Collective modes and interacting Majorana fermions in topological superfluids

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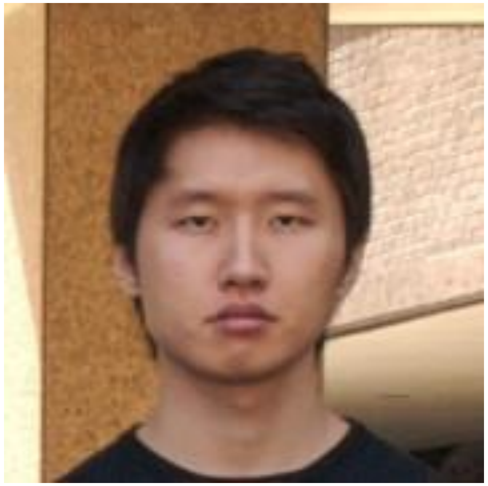
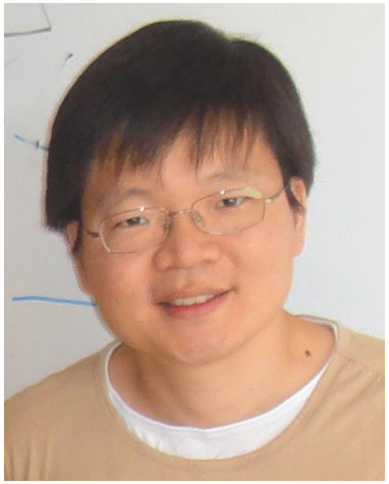


YeJe Park
(KAIST)



Suk Bum Chung
(IBS/SNU)

Y. J. Park, S. B. Chung, and JM, Phys. Rev. B 91, 054507 (2015)

“Free fermion” topological phases

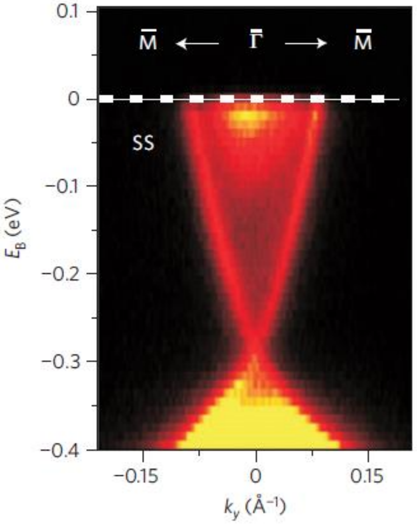
- Classification of free fermion topological phases is well understood (Kitaev, Schnyder, Ryu, Furusaki, Ludwig, …)

- Bulk: gapped, characterized by topological invariant (Z or Z_2) that corresponds (sometimes) to quantized physical observable

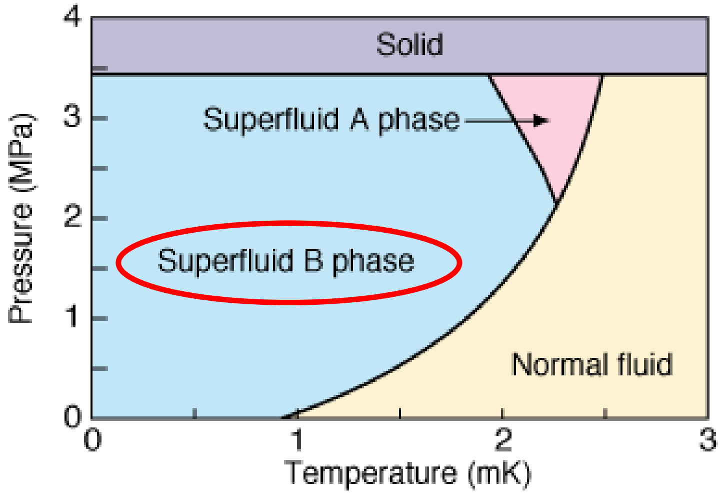
- Surface: gapless, protected by symmetry, cannot be realized by symmetry-preserving lattice model in same number of dimensions

“Free fermion” topological phases

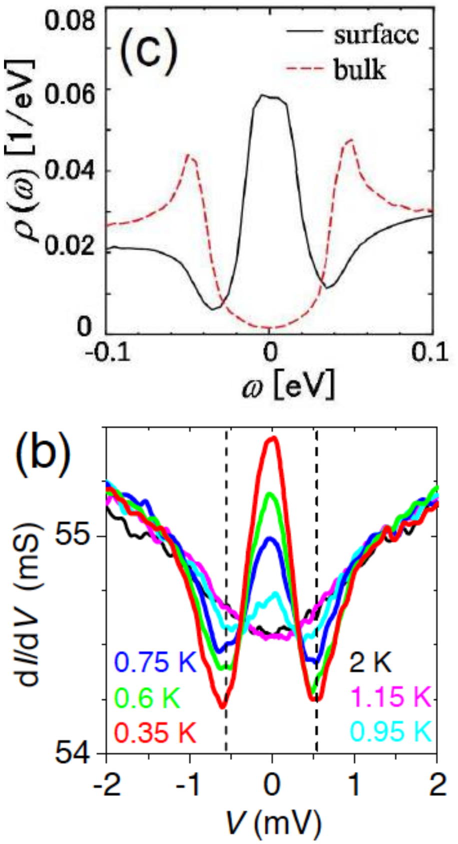
- 3D topological phases discovered experimentally via detection of 2D surface states



