Can we reconcile Gravity & Quantum Mechanics?
A CORRELATED WORLDLINE THEORY of QUANTUM GRAVITY

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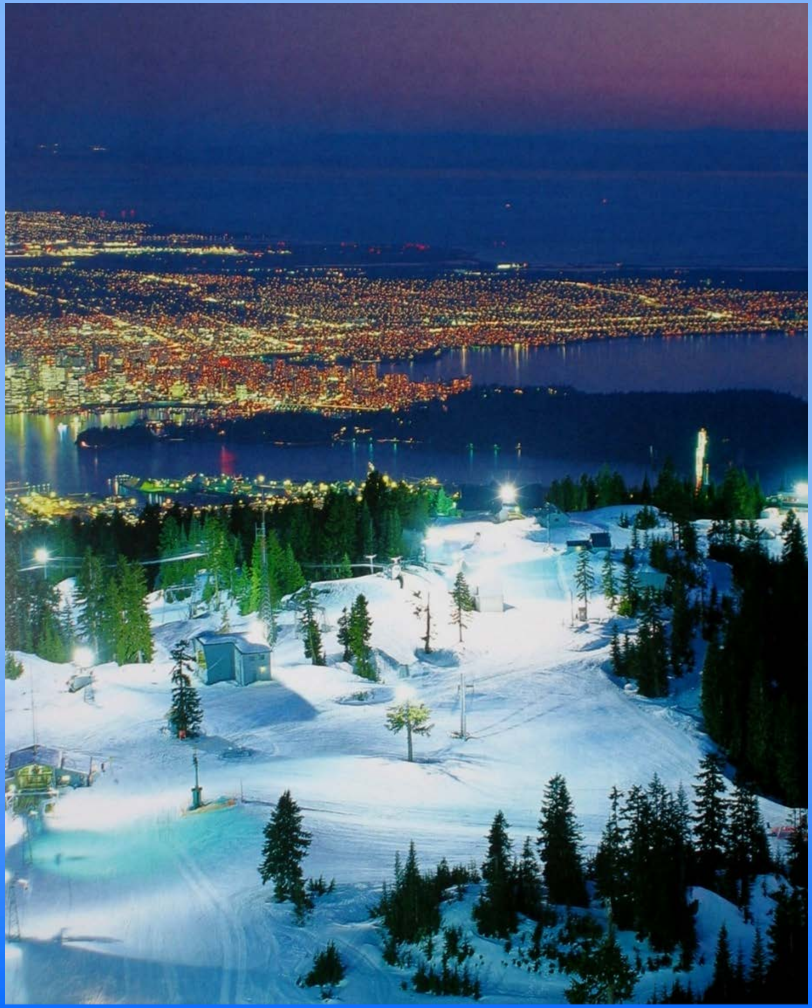
Edmonton, June 18th 2015



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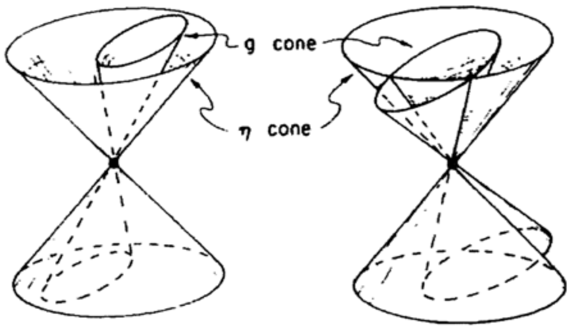
(1) WHAT is the ESSENCE of GRAVITY/GENERAL RELATIVITY ?

The ***general theory of relativity*** was established by Einstein (and finally formulated by him in 1916), and represents probably the most beautiful of all existing physical theories.

L.D. Landau, E.M. Lifshitz "The Classical Theory of Fields", sec.82



(1) CAUSAL STRUCTURE: As field strength goes up (eg., add gravitons), spacetime causal structure *changes*. The original gravitons become superluminal.

Causal structure is essential

(2) WEAK PRINCIPLE of EQUIVALENCE: identical coupling of all forms of energy to gravity, as expressed in the "minimal substitution", has overwhelming support from weak field tests and strong field observations.

So: we must use a metric structure to define spacetime – ie., $g^{\mu\nu}(x)$

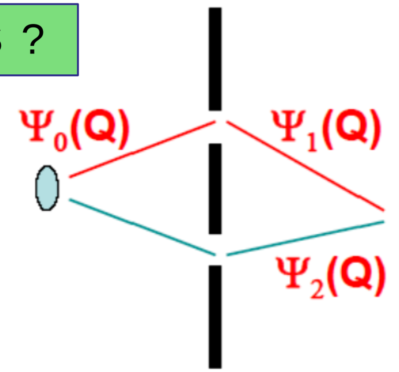
(3) WORLDLINES & CONNECTION FIELDS: We will assume what is at the heart of relativity – and also in QM – the idea of worldliness or worldsheets. In addition we assume that in curved spacetime the connection field can be defined in the usual way for a worldline. So: we need the connection (NB: a metric-affine formulation is OK).

(4) LOW-ENERGY EFFECTIVE THEORY: General Relativity is assumed to be good for quantization at low-energy. If spacetime is coupled to a quantized matter field, it must also go into a superposition

So: quantizing matter → spacetime must also be quantized.

So: The essence of GR is to be found in the metric, the connection, the associated causal structure, & the association with Quantum Mechanics

(2) WHAT is the ESSENCE of QUANTUM MECHANICS ?



