Experimental Test of the Leptonic Mixing (PMNS) matrix

Akira Konaka (TRIUMF) @CAP congress, June 2015
Neutrino Oscillation

\[ < \nu_l | \nu_i > = U_{li} : PMNS \text{ matrix} \]
\[ l = e, \mu, \tau : \text{weak eigenstates} \]
\[ i = 1, 2, 3 : \text{mass eigenstates} \]

Neutrinos are produced and detected as weak eigenstates and travels as mass eigenstates:

Interference

\[ P_{l \rightarrow l'} = | < \nu_{l'}(t) | \nu_l(t) > |^2 \]
\[ = | \sum_i < \nu_{l'} | \nu_i > e^{-iE_i t} < \nu_i | \nu_l > |^2 \]
\[ = \sum_i U_{li}^* U_{l'i} e^{-im_i^2 L/2E} = \sum_{i,j} U_{li}^* U_{l'i} (U_{lj}^* U_{l'j})^* e^{-i(m_i^2 - m_j^2)L/2E} \]
Unitarity of the PMNS matrix

- **Unitarity condition:** $U^\dagger U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- **Normalization**
  
  \[
  |U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1
  \]
  
  \[
  |U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1
  \]
  
  \[
  |U_{\tau 1}|^2 + |U_{\tau 2}|^2 + |U_{\tau 3}|^2 = 1
  \]

- **Unitarity triangle**

  \[
  U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0
  \]
  
  \[
  U_{\mu 1}U_{\tau 1}^* + U_{\mu 2}U_{\tau 2}^* + U_{\mu 3}U_{\tau 3}^* = 0
  \]
  
  \[
  U_{\tau 1}U_{e1}^* + U_{\tau 2}U_{e2}^* + U_{\tau 3}U_{e3}^* = 0
  \]

- **New physics can break the Unitarity of PMNS**
  - sterile neutrino or new interactions
e-μ unitarity of PMNS matrix

- e-μ part of unitarity can be studied the best:

\[
|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1 \\
|U_{\mu1}|^2 + |U_{\mu2}|^2 + |U_{\mu3}|^2 = 1 \\
U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 0
\]

Normalization

Unitarity triangle
• Disappearance tests normalization part of unitarity and the sides of the unitarity triangle:

\[ P_{\text{disapp}} = 1 - P_{\ell \rightarrow \ell} \]

\[ = 4 |U_{\ell 1}|^2 |U_{\ell 2}|^2 \sin^2 \Delta_{12} + 4 |U_{\ell 2}|^2 |U_{\ell 3}|^2 \sin^2 \Delta_{23} + 4 |U_{\ell 3}|^2 |U_{\ell 1}|^2 \sin^2 \Delta_{31} \]

\[ \Delta_{12} = \Delta m_{12}^2 L/4E \]
\[ \Delta_{23} = \Delta m_{23}^2 L/4E \]
\[ \Delta_{31} = \Delta m_{31}^2 L/4E \]

\[ |U_{e 1}|^2 + |U_{e 2}|^2 + |U_{e 3}|^2 = 1 \]
\[ |U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1 \]
\[ |U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1 \]

- **Solar neutrino**: \( |U_{e2}|^2 \)
  - MSW conversion in the sun: \( \nu_e \rightarrow \nu_2 \) and detect \( \nu_e \)
- **KamLand**: \( |U_{e1}|^2 |U_{e2}|^2 \)
  - Reactor \( \nu_e \) disappearance at \( \Delta_{12} \) scale of L/E
- **Reactor \( \theta_{13} \)**: \( |U_{e3}|^2 (|U_{e1}|^2 + |U_{e2}|^2) \)
  - Reactor \( \nu_e \) disappearance at \( \Delta_{13} \) scale of L/E

- **All the parameters are already measured**:
  - Better precision for \( |U_{e1}|^2 \) and \( |U_{e2}|^2 \) are expected by JUNO, untangling \( \Delta_{23} \) and \( \Delta_{31} \) contributions
\[ |U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1 \]

- LBL $\nu_\mu$ disappearance: $|U_{\mu 3}|^2 (|U_{\mu 1}|^2 + |U_{\mu 2}|^2)$
  - T2K/MINOS/NOvA/SK/ICECUBE measure this.
- $\nu_\mu \rightarrow \nu_e$ atm. matter resonance ($\sim 6\text{GeV}$): $(r|U_{\mu 3}|^2 - 1)$
  - Amplitude of the atm. $\nu$ mass hierarchy study [SK/HK]
- $\Delta_{12}$ scale atm. $\nu_\mu$ disapp. : $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
  - sub-GeV up-going atm. $\nu_\mu$ disapp. [HK]

