Dynamics of Gravitational Collapse in AdS Space-Time

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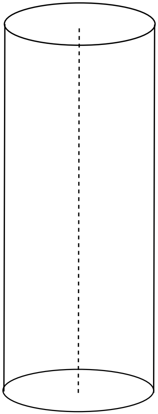
arXiv:1410.1869 (PRL) with N. Deppe, A. Kolly, and G. Kunstatter

work in progress with all of the above

Motivation

Anti-de Sitter Spacetime

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2$$



AdS has a boundary

- Massless waves to $r = \infty$ in $t = \pi/2$
- Bounce back to origin
- Collapse is boundary value problem

Contrast with asymptotically flat

- Forms horizon or disperses
- Critical behavior near transition

(*Choptuik 1993*)

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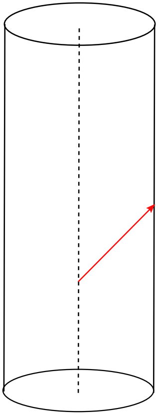

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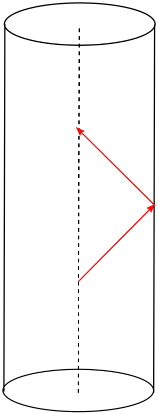

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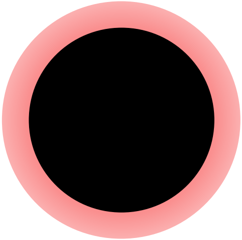
AdS/CFT Correspondence



Boundary conditions control solutions

- Build field theory on boundary
- Holographic correspondence
  Relates strong and weak coupling
- Best known $\mathrm{AdS}_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ SYM
  Many examples
- Extremely well tested

What does collapse represent?

- Black holes have temperature (Hawking)
- BH $\Leftrightarrow$ thermal state on boundary sphere
- Collapse $\Leftrightarrow$ thermalization of initial energy pulse
- Insight into dynamics far from equilibrium

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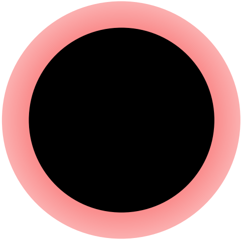

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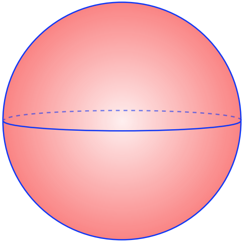

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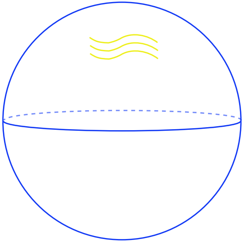

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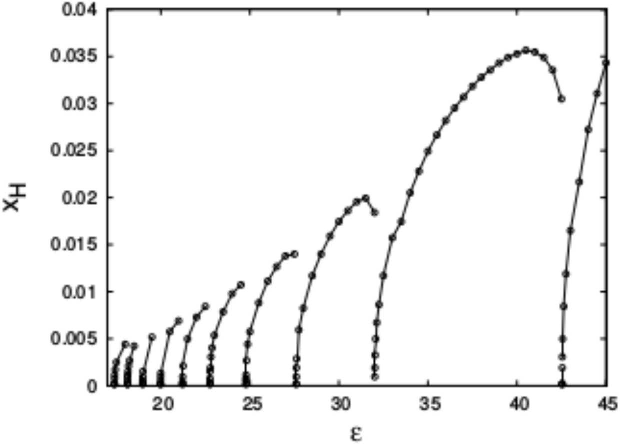
Instability of AdS

Numerics suggest small perturbations lead to collapse



(*Bizoń & Rostworowski, arXiv:1104.3702, PRL107, 031102 (2011)*)

Massless Scalars and Turbulence in AdS

Preliminaries

Maybe not surprising that contained energy thermalizes

- Schwarzschild-like coordinates

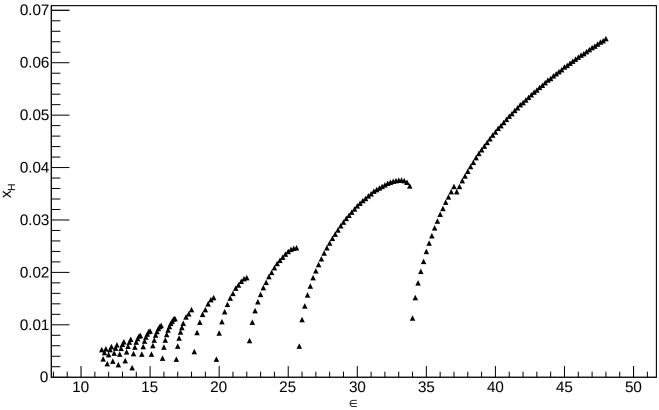
$$ds^2 = \sec^2(x)\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \sin^2(x)\,d\Omega^2\right)$$