(1) **According to Feynman (1965), ‘ the fundamental mystery of QM’ is encapsulated in the ‘2-slit’ experiment:**

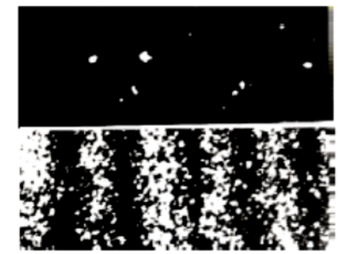
$\Psi_0(q)$ **evolves according to**

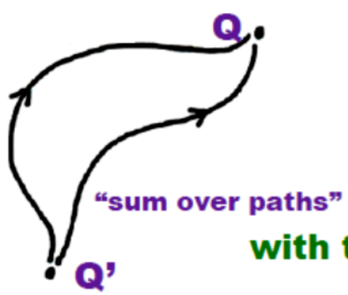
$$\Psi_o(q) \rightarrow [a_1 \Psi_1(q) + a_2 \Psi_2(q)]$$

The probability of seeing particle at position Q on screen:

$$P(Q) = |a_1 \Psi_1(Q) + a_2 \Psi_2(Q)|^2 = P_1 + P_2 + 2P_{12}$$

with cross-term $P_{12}(Q) \sim |a_1 a_2 \Psi_1(Q)\, \Psi_2(Q)|$





Feynman gave a beautiful formulation of QM, perfectly encapsulating this ‘superposition’. He writes

$$\psi(Q,t) = \int dQ'\, G(Q,Q';t,t')\, \psi(Q',t')$$

with the ‘path integral’ sum:

$$G(Q,Q';t,t') = \int_{q(t')=Q'}^{q(t)=Q} \mathcal{D}q(\tau)\, e^{\frac{i}{\hbar}S[q,\dot{q}]}$$

Notice that the path integral captures the relation between phase & action along the worldline

Actually, the path integral formulation gives us much more than the wave-function description:

$$G_o(2,1) = \int_1^2 \mathcal{D}\mathbf{r}(\tau)\, e^{\frac{i}{\hbar}S_{21}[\mathbf{r}(\tau)]} = \sum_\alpha \chi(\alpha) G_o^\alpha(2,1)$$

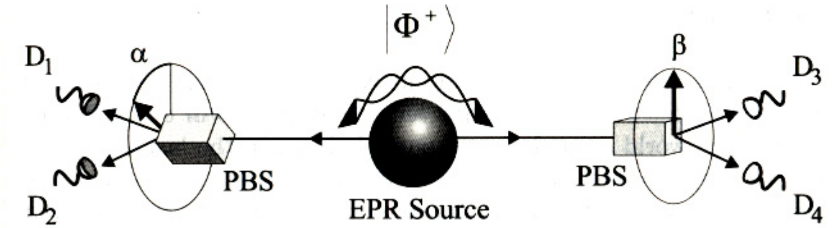
(fractional statistics!)

Thus, Feynman’s formalism gives directly an unambiguous answer to global problems. Other formalisms use ad hoc, extraneous conditions to deal with global problems, such as boundary conditions on wave functions, symmetry or antisymmetry property of the wave function, etc. … and their answers are not necessarily identical with Feynman’s.

C. Morette-DeWitt,
Comm. Math. Phys. 28, 47 (1972)

(2) Long before Feynman, Einstein & Schrodinger (1935) fingered "ENTANGLEMENT" as the real essence of QM – embodied in states like

$$\Psi = [\phi_+(A)\phi_-(B) + \phi_-(A)\phi_+(B)]$$



for which the quantum state of either individual system is literally meaningless!
NB: In the path integral formulation, entanglement is a CONSEQUENCE of superposition.

(3) Another thing that is often forgotten, but is also essentially quantum-mechanical, is the idea of INDISTINGUISHABILITY, which leads to particle statistics. Laidlaw & Morette-deWitt (1977) showed we need path integrals to truly understand this (for example, for fractional statistics, or any topological quantum state)

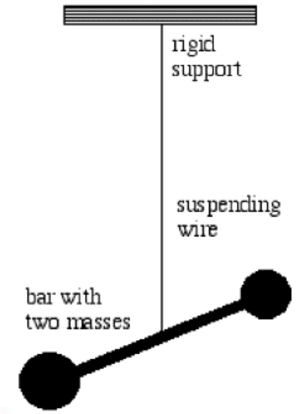
(4) The flat space field generating functional is a generalization of sourced QM (path integral form). Thus, eg., for QED, we have the 'in-out' functional:

$$\mathcal{Z}[\bar\eta,\eta;j^\mu] = \int_{in}^{out} \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}A^\mu \quad \exp\frac{i}{\hbar}\int d^4x\ [(\bar\psi\eta+\bar\eta\psi+j_\mu A^\mu)]$$
$$\times\ \exp\frac{i}{\hbar}\int d^4x\ \left[L_A^o(A^\mu) - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \bar\psi(i\gamma_\mu\partial^\mu - m_o - e\gamma_\mu A^\mu)\psi\right]$$

These are the basis of contemporary QFT – they are NECESSARY TO CAPTURE GLOBAL EFFECTS. Again, we need a path integral. Likewise in curved spacetime.

