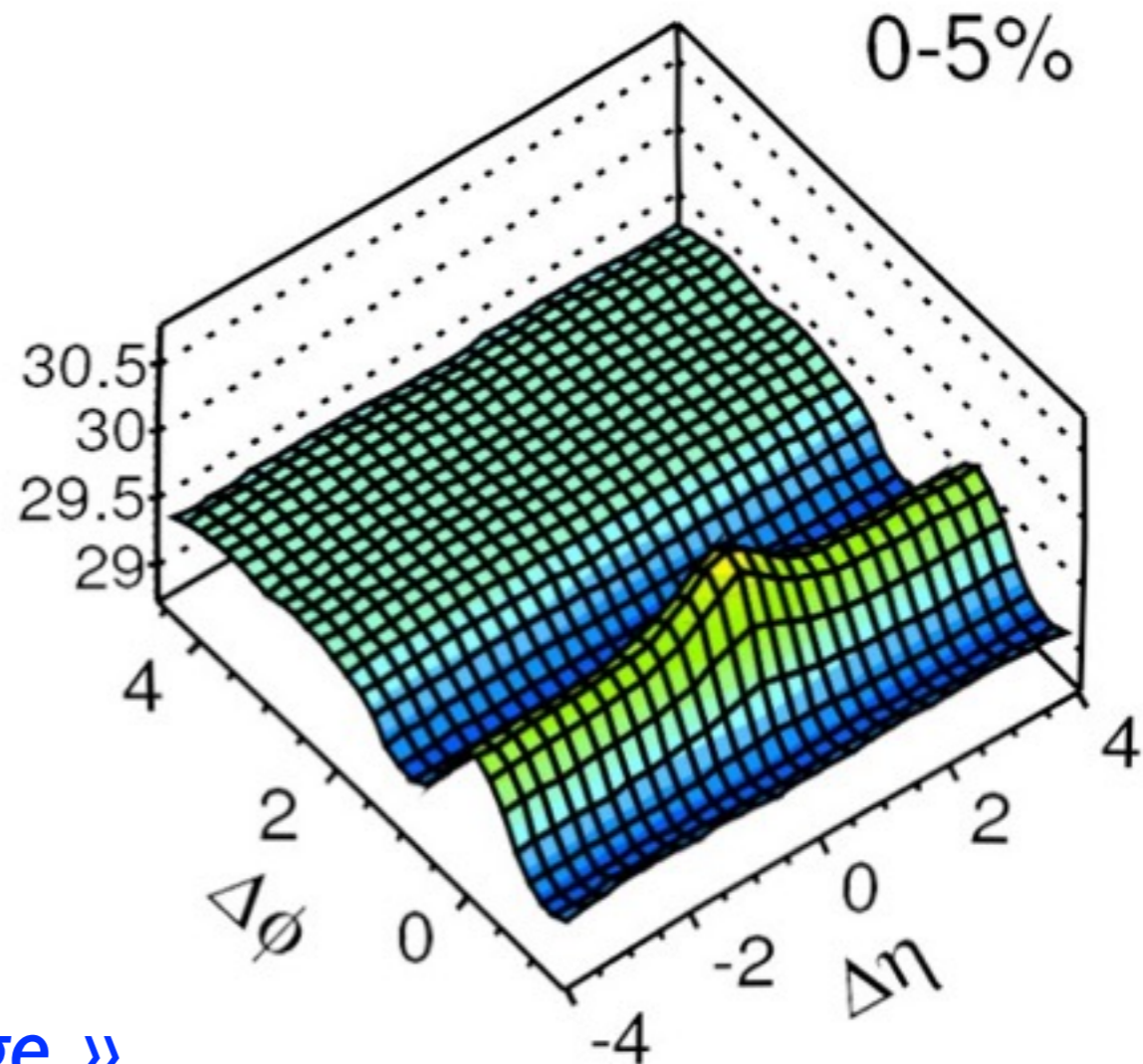
*Away-side* ( $\Delta\varphi \sim \pi$ ) back-to-back jet correlations:  
the two jets may have different rapidities:  $\Delta\eta \neq 0$

# Correlations in Pb+Pb collisions

More is different:  
Simplicity emerges!



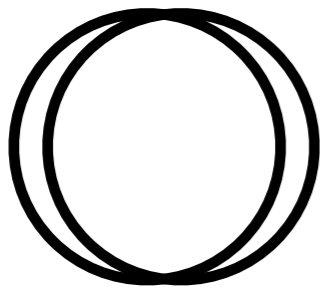
$$\frac{1}{N_{\text{trg}}} \frac{d^2 N^{\text{pair}}}{d\Delta\eta d\Delta\phi}$$



*Ripple in a pond, aka « ridge »  
=the cleanest signature of fluid  
behavior.*

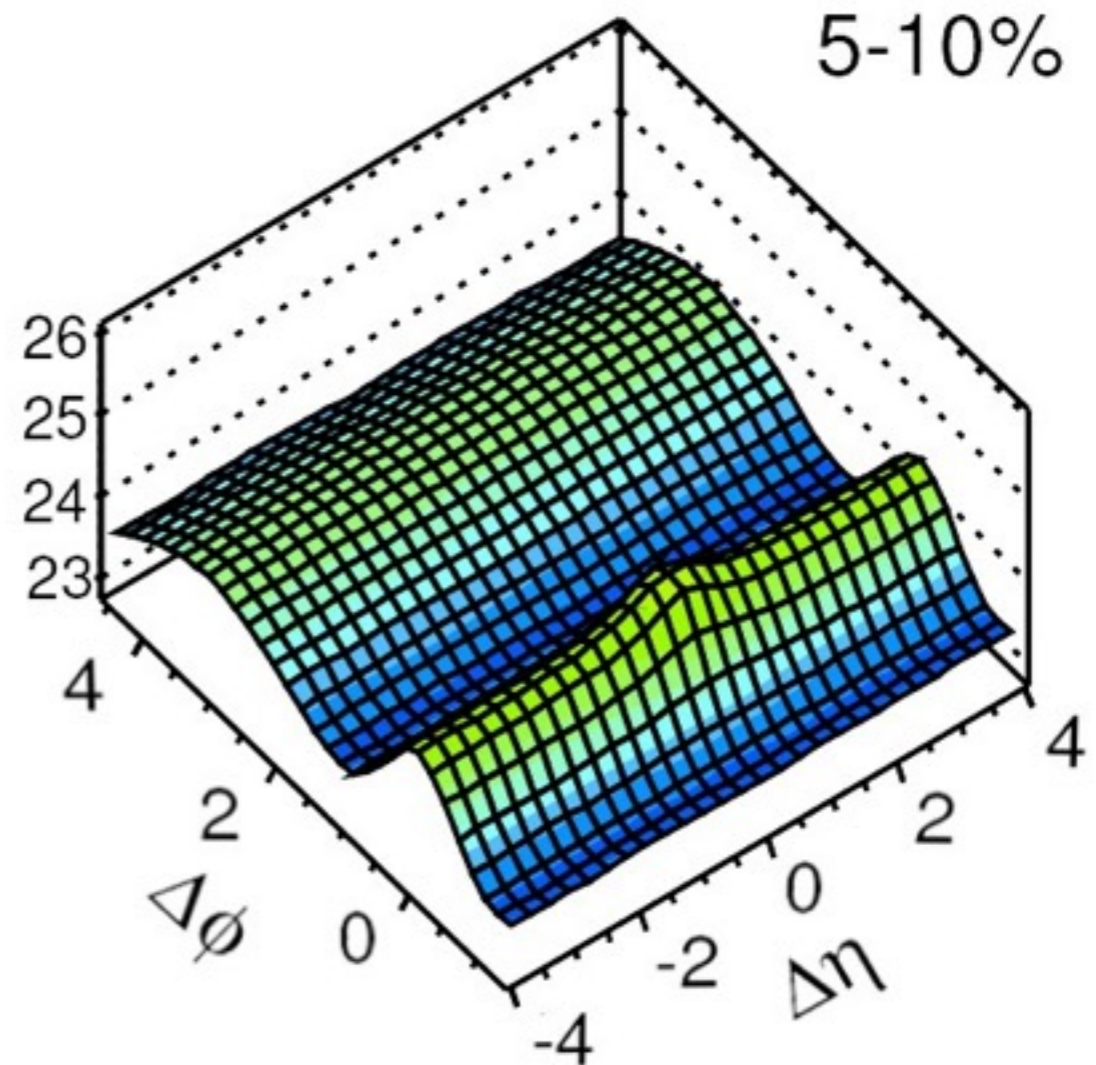
CMS 1201.3158

# Correlations in Pb+Pb collisions



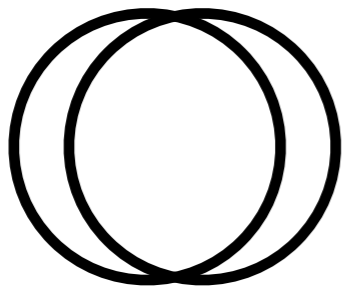
Similar regular pattern,  
independent of  $\Delta\eta$

$$\frac{1}{N_{\text{trg}}} \frac{d^2 N^{\text{pair}}}{d\Delta\eta d\Delta\phi}$$



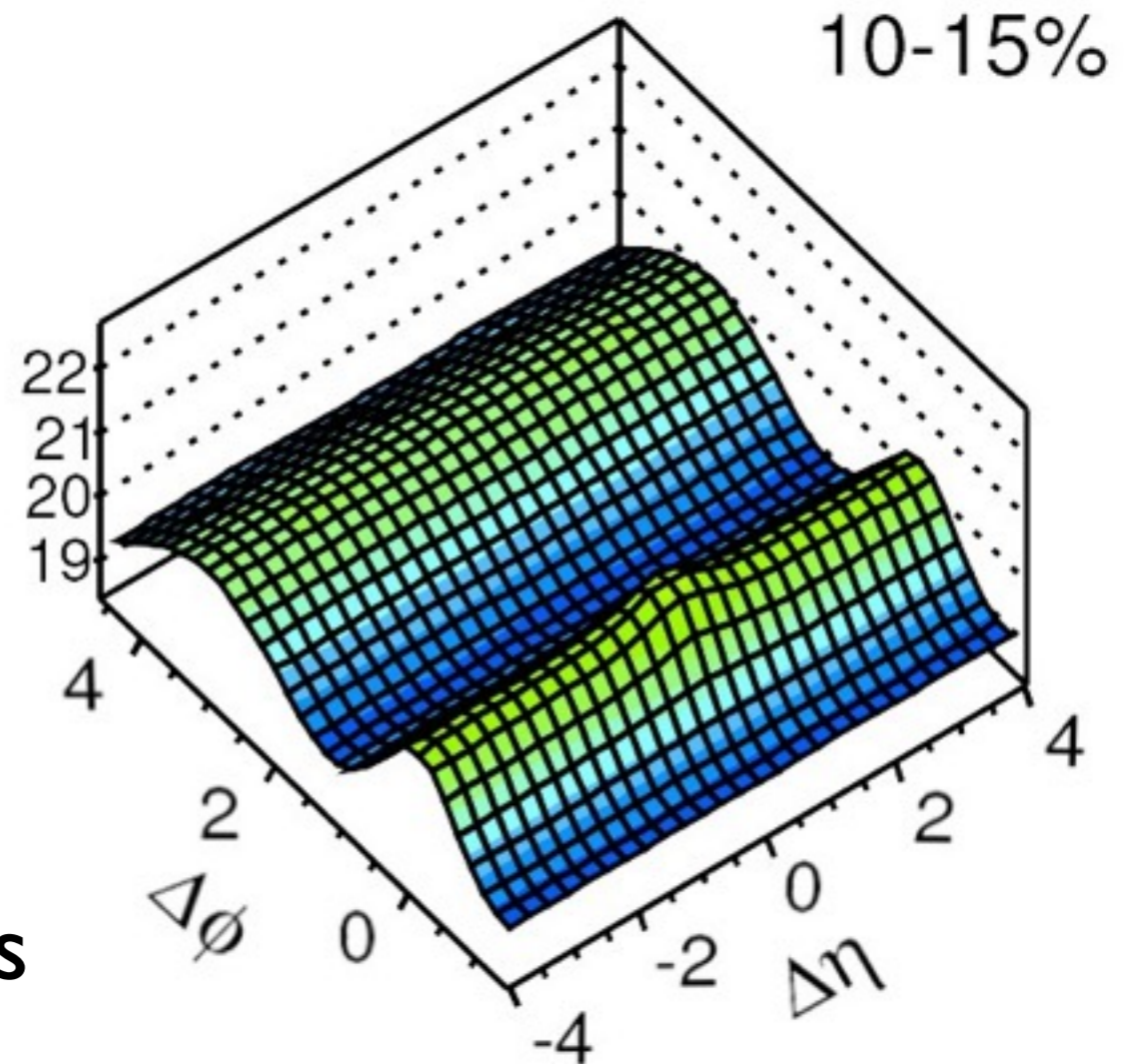
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# Correlations in Pb+Pb collisions



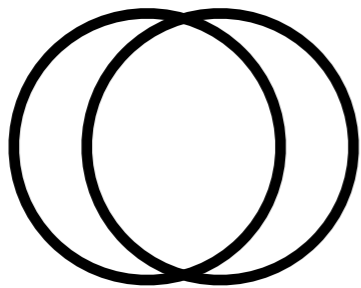
$$\frac{1}{N_{\text{trg}}} \frac{d^2 N^{\text{pair}}}{d\Delta\eta d\Delta\phi}$$

As the collision becomes less and less central, only the structure in  $\Delta\phi$  evolves.