Bi_2Se_3 *(Xia et al., Nat. Phys. 2009)*



^{3}He-B?
(Murakawa et al., JPSJ 2011)



$Cu_xBi_2Se_3$?
(Sasaki et al., PRL 2011)

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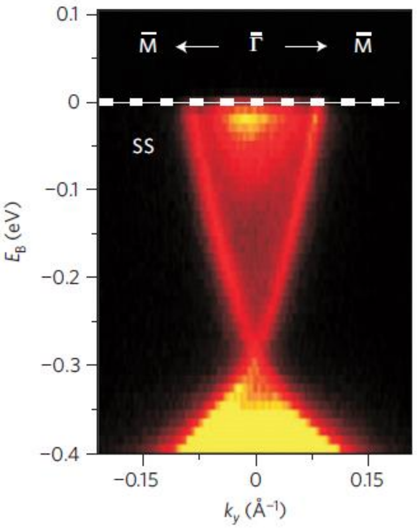

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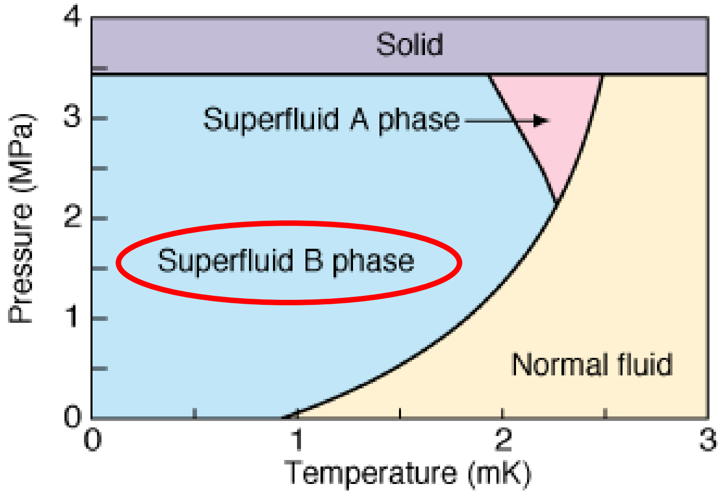

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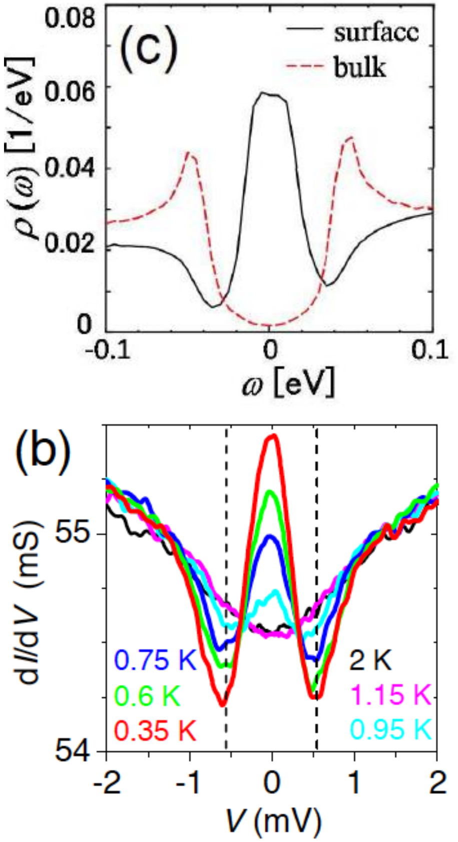
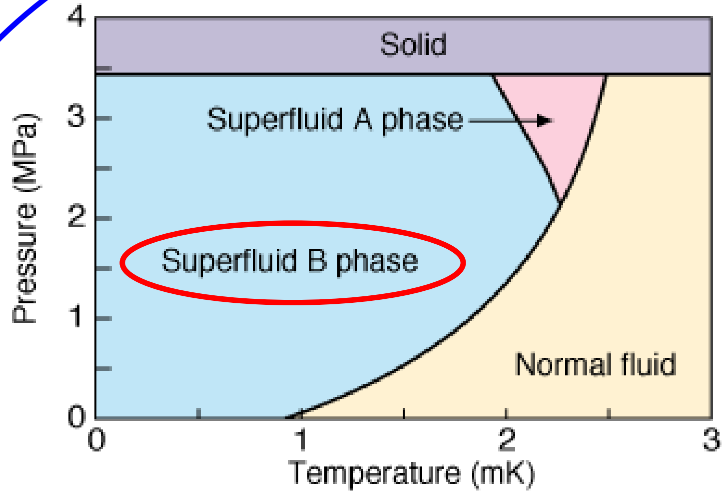


$Cu_xBi_2Se_3$? *(Sasaki et al., PRL 2011)*

- **Are these surface states truly free fermion systems? If not, how to describe/measure their interactions?**

“Free fermion” topological phases

- 3D topological phases discovered experimentally via detection of 2D surface states



^{3}He-B?

(Murakawa et al., JPSJ 2011)

Outline

- Motivation
- Interacting 3D topological superfluids: Surface Majorana fermions and bulk collective modes in ^{3}He-B
  - Y. J. Park, S. B. Chung, and JM, Phys. Rev. B 91, 054507 (2015)
- Conclusion

Theory of topological SF/SC

- Theory of topological superfluids/superconductors was developed by analogy with topological insulators (Volovik, Read, Green, Roy, Schnyder, Ryu, Furusaki, Ludwig, Kitaev, Qi, Hughes, Raghu, Zhang, …)

Topological Insulator (TI)		**Topological Superfluid/Superconductor (TSF/TSC)**
electrons	→	Bogoliubov QPs
Bloch Hamiltonian	→	BdG Hamiltonian
band gap	→	pairing gap

Theory of TSF/TSC: beyond BdG?