- Horizon at $A = 0$ (infinite boundary time)
  Effectively formed at fixed cut-off (mostly thermalized)
- $r = \tan x$ so boundary at $x = \pi/2$
- Mass function M' gives mass in shell of radius R
- Original studies in AdS$_4$; here AdS$_5$

Massless Scalars and Turbulence in AdS

Many Bounces

Interesting patterns emerge at strong coupling



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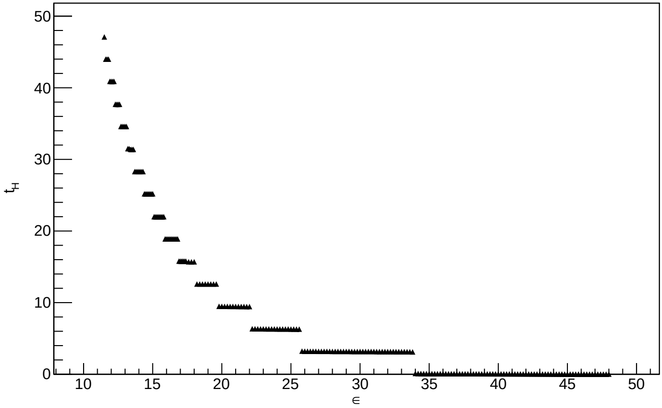

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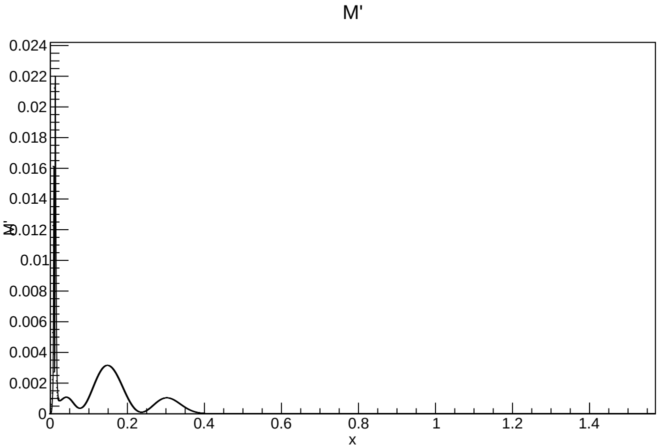
Many Bounces

▸ Mass Evolution



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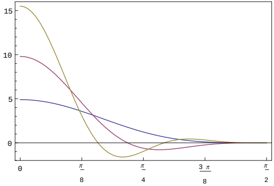
Turbulence & Perturbation Theory

Insight from perturbation theory?

Scalar eigenmodes of AdS have integrally-spaced frequencies



- Secular growth beyond 1st order
  Maximal resonance
- But many removed by frequency shifts
- Improved perturbation theory

  (*Balasubramanian, et al.; Craps, Evnin, Vanhoof*)
- Some simple modes quasi-periodic
  But which ones & for how long?
- Technically difficult to answer

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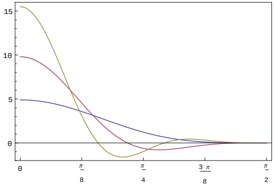

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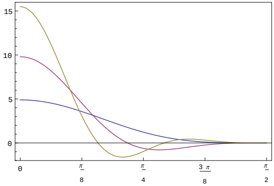

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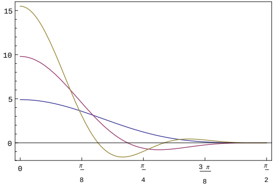

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Another Scale

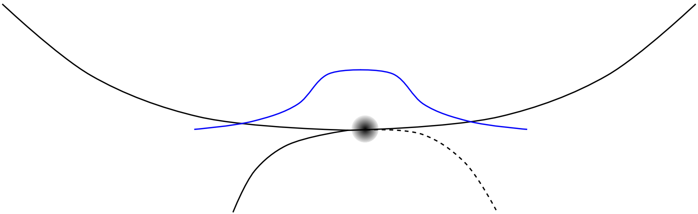
The Importance of Scales

Two scales for massless scalar: AdS radius & pulse width

- Planck scale factors out
- AdS radius important globally, not locally
- Possible interplay for very wide pulses

A new scale gives richer physics

- Ratios of scales important
- Possible local importance at horizon size



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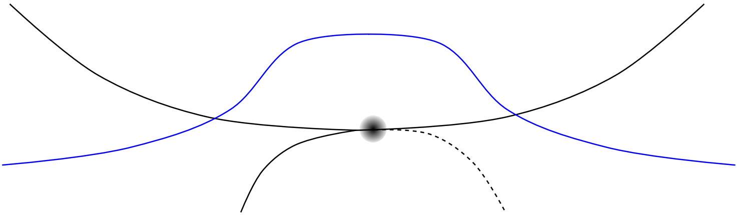

