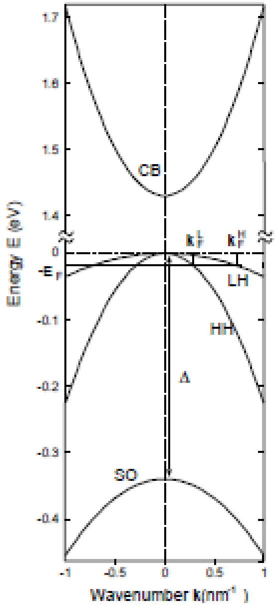
Nematic and non-Fermi liquid phases of systems with quadratic band crossing

Igor Herbut

(Simon Fraser University, Vancouver)

Lukas Janssen (SFU)
Balazs Dora (Budapest)
Roderich Moessner (MPI PKS))



IH and Lukas Janssen,
Phys. Rev. Lett. 113, 106401 (2014)

CAP 2015, Edmonton

Quadratic band crossing in 2D: (e. g. bilayer graphene)

Irreducible Hamiltonian: $(c = \cos(2\alpha),\ s = \sin(2\alpha)\ )$

$$H_0 = -\frac{p^2}{4m'}\mathbb{I} - \frac{p^2 c}{4m}\sigma_3 - \frac{s}{4m}\left[\sigma_1(p_x^2 - p_y^2) + \sigma_2 2p_x p_y\right]$$

With short-range interaction:

$$H = \int d\mathbf{r}\left[\mathbf{\Psi}^+(\mathbf{r})H\mathbf{\Psi}(\mathbf{r}) + U\delta n_1(\mathbf{r})\delta n_2(\mathbf{r})\right]$$

has an instability at weak coupling:

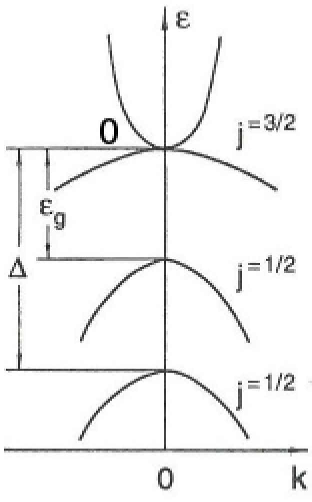
$$\frac{dU}{d\ln s} = U^2\rho_0 + O(U^3)$$

towards QAH (gapped) ($|\sin(2\alpha)| > \sqrt{2/3}$) or nematic (gapless) phase. (Sun et al, 2010, Dora, IH, Moessner, 2014)

Three dimensions: gapless semiconductors (gray tin, HgTe,…)

Luttinger spin-orbit Hamiltonian (J= 3/2) (Luttinger, 1956)

$$H = \frac{1}{2m}\left((\gamma_1 + \frac{5}{2}\gamma_2)k^2 - 2\gamma_2(\mathbf{k}\cdot\mathbf{S})^2 \right)$$



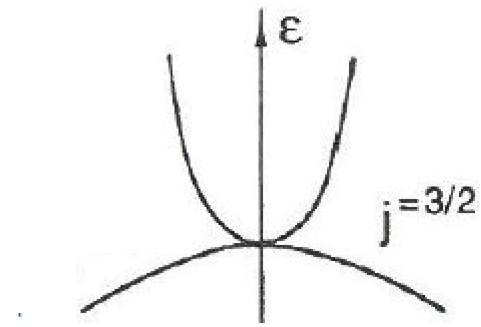
with (twice degenerate) eigenvalues:

$$E_L(k) = \frac{\gamma_1 + 2\gamma_2}{2m}k^2 \quad , \quad E_H(k) = \frac{\gamma_1 - 2\gamma_2}{2m}k^2$$

Density of states now vanishes at the QTP: short-range interactions are irrelevant, but there is no screening.

What is the effect of long-range Coulomb interaction?

Without the hole band empty, at ``zero” (low) density:



Wigner crystal !

With the hole band filled and particle band empty: the system is

``critical”

In the RG language, changing the cutoff causes the charge to ``flow”

$$\frac{de^2}{d\ln b} = (z+2-d)e^2 - 4e^4$$

with the dynamical critical exponent: $z = 2 - \frac{16}{15}e^2$

(Coulomb interaction $\sim 1/p^2$.) (Abrikosov, JETP 1974)

Below and near the upper critical dimension, $d_{up} = 4$, the system is in the non-Fermi liquid interacting phase, with the charge at the fixed point value:

$$e_*^2 = 15\epsilon/76 + \mathcal{O}(\epsilon^2)$$

with the small parameter

$$\epsilon = 4 - d$$

and the dynamical critical exponent $z < 2$.