- But TI and TSF/TSC are fundamentally different: pairing comes from interactions!

- SF/SC pairing gap comes from a **dynamical order parameter**, while insulating band gap is **static**

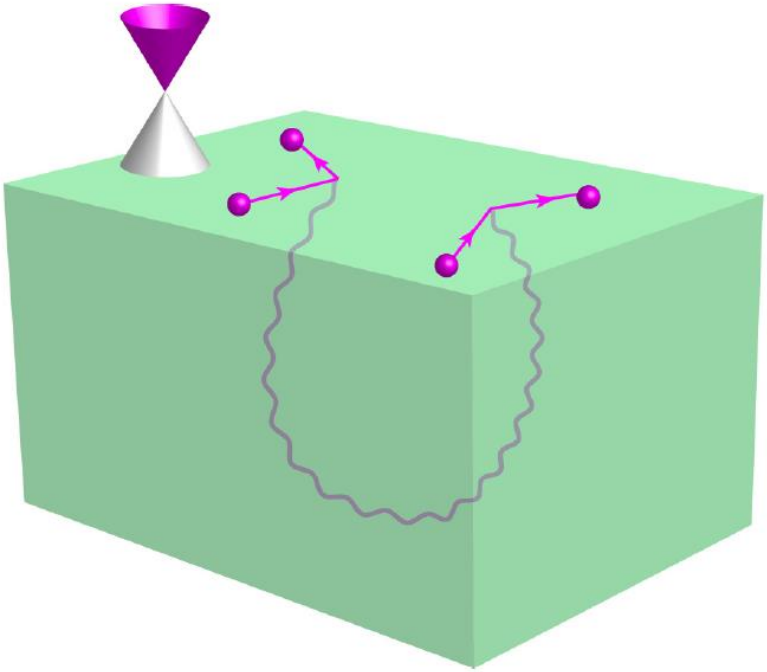
- BdG formalism = mean-field theory, ignores order parameter fluctuations (thermal and quantum)

- Bogoliubov QPs are “free fermions” in the BdG description

- How do OP fluctuations affect the physics of TSF/TSC?

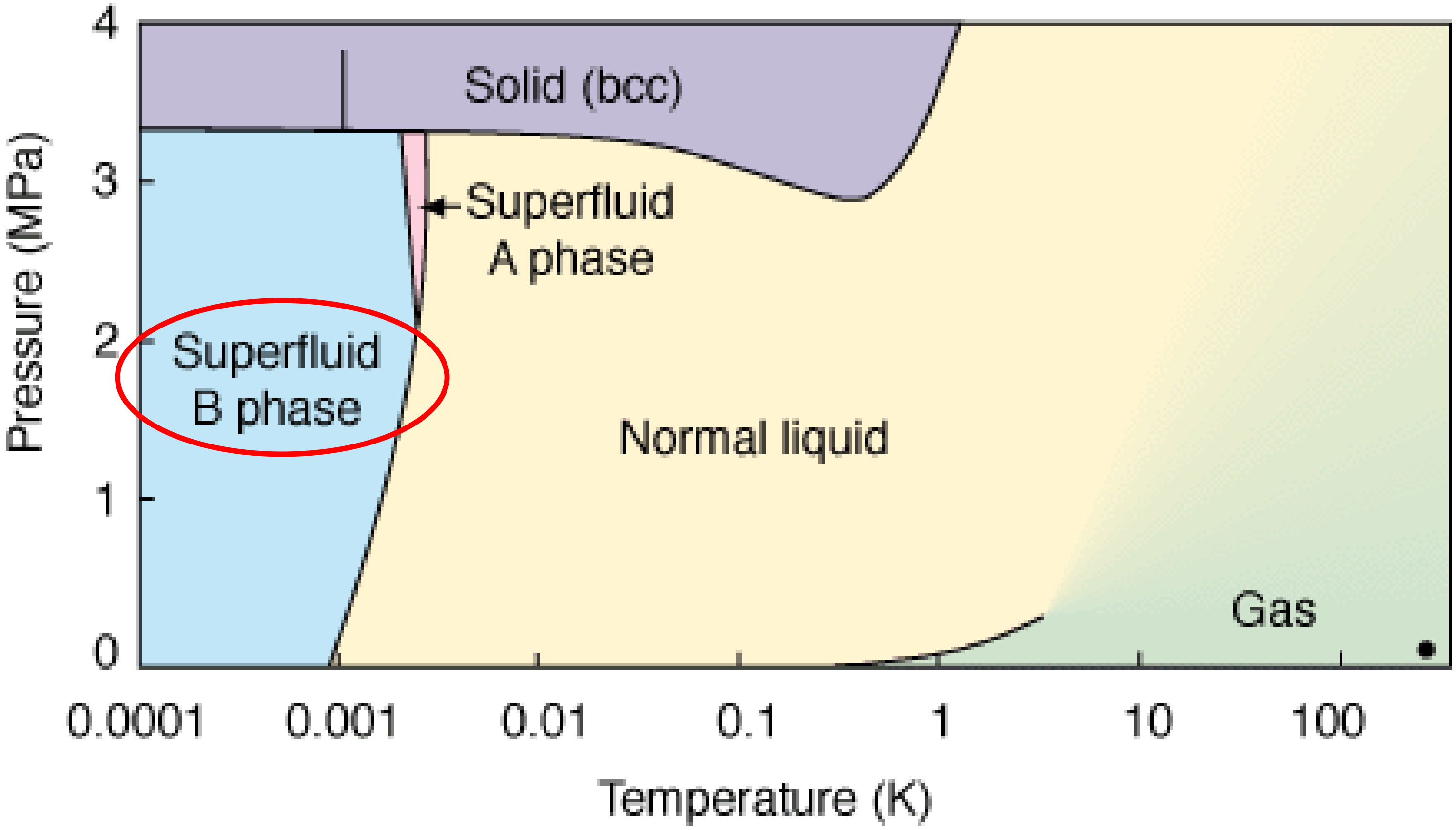
Theory of TSF/TSC: beyond BdG?

