Experimental Test of the Leptonic Mixing (PMNS) matrix

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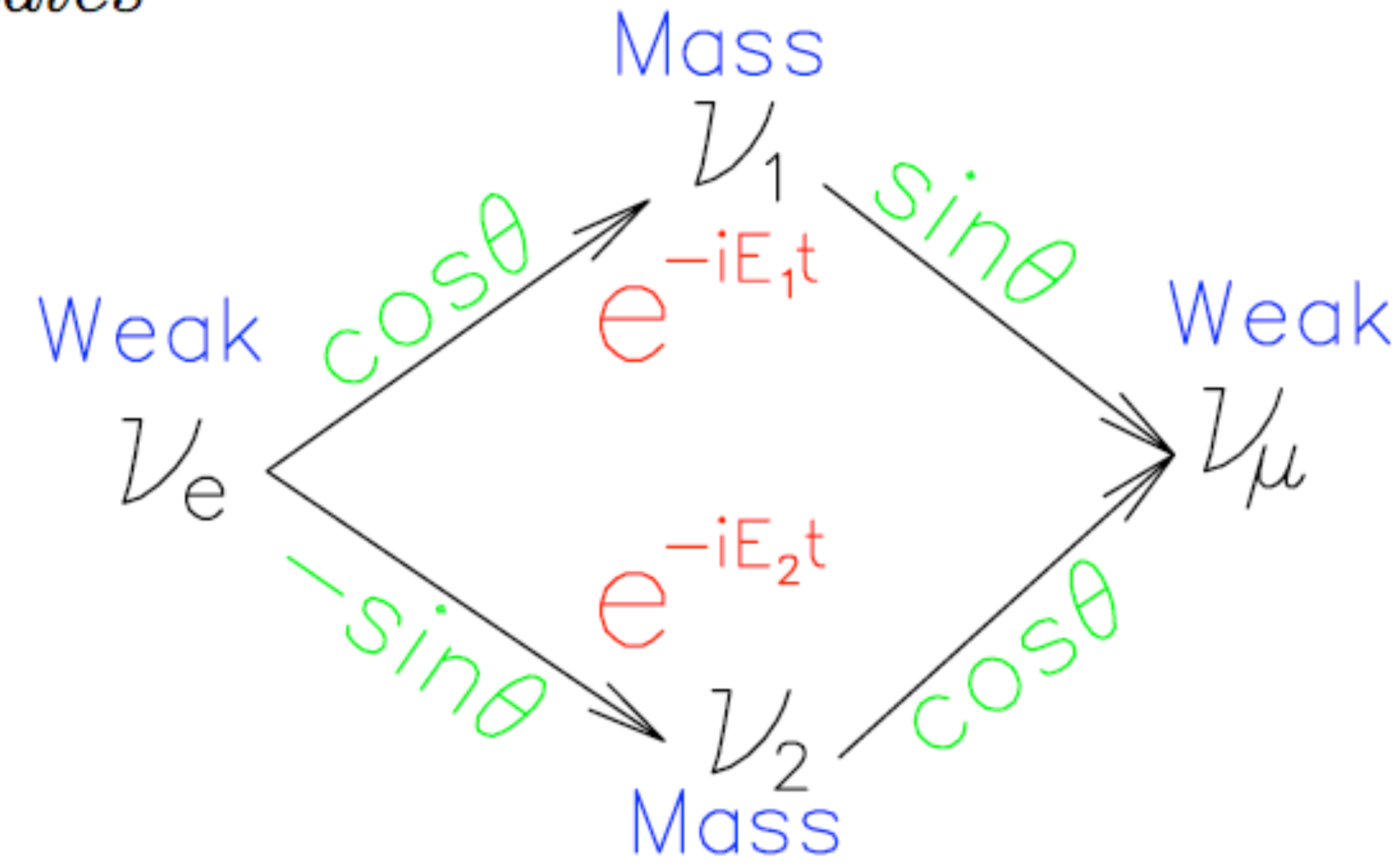
Neutrino Oscillation

$$
\begin{aligned}
&< \nu_l|\nu_i >= U_{li} : PMNS\ matrix \\
&\quad l = e, \mu, \tau : weak\ eigenstates \\
&\quad i = 1, 2, 3\ \ : mass\ eigenstates
\end{aligned}
$$

(Unitary matrix)

Neutrinos are produced and detected as weak eigenstates and travels as mass eigenstates:

Interference

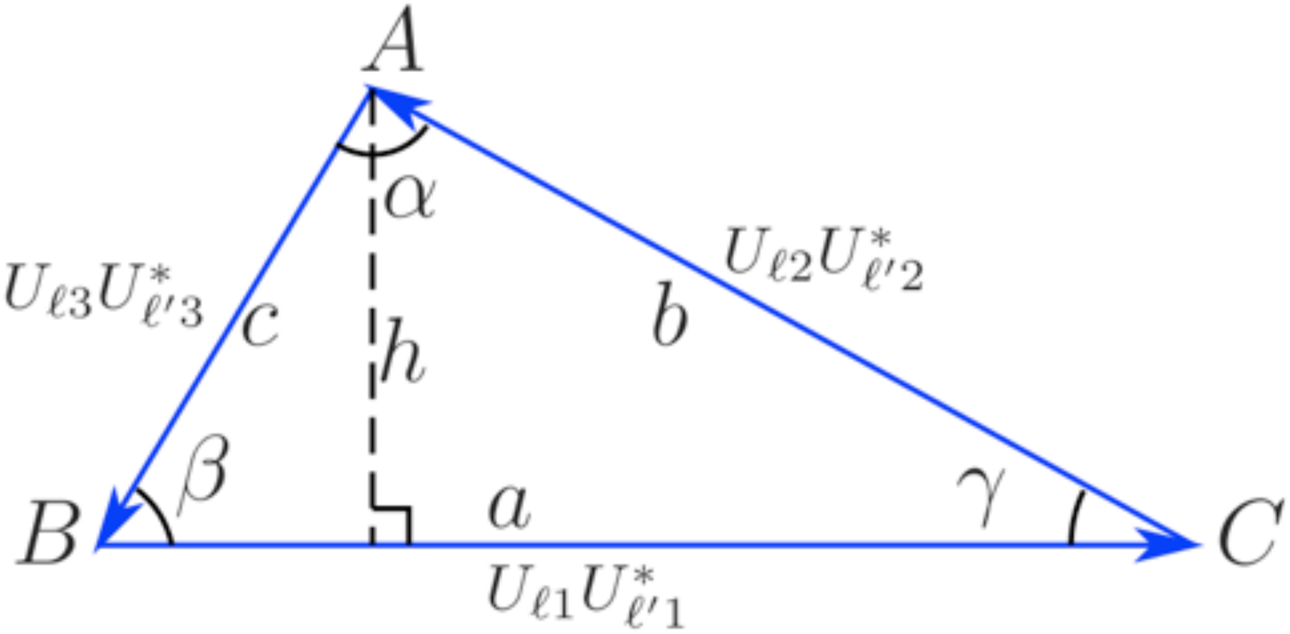


$$
\begin{aligned}
P_{l\to l'} &= |< \nu_{l'}(t)|\nu_l(t) > |^2 \\
&= |\sum_i < \nu_{l'}|\nu_i > e^{-iE_it} < \nu_i|\nu_l > |^2 \\
&= |\sum_i U^*_{li}U_{l'i}e^{-im_i^2L/2E}|^2 = \sum_{i,j} U^*_{li}U_{l'i}(U^*_{lj}U_{l'j})^* e^{-i(m_i^2-m_j^2)L/2E}
\end{aligned}
$$

Unitarity of the PMNS matrix

- Unitarity condition: $U^\dagger U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
  - Normalization

$$|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2 = 1$$
$$|U_{\mu 1}|^2+|U_{\mu 2}|^2+|U_{\mu 3}|^2 = 1$$
$$|U_{\tau 1}|^2+|U_{\tau 2}|^2+|U_{\tau 3}|^2 = 1$$