So – we conclude that the essence of QM can be captured by path integrals over worldlines, incorporating indistinguishability

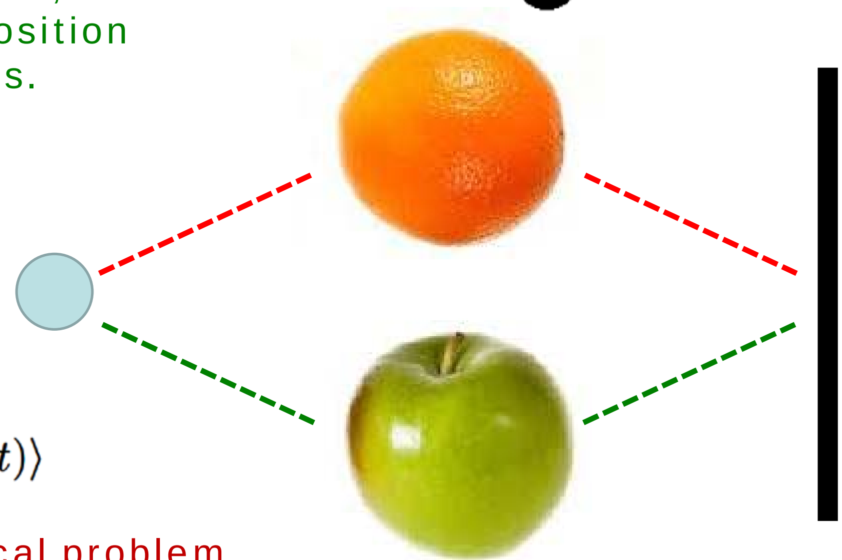
The low-E INCOMPATIBILITY of QM & GR



It is commonly asserted (usually by high-energy theorists) that the conflict between QM and gravity only exists at high energy (at energies approaching the Planck scale), where it is supposed to be resolved in favour of QM or QFT.

This argument is wrong.

Feynman 1957, Karolhazy 1966, Eppley-Hannah 1977, Kibble 1978-82, Page 1981, Unruh 1984, Penrose 1996, showed there is a basic conflict between the superposition principle & GR at ordinary 'table-top' energies.



Consider a 2-slit experiment with a mass M.
Suppose we assume a 'wave-fn':

$$|\Psi\rangle \;=\; a_1|\Phi_1;\tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2;\tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$

In a non-relativistic treatment we write

$$\Phi(\mathbf{r},t) \equiv \langle \mathbf{r}|\Phi(t)\rangle \;=\; a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

and then: $\langle \Phi_1(t)|\Phi_2(t)\rangle \;=\; \int d^3r \; \langle \Phi_1^*(\mathbf{r},t)|\Phi_2(\mathbf{r},t)\rangle$

But now we have both a formal and a physical problem.

(i) FORMAL PROBLEM: There are 2 different coordinate systems, $(\mathbf{r}_j, t_j)$, defined by the 2 different metrics: $\tilde{g}^{\mu\nu}_{(j)}(x)$, & in general we cannot relate these.

(ii) PHYSICAL PROBLEM: A "wave-function collapse" causes non-local changes, which if linked to the metric cause drastically unphysical changes in the metric.

This is quite apart from all the usual problems of Quantum Gravity !

So, what do we do? We must weigh our options here....

We can't just drop one or the other theory – they both work incredibly well at low E.

Neither QM nor GR has ever failed an experimental test; and both have shown a shocking ability to predict and explain an amazing variety of new (very counter-intuitive) physical phenomena. EACH is JUST as INCREDIBLY SUCCESSFUL as the OTHER.

Obviously we need a new theory that combines the virtues of each one.....

This is very hard; they are both very difficult to modify

SO LET'S GO.....

RULES of the GAME

First, the following question – basically a question about DIETARY RESTRICTIONS:

Q1: What is the most general modification we can make to QM/QFT, consistent with those features we wish to keep?

Remember what these features are:

(i) connection between phase (+ connection), and action on worldlines (paths)

(ii) indistinguishability for multiple particles and/or fields

(iii) fully relativistic – obeying the weak principle of equivalence, no violation of causal structure, well-defined metric.

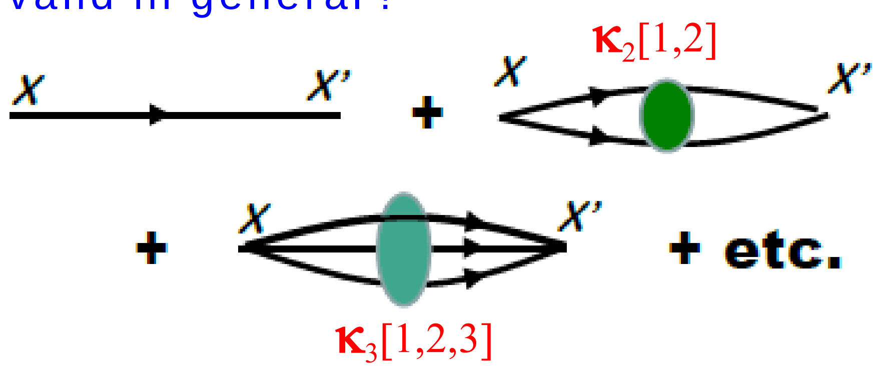
(iv) gravity/spacetime is treated as a quantum field as well as matter

The answer goes as follows; we change the mathematics to:

$$G_o(2,1) = \int_1^2 \mathcal{D}q(\tau) e^{\frac{i}{\hbar}S(2,1)} \longrightarrow \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\ \kappa_n[\{q_k\}]\ e^{\frac{i}{\hbar}S[q_k;2,1]}$$

In other words, we allow arbitrary correlations between any number of different paths. Since the paths are no longer independent, the superposition principle is no longer valid in general !

A diagrammatic view of this is: G(x,x') =



But – this is only a mathematical framework !

The answer to the 1st question gave us a framework with almost infinite freedom to choose different correlators – in this sense it is almost completely useless.

Now a 2nd question, which is about CULINARY CHOICE

Q2: If the correlation between paths is "gravitational", what does this imply for the correlators $\kappa_n[q_1,....q_n]$?