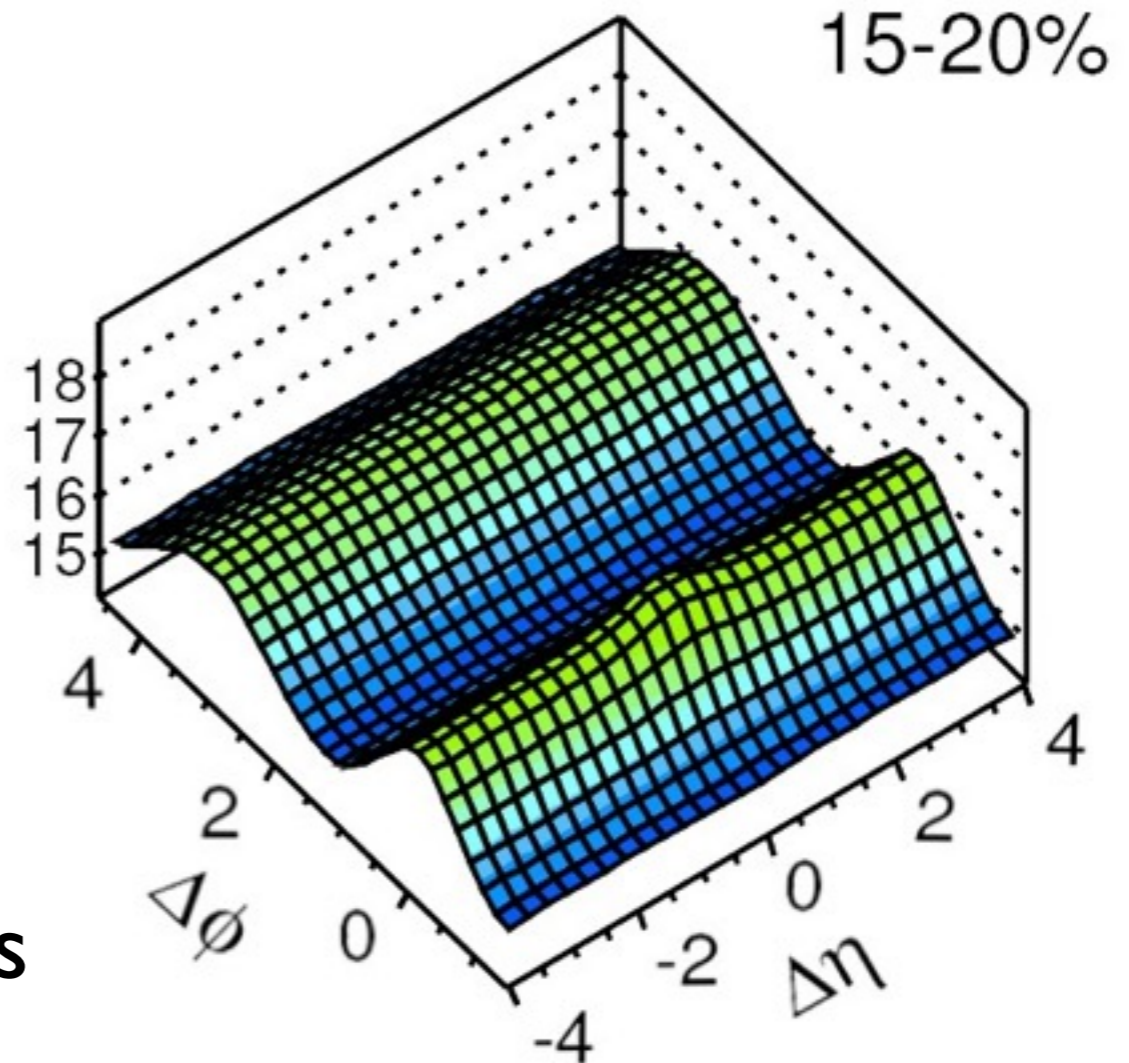
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# Correlations in Pb+Pb collisions



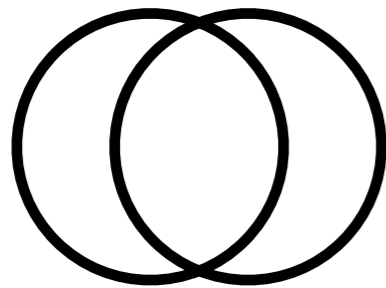
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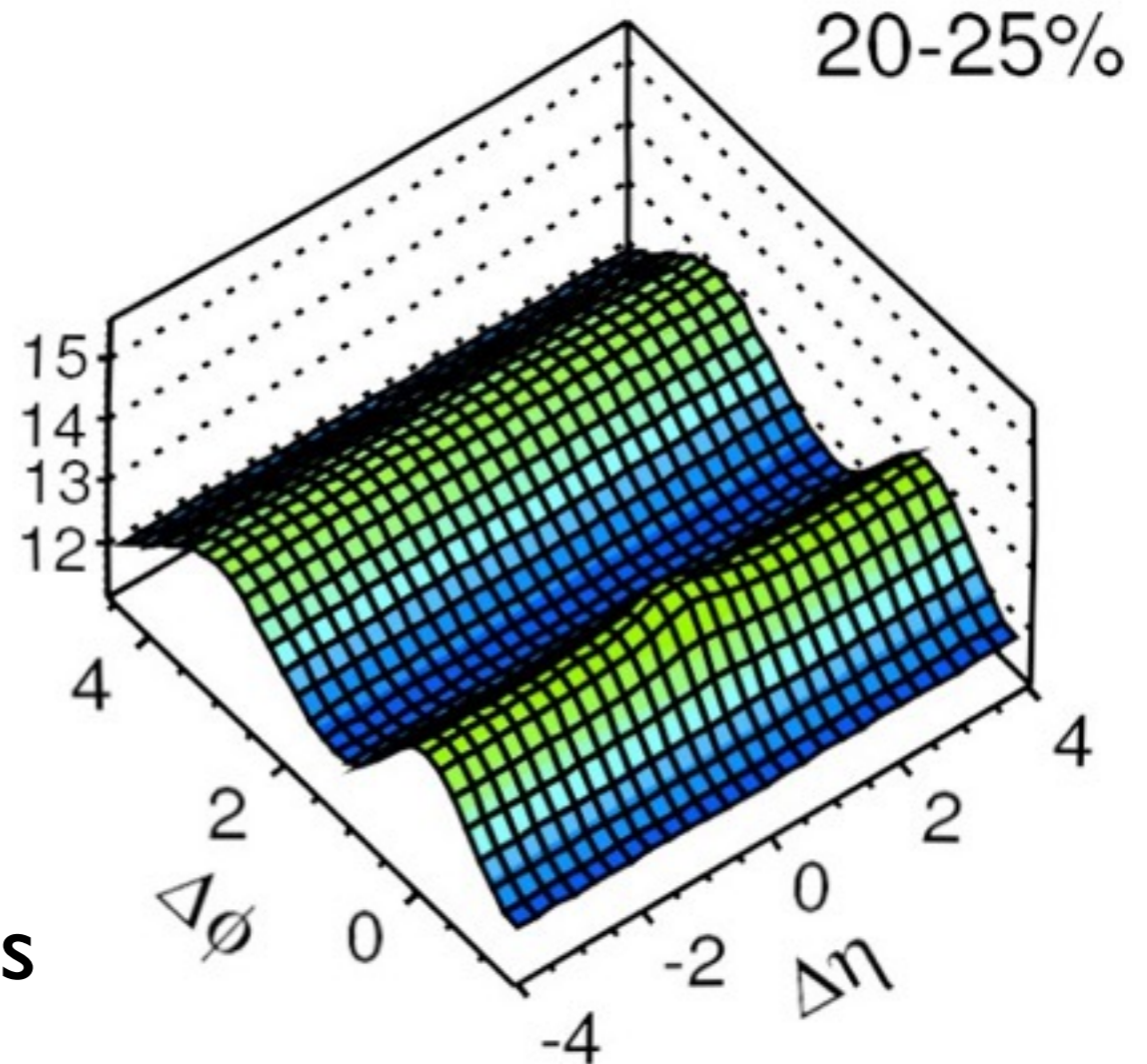


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# Correlations in Pb+Pb collisions



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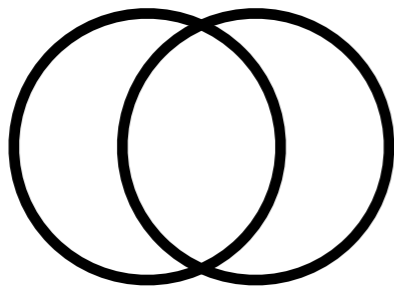


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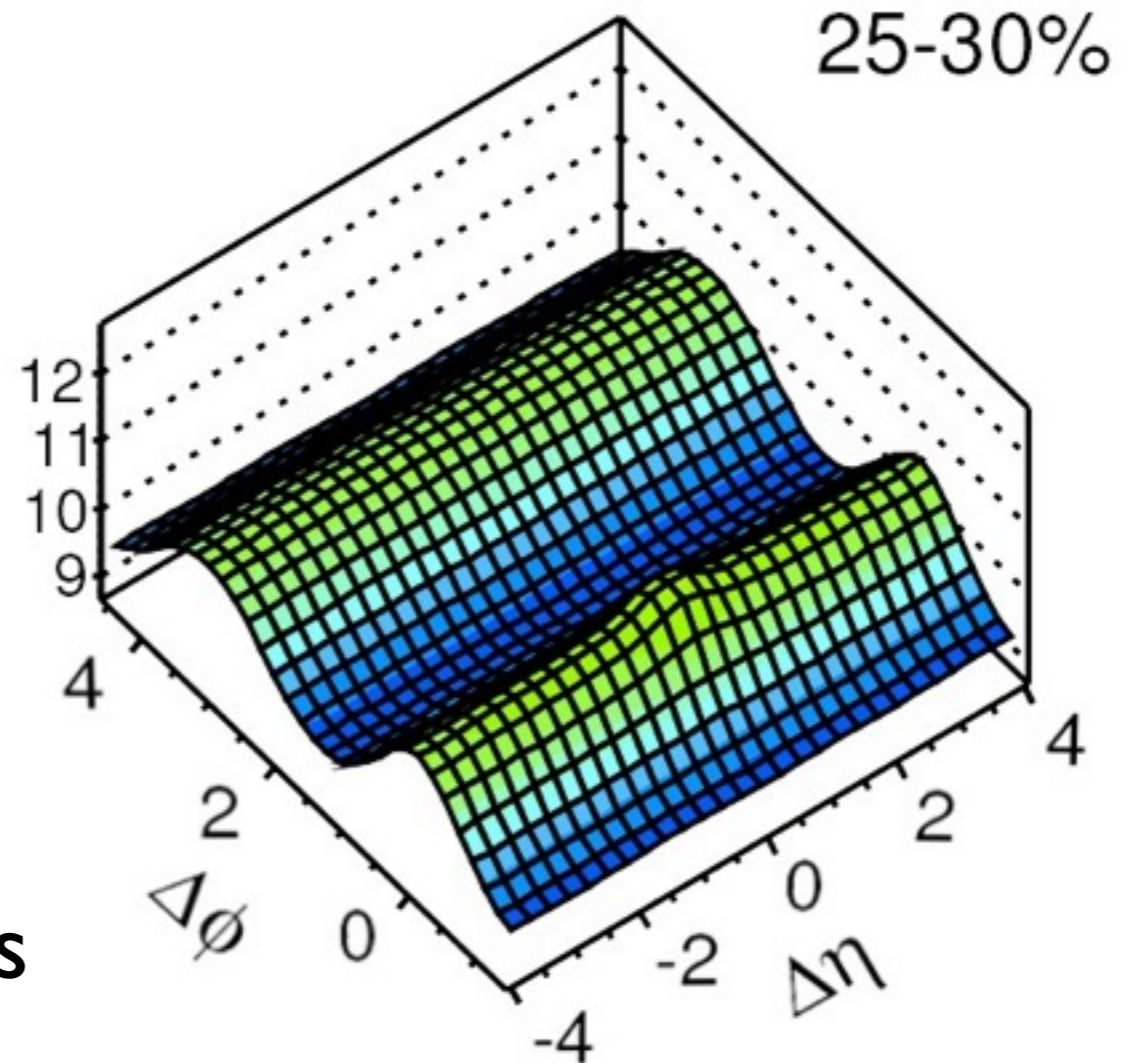