This implies power-laws in various responses, such as specific heat:

$$c_v \sim T^{d/z} \approx T^{1.7}$$

(Abrikosov, JETP 1974, Moon, Xu, Kim, Balents, PRL 2014)

Easy way to get a NFL phase in 3D!

Or not?

The picture must somehow break down before the dimension reaches d = 2; a short range coupling flows like

$$\frac{dg_1}{d\ln b} = (z - d)g_1 \text{ + high. ord. term.}$$

and becomes marginal in d=2.

What can happen to the NFL stable fixed point?

The mechanism : collision of UV and IR fixed points (Kaveh, IH, 2005, Gies, Jaeckel 2006, Kaplan, Lee, Son, Stephanov, 2009). First we rewrite the Luttinger Hamiltonian as :

$$H(k) = \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_a \Gamma^a$$

where,

$$\epsilon(\mathbf{k}) = \frac{\gamma_1}{2m} k^2, \;\; d_a(\mathbf{k}) = -3\xi_a^{ij} k_i k_j ,$$
$$d_1 = -\sqrt{3} k_y k_z , \;\; d_2 = -\sqrt{3} k_x k_z , \;\; d_3 = -\sqrt{3} k_x k_y$$
$$d_4 = -\frac{\sqrt{3}}{2}(k_x^2 - k_y^2),$$
$$d_5 = -\frac{1}{2}(2k_z^2 - k_x^2 - k_y^2).$$

and the (five!) Dirac matrices satisfy:

$$\{\Gamma^a, \Gamma^b\} = 2\delta_{ab}$$

The full interacting theory, with long-range and short-range interactions is then: (IH and Lukas Janssen, PRL 2014)

$$L = \Psi^\dagger \left(\partial_\tau + ia + d_i(-i\nabla)\gamma_i\right) \Psi + g_1(\Psi^\dagger\Psi)^2 + g_2(\Psi^\dagger\gamma_i\Psi)^2 + \frac{1}{2e^2}(\nabla a)^2$$

and is O(3) symmetric. Change of the cutoff now amounts to

$$\frac{dg_1}{d\ln b} = (z-d)g_1 - \frac{1}{2}g_1 g_2 - \frac{5}{2}g_2^2 - 4e^2 g_2$$

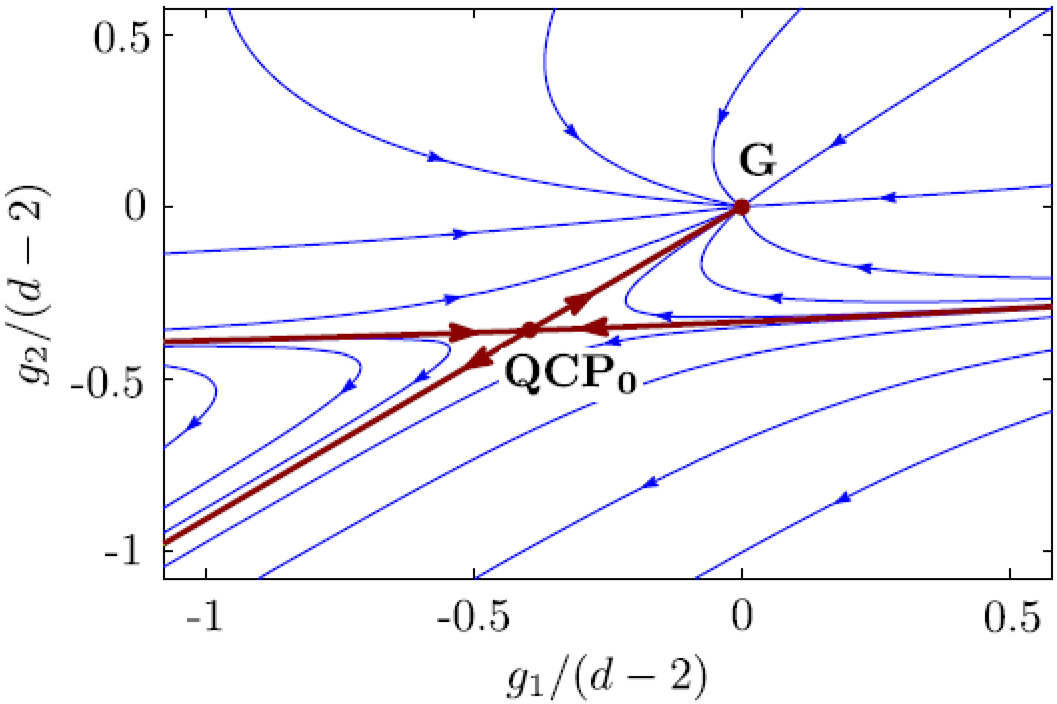
$$\frac{dg_2}{d\ln b} = (z-d)g_2 + \frac{2}{5}g_1 g_2 - \frac{1}{20}g_1^2 - \frac{63}{20}g_2^2 - \frac{4}{5}e^2 g_1 + \frac{16}{5}e^2 g_2 - \frac{16}{5}e^4$$