- Main message: bulk OP fluctuations can induce **interactions** among boundary Majorana fermions



- Focus on quantum (T=0) OP fluctuations in ^{3}He-B (class DIII TSF with ν=1)

^{3}He phase diagram



(credit: Aalto University)

Paired superfluids

- ^{3}He-B = superfluid of paired fermionic ^{3}He atoms

$$H = \sum_{\boldsymbol{k},\sigma} (\epsilon_{\boldsymbol{k}} - \mu) c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} + \frac{1}{2} \sum_{\boldsymbol{k},\sigma,\sigma'} (\Delta_{\sigma\sigma'}(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}\sigma} c^\dagger_{-\boldsymbol{k}\sigma'} + \mathrm{H.c.})$$

- In general, order parameter can have singlet and triplet components:

$$\Delta(\boldsymbol{k}) = (\Delta_s(\boldsymbol{k}) + \boldsymbol{d}_t(\boldsymbol{k}) \cdot \boldsymbol{\sigma})\, i\sigma^y$$

- Fermi statistics implies:

$$\Delta_s(-\boldsymbol{k}) = \Delta_s(\boldsymbol{k})$$ even-parity (s,d,…)

$$\boldsymbol{d}_t(-\boldsymbol{k}) = -\boldsymbol{d}_t(\boldsymbol{k})$$ odd-parity (p,f,…)

Balian-Werthamer state

- Bogoliubov QPs in ^{3}He-B are described by the Balian-Werthamer state = spin-triplet p-wave pairing (Balian & Werthamer 1963; Vdovin 1963)

$$\boldsymbol{d}_t(\boldsymbol{k}) = \frac{\Delta_0}{k_F} e^{i\phi} \boldsymbol{k}$$

$$k_x \propto Y_1^1(\hat{\boldsymbol{k}}) + Y_1^{-1}(\hat{\boldsymbol{k}})$$

$$k_y \propto i[Y_1^1(\hat{\boldsymbol{k}}) - Y_1^{-1}(\hat{\boldsymbol{k}})]$$

$$k_z \propto Y_1^0(\hat{\boldsymbol{k}})$$

Balian-Werthamer state

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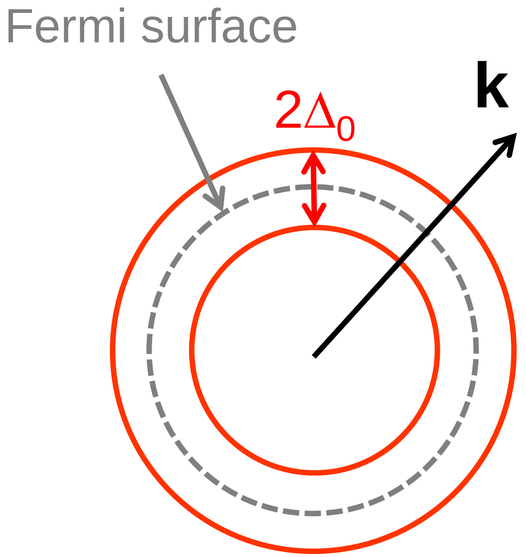
$$\boldsymbol{d}_t(\boldsymbol{k}) = \frac{\Delta_0}{k_F} e^{i\phi} \boldsymbol{k}$$

- QP spectrum is fully, isotropically gapped (for $\mu \neq 0$)

$$H_{\mathrm{BdG}}(\boldsymbol{k}) = \begin{pmatrix} \epsilon_{\boldsymbol{k}} - \mu & \frac{\Delta_0}{k_F} e^{i\phi} \boldsymbol{\sigma} \cdot \boldsymbol{k} \\ \frac{\Delta_0}{k_F} e^{-i\phi} \boldsymbol{\sigma} \cdot \boldsymbol{k} & -(\epsilon_{\boldsymbol{k}} - \mu) \end{pmatrix}$$

$$E_{\boldsymbol{k}} = \pm \sqrt{(\epsilon_{\boldsymbol{k}} - \mu)^2 + \left(\frac{\Delta_0}{k_F}\right)^2 \boldsymbol{k}^2}$$

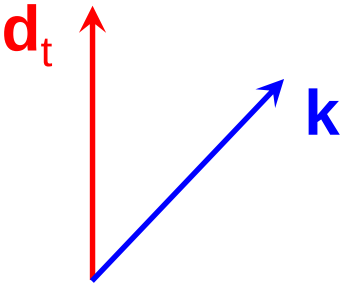


Balian-Werthamer state

$$E_{\boldsymbol{k}} = \pm\sqrt{(\epsilon_{\boldsymbol{k}} - \mu)^2 + \left(\frac{\Delta_0}{k_F}\right)^2 \boldsymbol{k}^2}$$