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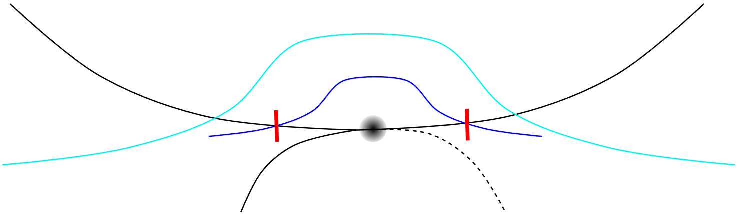

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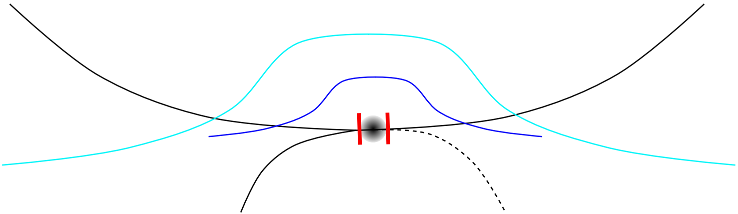


Another Scale

Massive Scalars

Modify scalar dynamics: easiest is adding a mass

- Pulse width vs Compton $\lambda_C = 1/\mu$
- Wide pulses lead to radius gap
- Common behavior w/extra scales

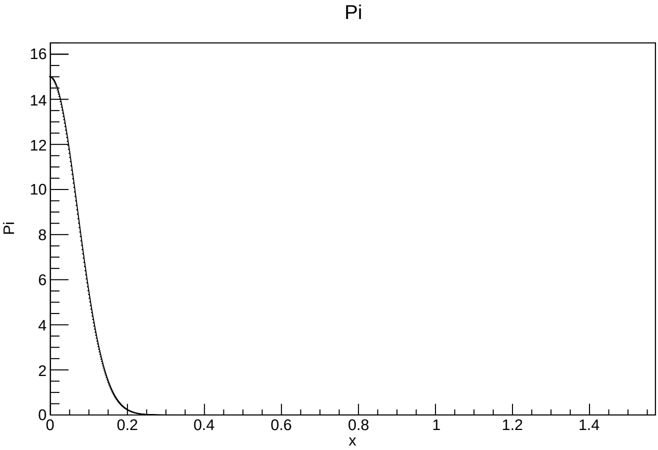
What happens in AdS? To appear on arXiv soon!

(*see also Okawa, Lopes, Cardoso arXiv:1504.05203*)

- Related to CFT irrelevant operators
- Comparison to AdS scale also
- Massive scalars still confined by gravitational potential
- Initial width important? Does bouncing change behavior?
- Most ratios of scales similar to massless

Another Scale
Massive Scalars

Momentum Evolution



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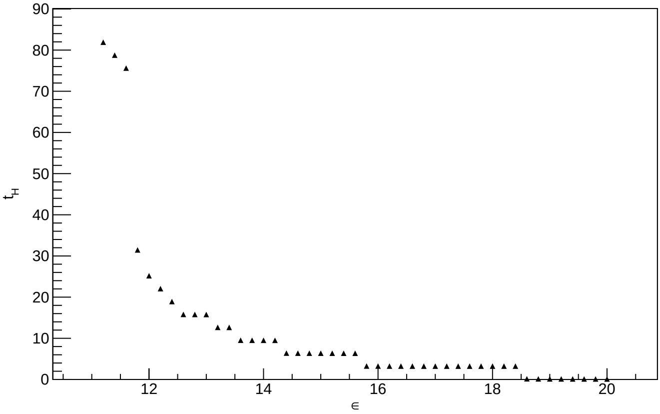
An intriguing example (AdS scale) $>$ width $>$ Compton:



Hints of a new class of quasi-stable solutions

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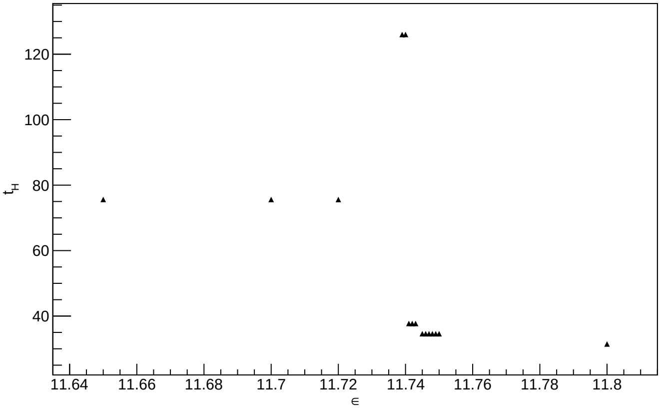