  - Unitarity triangle

$$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3} = 0$$
$$U_{\mu 1}U^*_{\tau 1}+U_{\mu 2}U^*_{\tau 2}+U_{\mu 3}U^*_{\tau 3} = 0$$
$$U_{\tau 1}U^*_{e1}+U_{\tau 2}U^*_{e2}+U_{\tau 3}U^*_{e3} = 0$$

sum of three complex numbers (vectors) =0

- New physics can break the Unitarity of PMNS
  - sterile neutrino or new interactions

e-μ unitarity of PMNS matrix

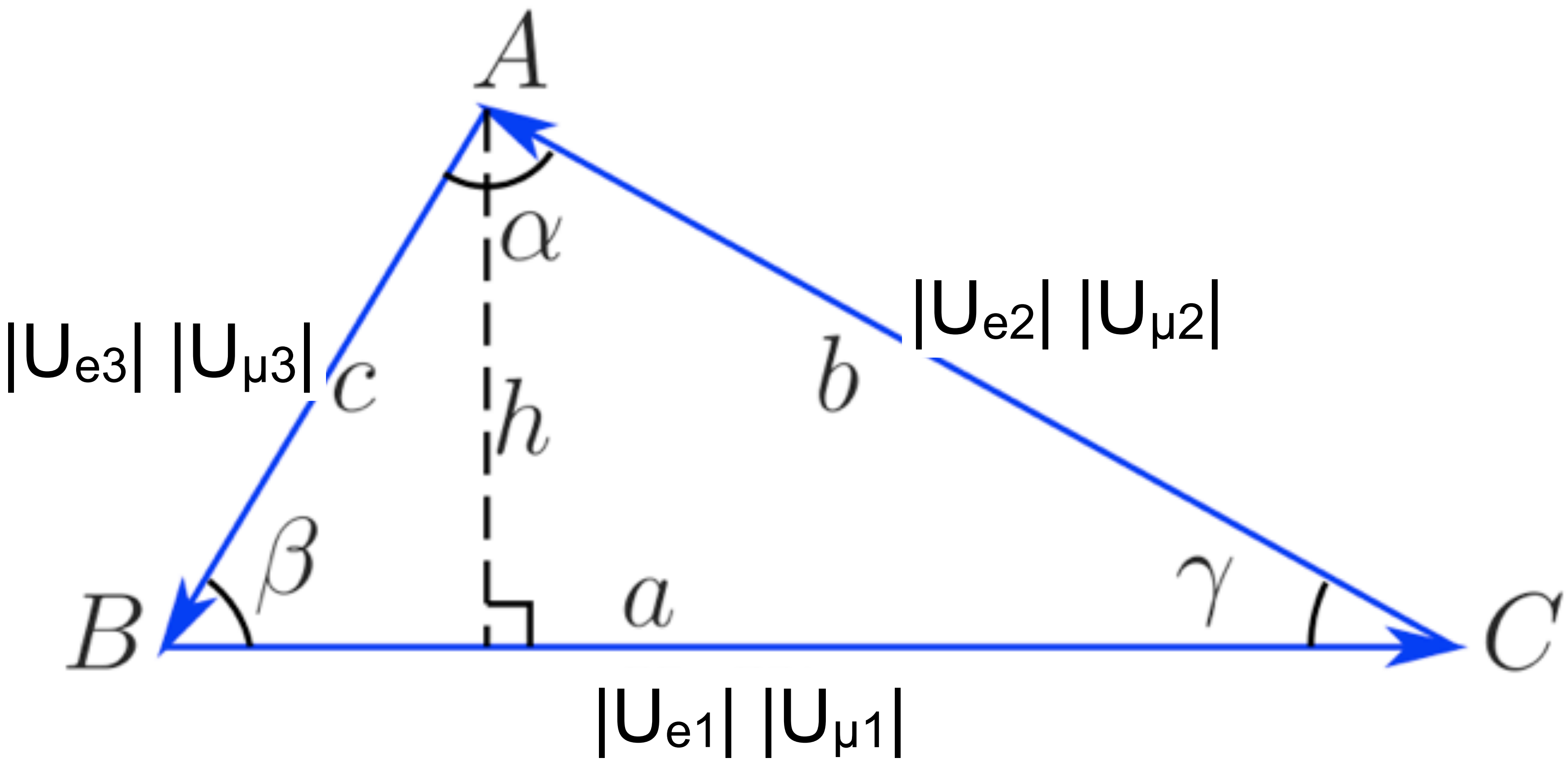
- e-μ part of unitarity can be studied the best:

$$|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2 = 1$$
$$|U_{\mu 1}|^2+|U_{\mu 2}|^2+|U_{\mu 3}|^2 = 1$$

Normalization

$$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3} = 0$$

Unitarity triangle



Disappearance

- Disappearance tests normalization part of unitarity and the sides of the unitarity triangle:

$$\boxed{\begin{aligned} P_{\mathrm{disapp}} &= 1-P_{\ell\to\ell} \\ &= 4|U_{\ell1}|^2\,|U_{\ell2}|^2\,\sin^2\Delta_{12} \\ &\quad + 4|U_{\ell2}|^2\,|U_{\ell3}|^2\,\sin^2\Delta_{23} \\ &\quad + 4|U_{\ell3}|^2\,|U_{\ell1}|^2\,\sin^2\Delta_{31} \end{aligned}}$$

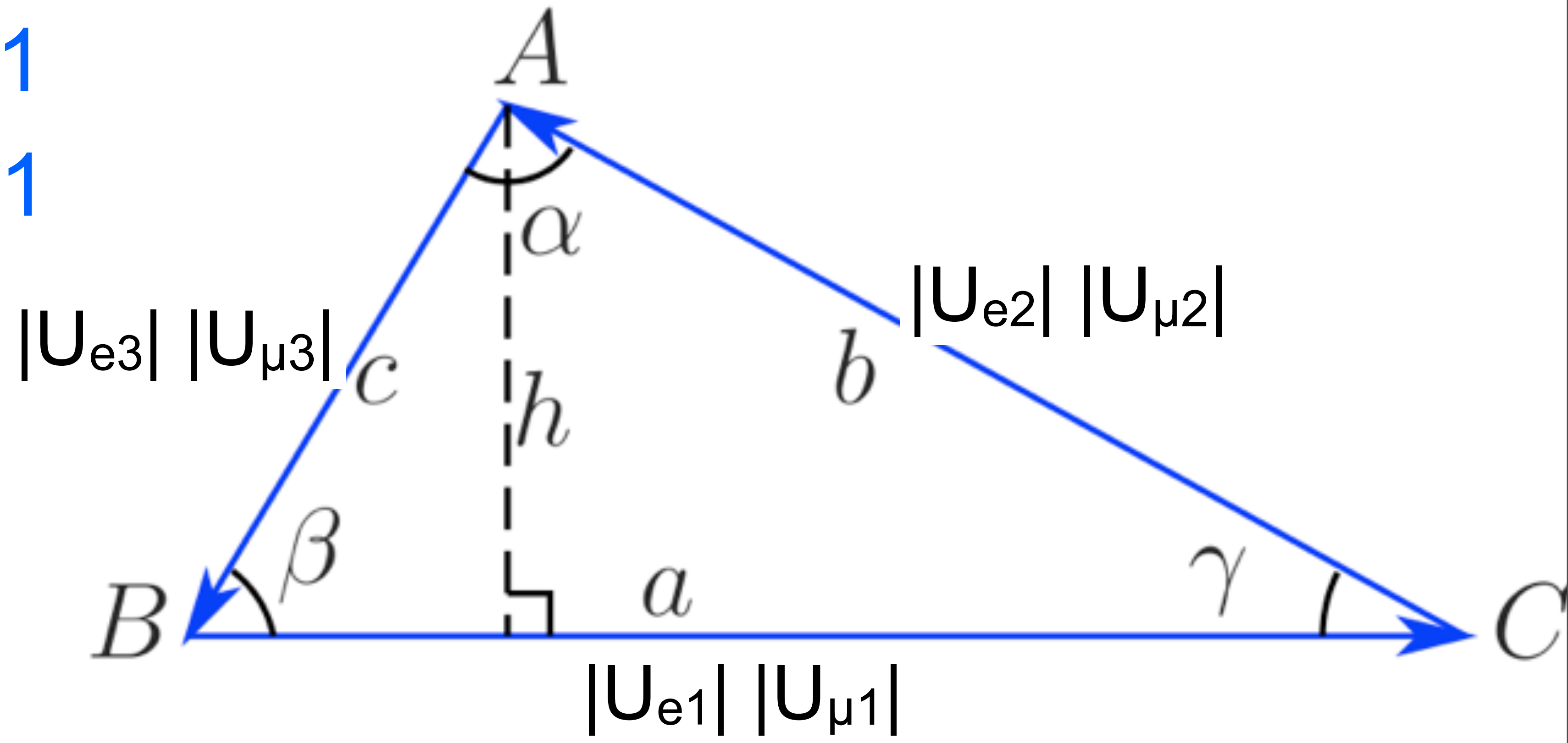
$\Delta_{12}=\Delta m^2_{12}L/4E$

$\Delta_{23}=\Delta m^2_{23}L/4E$

$\Delta_{31}=\Delta m^2_{31}L/4E$

$|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2 = 1$

$|U_{\mu1}|^2+|U_{\mu2}|^2+|U_{\mu3}|^2 = 1$



$|U_{e1}|^2+|U_{e2}|^2+|U_{e3}|^2 = 1$

- Solar neutrino: $|U_{e2}|^2$
  - MSW conversion in the sun: $\nu_e \rightarrow \nu_2$ and detect ν_e
- KamLand: $|U_{e1}|^2|U_{e2}|^2$
  - Reactor ν_e disappearance at Δ_{12} scale of L/E
- Reactor θ_{13}: $|U_{e3}|^2 \left(|U_{e1}|^2 + |U_{e2}|^2\right)$
  - Reactor ν_e disappearance at Δ_{13} scale of L/E