- Spectrum remains invariant if we rotate **k** by an arbitrary 3x3 rotation matrix $R^{(0)}$:

$$\boldsymbol{d}_t(\boldsymbol{k}) = \frac{\Delta_0}{k_F} e^{i\phi} R^{(0)} \boldsymbol{k}$$



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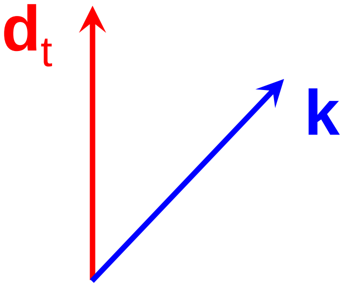


$$\boldsymbol{k}^2 \to \boldsymbol{k}^T [R^{(0)}]^T R^{(0)} \boldsymbol{k} = \boldsymbol{k}^2$$

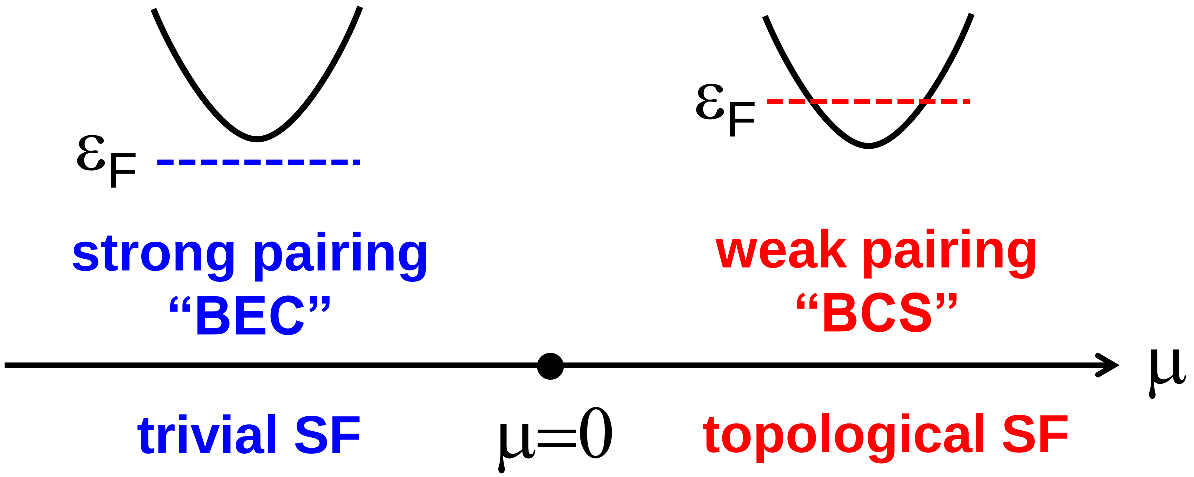
Balian-Werthamer state

- Order parameter: amplitude Δ_0 and Josephson phase ϕ, but also 3x3 matrix R of relative spin-orbit rotations

$$\Delta(\boldsymbol{k}) = \frac{\Delta_0}{k_F} e^{i\phi} \sigma^{\mu} i \sigma^{y} \boxed{R^{(0)}_{\mu j}} k_j$$

Strong vs weak pairing

- Can show that gapless point $\mu=0$ corresponds to a topological quantum phase transition between a trivial SF and a topological SF



- Unlike the BEC-BCS crossover for s-wave SF, in the p-wave case this is a genuine thermodynamic phase transition

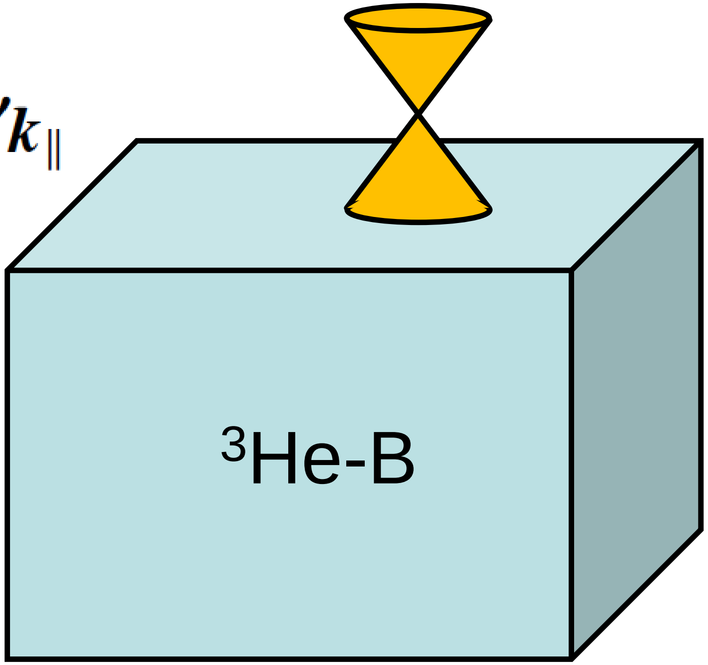
- ^{3}He-B corresponds to $\mu>0$: topological SF