Now the general answer turns out to be rather messy. However for all situations we will ever face on earth (and in most astrophysical situations) the following works:

(1) Use the action: $S_G = \frac{1}{\lambda^2}\int d^4x\, [\tilde{g}^{\mu\nu} R_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \tilde{g}^{\mu\nu})^2]$ with gauge-fixing term

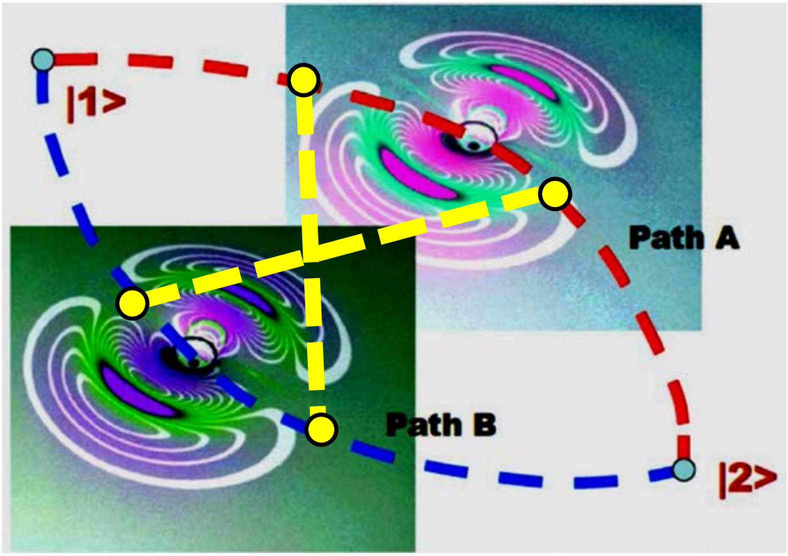
(2) Use the correlator (only valid for energies << Planck scale):

$$\kappa_n = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)\; e^{\frac{i}{\hbar}S_G}\,\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$$

ie., integrate over different spacetimes with a weighting factor

metric density → $\mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)$; gravitational action → S_G; Faddeev-Popov determinant → $\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$

Now what this does is COMMUNICATE BETWEEN PATHS the information about each path's spacetime status (and what the object is doing to spacetime).



We now have a PREDICTIVE THEORY with NO ADJUSTABLE PARAMETERS!!

GENERAL FORM of the THEORY

We assume a gravitational action: $$S_G = \frac{1}{\lambda^2}\int d^4x\,[\tilde{g}^{\mu\nu}R_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \tilde{g}^{\mu\nu})^2]$$

We then define a "CWL ring functional" of form:

Single Particle: we have

$$\mathbb{Q}[j] = \oint \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x) e^{iS_G/\hbar}\,\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)] \sum_{n=1}^{\infty}\prod_{k=1}^{n}\oint \mathcal{D}q_k\; e^{\frac{i}{\hbar}\sum_k [S[q_k]+jq_k]}$$

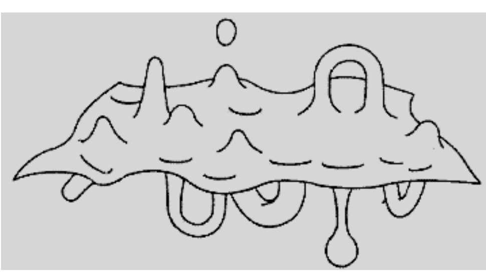
With matter action: $S_M = \int d^4x\,\frac{m}{2}g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu\;\delta(x^\lambda - q^\lambda(s))$ (depends on the metric)

Scalar Field: the action is: $S[\Phi] = \frac{1}{2}\int d^4x g^{1/2}\,[g^{\mu\nu}\nabla_\nu\Phi\nabla_\mu\Phi - m^2\Phi^2]$

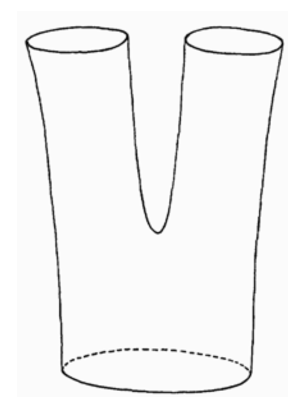
so that: $$\mathbb{Q}[J] = \oint \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x) e^{iS_G/\hbar}\,\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)] \sum_{n=1}^{\infty}\prod_{k=1}^{n}\oint \mathcal{D}\phi_k\; e^{\frac{i}{\hbar}\sum_k [S[\phi_k(x)]+\int d^4x\; J(x)\phi_k(x)]}$$

and so on for higher fields.

The definition of the measure of the path integral is as usual a non-trivial involving topological fluctuations of the metric – much ink has been expended on this.



However, we will be treating this as an EFFECTIVE LOW-E theory – such problems do not then occur



INTERPRETATION: The UNIQUE ROLE of GRAVITATION

(1) The comparison/communication between different spacetimes, in a superposition of different matter states, is achieved – is DEFINED **- by** GRAVITY ITSELF**.**

(2) This is why gravity couples universally to matter – and in the same universal way between paths

(3) The key fundamental quantity is PHASE**. It is defined in the comparison between wordlines by the metric, as a RELATIVE PHASE, and along a given worldline by the connection (INTERNAL or GAUGE PHASE). This means we are <u>DEFINING</u> spacetime via the notion of quantum phase, and via phase comparisons.**

Recall that a fundamental problem in Quantum Gravity is that there is no sensible way, in GR, to superpose spacetimes; different spacetimes exist on different manifolds, with no way of mapping between them. As we saw, this is very serious, since it means we have no proper way, in such a formulation, of even DEFINING a superposition in ordinary QM (it requires a 'background' spacetime).