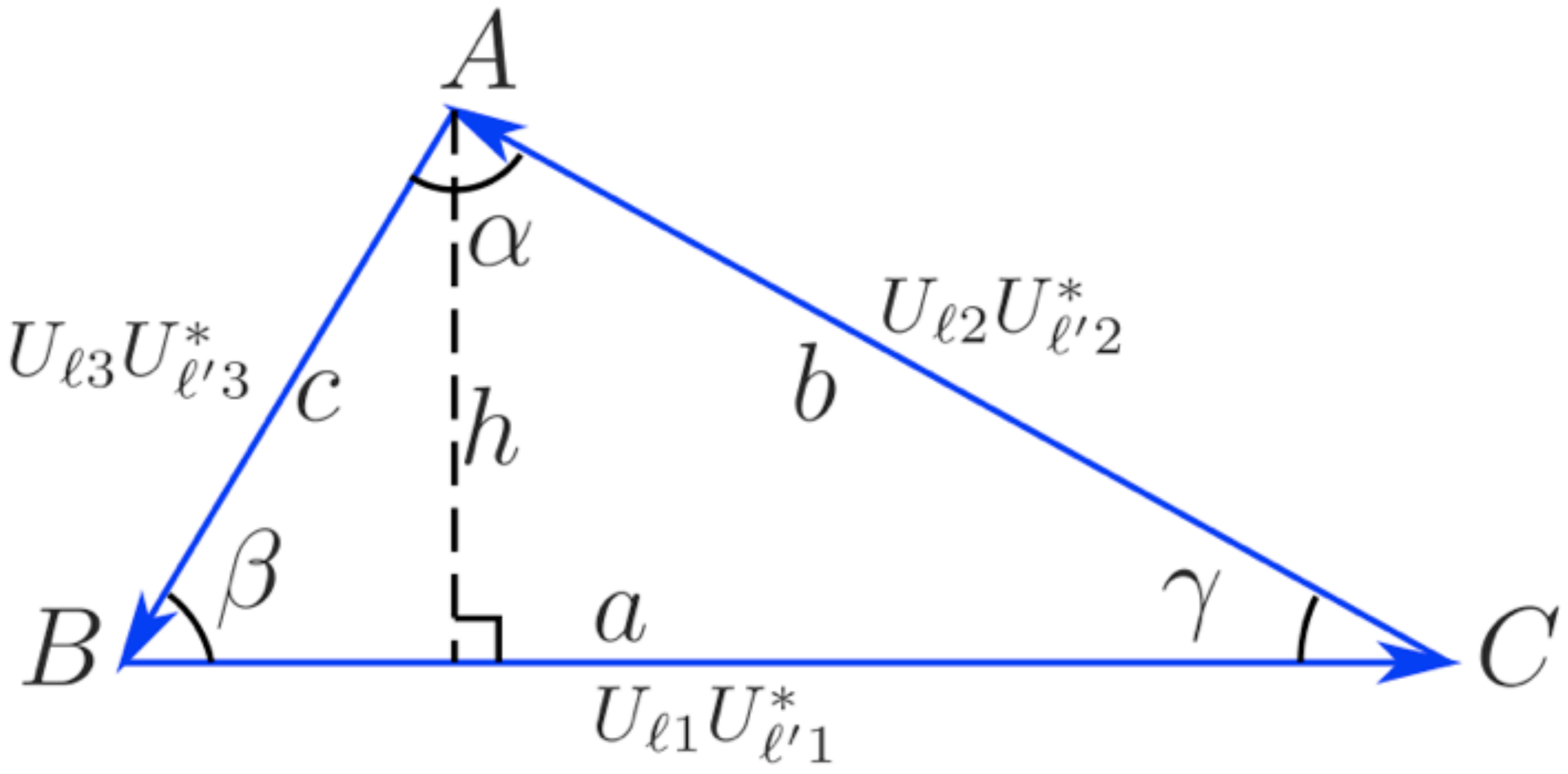
- All the parameters are already measured:
  - Better precision for $|U_{e1}|^2$ and $|U_{e2}|^2$ are expected by JUNO, untangling Δ_{23} and Δ_{31} contributions

$|U_{\mu 1}|^2+|U_{\mu 2}|^2+|U_{\mu 3}|^2 = 1$

- LBL ν_μ disappearance: $|U_{\mu 3}|^2$ $(|U_{\mu 1}|^2 + |U_{\mu 2}|^2)$
  - T2K/MINOS/NOvA/SK/ICECUBE measure this.
- $\nu_\mu \rightarrow \nu_e$ atm. matter resonance (~6GeV): $(r|U_{\mu 3}|^2 - 1)$
  - Amplitude of the atm. ν mass hierarchy study [SK/HK]
- Δ_{12} scale atm. ν_μ disapp. : $|U_{\mu 1}|^2|U_{\mu 2}|^2$
  - sub-GeV up-going atm. ν_μ disapp. [HK]

- Good potential to untangle all three parameters: HyperK will play a major role

Appearance (PRD89(2014)073002)



$a=|U_{\ell 1}|\,|U_{\ell' 1}|$
$b=|U_{\ell 2}|\,|U_{\ell' 2}|$
$c=|U_{\ell 3}|\,|U_{\ell' 3}|$

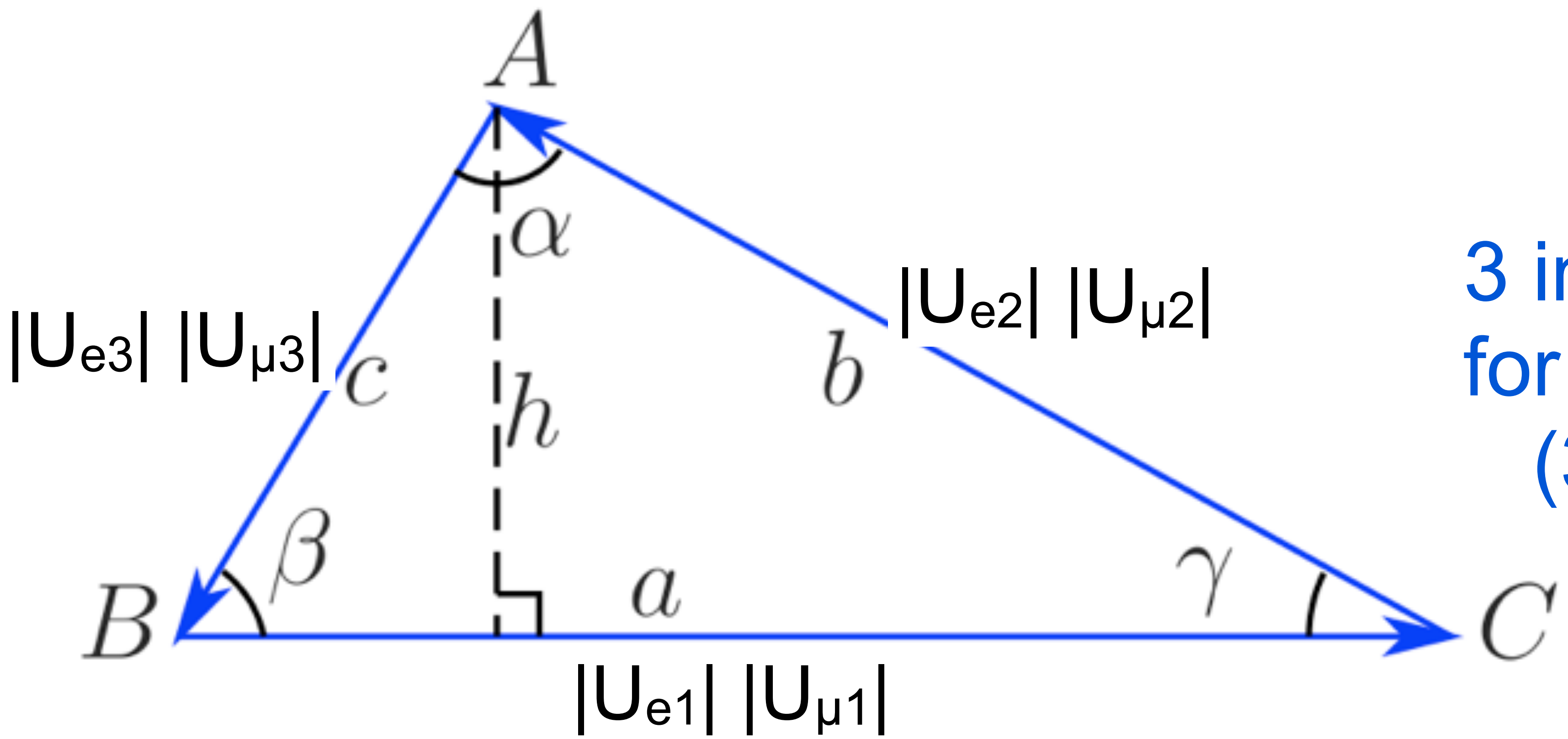
$\Delta_{12}=\Delta m^2_{12}L/4E$
$\Delta_{23}=\Delta m^2_{23}L/4E$
$\Delta_{31}=\Delta m^2_{31}L/4E$

