Consistency Conditions for an AdS/MERA Correspondence

arXiv:1504.06632

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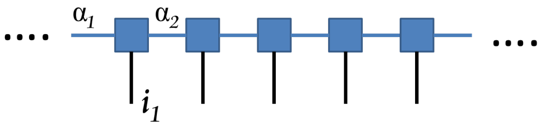
CAP Congress 2015, University of Alberta

Outline

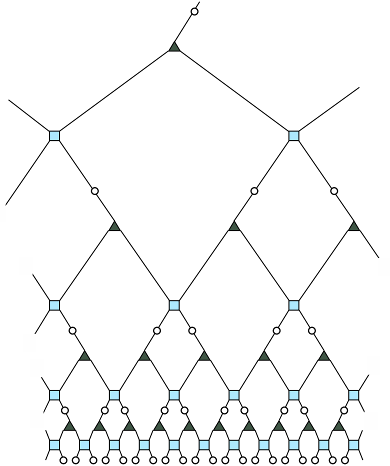
Tensor Networks

- Powerful tools for many body quantum systems (e.g. reviews arXiv:1306.2164; arXiv:0907.2796)
- Low lying energy states occupy a small part of the Hilbert Space
- Consider a 1d lattice with N sites
- Assign some Hilbert Space $\mathcal{H}_j$ to each lattice, $\mathcal{H} = \bigotimes_j \mathcal{H}_j$
- Generic State $|\psi\rangle = \sum C_{i_1 i_2 \ldots i_N} |i_1, i_2, \ldots, i_N\rangle$
- Break C into contraction of smaller tensors. e.g. $(C_{i_1 \ldots i_N} = A_{i_1}^{\alpha_1 \alpha_2} A_{i_2}^{\alpha_2 \alpha_3} \ldots A_{i_N}^{\alpha_N \alpha_1})$

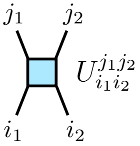


Multiscale Entanglement Renormalization Ansatz (MERA)

- Can write down tensor network for a state at criticality (G. Vidal arXiv:cond-mat/0512165)



Binary MERA (k=2)



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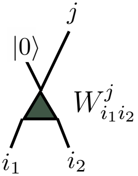


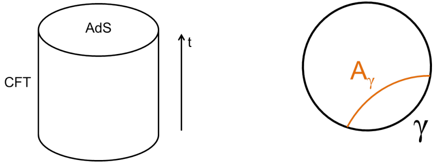
isometry

Multiscale Entanglement Renormalization Ansatz (MERA)

- Each tensor index can take values from 1 to χ where χ is called the bond dimension
- Computationally efficient ansatz (Vidal cond-mat/0512165, Pfeifer, Evenbly, Vidal arXiv:0810.0580)
- Good approximation for ground state of quantum systems at criticality (Rel. Err $< 3 \times 10^{-4}$).
- Produce the correct 2 point and 3 point functions.
- Can numerically find the central charge, OPE coefficients and scaling dimensions.

AdS/CFT Correspondence

- Conformal Field Theory (CFT) in d+1 dimensions is dual to theory of (quantum) gravity in d+2 Anti-de Sitter space.



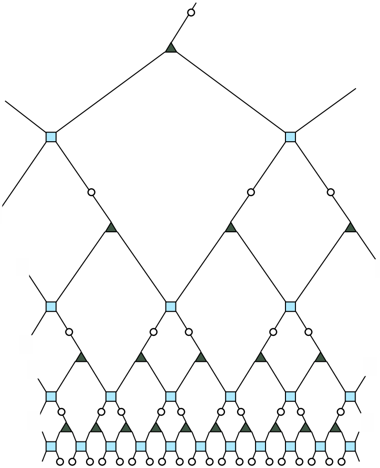
- Ryu-Takayanagi Formula

$$S_{CFT}(\gamma) = \frac{A_\gamma}{4G} \quad (1)$$

where A_γ is the minimal surface bounding region γ on the boundary.

Is MERA related to AdS/CFT?

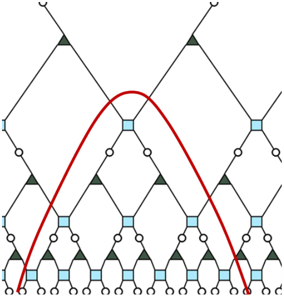
- MERA is good at approximating CFT ground states, is it related in any way to AdS?
- The network "looks like" a discretization of AdS with some UV cut off at a lattice spacing.