# Correlations in Pb+Pb collisions



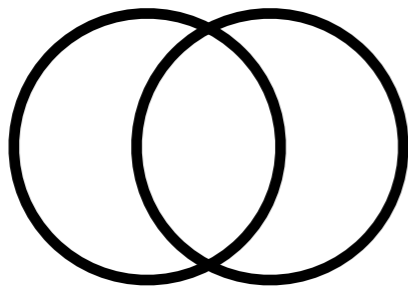
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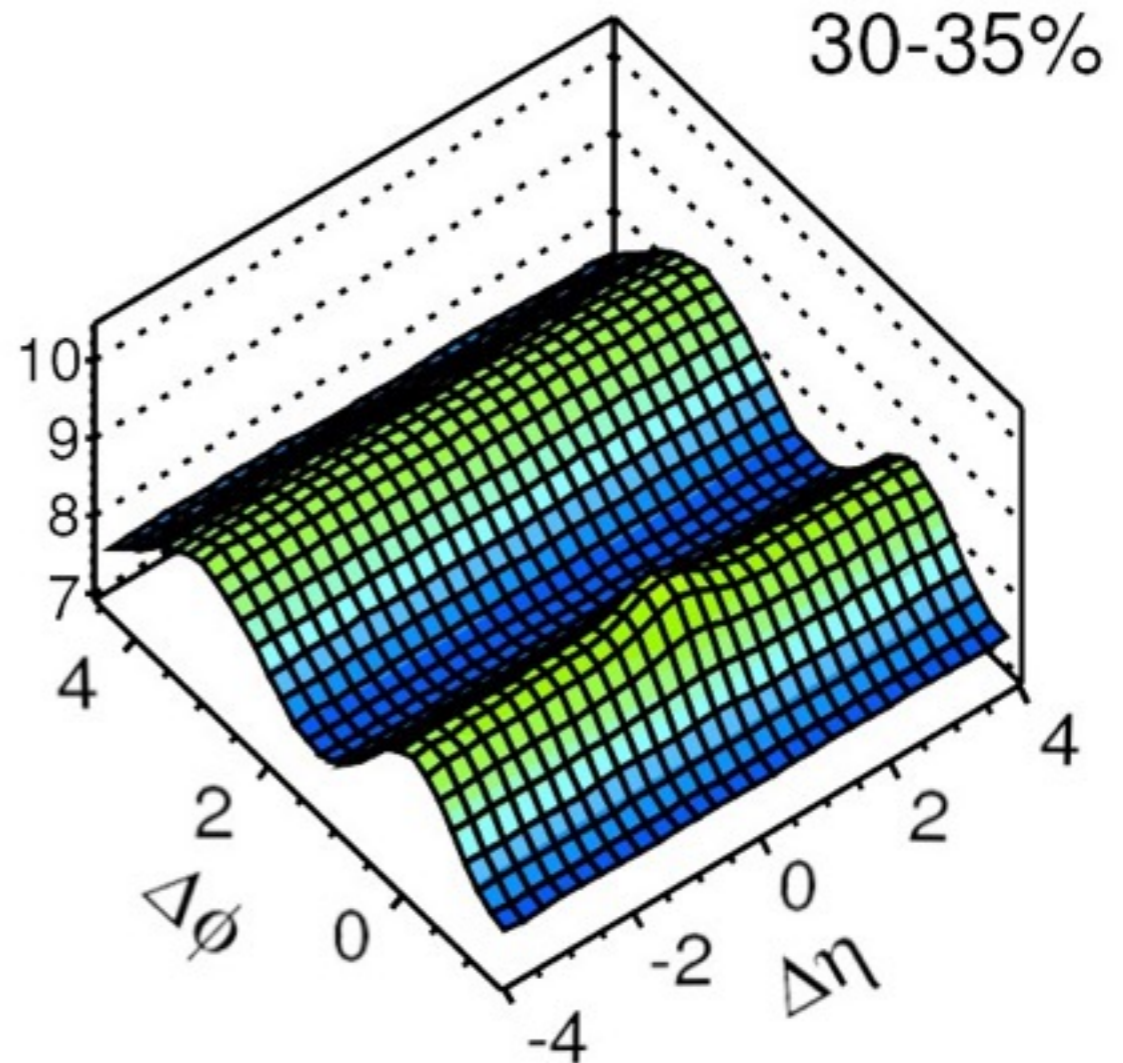


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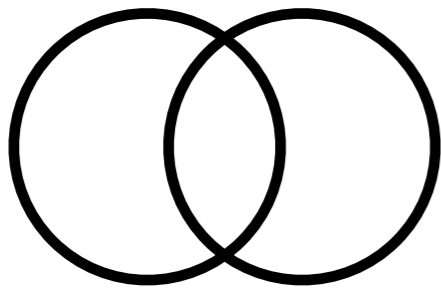


$$\frac{1}{N_{\text{trg}}} \frac{d^2 N^{\text{pair}}}{d\Delta\eta d\Delta\phi}$$

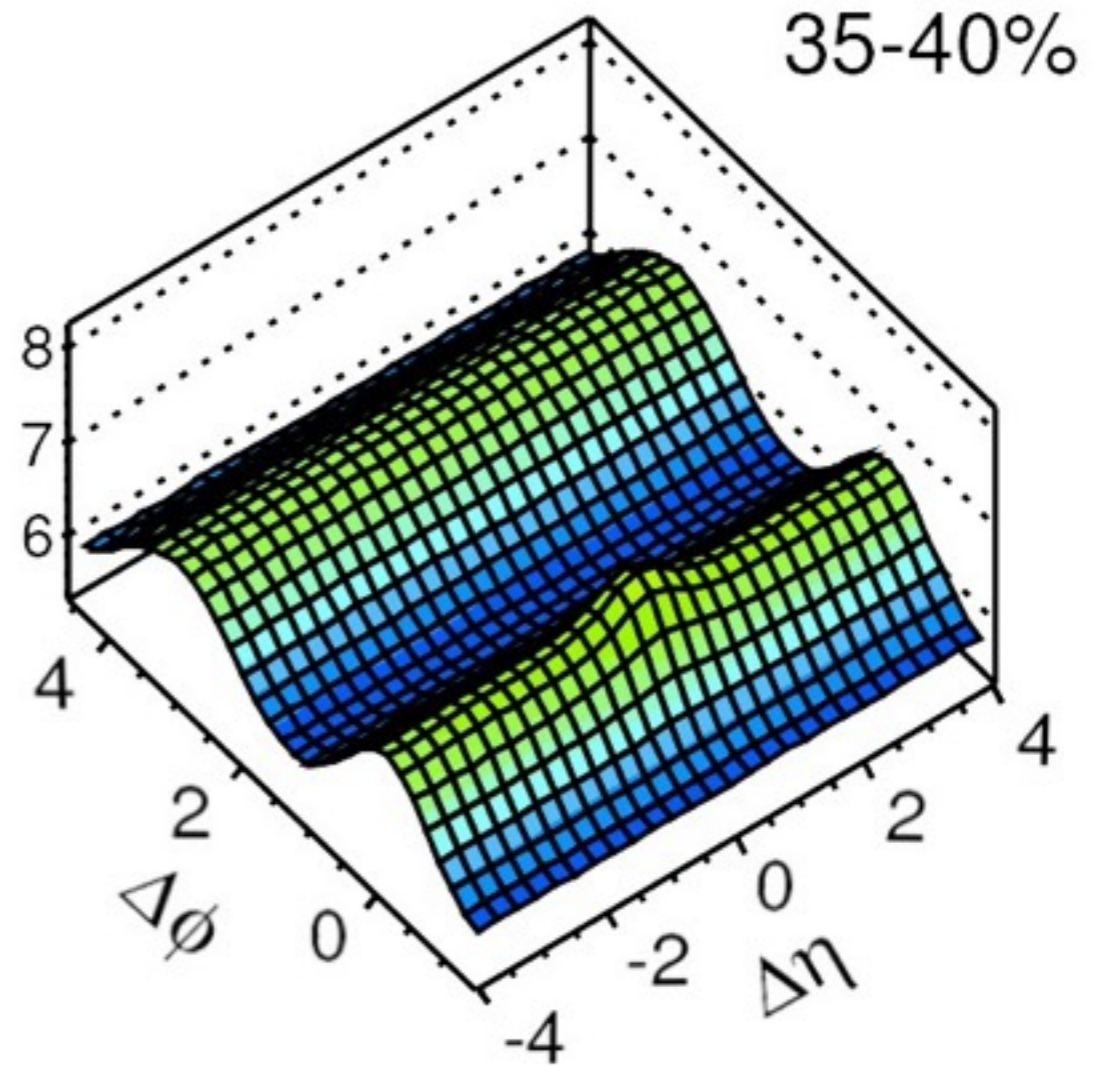


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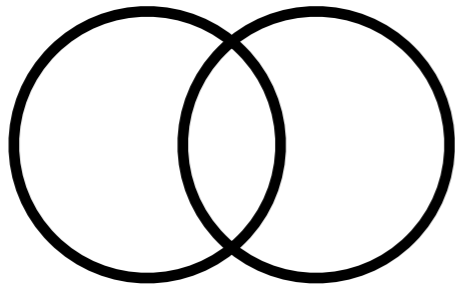


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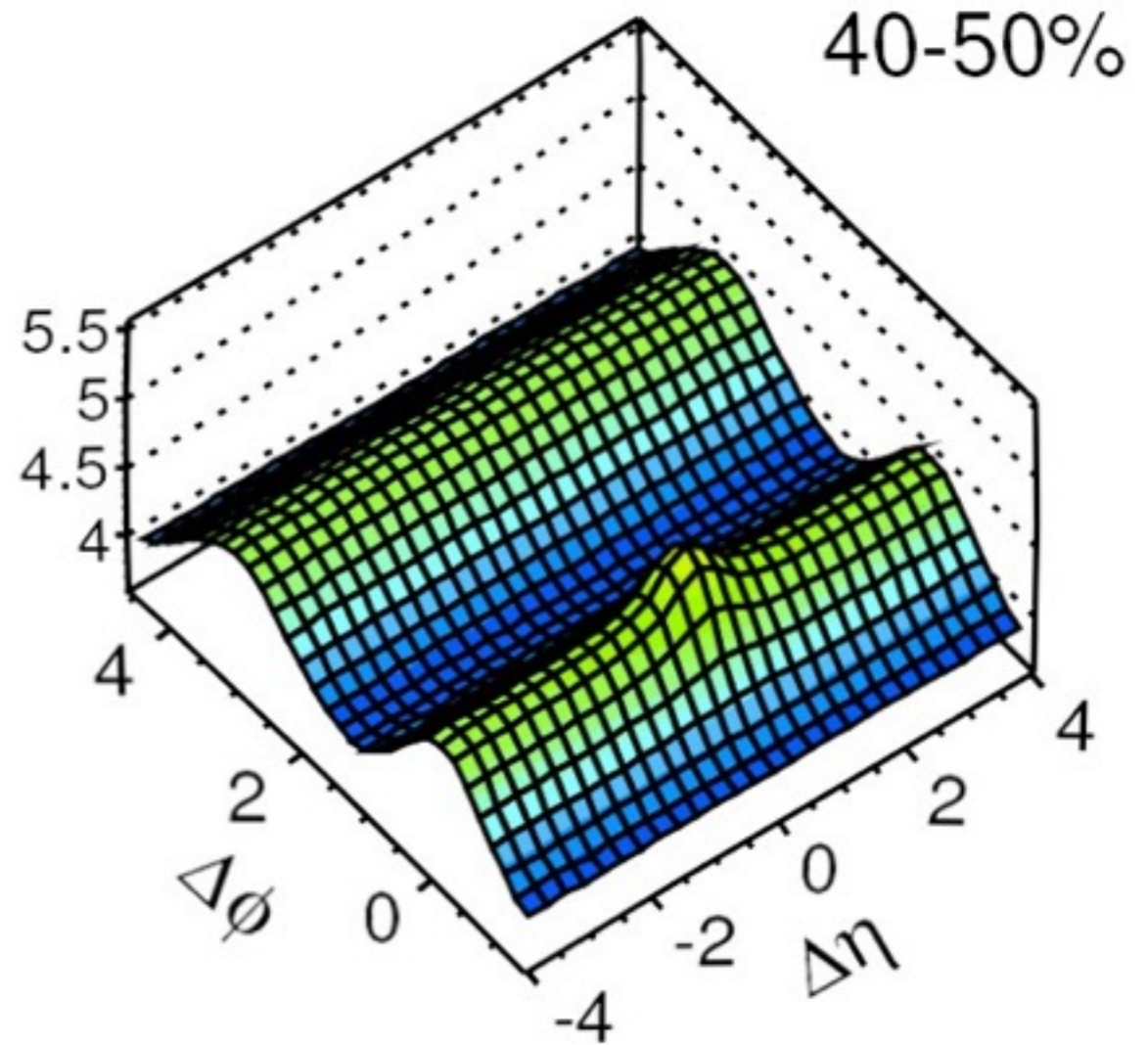


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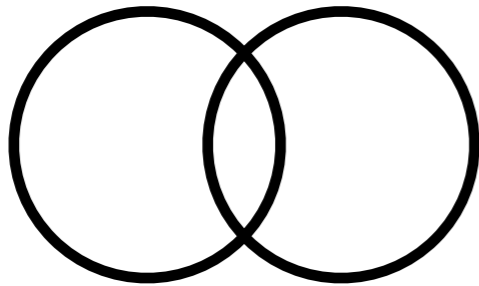