in addition to:

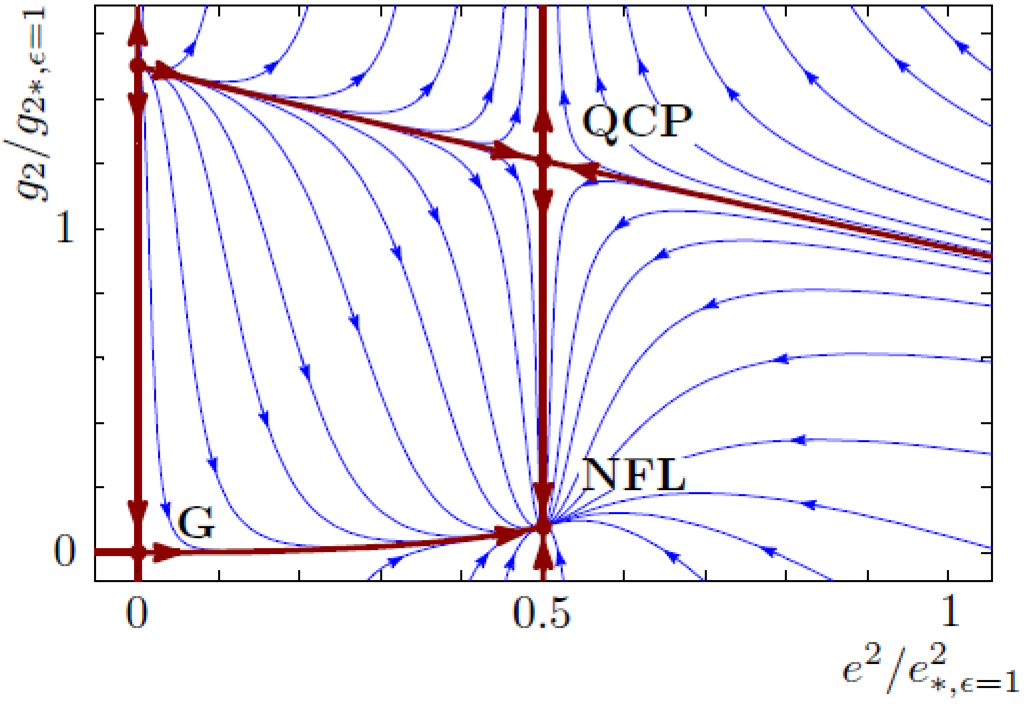
$$\frac{de^2}{d\ln b} = (z+2-d)e^2 - 4e^4$$

Dimensionless charge: $e^2 = 2m e_{\mathrm{el}}^2/(4\pi\hbar^2\varepsilon)$

Without the long-range interaction (e=0), the theory possesses a quantum critical point (QCP_0); weakly coupled close to d=2:

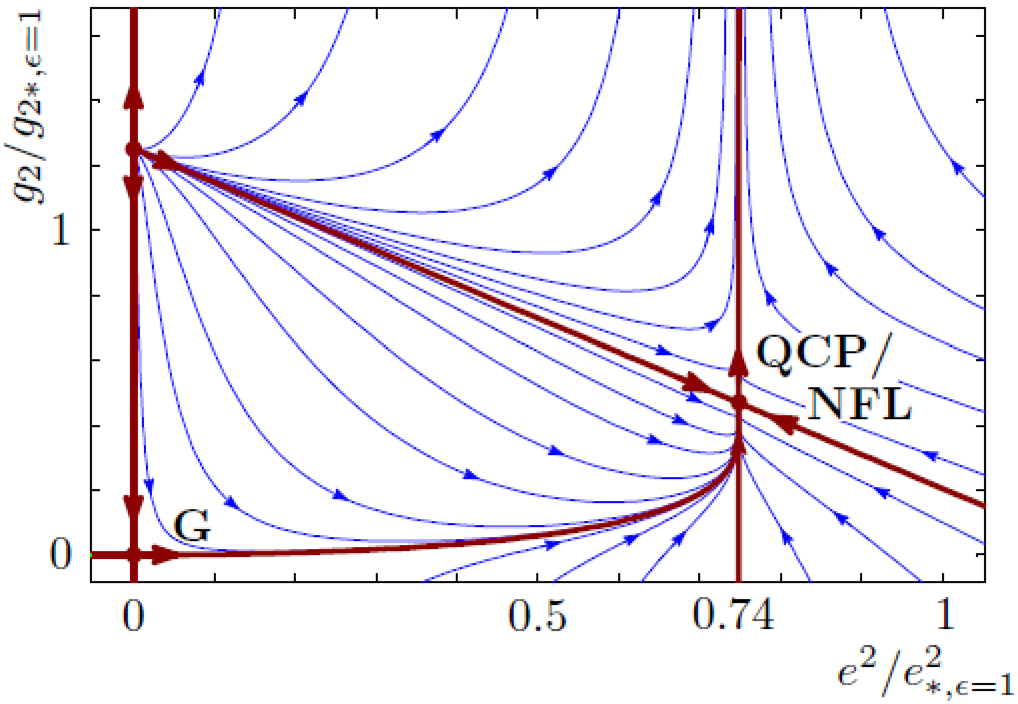


Close to and below d=4 there is a (IR stable) NFL fixed point, but also a (UV stable) quantum critical point at strong interaction: (d=3.5)



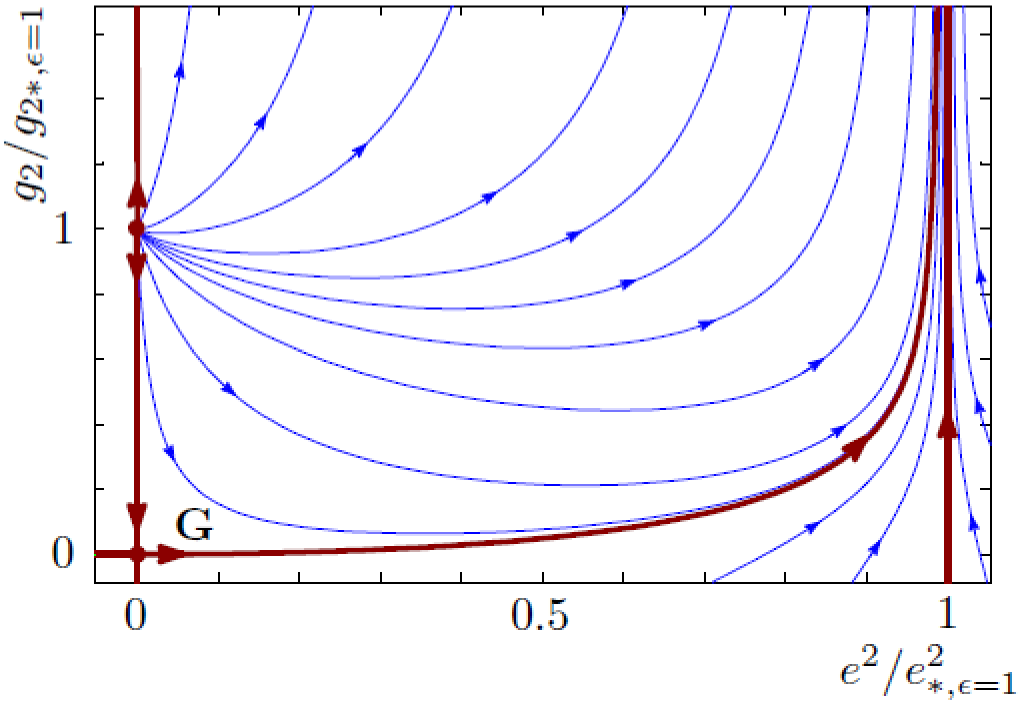
They get closer, but remain separated in the coupling space!