Here we avoid this problem – spacetime is now defined via superpositions themselves, and via Quantum Phase

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Now, most experimentalists want more than this BLA-BLA-BLA
They want testable non-trivial predictions

SO – WHAT DOES THIS ALL MEAN in the REAL WORLD?

PERTURBATIVE EXPANSION (WEAK FIELDS)

We expand the metric density as $\tilde{\mathfrak{g}}^{\mu\nu}(x) = \eta^{\mu\nu} + \lambda h^{\mu\nu}(x)$

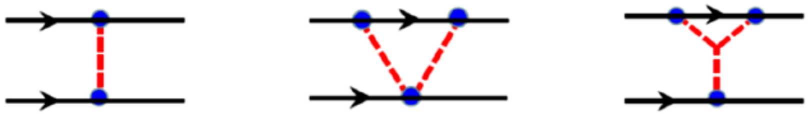
Then split off the non-linear parts of the gravitational action: $L_G = L_o - \int d^4x U(h^{\mu\nu})$

where $U(h^{\mu\nu}) \sim O(\lambda)$

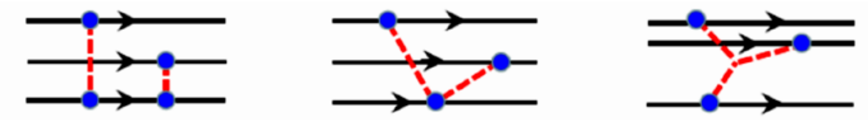
We can now calculate the generating functional and all the correlators in 'graviton expansions', either around a flat metric or around some background curved metric. This is nothing but the Schwinger-DeWitt/Fradkin-Vilkovisky/Donoghue background field method, adapted to the CWL theory.

The lowest order irreducible graphs for the ring correlators are

$\kappa_2[q_1(\tau), q_2(\tau)]$



$\kappa_3[q_1(\tau), q_2(\tau), q_3(\tau)]$



Consider now a calculation of the 4-point correlator for the dynamics of the density matrix. We have

$$\mathcal{K}_{2,2';1,1'} = lim_{\hat{h}=0} \{ e^{\frac{i}{2\hbar}(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'})} e^{\frac{-i}{\hbar}\int U(\hat{h})} \mathcal{G}_{2,1}[\hat{h}(x)]\mathcal{G}_{1',2'}[\hat{h}(x')]\}$$

where we have defined $(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'}) = \int d^4x d^4x' \frac{\delta}{\delta\hat{h}(x)} \hat{D}(x,x') \frac{\delta}{\delta\hat{h}(x')}$

and where $\mathcal{G}_{2,1}[\hat{h}(x)] = \sum_{n=1}^{\infty} \prod_{j=1}^{n} G_j(2,1|\hat{h}(x))$ is the total propagator for the particle in the background field h(x)

WEAK FIELD EXPANSON for an INTERFERENCE EXPERIMENT

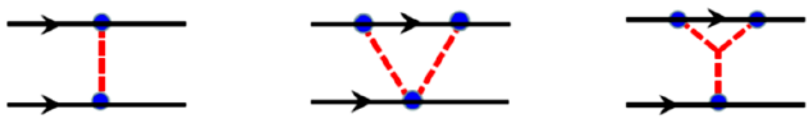
We can calculate the 4-point correlator for the density matrix dynamics, but it is easier to just find the 2-point propagator. Again, recall the form this will take – after integrating over the field *h*(x) we have

$$\mathcal{G}(2,1) = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\ \kappa_n[\{q_k\}]\ e^{\frac{i}{\hbar}S[q_k;2,1]}$$

The lowest correction to QM goes like:

$$\Delta\mathcal{G}(2,1) = \int_1^2 \mathcal{D}q \int_1^2 \mathcal{D}q'\ \kappa_2[q,q']e^{\frac{i}{\hbar}(S[q]+S[q'])} + \ \dots$$

The lowest order irreducible diagrams for this first correction are at right. In de Donder gauge the graviton propagator is



$\kappa_2[q_1(\tau), q_2(\tau)]$

$$\mathcal{D}^o_{\mu\nu\lambda\rho}(q) \ = \ \frac{1}{2q^2}[\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}]$$

and we get: $\kappa_2[x,x'] \ = \ \exp\left[\frac{i\lambda^2}{2\hbar}\int d^4x \int d^4x'\ T^{\mu\nu}(x)\mathcal{D}^o_{\mu\nu\lambda\rho}(x-x')T^{\lambda\rho}(x')\right] - 1 \qquad + \ etc.$

Let's write this as $\kappa_2[q,q'] = e^{i\chi_2[q,q']} - 1$ and take the 'slow-moving' limit where v << c.

Then $q \rightarrow (\mathbf{q}, t)$; define the relative coordinate $\mathbf{r} = \mathbf{q} - \mathbf{q}'$

and we find $\chi_2[q,q'] \ = \ \int^t \frac{d\tau}{\hbar}\frac{m^2\lambda^2}{r(\tau)}\left[1 - \frac{R_s}{2r(\tau)} - \frac{17}{20}\frac{L_p^2}{r^2(\tau)} + \dots\right]$

$L_p = (\hbar G/c^3)^{1/2}$

$R_s \ = 2Gm/c^2$

SLOW DYNAMICS

In any lab experiment involving massive objects, we will also be able to assume velocities << c. The correlator then simplifies further, to

$$\kappa_2[\mathbf{q},\mathbf{q}'] \;=\; \exp\frac{i}{\hbar}\int^t d\tau\,\frac{4\pi G m^2}{|\mathbf{q}(\tau)-\mathbf{q}'(\tau)|} \;-\; 1$$

so the path integral looks like that for a Coulomb attraction, with charges m. The key scales are