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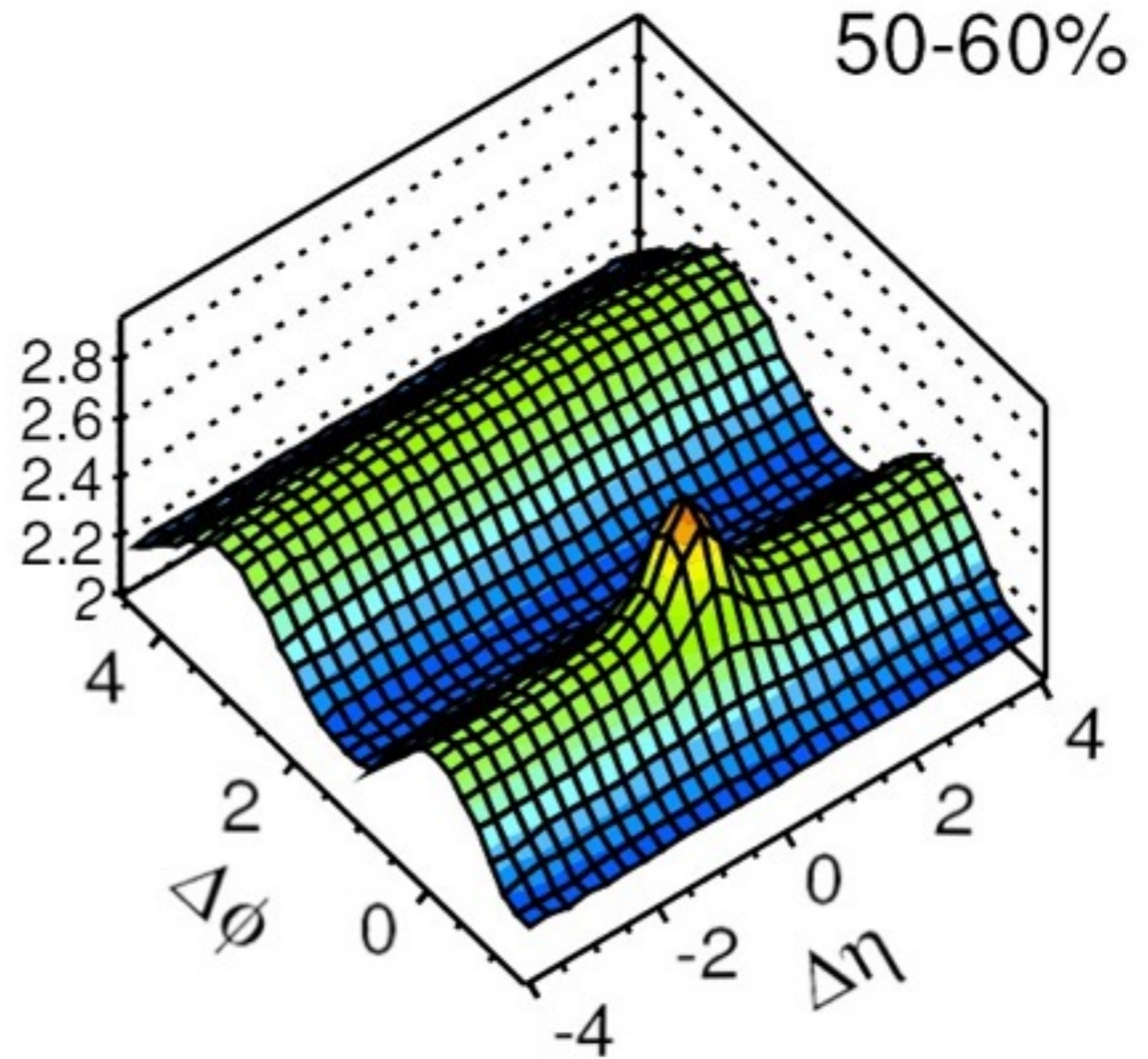


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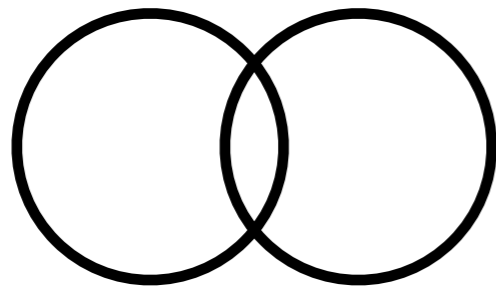


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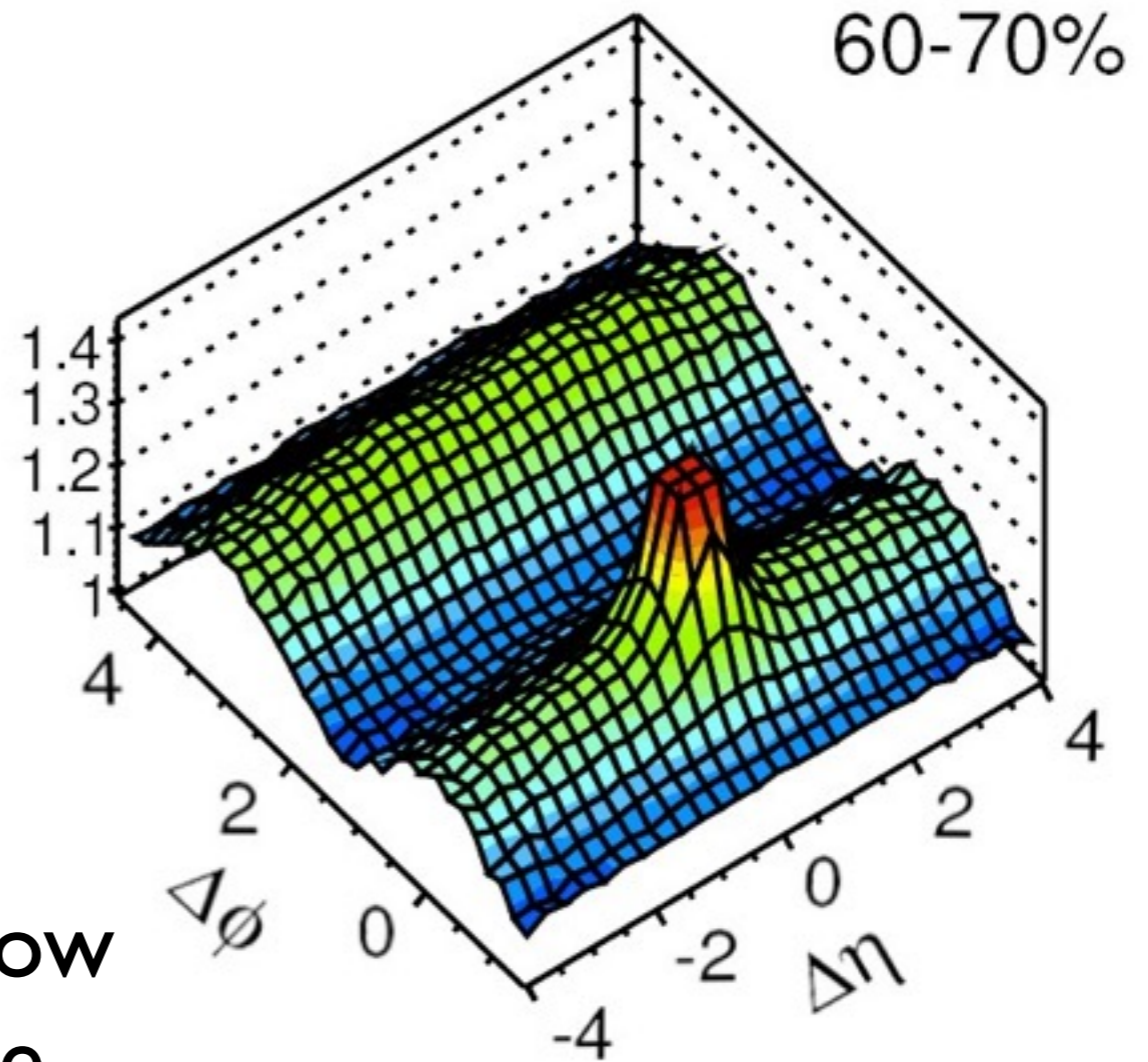


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# Correlations in Pb+Pb collisions



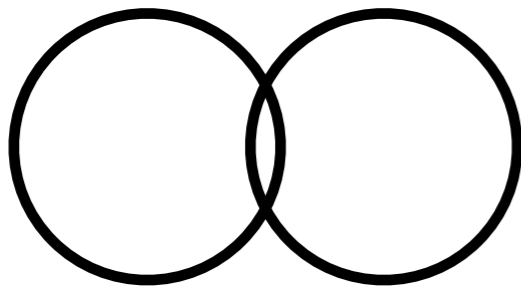
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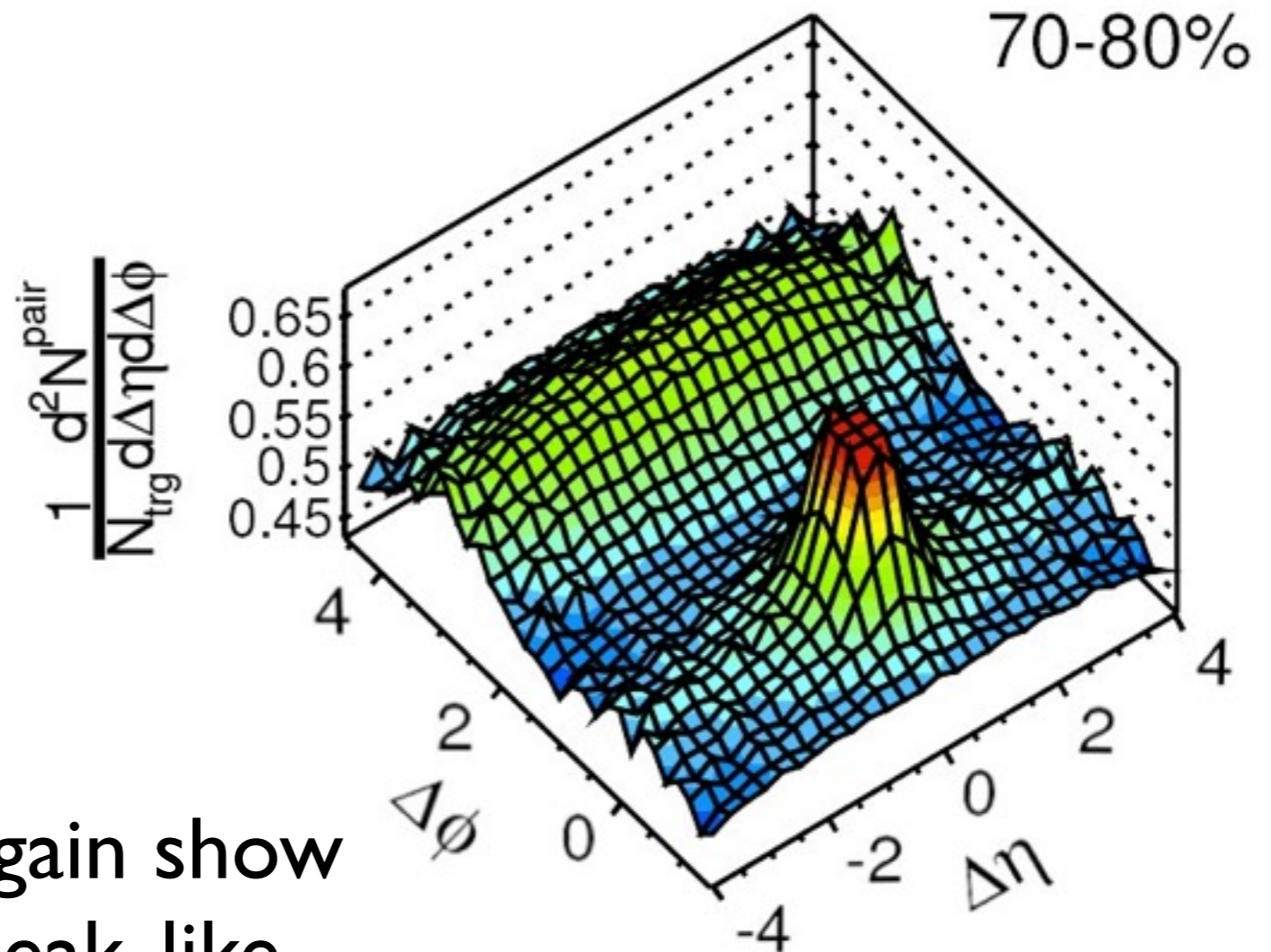
Peripheral collisions again show a large « near side » peak, like proton-proton collisions.

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# Correlations in Pb+Pb collisions



Peripheral collisions again show a large « near side » peak, like proton-proton collisions.