Appearance

$$\begin{aligned}P_{\ell\to\ell'} &= 4ab\sin(\Delta_{12}\pm\gamma)\sin\Delta_{12}\\ &+4bc\sin(\Delta_{23}\pm\alpha)\sin\Delta_{23}\\ &+4ac\sin(\Delta_{31}\pm\beta)\sin\Delta_{31}\end{aligned}$$

Each of the CP angles (α,β,γ) correspond to each L/E oscillation length; Δ_{23}, Δ_{31}, Δ_{12}

$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3} = 0$

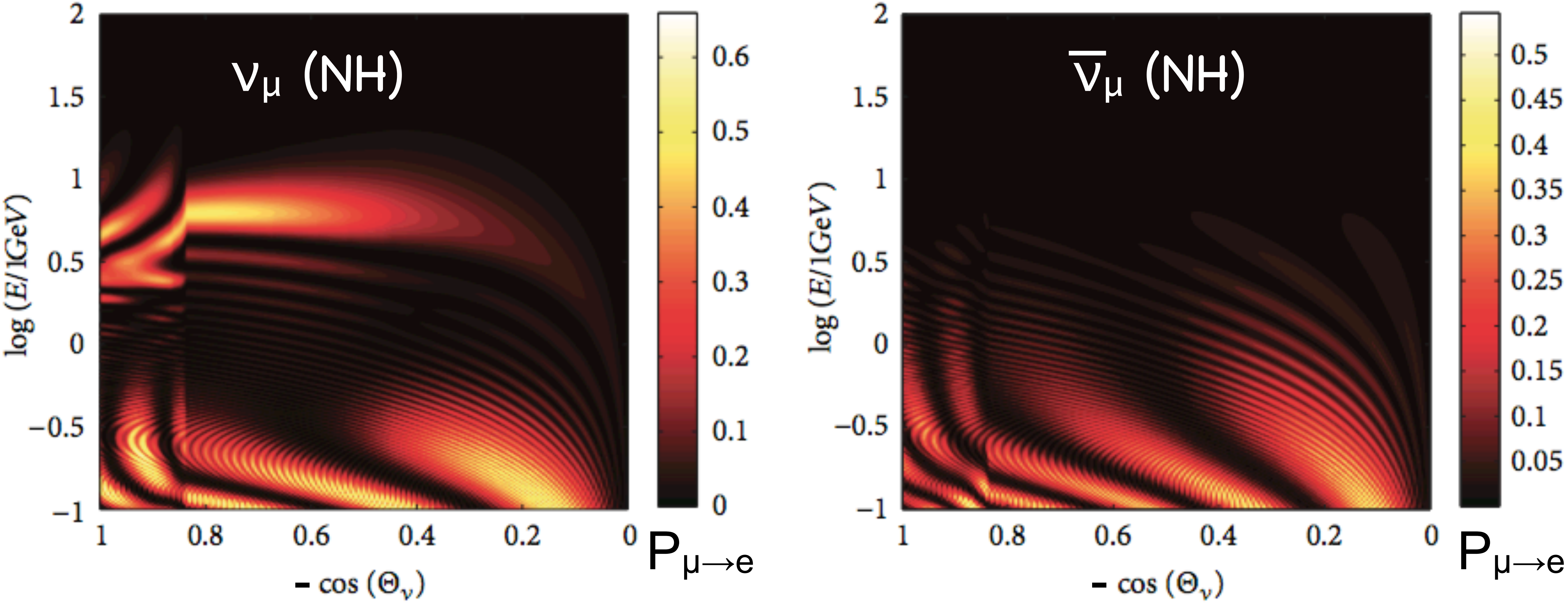


3 independent parameter for 6 observables
(3 constraints)

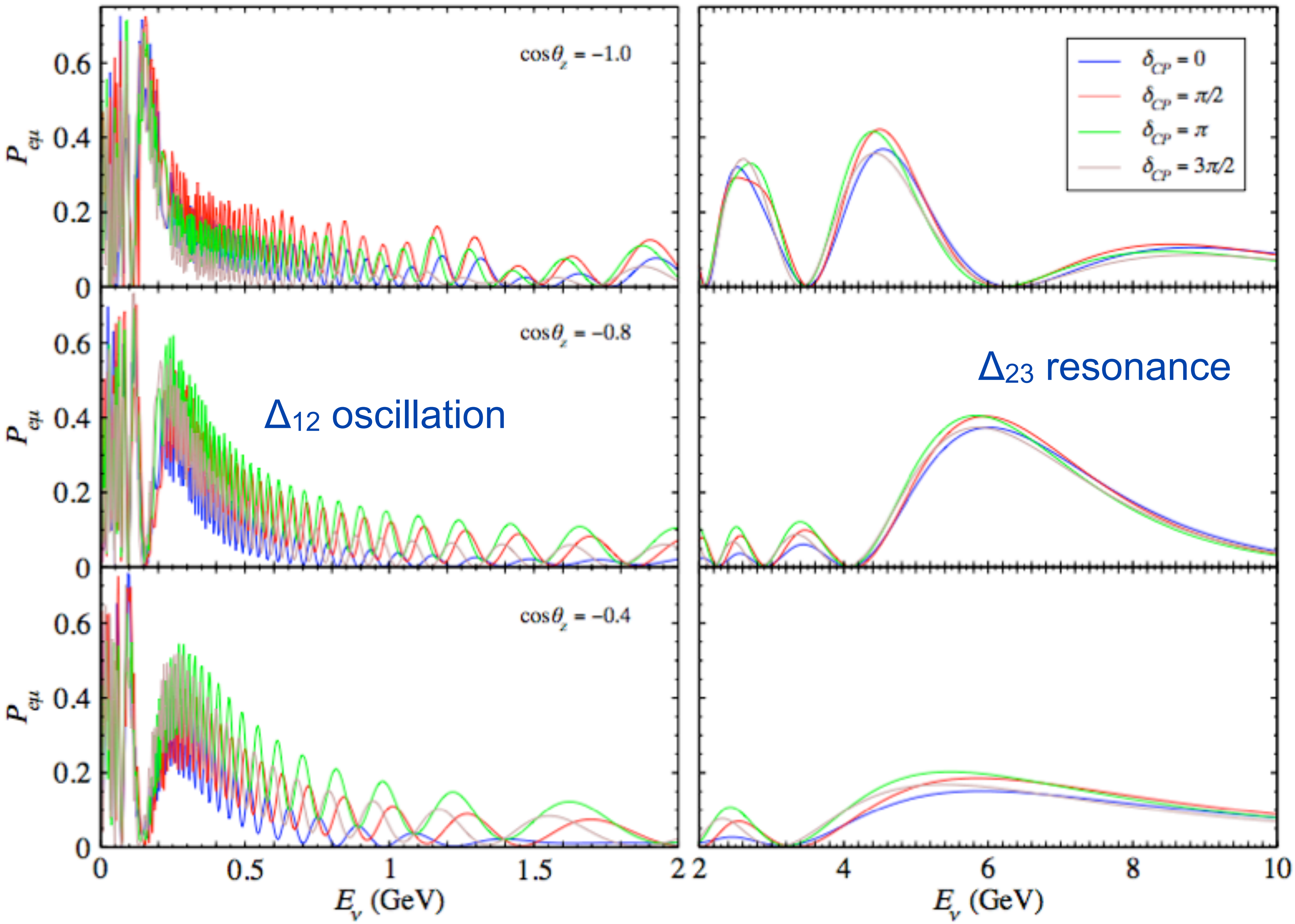
- a,b,c can be determined by disappearance
- α,β from LBL $\nu_\mu \rightarrow \nu_e$ CP violation, γ from atm. $\nu_\mu \rightarrow \nu_e$ CP
  - T2K/T2HK/DUNE will determine combination of α and β [T2K/HK]
  - γ can be determined by the 0.5-0.8GeV atm. $\nu_\mu \rightarrow \nu_e$ [SK/HK]
  - Atm. $\nu_\mu \rightarrow \nu_e$ (2nd to 10th max.) untangles $\beta(\Delta_{31})$ and $\alpha(\Delta_{23})$ [HK]