Surface Majorana fermions

- Surface Andreev bound states = solution of BdG Hamiltonian in semi-infinite geometry, with **uniform**, **constant** background OP Δ_0, ϕ, R
- Gives free Majorana fermions on the surface, linearly dispersing inside bulk gap (Nagato, Higashitani, Nagai 2009; Chung & Zhang, 2009)

$$H_0 = \frac{\Delta_0}{2k_F} \sum_{\boldsymbol{k}_\parallel} \gamma^T_{-\boldsymbol{k}_\parallel} (\boldsymbol{k}_\parallel \cdot \boldsymbol{\sigma}) \gamma_{\boldsymbol{k}_\parallel}$$

$$E(\boldsymbol{k}_\parallel) = \Delta_0 \frac{|\boldsymbol{k}_\parallel|}{k_F}$$



Collective modes

- But OP Δ_0, ϕ, R = dynamical variables, with quantum and thermal fluctuations

- Focus on T=0 limit: only quantum OP fluctuations

- Amplitude modes have a gap ~ bulk QP gap, ignore in low-energy limit

- Four gapless bosonic Goldstone modes:

  - 1 phase mode: fluctuations of ϕ
  - 3 spin-orbit modes: fluctuations of R (Brinkman & Smith 1974)

Fermion-boson coupling

- Replace static OP in BdG Hamiltonian by **position/time-dependent OP (Goldstone) fields**

$$\Delta(\boldsymbol{k};\boldsymbol{R}) \simeq \frac{\Delta_0}{k_F}(1+i\varphi(\boldsymbol{R}))\sigma^{\mu} i\sigma^{y} R_{\mu j}(\boldsymbol{R})k_j$$

$$R_{\mu j}(\boldsymbol{R}) \simeq \left(\delta_{\mu\nu}+i\theta_{\alpha}(\boldsymbol{R})S^{(\alpha)}_{\mu\nu}\right)\delta_{\nu j}$$

$$H_{\text{coupling}} = \frac{1}{2V}\sum_{\boldsymbol{k},\,\boldsymbol{Q}} c^{\dagger}_{\boldsymbol{k}+\boldsymbol{Q}/2,\sigma}c^{\dagger}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\sigma'}\Delta_{\sigma\sigma'}(\underset{\substack{\nearrow \\ \text{relative} \\ \text{momentum}}}{\boldsymbol{k}};\ \underset{\substack{\nwarrow \\ \text{center-of-mass} \\ \text{momentum}}}{\boldsymbol{Q}})+\text{H.c.}$$

Surface-bulk coupling

- In a semi-infinite geometry, this implies a coupling between **surface Majorana fermions** and **bulk Goldstone modes**

- Of the phase mode ϕ and the spin-orbit modes θ_x, θ_y, θ_z, only θ_x couples to the Majorana fermions

$$H_{\text{coupling}} = \frac{\Delta_0}{V} \sum_{\boldsymbol{Q}} \theta_x(-\boldsymbol{Q}) \rho(\boldsymbol{Q})$$

$$\rho(\boldsymbol{Q}_{\parallel}) = \frac{1}{2k_F} \sum_{\boldsymbol{k}_{\parallel}} \gamma^T_{-\boldsymbol{k}_{\parallel}+\boldsymbol{Q}_{\parallel}/2} [\hat{\boldsymbol{x}} \cdot (\boldsymbol{k}_{\parallel} \times \boldsymbol{\sigma})] \gamma_{\boldsymbol{k}_{\parallel}+\boldsymbol{Q}_{\parallel}/2}$$

☐ ϕ does not couple because Majorana fermions are neutral

- Only one component of SO modes couples because spin of Majorana fermions is “Ising” (Nagato, Higashitani, Nagai 2009; Chung & Zhang 2009)

Effective surface theory

- Derive an effective surface theory by integrating out bulk SO mode θ_x

$$\int \mathcal{D}\theta_x \, e^{-S_B[\theta_x,\rho]} \propto e^{-S_I[\rho]}$$

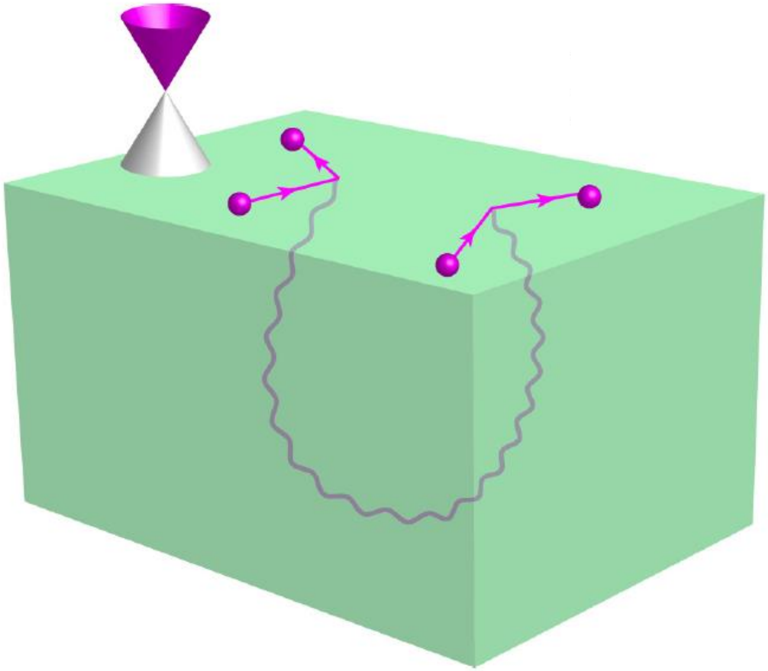
□ θ_x is not truly gapless because of dipole-dipole interaction: quadratic energy cost for θ_x deviating from preferred value θ_L = Leggett angle (Leggett 1973; Brinkman & Smith 1974)