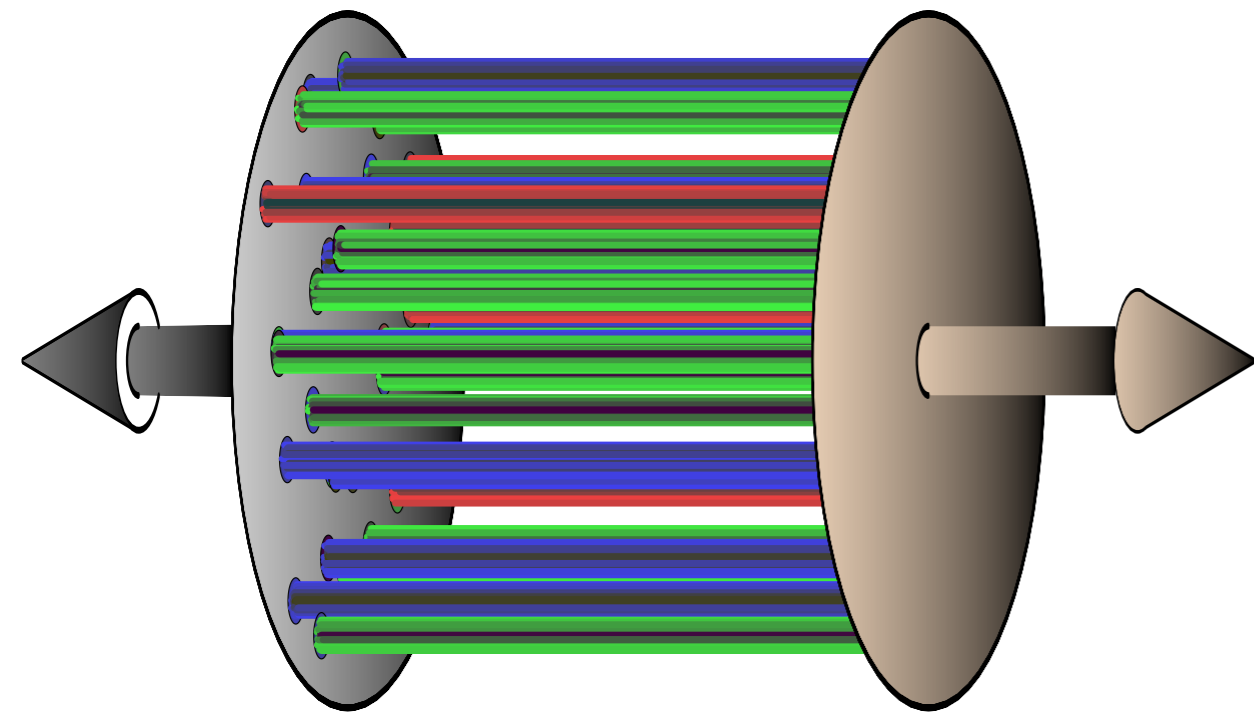
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Next:

why we understand these **waves** in  
measured pair correlations as the  
signature of a *little fluid*



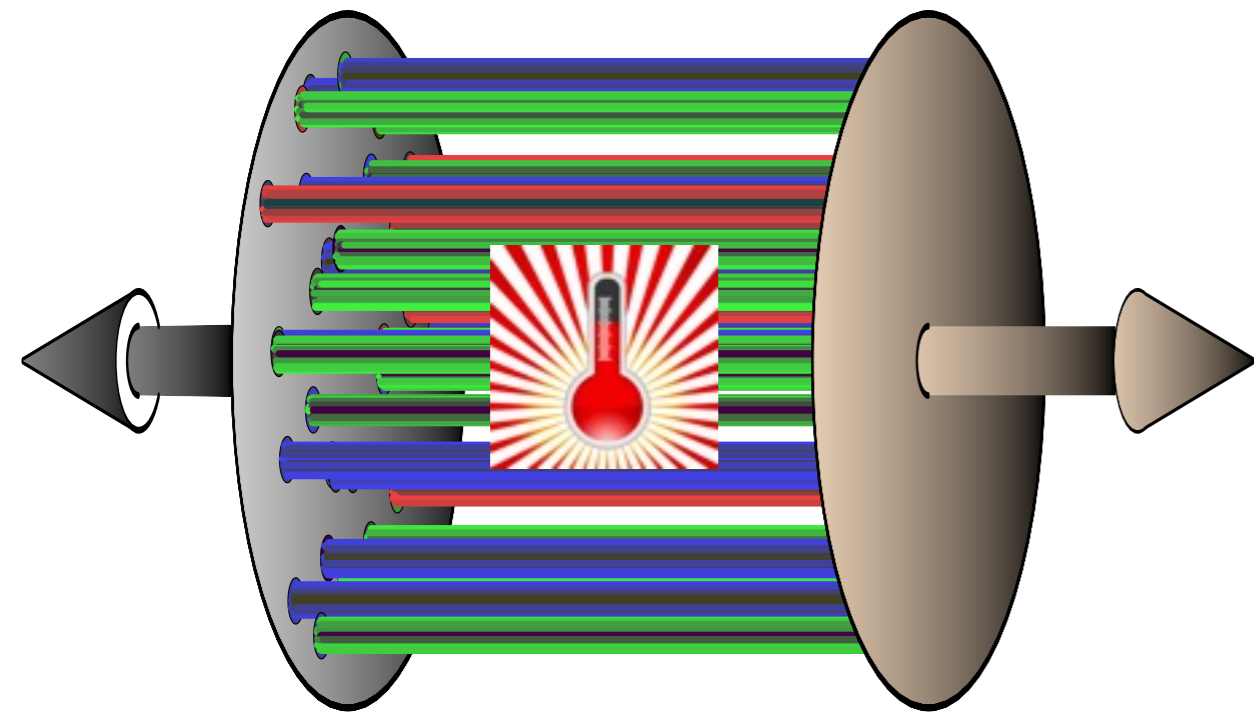
# Symmetries



Gelis 1110.1544

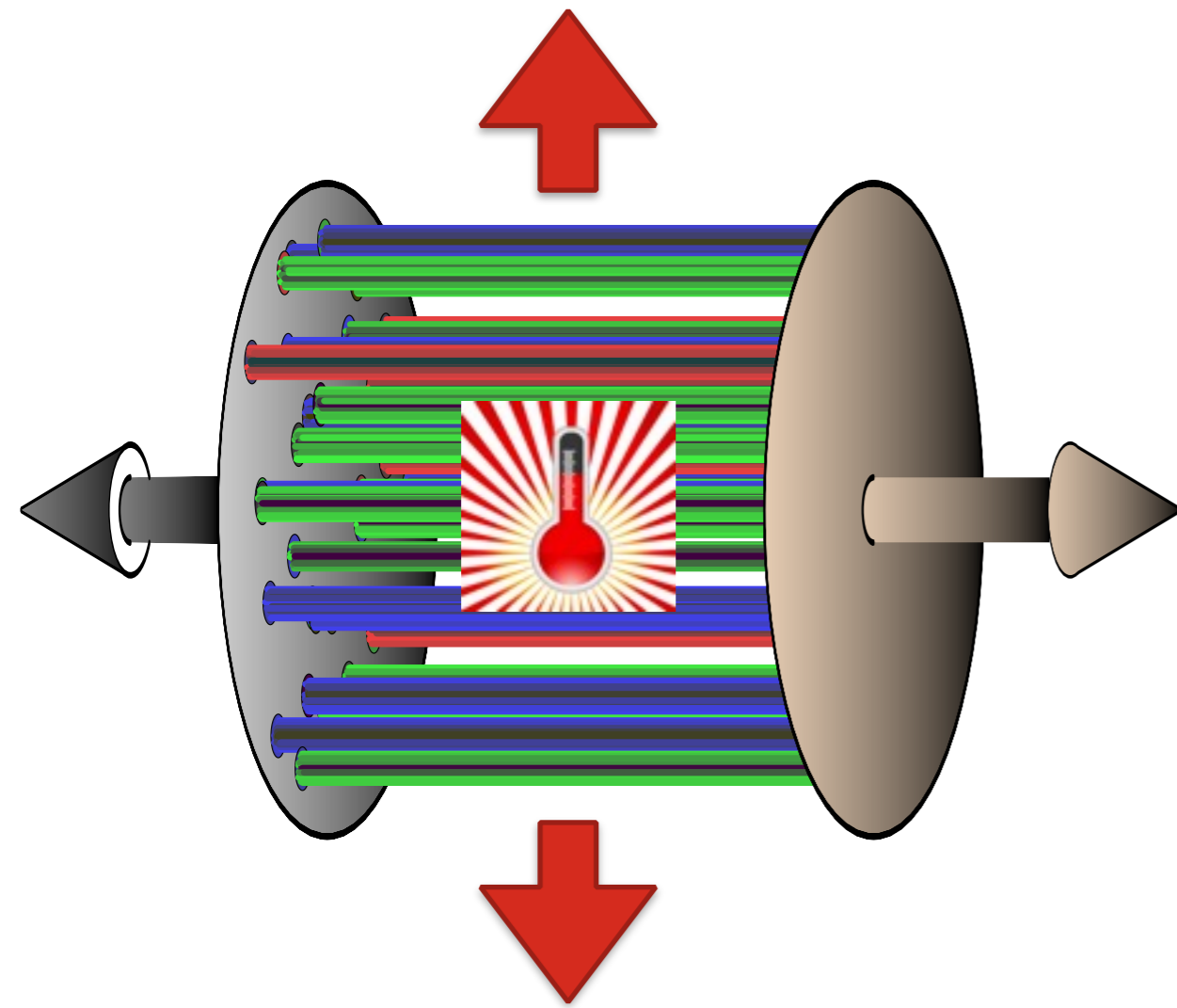
- Pb nucleus=208 nucleons
- **Lorentz contraction** projects the nuclear sphere on the transverse plane
- Elementary collisions deposit energy **at the transverse location** of the nucleons
- Thus the initial density profile is typically **uniform longitudinally**, but with a **bumpy transverse profile**

# Symmetries



Thermalization creates a **little fluid** with the **same symmetry** as the initial state: longitudinally invariant, with large transverse fluctuations.

# Symmetries

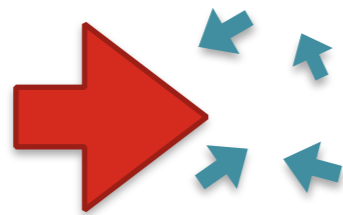


Transverse fluid velocity is generated by expansion into the vacuum (pressure gradient)

The fluid velocity depends on the azimuthal direction  $\varphi$

# Fluid to particles

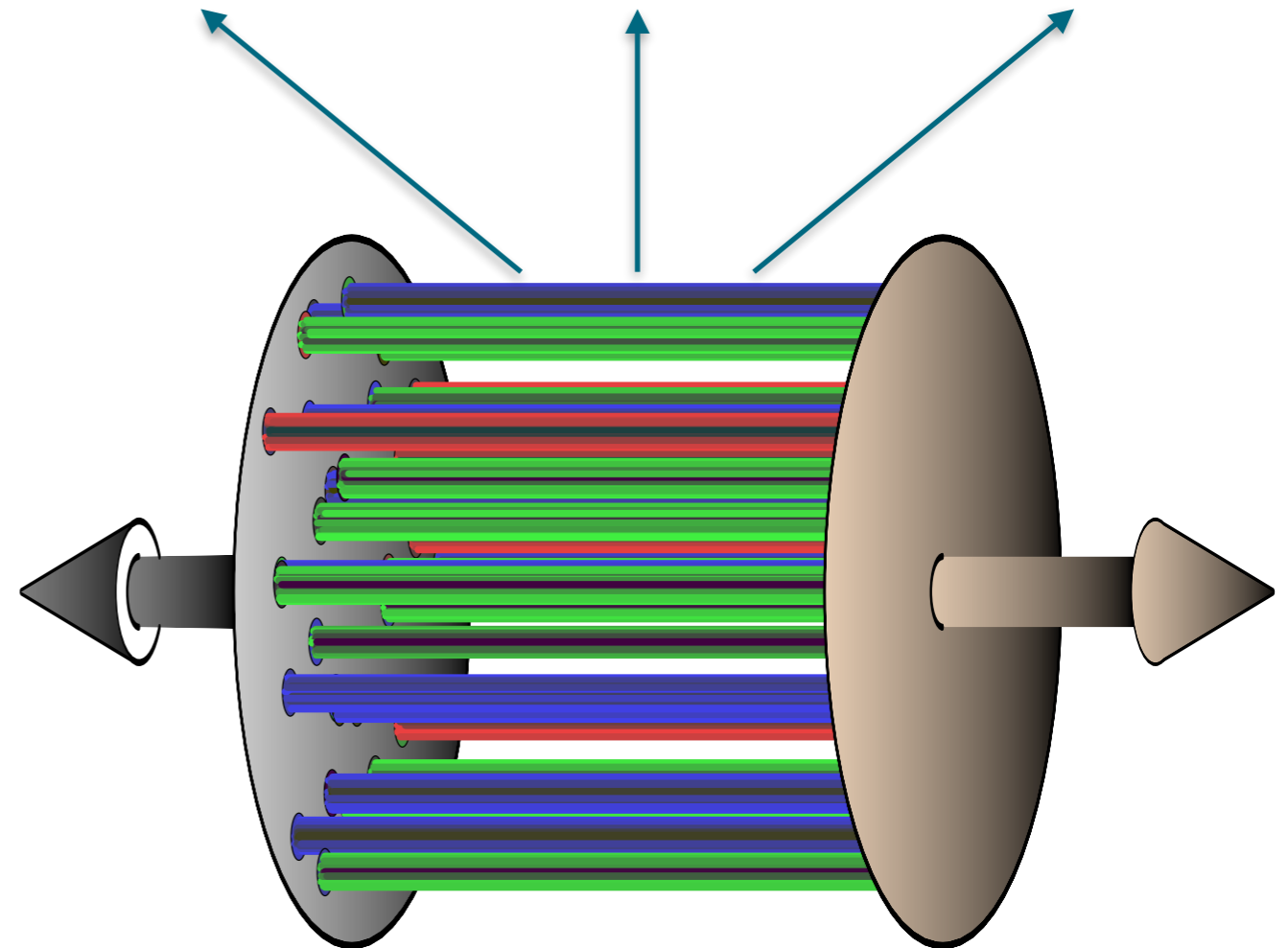
As the **fluid** cools down  
it becomes a gas of  
**independent** particles