Atmospheric $\nu_\mu \to \nu_e$

- $\nu_\mu \to \nu_e$ at several GeV: matter resonance
  - Mass hierarchy determination
- Δ_{12} oscillation in the sub-GeV region:
  - large θ_{12} effect



arXiv:1406.1407 (Smirnov et.al.)



Δ_{12} oscillation

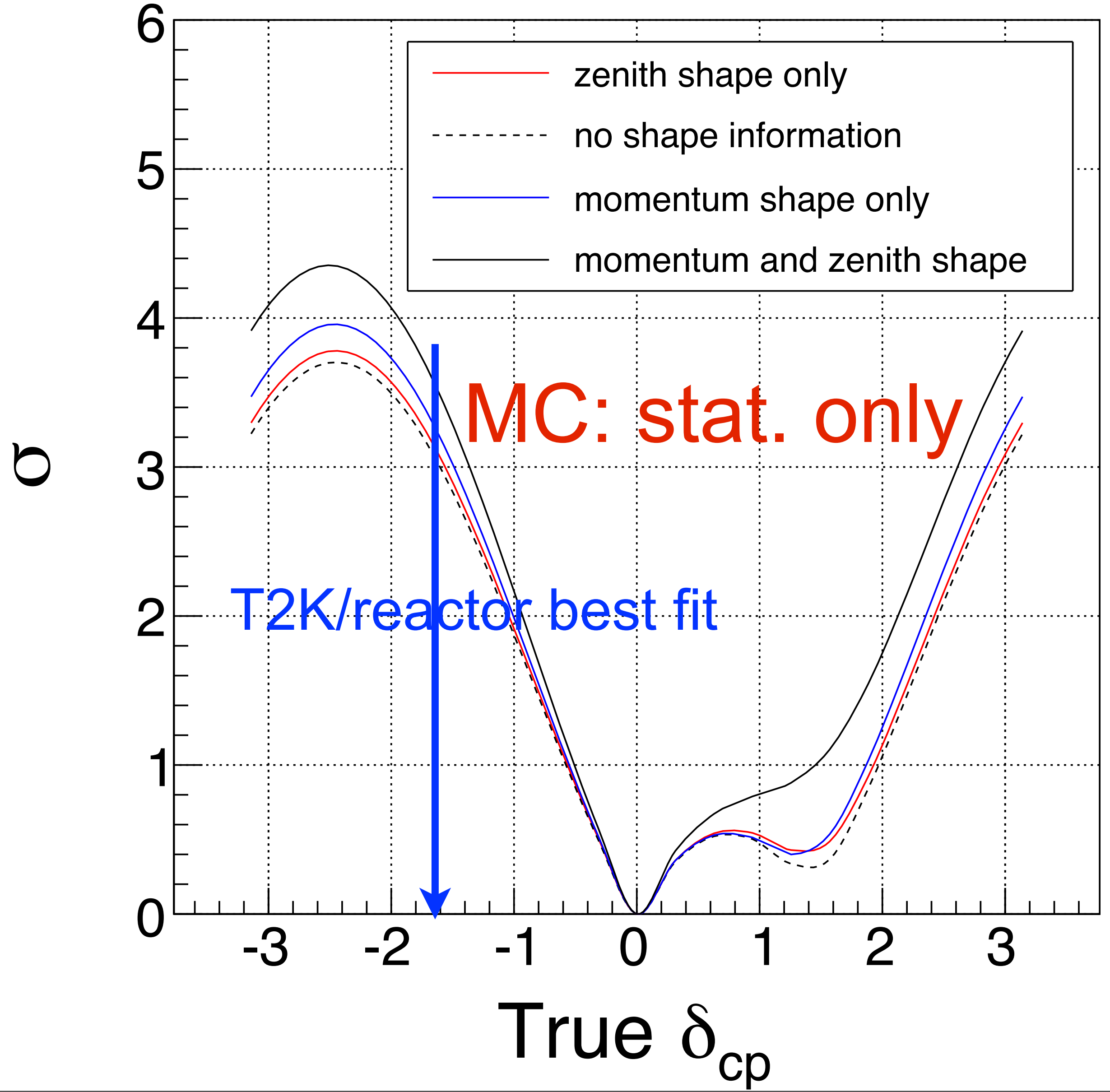
Δ_{23} resonance

CP violation effect is large! $P_{\mu e}$ ~ up to 20%

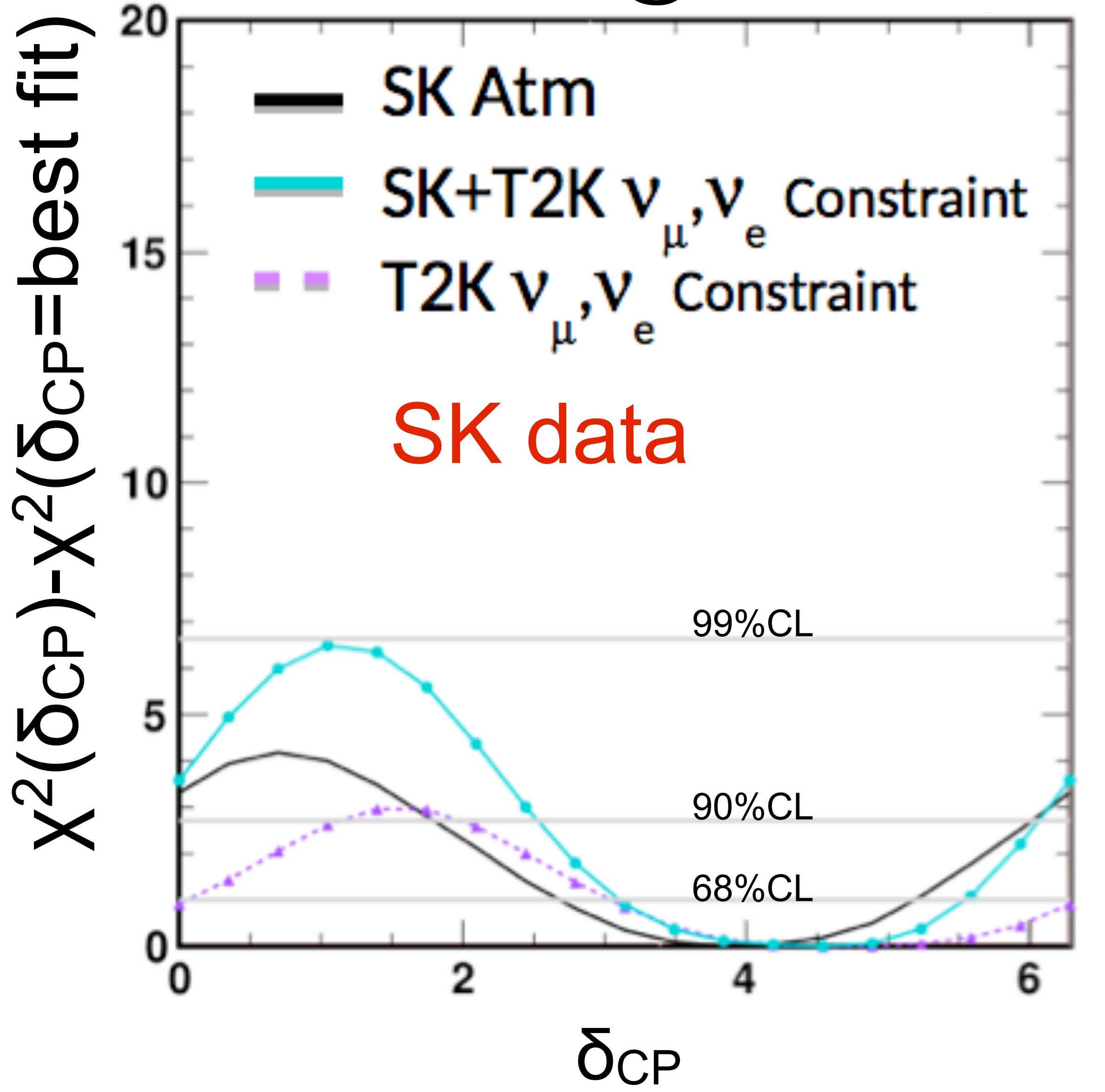
Sub-GeV CP sensitivity of SK

- Sensitivity to reject $\delta_{CP}=0$ with existing data:
  - Statistical: 3.6σ at the T2K-reactor best fit point
  - Current SK analysis with systematics: 1.6σ