$l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$ Newton radius (gravitational analogue of the Bohr radius)

$\epsilon_G(m) = G^2 m^2 / l_G(m) \equiv E_p(m/M_p)^5$ Mutual binding energy for paths

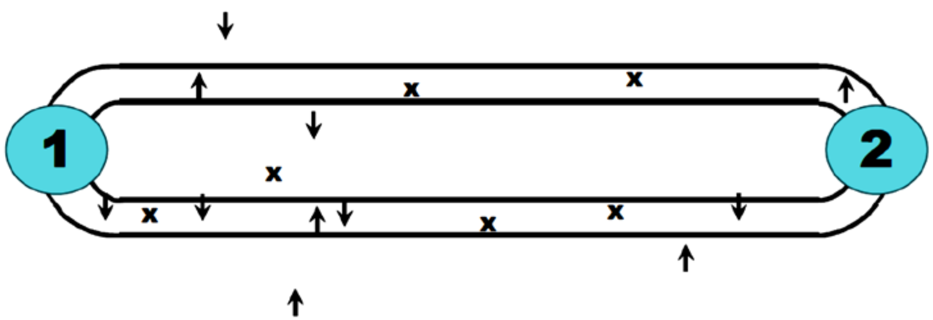
} (QM)

$R_s = 2Gm/c^2$ Schwarzchild radius for the particle (Classical)

Intuition about this result is best obtained by imagining it as the 'binding' of 2 paths in the potential well created by this 'Coulomb-Newton' attraction. We see that in this simple picture, the 2 paths will bind if

$$\varepsilon_G > E_Q$$

where E_Q is the energy scale associated with any other perturbations in the problem – we are thinking here of impurities, phonons, photons, imperfections in any controlling potentials in the systems, and, worst of all, dynamics localized modes likes defects, dislocations, paramagnetic or nuclear spins, etc.



But be careful!

As soon as a pair of paths starts to bind, then ALL paths will begin to bind – it is no longer a 2-path problem

COMPARISON with OTHER WORK

COMPARISON with PENROSE RESULT: Penrose argues that the 2 proper times elapsed in a 2-branch superposition cannot be directly compared; there is a time uncertainty, related to an energy uncertainty given in weak field by

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}.$$

$$E_{i,j} = -G \int\int d\vec{r}_1 d\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

R Penrose Gen Rel Grav 28, 581 (1996)

There are 2 problems here:

(i) The density is fed in by hand – it should be calculated from the theory itself, and will depend on the UV cutoff

(ii) It is only the first term in an exponential.

W Marshall et al., PRL 91, 130401 (2003)
D Kleckner et al., NJ Phys 10, 095020 (2008)

We can't expand the exponential: each term gives a divergent contribution...

$$\kappa_2[\mathbf{r}, \mathbf{r}'] = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \int^{t_j} d\tau_j \, \theta(\tau_j - \tau_{j-1}) \delta(t - \tau_n) \frac{(4\pi i G m^2)^n}{|\mathbf{r}(\tau_j) - \mathbf{r}'(\tau_j)|}$$

$$= \int^t d\tau \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} + \int^t d\tau \int^{\tau} d\tau' \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} \frac{4\pi i G m^2}{|\mathbf{r}(\tau') - \mathbf{r}'(\tau')|} + \ldots$$

If we put in the density by hand, the role of a UV cutoff is obvious from the results:

$$\Delta E = \frac{Gmm_1}{x_0}\left(\frac{24}{5} - \frac{1}{\sqrt{2\kappa}}\right)$$ "Zero point" estimate

$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right)$$ "nuclear radius" estimate

These numbers differ by ~ 1000 !

Thus this theory does not make unambiguous predictions

The BOTTOM LINE

The right theory will be decided by experiment – these experiments will not be easy. For more on all this:

PCE Stamp, Phil Trans Roy Soc 370, 4429 (2012)
PCE Stamp, New J Phys (in press)
PCE Stamp, Phys Rev Lett (submitted)

D Carney, A Gomez, PCE Stamp, in preparation
F Queisser, G Semenoff, PCE Stamp, in preparation


THANK YOU TO:
A Gomez
R Penrose
G Semenoff
H Brown
D Carney
C Gooding
F Quiesser
WG Unruh
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John
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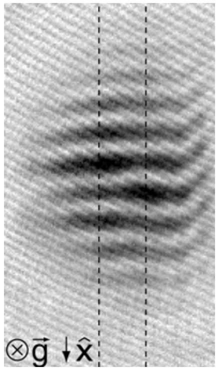
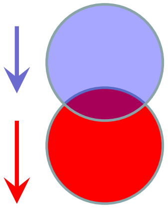
IDEAS for EXPERIMENTS . . .



(1) One idea is to just use straight interference between two entangled BECs. Such experiments are standard, and in principle could work very nicely. The problem is that we need a large fraction of the centre-of-mass coordinate of the BEC to be involved in the entangled wave-function – and this will be very hard to do.

(2) Another idea is to look at interference between 2 separate states of a moving object (this is the current Vienna idea). The simplest is to imagine a freely-falling object – the 2 paths here, corresponding to the 2 different positions of the mass, will interact gravitationally according to what we have seen.