Particle velocity =  
**fluid velocity** +  
(small) **thermal motion**

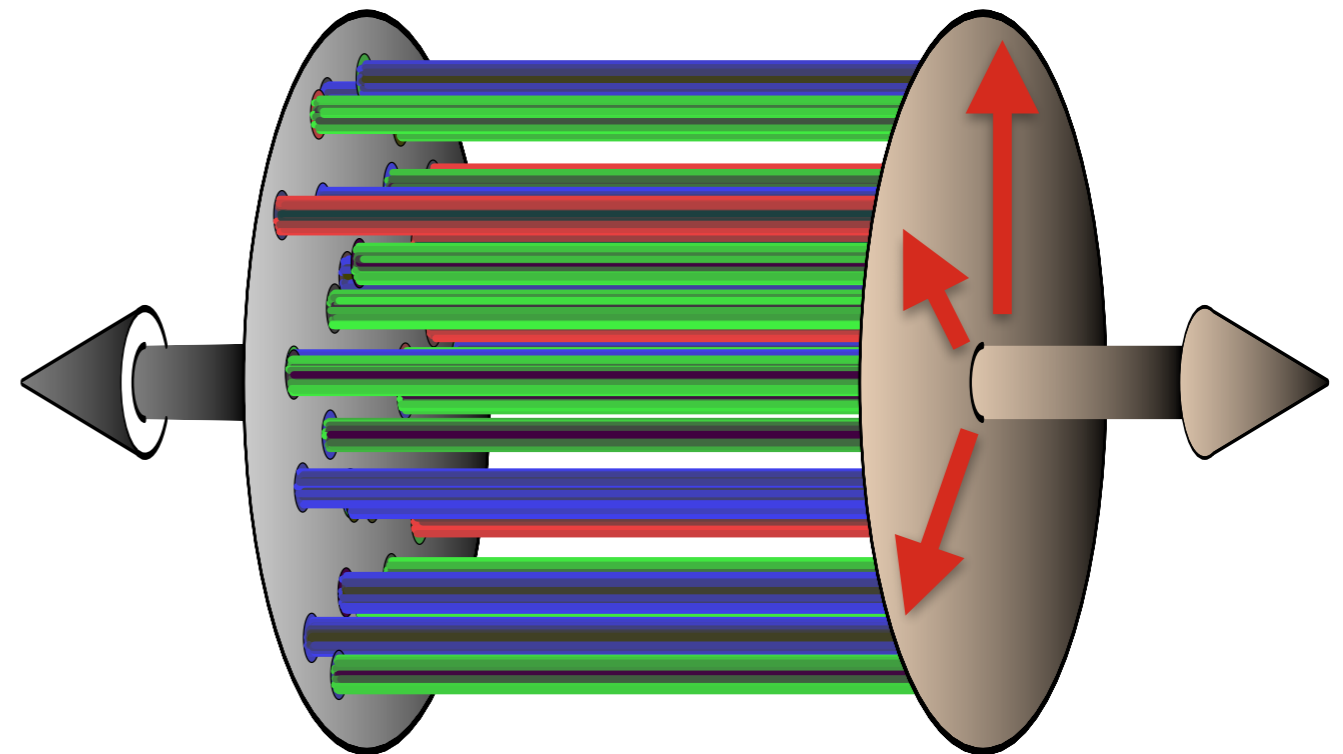
# Fluid to particles

The probability of emitting a particle is independent of the rapidity  $\eta$  (longitudinal invariance)



# Fluid to particles

But the probability depends on azimuthal angle  $\varphi$  (transverse fluctuations)

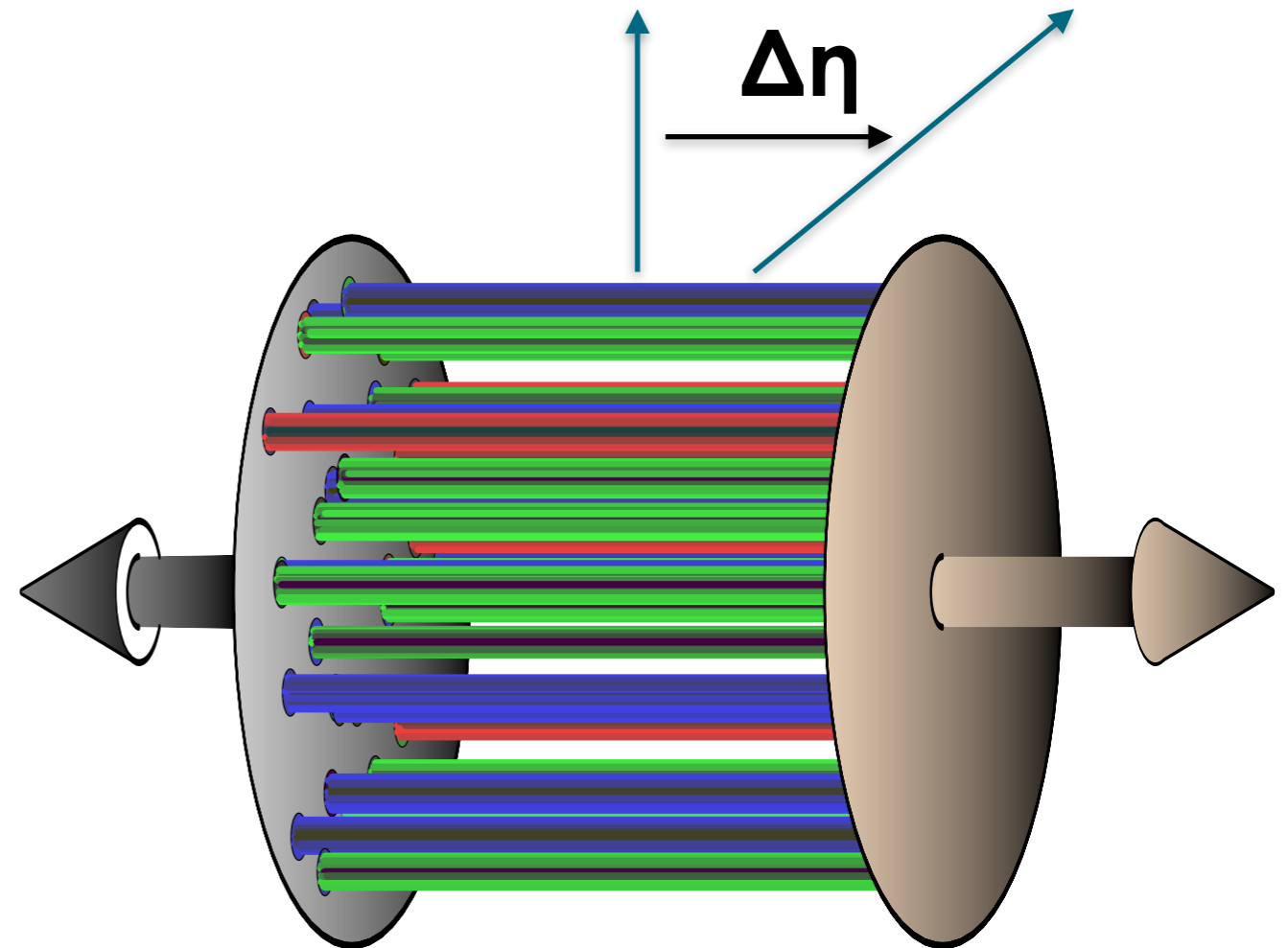


# Fluid to particles

Therefore the number of pairs is independent of the relative rapidity

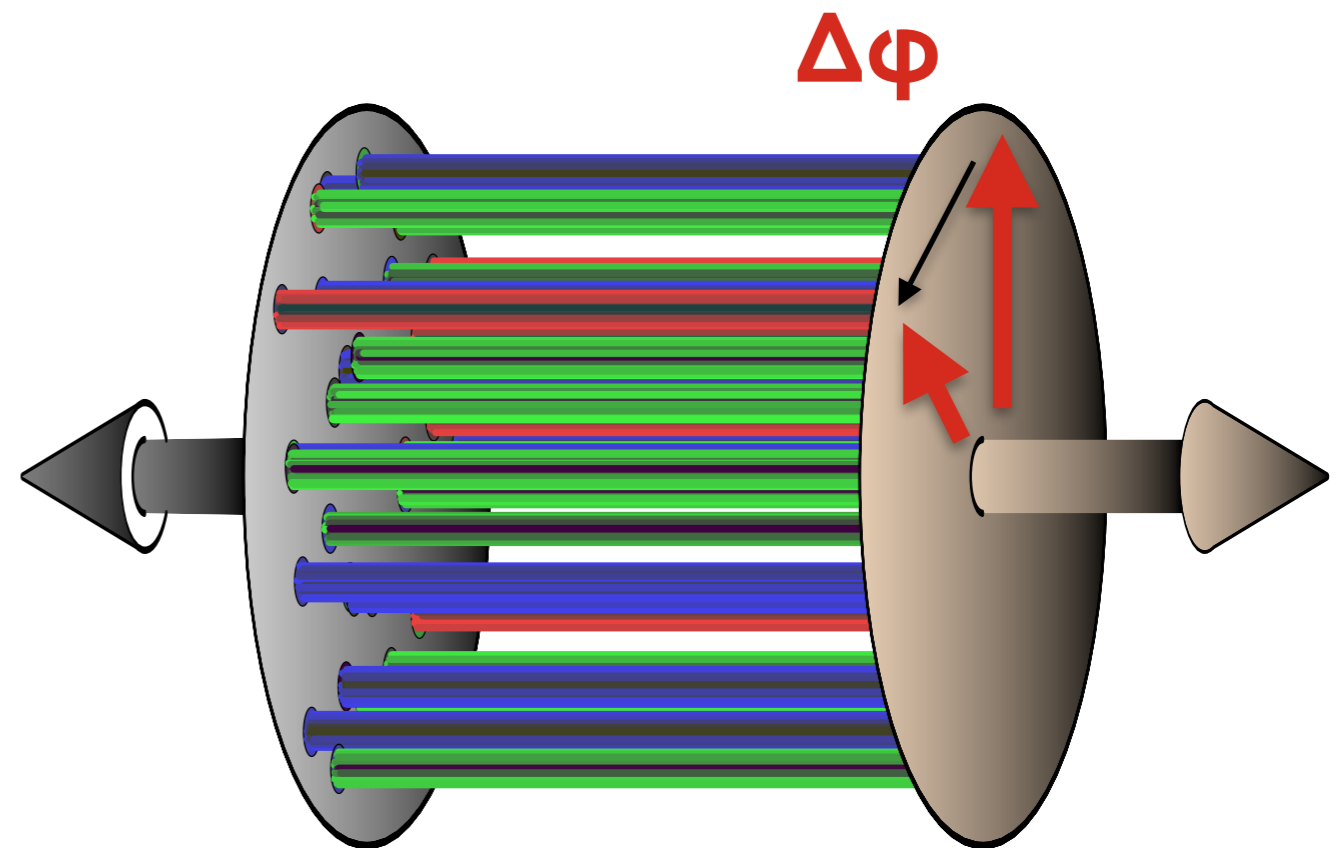
$\Delta\eta$

(longitudinal invariance)



# Fluid to particles

But depends on the relative azimuthal angle  $\Delta\phi$





I would like to know the current understanding of the long-range rapidity correlations observed at RHIC and LHC (the « ridge » structure) :

- Is it a flow effect described by the hydrodynamics?
- Or, is it an initial state physics described by the CGC + pQCD framework?
- Or, is it a remnant of flux tube structures just after a collision (so called ``glasma"?)

- A little bit of everything:
- Flux tube structures in the initial state explain why it is flat in  $\Delta\eta$
- Transverse flow explains why it is not flat in  $\Delta\phi$

# The *flow hypothesis*

- Particles are emitted **independently**, with an *underlying probability distribution*  $P(\varphi)$  that is not isotropic in  $\varphi$ , and is different in every event.
- Fourier decomposition :  $P(\varphi) = \sum_n V_n e^{-in\varphi}$
- $v_n \equiv |V_n| = \text{anisotropic flow}$  [phase of  $V_n \equiv \psi_n = \text{event plane}$ ]  
 $v_2 \equiv \text{elliptic flow}$   
 $v_3 \equiv \text{triangular flow...}$
- Caveat: what we see is a *finite sample*, too small to measure  $v_n$  event by event.

It would be nice to learn more about a quadrupole moment evolution through a collision and similarly about further multipole moments. What is known from theoretical and experimental sides?

- Anisotropic flow,  $v_n$ , is the multipole moment.
- Experimentally, one only sees outgoing particles, therefore one cannot follow the evolution of  $v_n$  through the collision
- In a model, one can follow how  $v_n$  develops in time.

