Dynamics of Gravitational Collapse in AdS Space-Time

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work in progress with all of the above
Motivation

Anti-de Sitter Spacetime

\[ ds^2 = -(1 + r^2) dt^2 + \frac{dr^2}{1 + r^2} + r^2 d\Omega^2 \]

AdS has a boundary

- Massless waves to \( r = \infty \) in \( t = \pi/2 \)
- Bounce back to origin
- Collapse is boundary value problem

Contrast with asymptotically flat

- Forms horizon or disperses
- Critical behavior near transition

(*Choptuik 1993*)
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AdS/CFT Correspondence

Boundary conditions control solutions
- Build field theory on boundary
- Holographic correspondence
  Relates strong and weak coupling
- Best known $\text{AdS}_5 \times S^5 \leftrightarrow \mathcal{N} = 4 \text{ SYM}$
  Many examples
- Extremely well tested

What does collapse represent?
- Black holes have temperature (Hawking)
- BH $\leftrightarrow$ thermal state on boundary sphere
- Collapse $\leftrightarrow$ thermalization of initial energy pulse
- Insight into dynamics far from equilibrium
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Instability of AdS

Numerics suggest small perturbations lead to collapse

Massless Scalars and Turbulence in AdS

Preliminaries

Maybe not surprising that contained energy thermalizes

- Schwarzschild-like coordinates

\[ ds^2 = \sec^2(x) \left( -Ae^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2(x) d\Omega^2 \right) \]

- Horizon at \( A = 0 \) (infinite boundary time)
  Effectively formed at fixed cut-off (mostly thermalized)

- \( r = \tan x \) so boundary at \( x = \pi/2 \)

- Mass function \( M' \) gives mass in shell of radius \( R \)

- Original studies in AdS\(_4\); here AdS\(_5\)
Massless Scalars and Turbulence in AdS

Many Bounces

Interesting patterns emerge at strong coupling
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Mass Evolution

\[ M' \]
Massless Scalars and Turbulence in AdS

Turbulence & Perturbation Theory

Insight from perturbation theory?

Scalar eigenmodes of AdS have integrally-spaced frequencies

- Secular growth beyond 1st order
- Maximal resonance
- But many removed by frequency shifts
- Improved perturbation theory
  
  (Balasubramanian, et al.; Craps, Evnin, Vanhoof)

- Some simple modes quasi-periodic
- But which ones & for how long?
- Technically difficult to answer
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Another Scale
The Importance of Scales

Two scales for massless scalar: AdS radius & pulse width
- Planck scale factors out
- AdS radius important globally, not locally
- Possible interplay for very wide pulses

A new scale gives richer physics
- Ratios of scales important
- Possible local importance at horizon size
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Turbulence
Another Scale
Importance
Massive Scalars
Curvature²
Future
Another Scale
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Modify scalar dynamics: easiest is adding a mass

- Pulse width vs Compton $\lambda_C = 1/\mu$
- Wide pulses lead to radius gap
- Common behavior w/extra scales

What happens in AdS? To appear on arXiv soon!

(see also Okawa, Lopes, Cardoso arXiv:1504.05203)

- Related to CFT irrelevant operators
- Comparison to AdS scale also
- Massive scalars still confined by gravitational potential
- Initial width important? Does bouncing change behavior?
- Most ratios of scales similar to massless
Another Scale
Massive Scalars

 mostrum Evolution
An intriguing example (AdS scale) > width > Compton:

Hints of a new class of quasi-stable solutions
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Hints of a new class of quasi-stable solutions
Another Scale
Higher-Curvature Gravity

Modify gravitational dynamics
- Higher powers of curvature expected from QM
- May impact formation of small horizons
- Represents non-infinite coupling in dual theory
  \( \text{Curvature}^2 \Leftrightarrow \text{distinct central charges} \)

We will consider adding Gauss-Bonnet term in 5D
- Still second-order equations of motion
- No black holes allowed below critical mass
- Also dynamical radius gap
- Small-scale “anti-gravity”
- Similarities and differences vs AdS\(_3\)
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Higher-Curvature Gravity

Complex behavior near critical points

![Graph showing complex behavior near critical points](image-url)
Another Scale
Higher-Curvature Gravity

Complex behavior near critical points
**Another Scale**

**Higher-Curvature Gravity**

Complex behavior near critical points

- No Bounces
- One Bounce
Another Scale
Higher-Curvature Gravity

Mass Evolution
Future Directions

- Further analysis of EGB gravity & massive scalars
- Also, conformally coupled and tachyonic scalars
- Other boundary conditions ⇔ new operators in boundary Hamiltonian
- Interpretation in boundary theory
- Incorporating Hawking radiation
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THANK YOU