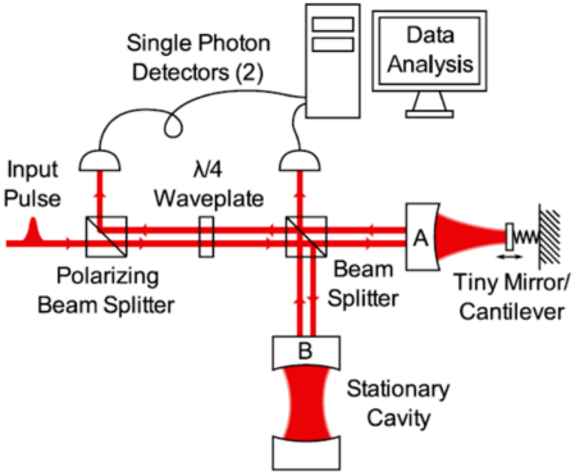
The difficulty here is to reduce environmental decoherence effects – coming from the interaction with photons, or between, eg., charged defects in the system (or spin defects/nuclear spins) and EM fields.

(3) Another idea is to look at interference between the 2 paths of a heavy mass which is oscillating. One starts a photon off entangled with a heavy mirror, and then looks for gravitational effects. Starting from a state

$$|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$$

we evolve to

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_C + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B \; |\kappa(1 - e^{-i\omega_m t})\rangle_m]_C$$

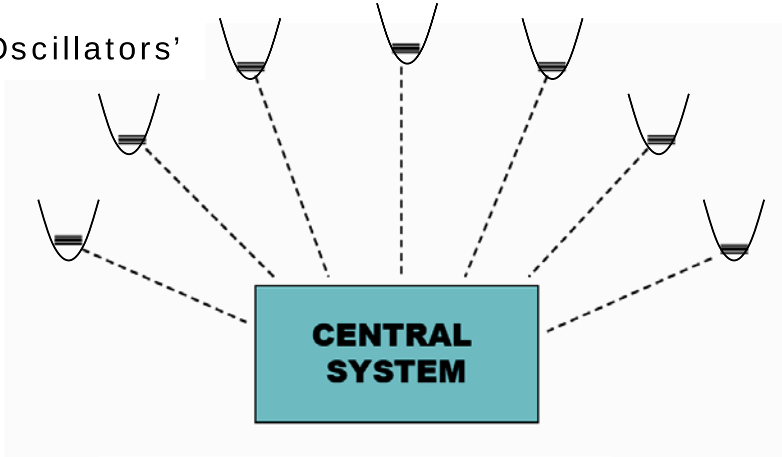


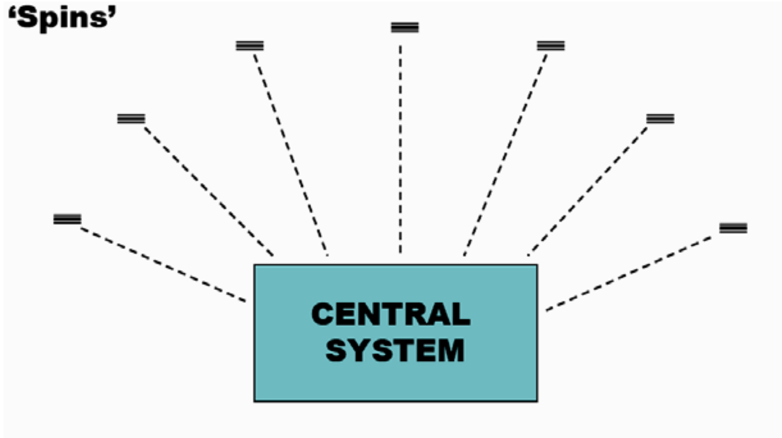
For more on this, see, eg.

D Kleckner et al., N J Phys 10, 095020 (2008)

I Pikowski et al., Nat Phys 8, 393 (2012)

REMARKS on ENVIRONMENTAL DECOHERENCE





$$H_{\text{eff}}^{\text{osc}} = H_0 + H_{\text{int}} + H_{\text{env}}^{\text{osc}}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left(\frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2\right)$$

Int: $$H_{\text{int}}^{\text{osc}} = \sum_{q=1}^{N} [F_q(Q) x_q + G_q(P) p_q]$$

Very SMALL (~ $O(1/N^{1/2})$)

Phonons, photons, magnons, spinons,
Holons, Electron-hole pairs, gravitons,..

Feynman & Vernon, Ann. Phys. 24, 118 (1963)

Caldeira & Leggett, Ann. Phys. 149, 374 (1983)
AJ Leggett et al, Rev Mod Phys 59, 1 (1987

DELOCALIZED BATH MODES
↓
OSCILLATOR BATH

$$H_{\text{eff}}^{\text{sp}}(\Omega_0) = H_0 + H_{\text{int}}^{\text{sp}} + H_{\text{env}}^{\text{sp}}$$

Bath: $$H_{\text{env}}^{\text{sp}} = \sum_k^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^{\alpha} \sigma_{k'}^{\beta}$$

Interaction: $$H_{\text{int}}^{\text{sp}} = \sum_k^{N_s} \boldsymbol{F}_k(P, Q) \cdot \boldsymbol{\sigma}_k$$

NOT SMALL !

Defects, dislocation modes, vibrons,
Localized electrons, spin impurities,
nuclear spins, ...

LOCALIZED BATH MODES
↓
SPIN BATH

(1) P.C.E. Stamp, PRL 61, 2905 (1988)
(2) NV Prokof'ev, PCE Stamp, J Phys CM5, L663 (1993)
(3) NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

FORMAL ASPECTS of ENVIRONMENTAL DECOHERENCE

density matrix propagator:

$$K(Q_2, Q_2'; Q_1, Q_1'; t, t') = \int_{Q_1}^{Q_2} \mathscr{D}q \int_{Q_1'}^{Q_2'} \mathscr{D}q' \, \mathrm{e}^{-\mathrm{i}/\hbar(S_0[q]-S_0[q'])} \mathscr{F}[q, q'],$$

with $$\mathcal{F}[Q, Q'] = \prod \langle \hat{U}_k(Q, t) \hat{U}_k^{\dagger}(Q', t) \rangle$$

Here the unitary operator $\hat{U}_k(Q, t)$ describes the evolution of the kth environmental mode, given that the central system follows the path $Q(t)$ on its 'outward' voyage, and $Q'(t)$ on its 'return' voyage; and $\mathcal{F}[Q, Q']$ acts as a weighting function, over different possible paths $(Q(t), Q'(t'))$.