MERA and AdS/CFT

- "RT formula" in MERA,

$$S(\ell)_{\mathsf{MERA}} \sim \log(\ell) \sim \# \text{ bond} \times \log \chi \qquad (2)$$



- The minimal surface is given by the one with least number of bond cuts

Consistency Conditions

- Suppose MERA describes a valid AdS/CFT correspondence
- Needs to satisfy at least certain properties of CFT
  e.g. for some 1+1d CFT with central charge c

$$S(\ell)_{\mathrm{MERA}} = S_{\mathrm{CFT}}(\ell) = \frac{c}{3} \log \ell \tag{3}$$

  in the limit for $\ell = x_0/a$ large (a is UV cut off for CFT)
- Needs to satisfy certain properties of AdS
  e.g. the graph discretization has to capture features of AdS
- In particular, consistent with our current understanding of gravity (satisfies covariant entropy bound)

Consistency Condition on Central Charge

- Construction:
  - k to 1 MERA
  - Assume scale and translational invariance
  - Hilbert space V with dim $V = \chi$ for each boundary lattice site
  - Same bond dimension χ and tensors
- Can obtain an upper bound for $S_{MERA}(\ell)$ following 1310.8372

$$S_{MERA}(\ell) \leq 4(k-1)\log_k(\ell)\log(\chi) \tag{4}$$

- Combined with $S_{MERA}(\ell) = S_{CFT}(\ell) = \frac{c}{3}\log(\ell)$ for ℓ large
- Then

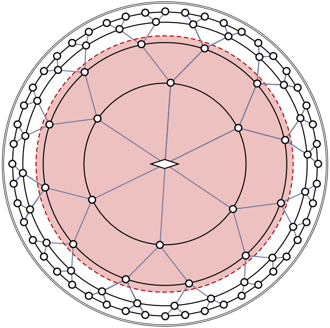
$$c \leq 12\left(\frac{k-1}{\ln k}\right)\log\chi \tag{5}$$

Limits on sub-AdS Scale Physics

- Naively would want discretization to correspond to scale in the UV
- Intuitively, with the fixed discretization, bond length is necessarily L or larger. (Swingle 0905.1317, Hartman, Maldacena 1303.1080)
- In 1504.06632 we do a more rigorous matching of AdS and graph geodesics
- In order to have consistent bulk AdS ,
  - Each bond length has to be AdS scale L or larger
  - Get consistent bulk physics only on scales much larger than L.

Do we have consistent bulk Quantum States?

- Assume correspondence, contrain on bulk states in bulk Hilbert space $\mathcal{H}_{\text{bulk}} = \mathcal{H}_{\text{boundary}}$
- Roughly speaking, we can assign Hilbert space V_{bulk} where $\dim V_{\text{bulk}} = (k-1)\chi$ at each *bulk* lattice site.



- Trace out a ball-shaped region $\mathcal{B}$ in the bulk, find its entanglement entropy

Constraints from Bousso Bound

- For a static geometry, Bousso Bound (1404.5635, 1406.4545) reduces to $S(\mathcal{B}) \leq \frac{A}{4G}$.
- (Bousso hep-th/0203101) Entropy of a system which we only know boundary area A is $\ln \dim \mathcal{H}_{\mathcal{B}}$. So we have

$$\ln \dim \mathcal{H}_{\mathcal{B}} \leq \frac{A}{4G} \propto \frac{L}{G} \tag{6}$$

- Combined with the bound on central charge, we have

$$\frac{k^2 \ln(k)}{2(k-1)^2} \leq 1 \tag{7}$$

cannot be satisfied by any $k \geq 2$.

- If we allow ancillae to be entangled, the bound is relaxed and $k = 2, 3, 4$ are allowed.

Conclusions and Outlook

- MERA appears to offer a more controlled version of AdS/CFT
- bond dimension exponentially large in c, no natural requirement for large c in MERA to yield classical geometry
- No sub-AdS Physics
  - cMERA
  - Local expansion of tensors into TN of other discretizations
- Seems that there is no consistent AdS/MERA in this construction
  - Allow entangled ancillae and identify them as bulk dof
  - Exact Holography Mapping (Qi 1309.6282)
  - Is MERA the space of geodesics in AdS? (Czech et al. 1505.05515 & Upcoming paper)
  - cMERA (Nozaki et al 1208.3469, Mollabashi et al arXiv:1311.6095, Miyaji et al, 1506.01353)

Thank You

MERA and AdS/CFT

- MERA tensor network reproduces discretized bulk AdS. (B. Swingle 0905.1317, 1209.3304)
- $\sim L$ (AdS radius) for each bond in the network and match geodesics
- For a geodesic connecting points x, y on the boundary where $|x-y| \gg$ (lattice spacing), get $d(x,y) \sim 2\log|x-y|$ simply by bond counting
- On the other hand, from properties of MERA

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_\Delta(y)\rangle \approx |x-y|^{-2\Delta} \tag{8}$$

- From AdS/CFT: $\langle \mathcal{O}_\Delta(x)\mathcal{O}_\Delta(y)\rangle \sim \exp(-\Delta d(x,y))$ therefore $d(x,y) \sim 2\log|x-y|$, consistent with bond counting.

Other Evidence for AdS/MERA

- MERA as RG flow (Molina-Vilaplana 1109.5592)
- Finite temperature MERA and BTZ black hole (e.g. Swingle 0905.1317, Matsueda, Ishihara, Hashizume 1208.0206, Molina-Vilaplana, Prior 1403.5395)
- Continuous MERA (cMERA) (Nozaki et al 1208.3469, Mollabashi et al arXiv:1311.6095, Miyaji et al, 1506.01353)