At some “lower critical dimension” NFL and QCP collide:



In one loop calculation, this occurs at $d_l = 3.26240$, and thus above, but close to three dimensions.

Finally, below dı the NFL and QCP become complex, and there is only a runaway flow left:

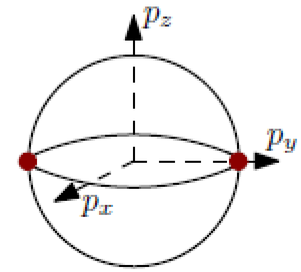


Non-Fermi liquid (scale invariant) phase is lost, and he system is unstable.

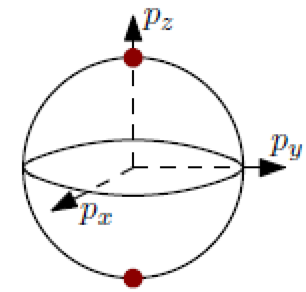
Order parameter for $d < d_{\text{low}}$ $\chi_i = 2g_2\langle\Psi^\dagger\gamma_i\Psi\rangle$

Out of the five $\chi_1, \ldots, \chi_5$ not all equivalent:

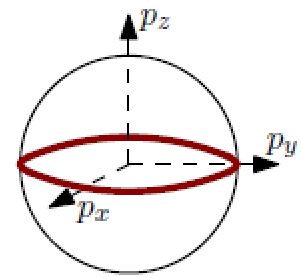
(1) $\chi_1 \neq 0$: $\varepsilon(\vec{p})$ gapped with minimal gap at two opposite points on equator



(2) $\chi_5 < 0$: $\varepsilon(\vec{p})$ gapless with gap closing at north and south pole



(3) $\chi_5 > 0$: $\varepsilon(\vec{p})$ gapped with minimal gap at entire equator



Energy $E = \int \frac{d\vec{p}}{(2\pi)^3}\varepsilon(\vec{p})$ is minimized for (3): $\chi_5 > 0$ (modulo O(3))

At large negative g_2 the system should develop anisotropic gap and,

$$\chi_5 > 0$$

The gap is minimal at the equator (in momentum space) at

$$p^2 = \chi_5 / 2$$

and the system looks as if under strain. The resulting ground state:

(topological) Mott insulator

(IH and Janssen, PRL 2014)

The state is equivalent in symmetry to ``uniaxial nematic".

The fate of NFL: if d_l is above but close to d=3, the flow becomes slow close to (complex!) NFL fixed point. The RG ``escape time" is long:

$$b_0 = e^{\frac{C}{\sqrt{d_{\mathrm{low}}-d}} - B + \mathcal{O}(d_{\mathrm{low}}-d)}$$

with non-universal constants C and B. There is wide crossover region of the NFL behavior within the temperature window

$$(T_{\mathrm{c}}, T_*)$$

with the critical temperature,

$$T_{\mathrm{c}} \approx T_* b_0^{-z}$$

And the characteristic energy scale for interaction effects as

$$k_{\mathrm{B}} T_* \sim \frac{e_{\mathrm{el}}^2}{\varepsilon L_*} = \frac{\hbar^2}{2mL_*^2} = \frac{4m}{m_{\mathrm{el}}\varepsilon^2} E_0$$

Assuming a small band mass

$$m/m_{\mathrm{el}} \approx 1/50$$

and a high dielectric constant

$$\varepsilon \approx 30$$

still gives a reasonable

$$T_* \sim 10\,\mathrm{K} - 100\,\mathrm{K}$$

and a detectable

$$T_{\mathrm{c}} \approx T_*/100$$

Conclusion:

1)Abrikosov's non-Fermi liquid phase at T=0 exists only in dimensions

$$d_{low} < d < d_{up} = 4$$

with lower critical dimension $d_{low} > 2$, and probably close to three.

2) Below d_{low} the system develops a gap, and most likely becomes a (topological) Mott insulator.

3) NFL shows up in a possibly wide crossover regime of energy scales.

4) Gray tin or mercury telluride should be a (topological) Mott insulator at T=0, and at zero doping!