R. Calland



Wendel@Neutrino2014

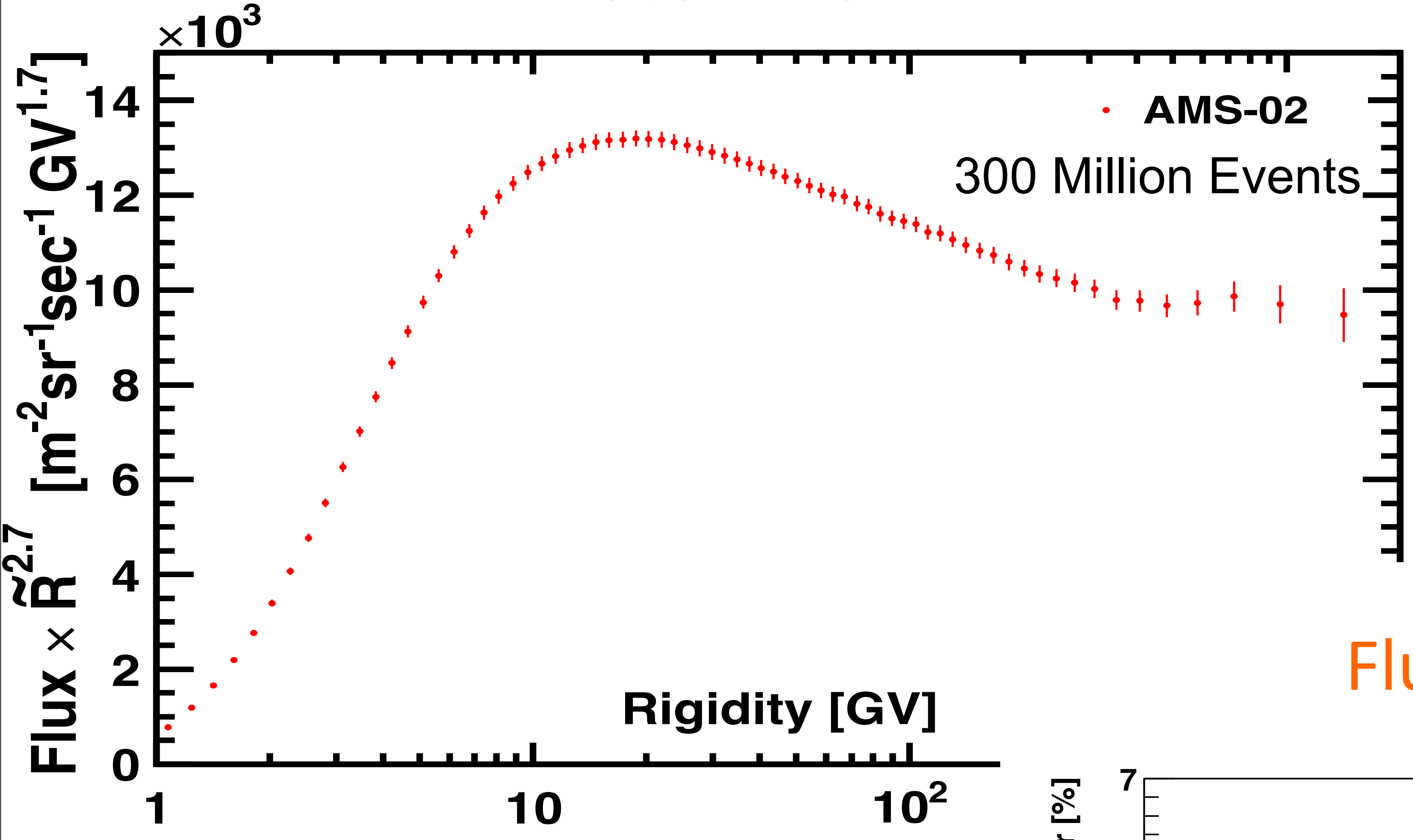


“Grain of Salt” ?

- Atm. ν offers wide range of L/E in oscillation
  - However, taken by the community with “grain of salt”
  - Good statistics, limited by “unknown” systematics
- Emerging opportunity for precision atm. ν: Control systematic errors by
  - Neutrino flux
    - **AMS** data provides precise primary cosmic ray flux
    - Hadron production studies developed by **CERN-NA61**
  - Neutrino cross section
    - Model independent cross section proposal by **nuPRISM**

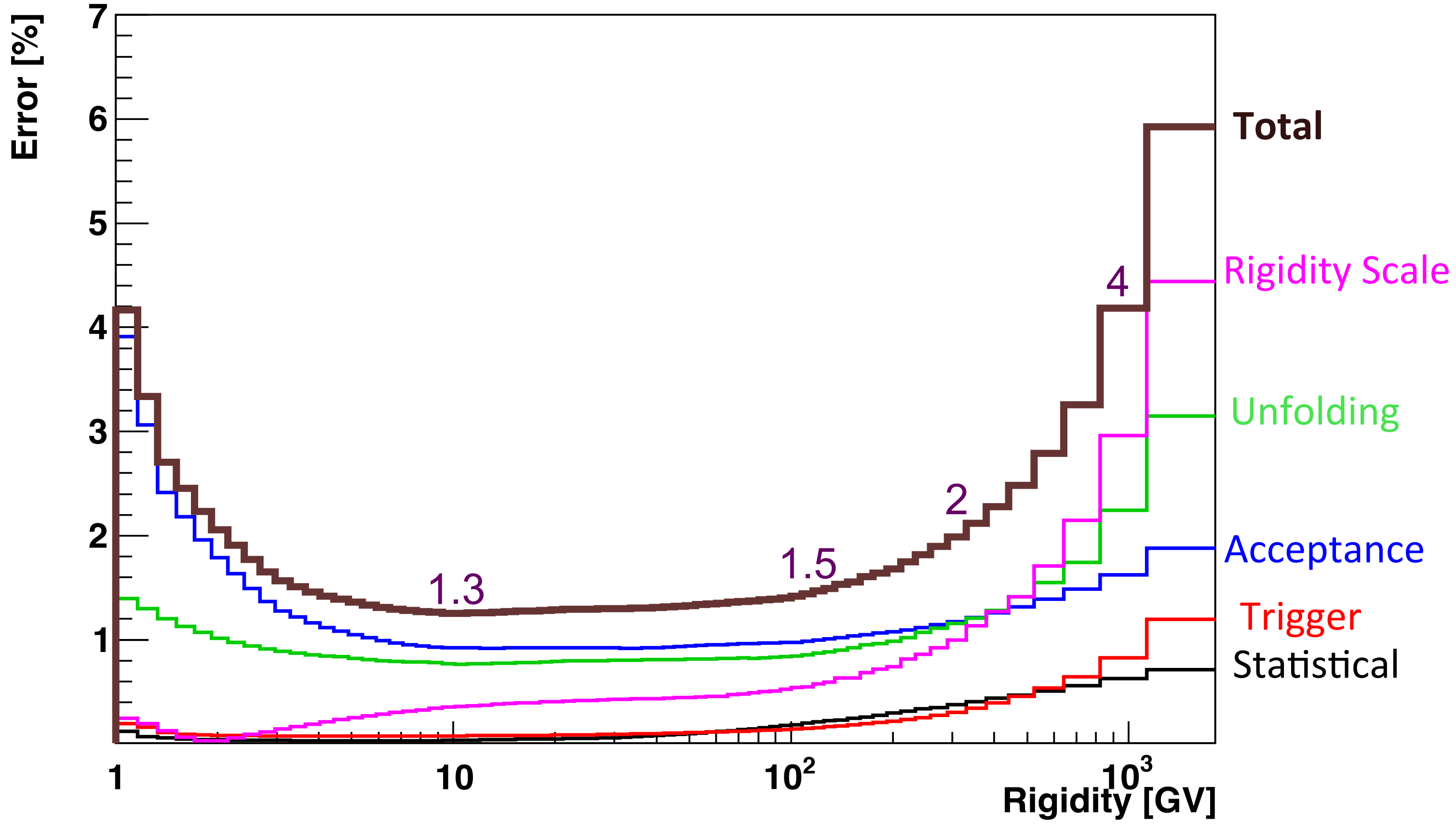
Primary Cosmic Ray Flux (AMS)