As an experimentalist, it sometimes seems to be that every theoretical model, especially surrounding QGP, can easily be tuned to match our data. What type of combination of measurements will be necessary to properly disprove for instance some of theories surrounding flow

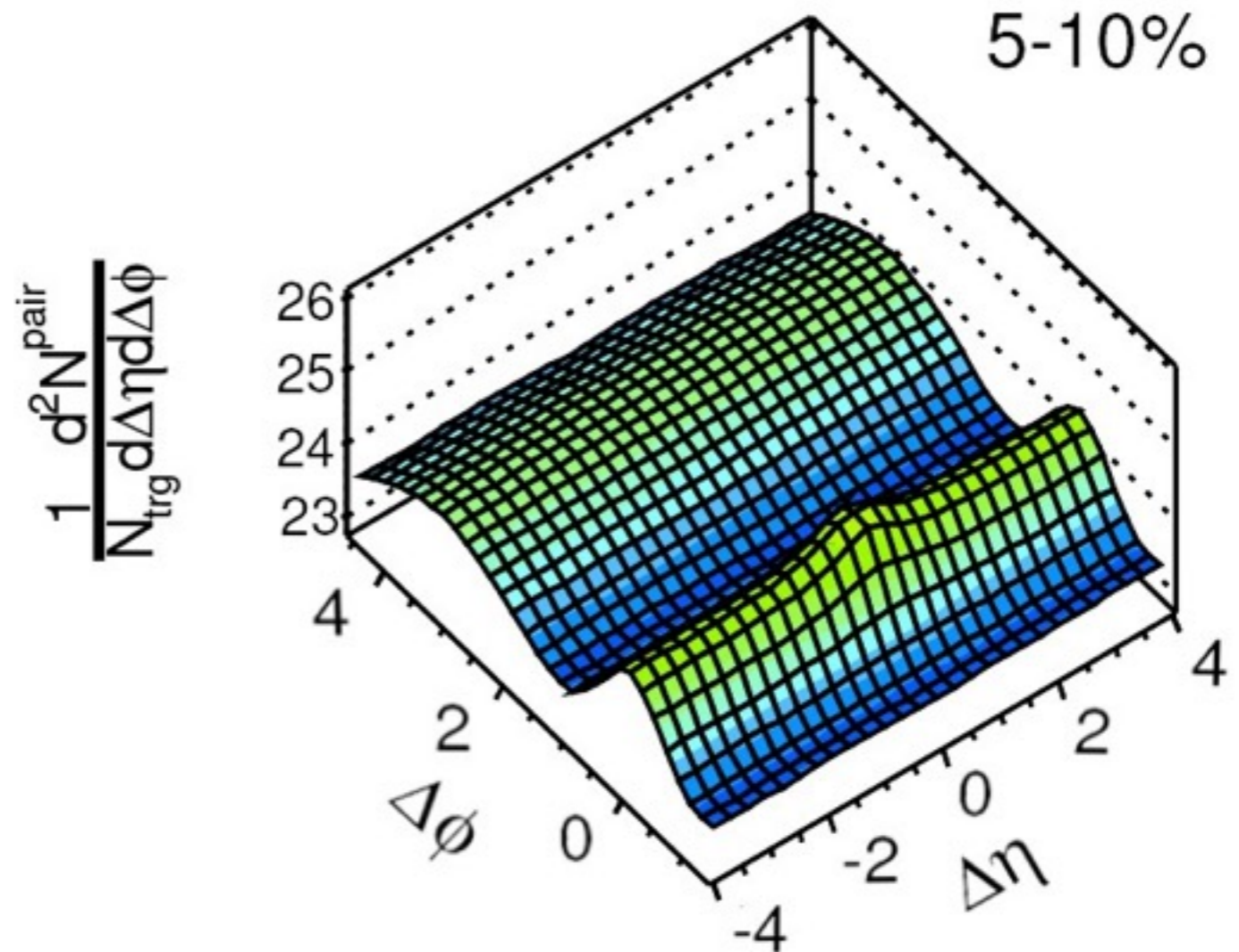
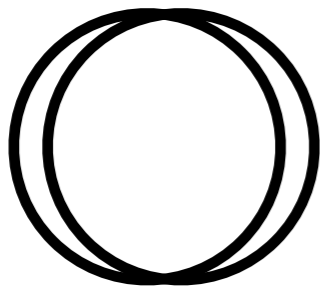
The « flow hypothesis » by itself, despite its simplicity, has an impressive predictive power and can easily be disproved.

# What the *flow hypothesis* implies

- Independence  $\rightarrow$  pair distribution in an event
$$N_{\text{pairs}}(\varphi_1, \varphi_2) = P(\varphi_1)P(\varphi_2)$$
- After  $\langle$ averaging over events $\rangle$ 
$$N_{\text{pairs}}(\Delta\varphi) = \sum_n \langle v_n^2 \rangle \cos(n\Delta\varphi)$$
- Fourier coefficients of the pair distribution are all **positive**. A non-trivial prediction which can be readily verified on data.
- Implies in particular : *absolute maximum at  $\Delta\varphi=0$*

# Flow hypothesis versus data

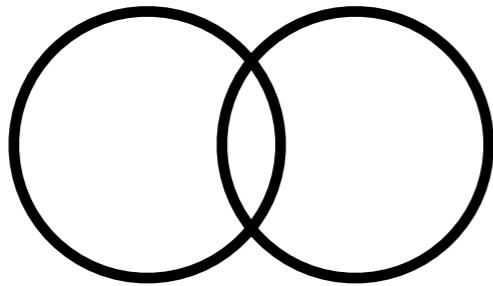
Central Pb+Pb



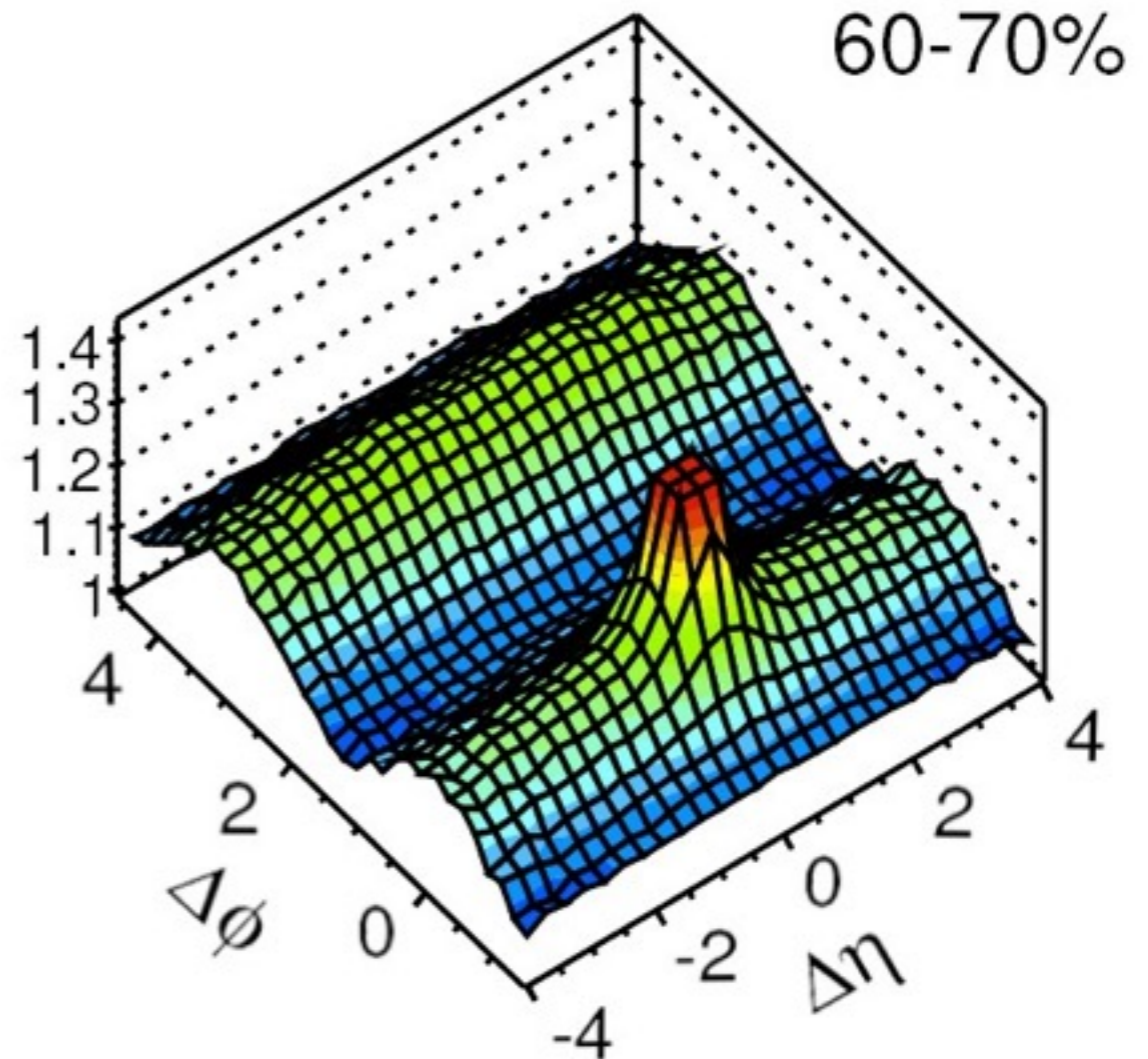
OK

# Flow hypothesis versus data

*Peripheral Pb+Pb*



$$\frac{1}{N_{\text{trg}}} \frac{d^2 N^{\text{pair}}}{d\Delta\eta d\Delta\phi}$$

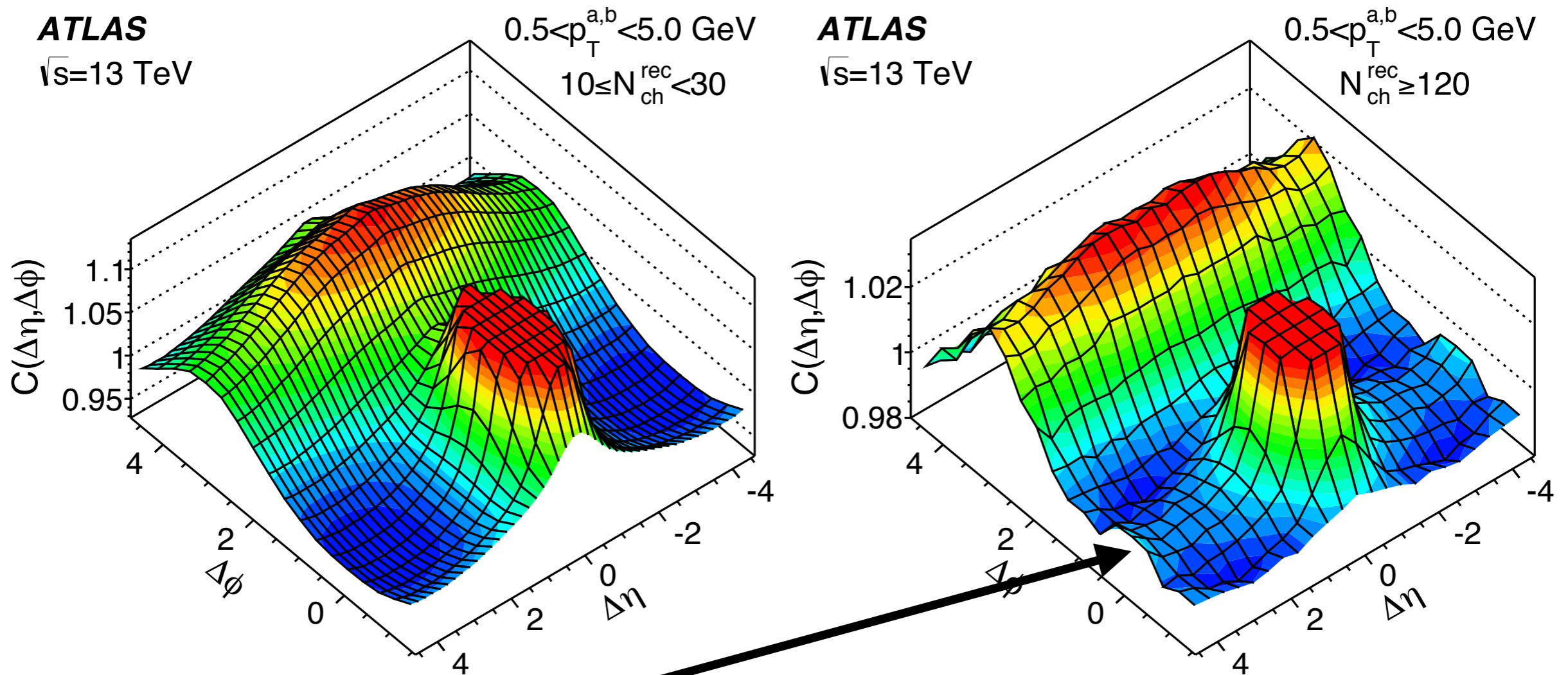


**Not just flow**

# Flow hypothesis versus data

*proton-proton collisions at LHC2*

ATLAS 1509.04776



A ridge is observed, which could have the same origin as in Pb+Pb.  
But flow here is at best a small correction.



# Measures of anisotropic flow

What are the relations among different definitions of  $v_2$  (and  $v_n$ )? And why different experiments don't use the same method to evaluate  $v_2$  ?