$$\mathcal{L}_{\mathrm{dipole}} = \frac{1}{2} g_D \, \theta_x^2$$

Effective surface interaction

- Integrating out θ_x yields effective surface interaction = interaction between Majorana fermions mediated by exchange of bulk (quasi-)Goldstone bosons

$$H = \frac{v}{2} \sum_{\boldsymbol{k}} \gamma_{-\boldsymbol{k}}^{T} (\boldsymbol{k} \cdot \boldsymbol{\sigma}) \gamma_{\boldsymbol{k}} - \frac{g_0}{2} \sum_{\boldsymbol{Q}} \rho(-\boldsymbol{Q}) \rho(\boldsymbol{Q})$$



$$g_0 \approx \frac{\Delta_0^2}{4 L_{\parallel}^2 \xi_D g_D}$$

Possible broken symmetries

- Interaction is perturbatively irrelevant at the free Majorana fermion fixed point, but here coupling constant is finite

- Investigate possible broken symmetry states

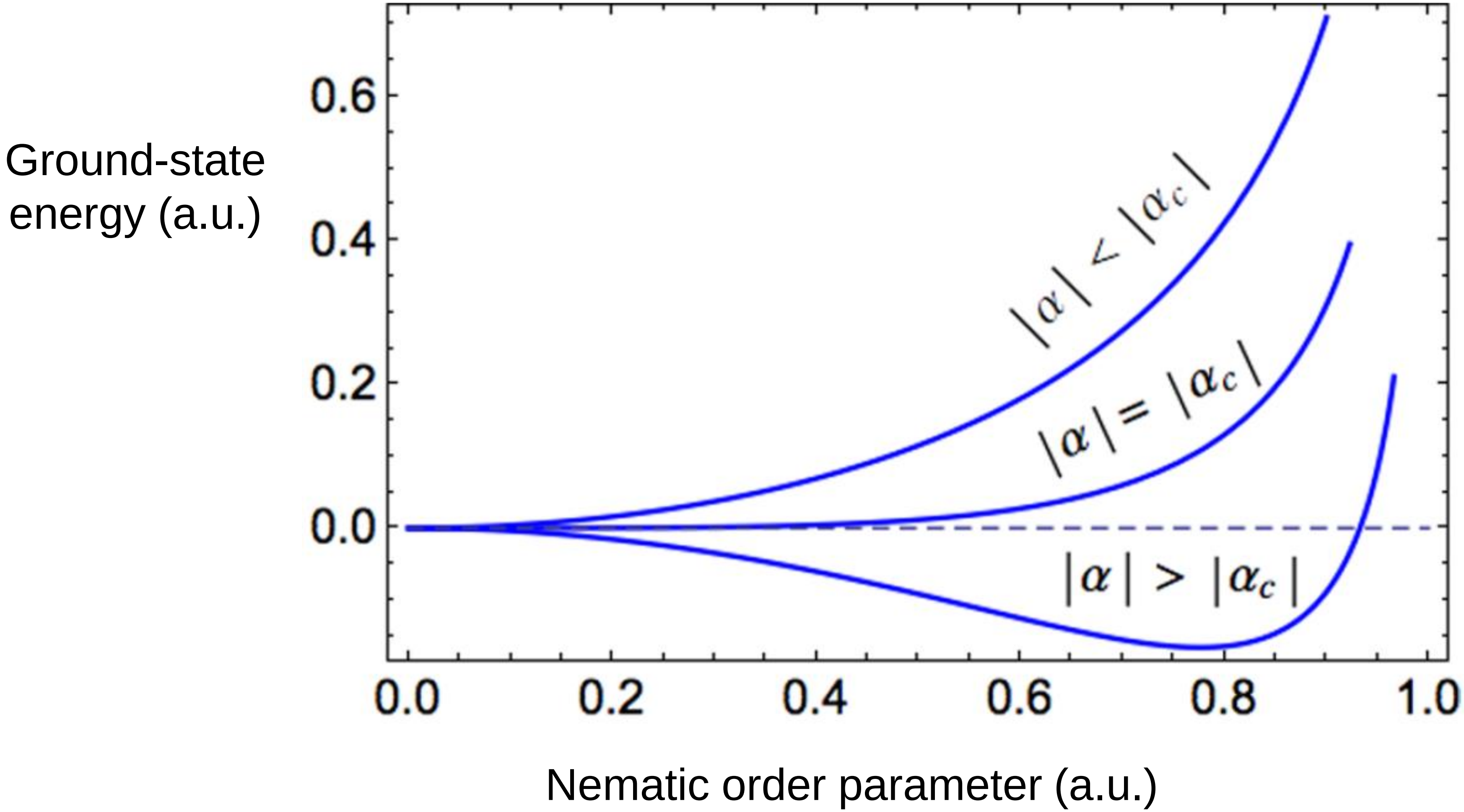
- To linear order in **k**, only allowed uniform order parameters are a T-breaking Ising OP (massive Majorana fermions) and a T-invariant nematic OP (gapless but anisotropic Majorana cone)

$$\mathcal{M} = \frac{1}{2}\sum_{\boldsymbol{k}} \gamma_{-\boldsymbol{k}}^{T} \sigma^{y} \gamma_{\boldsymbol{k}}$$

$$\mathcal{Q}_{ab} = \frac{1}{2k_F}\sum_{\boldsymbol{k}} \gamma_{-\boldsymbol{k}}^{T} (k_a \sigma^{b} + k_b \sigma^{a} - \delta_{ab} \boldsymbol{k} \cdot \boldsymbol{\sigma}) \gamma_{\boldsymbol{k}}$$