Proton Flux



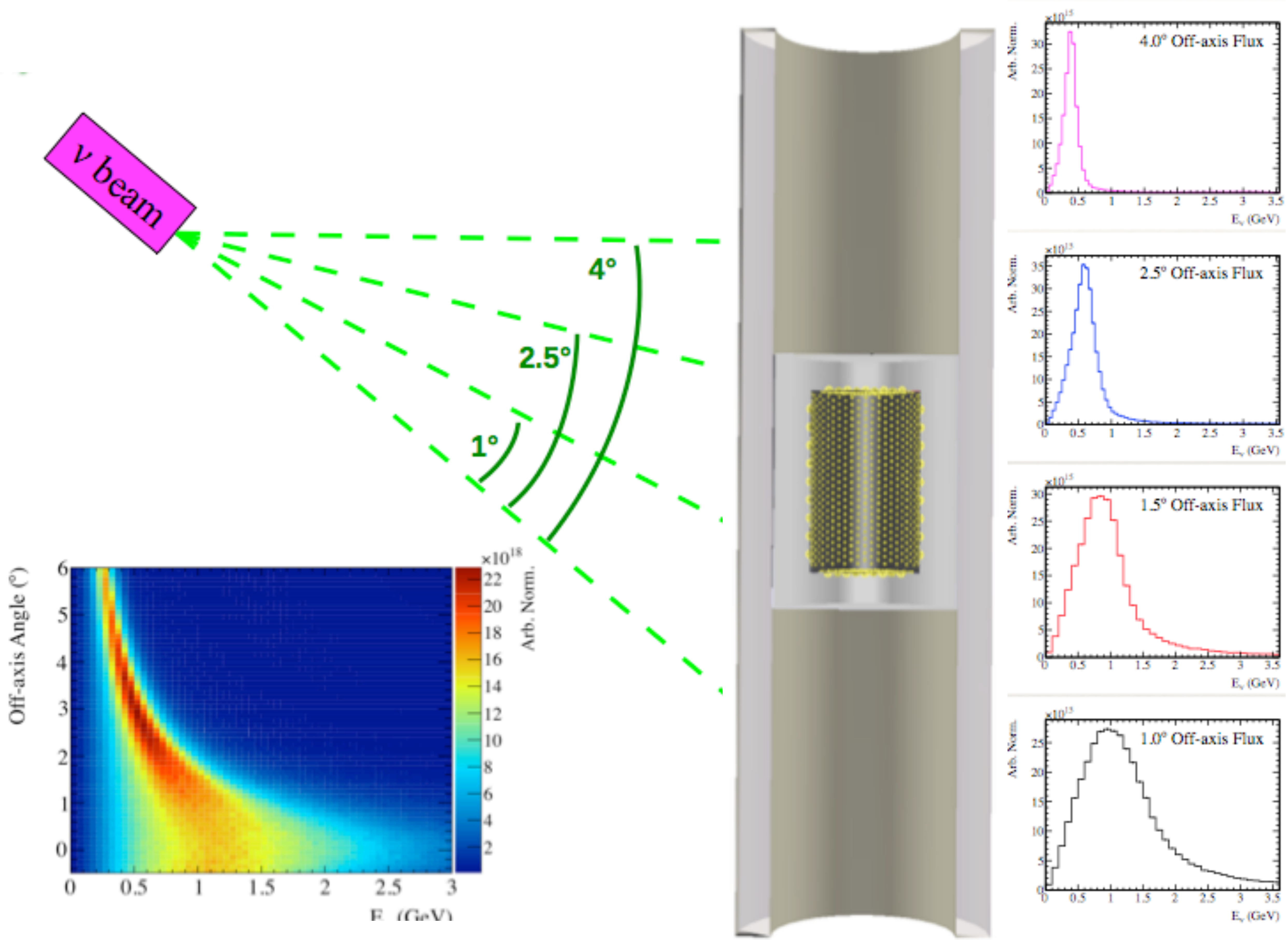
AMS: April 2015

Flux Errors Breakdown



- Primary cosmic ray flux is now measured at 1-2% level.
- Open the door for very precise atm.ν studies

νPRISM (T2K/HyperK ND) concept



Summary

- Atm. ν covers wide range of oscillation parameters
  - Realistic scenario emerges to control systematic uncertainties:
    - Precise AMS cosmic ray measurement
    - Precision hadron production study established by NA61: nuPRISM proposal for model independent ν cross section study
- Precision test of e-μ PMNS Unitarity
  - 9 parameters with 5 constraints to be tested
    - $|U_{e1}|^2, |U_{e2}|^2, |U_{e3}|^2$, $|U_{\mu 1}|^2, |U_{\mu 2}|^2, |U_{\mu 3}|^2$, α, β, γ
    - Unitarity triangle (3 constraints), normalization (2)
  - All 9 parameters can be measured:
    - Typical expected accuracy would be 10-20%
    - HyperK will lead the new measurements: $|U_{\mu i}|^2$, α, β, γ JUNO will improve $|U_{ei}|^2$
  - HyperK will play a leading role in establishing PMNS paradigm, similar to what B-factory did for CKM

Backup slides

Unitarity triangle

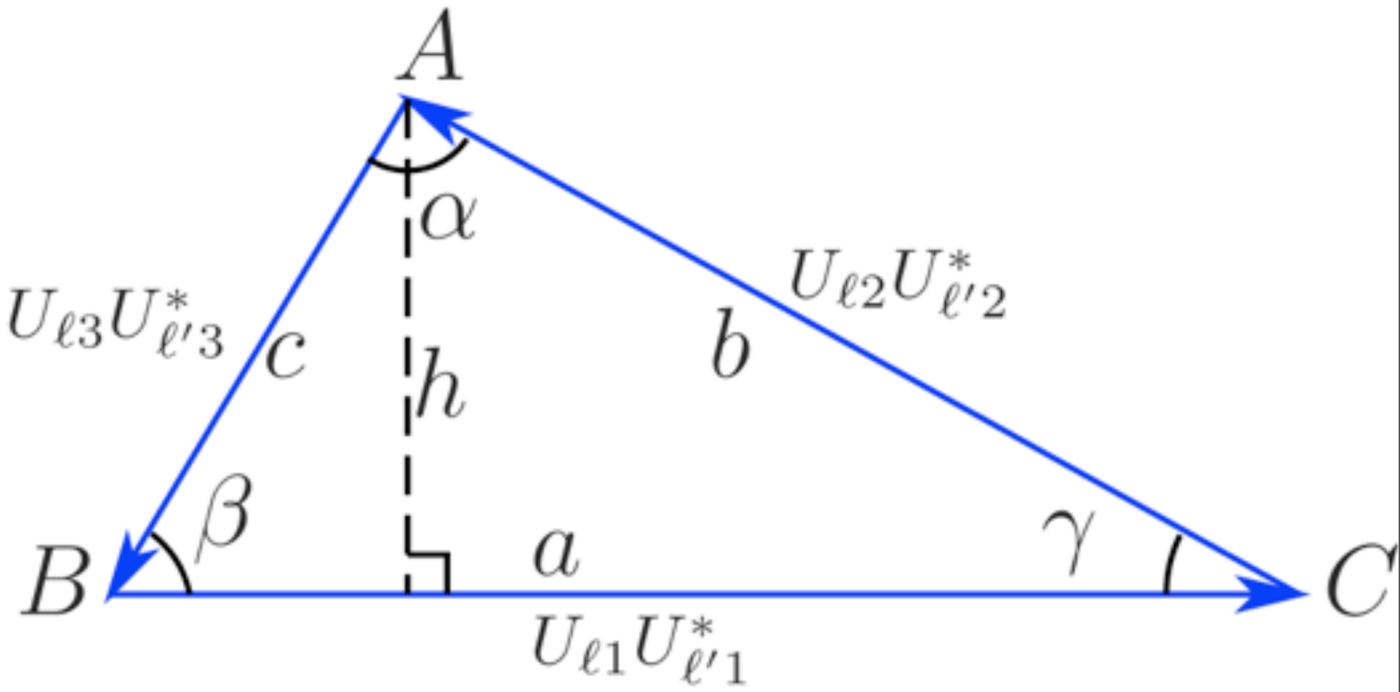
Calculation is simpler without Unitarity