- Good potential to untangle all three parameters: HyperK will play a major role
Appearance

Each of the CP angles \((\alpha, \beta, \gamma)\) correspond to each L/E oscillation length; \(\Delta_{23}, \Delta_{31}, \Delta_{12}\)

\[
P_{\ell \to \ell'} = 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}
\]

\[
a = |U_{\ell 1}| |U_{\ell' 1}|
\]
\[
b = |U_{\ell 2}| |U_{\ell' 2}|
\]
\[
c = |U_{\ell 3}| |U_{\ell' 3}|
\]

\[
\Delta_{12} = \Delta m_{21}^2 L/4E
\]
\[
\Delta_{23} = \Delta m_{23}^2 L/4E
\]
\[
\Delta_{31} = \Delta m_{31}^2 L/4E
\]
\( U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0 \)

- \( a, b, c \) can be determined by disappearance
- \( \alpha, \beta \) from LBL \( \nu_\mu \rightarrow \nu_e \) CP violation, \( \gamma \) from atm. \( \nu_\mu \rightarrow \nu_e \) CP
  - T2K/T2HK/DUNE will determine combination of \( \alpha \) and \( \beta \) [T2K/HK]
  - \( \gamma \) can be determined by the 0.5–0.8GeV atm. \( \nu_\mu \rightarrow \nu_e \) [SK/HK]
  - Atm. \( \nu_\mu \rightarrow \nu_e \) (2nd to 10th max.) untangles \( \beta(\Delta_{31}) \) and \( \alpha(\Delta_{23}) \) [HK]
Atmospheric $\nu_\mu \rightarrow \nu_e$

- $\nu_\mu \rightarrow \nu_e$ at several GeV: matter resonance
  - Mass hierarchy determination
- $\Delta_{12}$ oscillation in the sub-GeV region:
  - large $\theta_{12}$ effect
CP violation effect is large! $P_{\mu e} \sim$ up to 20%
Sub-GeV CP sensitivity of SK

- Sensitivity to reject $\delta_{CP}=0$ with existing data:
  - Statistical: $3.6\sigma$ at the T2K-reactor best fit point
  - Current SK analysis with systematics: $1.6\sigma$

R. Calland

Wendel@Neutrino2014
• Atm. $\nu$ offers wide range of L/E in oscillation
  • However, taken by the community with “grain of salt”
  • Good statistics, limited by “unknown” systematics

• Emerging opportunity for precision atm. $\nu$: Control systematic errors by
  • Neutrino flux
    • AMS data provides precise primary cosmic ray flux
    • Hadron production studies developed by CERN-NA61
  • Neutrino cross section
    • Model independent cross section proposal by nuPRISM
• Primary cosmic ray flux is now measured at 1-2% level.
• Open the door for very precise atm.ν studies
vPRISM (T2K/HyperK ND) concept
• Atm. \( \nu \) covers wide range of oscillation parameters
  
  • Realistic scenario emerges to control systematic uncertainties:
    
    • Precise AMS cosmic ray measurement
    
    • Precision hadron production study established by NA61:
      nuPRISM proposal for model independent \( \nu \) cross section study

• Precision test of e-\( \mu \) PMNS Unitarity
  
  • 9 parameters with 5 constraints to be tested
    
    • \( |U_{e1}|^2, |U_{e2}|^2, |U_{e3}|^2, |U_{\mu 1}|^2, |U_{\mu 2}|^2, |U_{\mu 3}|^2, \alpha, \beta, \gamma \)
    
    • Unitarity triangle (3 constraints), normalization (2)
  
  • All 9 parameters can be measured:
    
    • Typical expected accuracy would be 10–20%
    
    • HyperK will lead the new measurements: \( |U_{\mu i}|^2, \alpha, \beta, \gamma \)
      JUNO will improve \( |U_{ei}|^2 \)
  
• HyperK will play a leading role in establishing PMNS paradigm, similar to what B-factory did for CKM
\[ P_{l \rightarrow l'} = \left| \langle \nu_l(t) | \nu_{l'}(t) \rangle \right|^2 \]

\[ = \left| \sum_i < \nu_l' | \nu_i > e^{-iE_i t} < \nu_i | \nu_{l'} > \right|^2 \]

\[ = \left| \sum_i U_{l'i}^* U_{l'j} e^{-i \frac{m_i^2 L}{2E}} \right|^2 = \sum_{i,j} U_{l'i}^* U_{l'j} (U_{ij}^* U_{jj'}) e^{-i (m_i^2 - m_j^2) \frac{L}{2E}} \]

\[ \Delta_{12} = \Delta m_{12}^2 \frac{L}{4E} \]

\[ \Delta_{23} = \Delta m_{23}^2 \frac{L}{4E} \]

\[ \Delta_{31} = \Delta m_{31}^2 \frac{L}{4E} \]
νPRISM (T2K/HyperK ND) concept

-0.5  

+1.0  

-0.2  

ν beam  

Take linear combinations

Muon p-θ

νPRISM

4.0° Off-axis Flux

2.5° Off-axis Flux

1.0° Off-axis Flux

Neutrino Energy (GeV)