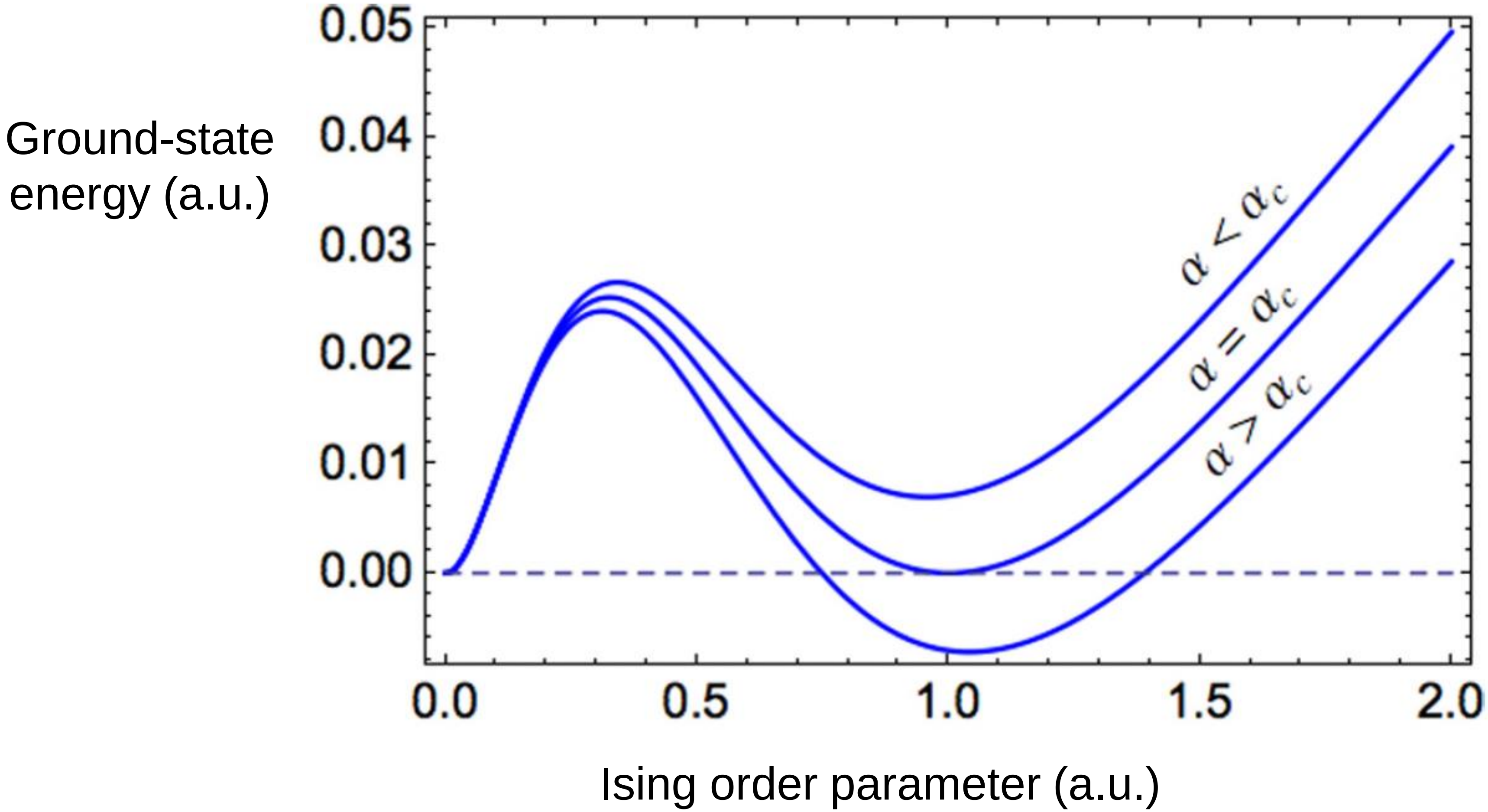
- Investigate those in mean-field theory

Nematic order



- Continuous nematic transition is possible for negative couplings, but here coupling is positive

Ising order



- First-order transition to T-breaking phase with gapped Majorana fermions is possible in principle

Summary

- Quantum fluctuations of the superfluid OP in ^{3}He-B can induce effective interactions among surface Majorana fermions
- First-order transition to T-breaking phase of gapped Majorana fermions is possible in this model
- Since evidence suggests gapless Majorana fermions in ^{3}He-B, may be in a regime with **metastable** T-breaking surface phase
- More exotic possibilities:
  - ➢ Fluctuations render T-breaking transition continuous → possibility of $\mathcal{N}$=1 SUSY quantum critical point (Grover, Sheng, Vishwanath 2014)
  - ➢ Symmetric strong-coupling phase with surface topological order (Fidkowski, Chen, Vishwanath 2013; Metlitski, Fidkowski, Chen, Vishwanath 2014)