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Higher-Curvature Gravity

Modify gravitational dynamics

- Higher powers of curvature expected from QM
- May impact formation of small horizons
- Represents non-infinite coupling in dual theory
  Curvature^2 $\Leftrightarrow$ distinct central charges

We will consider adding Gauss-Bonnet term in 5D

- Still second-order equations of motion
- No black holes allowed below critical mass
- Also dynamical radius gap
- Small-scale "anti-gravity"
- Similarities and differences vs AdS_3

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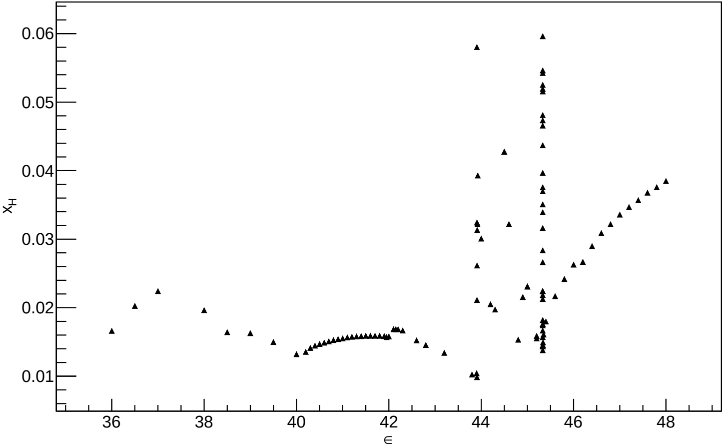
Complex behavior near critical points



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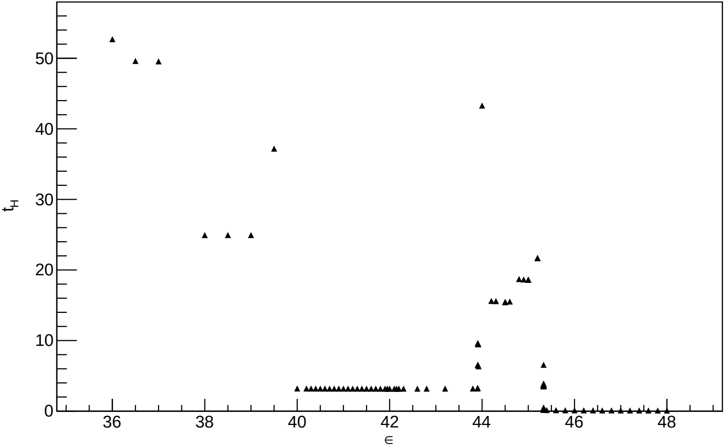

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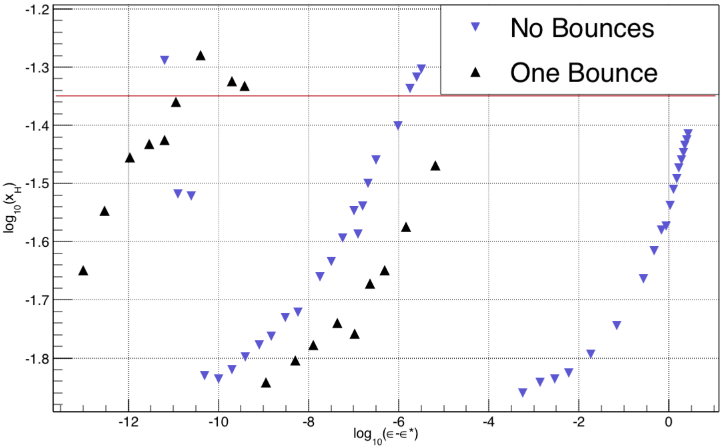

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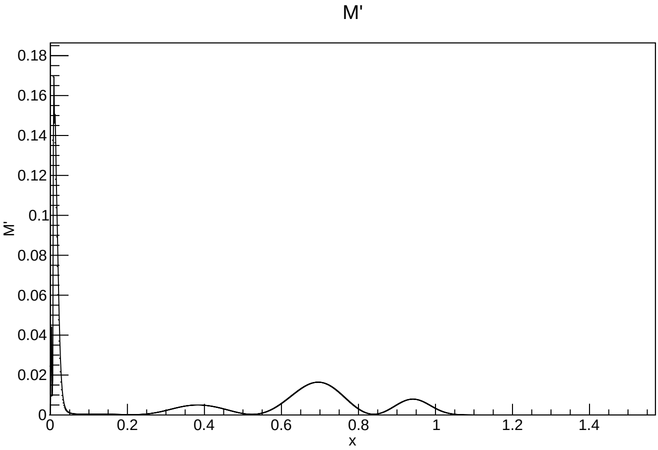

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- Also, conformally coupled and tachyonic scalars
- Other boundary conditions $\Leftrightarrow$ new operators in boundary Hamiltonian
- Interpretation in boundary theory
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