Neutrino Flux at 0.34° off-axis angle (degrees) = 1.06
Increasing the statistics

- **Accelerator upgrades**
  - beam power: 250kW → 700kW
  - by increased rep. rate:
    - every 3sec → 1sec
  - keeping the similar instantaneous rate

- **Far detector upgrade:** Hyper-Kamiokande
  - 1Mton water Cerenkov
  - x20 larger fiducial volume than SK
Systematic uncertainties

- Systematic uncertainty of 1–2% is required for the wide range of CP discovery
- Challenge: $\nu$ cross section systematics (multi-nucleon correlation, final state interaction)
T2K systematic uncertainties

- Systematic uncertainties in event rate at SK:

<table>
<thead>
<tr>
<th>Source</th>
<th>$\nu_e$ app.</th>
<th>$\nu_\mu$ disapp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{flux \times cross section}$</td>
<td>3.1%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Cross section</td>
<td>5.3%</td>
<td>5.8%</td>
</tr>
<tr>
<td>SK detector</td>
<td>2.7%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Total</td>
<td>6.8%</td>
<td>7.7%</td>
</tr>
</tbody>
</table>

- [Flux $\times$ cross section]: ND280 constrains
  - place near detector further away than 280m
- Cross section: Nuclear effects
  - water target with well controlled $\nu$ beam
- SK detector: SK detector efficiency
  - water Cherenkov calibration and $\nu$ beam test
• $\sigma(\nu_e)/\sigma(\nu_\mu)$ ratio
  • For $\nu_e$ appearance, the near/far detectors observe $\nu_e/\nu_\mu$, respectively
  • In the first order, it is the same: kinematical phase space, radiative correction
• $\sigma(\bar{\nu}_\mu)/\sigma(\nu_\mu)$ and $\sigma(\bar{\nu}_e)/\sigma(\nu_e)$ ratio
  • angular distribution is very different between neutrinos and anti-neutrinos
  • nuclear effect can be different
• nuPRISM to address these
  • nuSTORM ($\pi$ storage ring) is another proposal
Atmospheric neutrinos

Produced by cosmic ray proton interacting in earth atmosphere:

\[ r = \frac{\Phi(\nu_\mu)}{\Phi(\nu_e)} \sim 2 \]
for \( E_\nu < 3 \text{GeV} \)

arXiv:1102.2688
Cancellation of atm. $\nu_e$ appearance

$P_{ee} = 1 - P_{e\mu} - P_{e\tau} \approx 1 - 2P_{e\mu}$

$P_{e\mu} \approx P_{e\mu}$

$P_{e\tau} \approx P_{e\mu}$

Flux

$1 - 2P_{e\mu}$

$2P_{e\mu}$

$\nu_e$ $\nu_\mu$ $\nu_\tau$ $\nu_e$

$\nu_\mu$ $\nu_\mu$ $\nu_\mu$
atm. $\nu_e$ app. sensitive to T(CP) viol.

Flux

1

$\nu_e$

$P_{ee} = 1 - P_{e\mu} - P_{e\tau} \approx 1 - 2P_{e\mu}$

$P_{e\mu}$

$P_{e\tau} \approx P_{e\mu}$

$\cos^2 2\theta_{23} = 1$

$1 - 2P_{e\mu}$

2

$\nu_\mu$

$P_{\mu e} \approx P_{e\mu}$

$P_{\mu e} \approx P_{e\mu}$

$2P_{e\mu}$

$1$

T (CP) conservation
atm. $\nu_e$ app. sensitive to T(CP) viol.

$\nu_e$ app. sensitive to T(CP) viol.

$P_{ee} = 1 - P_{e\mu} - P_{e\tau} \approx 1 - 2P_{e\mu}$

$\cos^2 2\theta_{23} = 1$

Flux

1. $\nu_e$ to $\nu_e$
2. $\nu_\mu$ to $\nu_e$

$T$ (CP) conservation

$P_{\mu e} \approx P_{e\mu} + \Delta P_{e\mu}$

$\Delta P_{e\mu} = P_{\mu e} - P_{e\mu}$

$2P_{e\mu} + 2\Delta P_{e\mu}$

$1 + 2\Delta P_{e\mu}$