- $v_n$  cannot be measured in every event. One can only measure its statistical properties.
- There is only *one* definition of  $v_n$  but one can measure *several* statistical properties - typically *moments*.
- The confusion is due to the fact that all methods currently in use were devised before it was realized that  $v_n$  fluctuates

# Measures of anisotropic flow

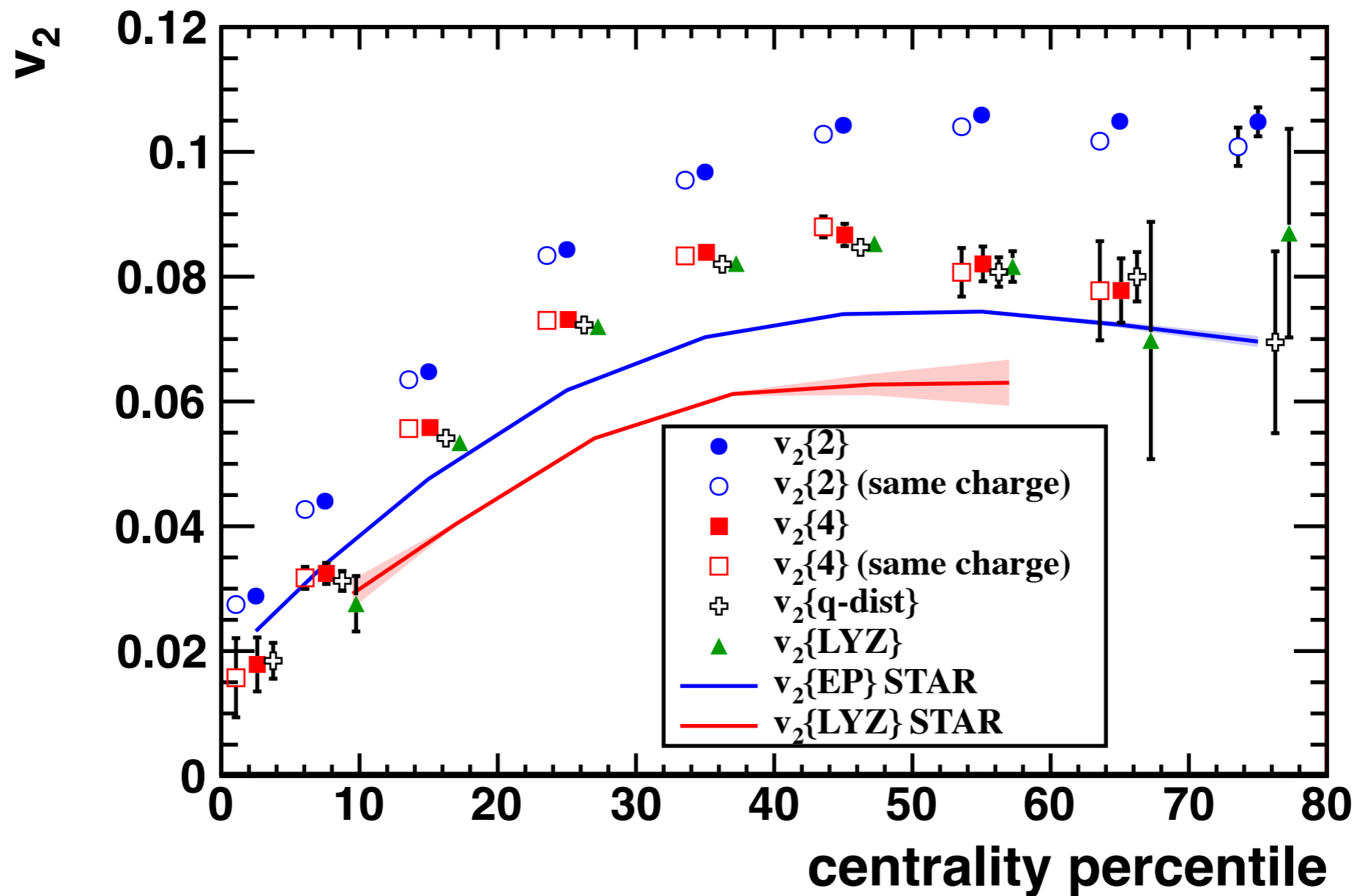
- We have seen that 2-particle **correlations** measure  $\langle v_n^2 \rangle$ . *By far the most common measure.*
- Similarly, a 4-particle correlation measures  $\langle v_n^4 \rangle$
- For historical reasons, experiments do not measure **moments** but **cumulants**

$$v_n\{2\} = (\langle v_n^2 \rangle)^{1/2}$$

$$v_n\{4\} = (2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle)^{1/4}$$

- $v_n\{2\} = v_n\{4\} = v_n$  if  $v_n$  does not fluctuate
- $v_n\{4\} < v_n\{2\}$  if  $v_n$  fluctuates

# Illustration: first measurement of elliptic flow at LHC



ALICE 1011.3916

# More detailed flow analyses

- Both the magnitude **and phase** of anisotropic flow depend on transverse momentum  $p_t$ , and even on pseudorapidity  $\eta$  even though this is a small effect.
- Measurements of  $v_n(p_t, \eta)$  are often presented, which involve averaging over one of the particles in the pair. Their relation to the fluctuating  $v_n$  defined above is not simple — in fact, not known to my knowledge.

# Hydrodynamic modeling: 3 steps

- **Initial conditions:** typically a model of the transverse density profile right after the collision+thermalization.
- **Relativistic fluid dynamics** describes the subsequent **expansion into the vacuum**
- The fluid « freezes » to independent **particles**.  
*in particular, one calculates the full probability distribution  $P(\varphi)$ .*

*What do we learn from fluid dynamics, beyond the « flow hypothesis » (independent-particle emission) ?*

What is the current state of knowledge regarding rescattering phase of heavy ion collisions? In particular, what do we know about hadronic interactions, especially between baryons and antibaryons? It is argued that e.g. antibaryon-baryon annihilation is important for describing particle yields, but is there a theoretical description for such interactions?

- Hadronic interactions are important in hydrodynamics because they determine the **viscous correction** to particle distributions at freeze-out, aka  $\delta f$ , which is unknown.
- This is one of the major sources of uncertainty in hydro: in particular, when making predictions for identified particles.

# Ideal fluid dynamics

- **Pressure** accelerates the fluid :

$$\rho \, dv/dt = -\nabla P$$

- The pressure **P** is related to the density  $\rho$  through the equation of state (EoS)
- By dimensional analysis, the **expansion time** is of the order of the **transverse radius R**.
- Note that flow probes the system at much shorter times in  $p+P_b$  than in  $P_b+P_b$

I'm not sure this is a proper question. Maybe many groups use lattice QCD results for EOS to run the Hydrodynamics calculation in AA collisions. On the other hand, is it possible to determine EOS directly by comparing the Hydrodynamic calculations with data at RHIC and the LHC?

- Yes, many groups use EOS from Lattice QCD.
- What you propose is an excellent research project.
- The equation of state relates the **temperature** to the **density**, that is, the **energy per particle (equivalently, pt spectra)** to the **number of particles**.
- For dimensional reasons, we are probing this relation at a time  $t \sim R$



# Viscous fluid dynamics

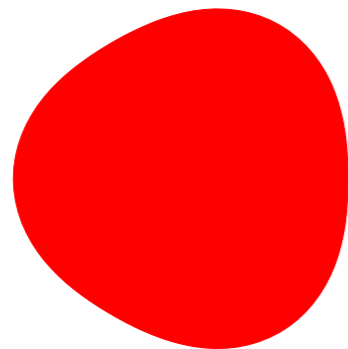
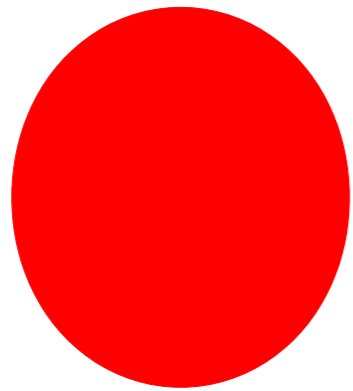
- **Viscosity** slows down the expansion
- Gradient expansion:  
$$\rho \, dv/dt = -\nabla P + \eta \Delta v + \ll 2\text{nd order terms} \gg$$
$$1/R + 1/R^2 + 1/R^3 + \dots$$
- Hydrodynamics applies only if the expansion converges, i.e., viscous terms are small.

Hydrodynamical simulations develop numerical instabilities, often in the external regions of low energy density that surround the dense collisions region but also in the presence of large gradients....my impression is that numerical schemes tend to fail in unphysical situation where hydrodynamics should not even be applied (low energy density/large gradients). Is there a rationale to solving hydrodynamics in region where it should not apply?

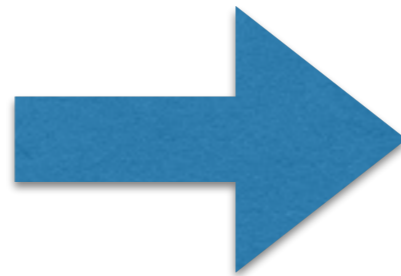
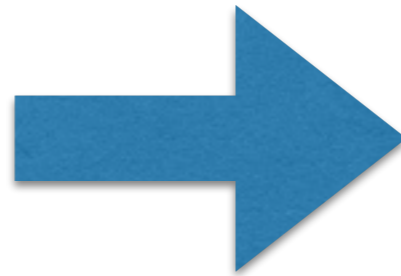
Not that I know of.

# Anisotropic flow and hydrodynamics

*Initial transverse density profile*



Expansion



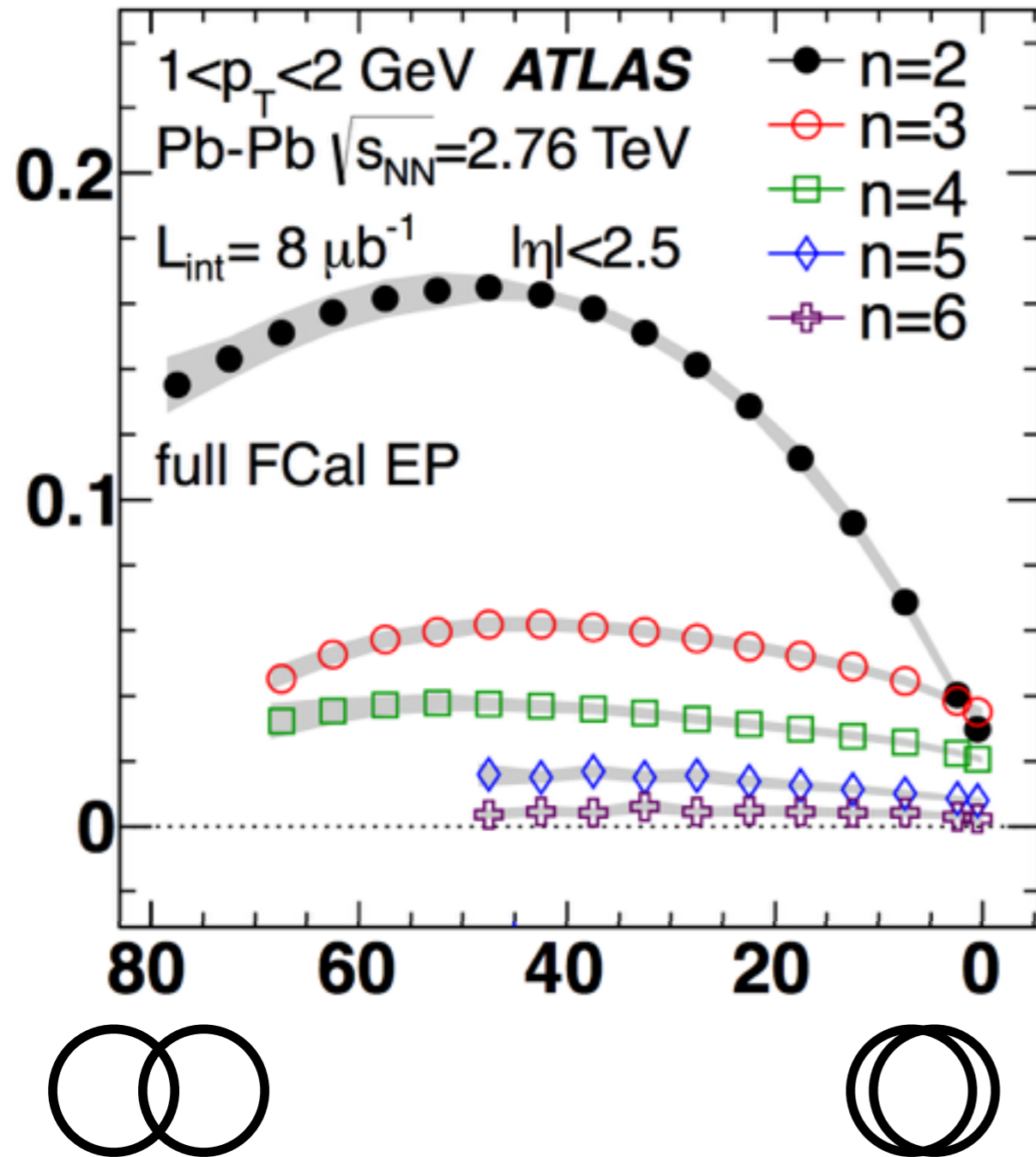
*Final distribution*

Elliptic flow  $v_2$

Triangular flow  $v_3$

*In hydrodynamics, anisotropic flow is a **response** to the **anisotropy of the initial density profile**. What about data?*

# The centrality dependence of $v_n$



- Steep decrease of  $v_2$  for central collisions: reflects the **elliptic geometry** of the overlap area
- Mild decrease of  $v_3$  : reflects the initial triangularity, due to **fluctuations**, smaller for larger systems
- For peripheral collisions, both  $v_2$  and  $v_3$  decrease:  $1/R$  **viscous suppression** becomes large.

What is the most accepted origin of  $v_2$  and  $v_3$ , not only in PbPb collisions, but also in high multiplicity events (pPb and pp)? Are they originated mainly by **initial state fluctuations**, **geometry of the collision** or **bulk properties**?

- the initial anisotropy responsible for  $v_2$  in PbPb is essentially the **collision geometry**.
- In all other cases ( $v_3$  in PbPb,  $v_2$  and  $v_3$  in pPb), the initial anisotropy is created by **initial state fluctuations**.
- The conversion of all initial anisotropies into a momentum anisotropy involves the **bulk properties** (viscosity, etc).

The QGP we obtain at LHC is a strongly interacting fluid. Do we expect that due to the asymptotic freedom QGP at some higher collision energies will be an almost ideal gas of quarks and gluons? If yes then how high should be such collision energy?

- At higher temperature, one expects a larger viscosity/entropy ratio, i.e., a less-strongly-interacting fluid.
- But in order to increase the temperature by a factor 2, you must increase the particle density by a factor 8 for dimensional reasons.
- $dN/d\eta$  in Pb+Pb collisions varies with energy roughly like  $s^{0.15}$ . A factor 8 in  $dN/d\eta$  implies a factor 1000 in the collision energy.

# Conclusions

- We have ample evidence that the matter produced in nucleus-nucleus collisions at the LHC behaves like a small fluid.
- There will be debates at this conference to figure out whether this description extends to smaller systems, pA or pp.
- I hope you will be able to follow the debate and ask the right questions.