Easy for oscillator baths (it is how Feynman set up quantum field theory); we integrate out a set of driven harmonic oscillators, with Lagrangians:

$$L = \frac{M}{2} \dot{x}^2 - \frac{M\omega^2}{2} x^2 - \gamma(t) x$$

Thus:

$$\mathcal{F}[Q, Q'] = \prod_q^{N_o} \int \mathcal{D}x_q(\tau) \int \mathcal{D}x_q(\tau') \, \exp\left[\frac{i}{\hbar} \int d\tau \frac{m_q}{2} [\dot{x}_q^2 - \dot{x}_q'^2 + \omega_q^2 (x_q^2 - x_q'^2)] + [F_q(Q) x_q - F_q(Q') x_q']\right]$$

Bilinear coupling →

$$F[q, q'] = \exp\left[-\frac{1}{\hbar} \int_{t_o}^{t} d\tau_1 \int_{t_o}^{\tau_1} d\tau_2 [q(\tau_1) - q'(\tau_2)][\mathcal{D}(\tau_1 - \tau_2) q(\tau_2) - \mathcal{D}^*((\tau_1 - \tau_2) q'(\tau_2)]\right]$$

Bath propagator

For spin baths it is more subtle:

$$\mathcal{F}[Q, Q'] = \prod_k^{N_s} \int \mathcal{D}\boldsymbol{\sigma}_k(\tau) \int \mathcal{D}\boldsymbol{\sigma}_k(\tau') \, \exp\left[\frac{i}{\hbar} (S_{int}[Q, \boldsymbol{\sigma}_k] - S_{int}[Q', \boldsymbol{\sigma}_k'] + S_E[\boldsymbol{\sigma}_k] - S_E[\boldsymbol{\sigma}_k'])\right]$$

$$S_{int}^{sp}(Q, \boldsymbol{\sigma}_k) = -\int d\tau \sum_k^{N_s} \boldsymbol{F}_k(P, Q) \cdot \boldsymbol{\sigma}_k$$

$$S_{env}^{sp} = \int d\tau \left[\sum_k^{N_s} (\mathcal{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{dt} - \mathbf{h}_k \cdot \boldsymbol{\sigma}_k) - \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^{\alpha} \sigma_{k'}^{\beta} \right]$$

Vector coupling

Berry phase coupling

MECHANISMS of ENVIRONMENTAL DECOHERENCE: a SIMPLE PICTURE

Easiest to visualize this in path integral theory:

(1) OSCILLATOR BATH Oscillator Lagrangian: $L_q(x_q, \dot{x}_q; t) = \frac{m_q \dot{x}_q^2}{2} - \Upsilon_q(t) x_q$

Each oscillator is subject to a force $\Upsilon_q(t) = m_q \omega_q^2 x_q - F_q(Q(t))$

Problem exactly solvable (Feynman). Each oscillator very weakly coupled to system, & slowly entangles with it…weak oscillator excitation, DISSIPATION

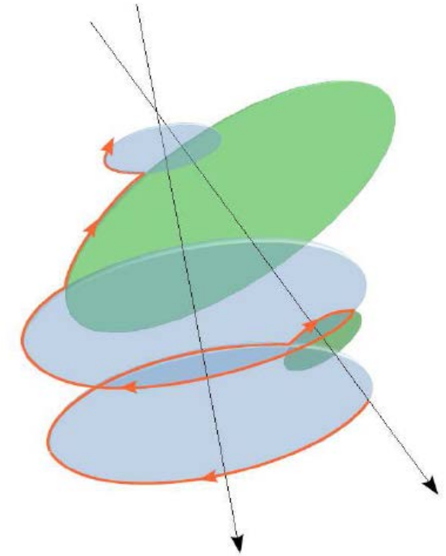
(2) SPIN BATH Each bath spin has the Lagrangian

$$L(\boldsymbol{\sigma}_k, \dot{\boldsymbol{\sigma}}_k; t) = \mathscr{A}_k \cdot \frac{d\boldsymbol{\sigma}_k}{d\tau} - \boldsymbol{\Upsilon}_k(t) \cdot \boldsymbol{\sigma}_k$$

with the force: $\boldsymbol{\Upsilon}_k(t) = \mathbf{h}_k + \mathbf{F}_k(t) + \xi_k(t)$

Entanglement with system via $F_k(P, Q)$ (not weak)

This problem is highly non-trivial (in general UNSOLVABLE even for spin-1/2 !).



Precessional path for bath spin

Example: Spin qubit Decoherence is precessional – NO DISSIPATION

$$\hat{H}_{QB} = H^0_{QB}(\vec{\tau}) + \sum_k (\vec{\gamma}_k + \xi_k) \cdot \vec{\sigma}_k$$

field: $\gamma_k^\alpha = h_k^\alpha + \sum_\beta \omega_k^{\beta\alpha} \tau_\beta$

Calculations here can become quite technical:

" *Only wimps specialize in the general case. Real scientists pursue examples.* "

MV Berry: Ann NY Acad Sci 755, 303 (1995)

BOTTOM LINE: all these contributions need to be separated out from the CWL effects