$$
\begin{aligned}
P_{l\to l'} &= |<\nu_{l'}(t)|\nu_l(t)>|^2 \\
&= |\sum_i <\nu_{l'}|\nu_i> e^{-iE_it} <\nu_i|\nu_l>|^2 \\
&= |\sum_i U^*_{li}U_{l'i}e^{-im_i^2L/2E}|^2 = \sum_{i,j} U^*_{li}U_{l'i}(U^*_{lj}U_{l'j})^*e^{-i(m_i^2-m_j^2)L/2E} \\
&= |U^*_{l1}U_{l'1}|^2 + |U^*_{l2}U_{l'2}|^2 + |U^*_{l3}U_{l'3}|^2 \\
&+ 2Re[U^*_{l1}U_{l'1}[U^*_{l2}U_{l'2}]^*e^{-i(m_1^2-m_2^2)L/2E} \\
&+ 2Re[U^*_{l2}U_{l'2}[U^*_{l3}U_{l'3}]^*e^{-i(m_2^2-m_3^2)L/2E} \\
&+ 2Re[U^*_{l3}U_{l'3}[U^*_{l1}U_{l'1}]^*e^{-i(m_3^2-m_1^2)L/2E} \\
&= a^2 + b^2 + c^2 \\
&\quad -2ab\cos(2\Delta_{12} + \gamma) \\
&\quad -2bc\cos(2\Delta_{12} + \alpha) \\
&\quad -2ca\cos(2\Delta_{12} + \beta) \\
&= 4ab\sin(\Delta_{12} + \gamma)\sin\Delta_{12} \\
&+ 4ab\sin(\Delta_{23} + \alpha)\sin\Delta_{23} \\
&+ 4ab\sin(\Delta_{31} + \beta)\sin\Delta_{31}
\end{aligned}
$$

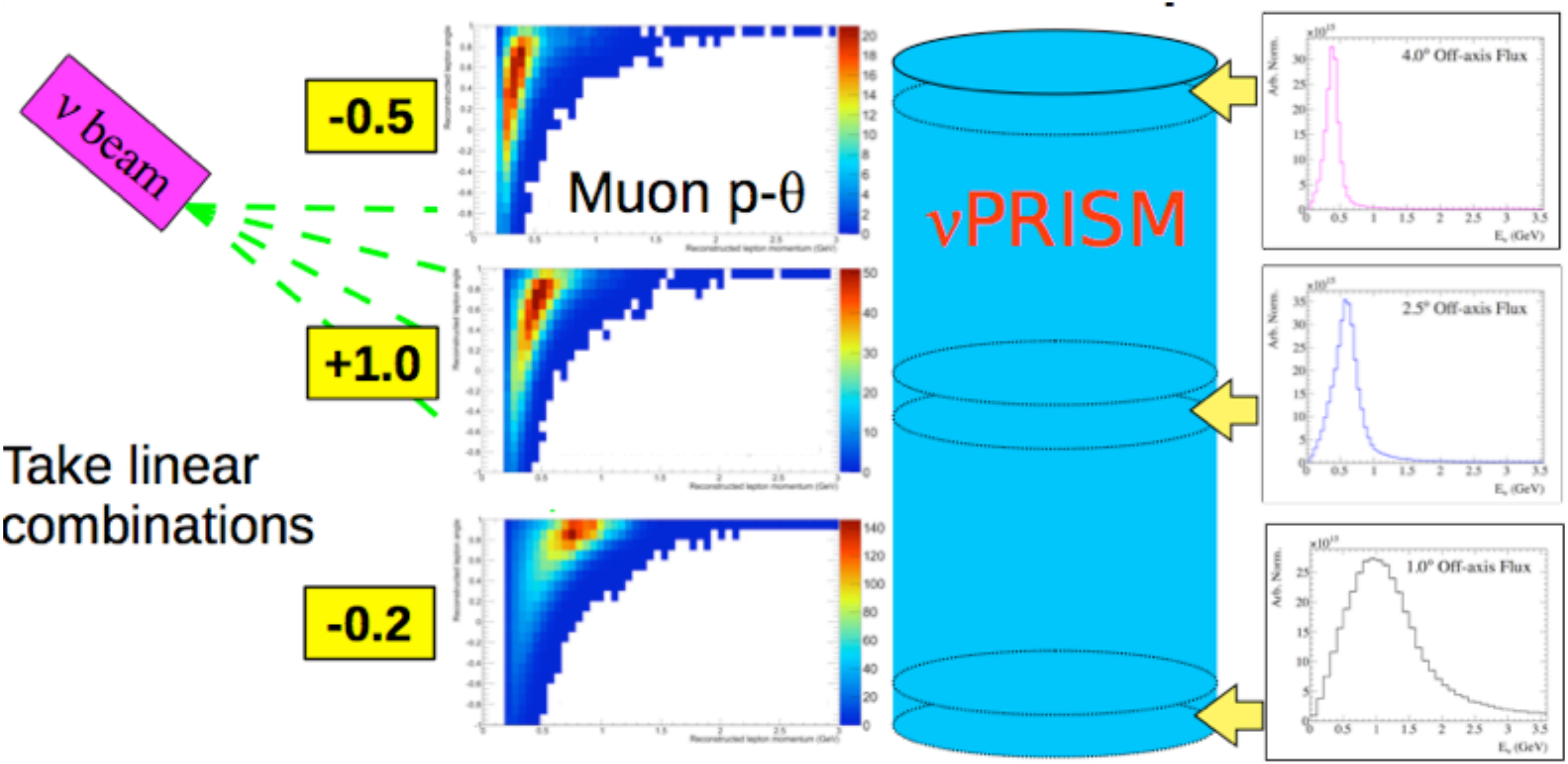
$\Delta_{12}=\Delta m^2_{12}L/4E$

$\Delta_{23}=\Delta m^2_{23}L/4E$

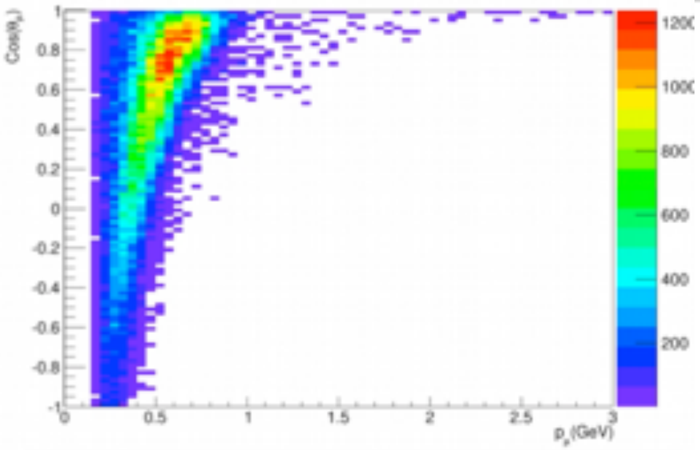
$\Delta_{31}=\Delta m^2_{31}L/4E$

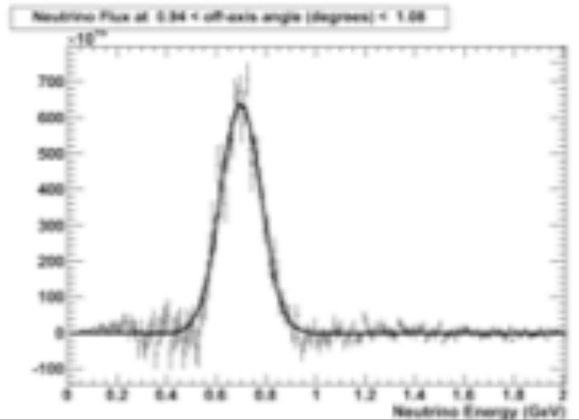


νPRISM (T2K/HyperK ND) concept



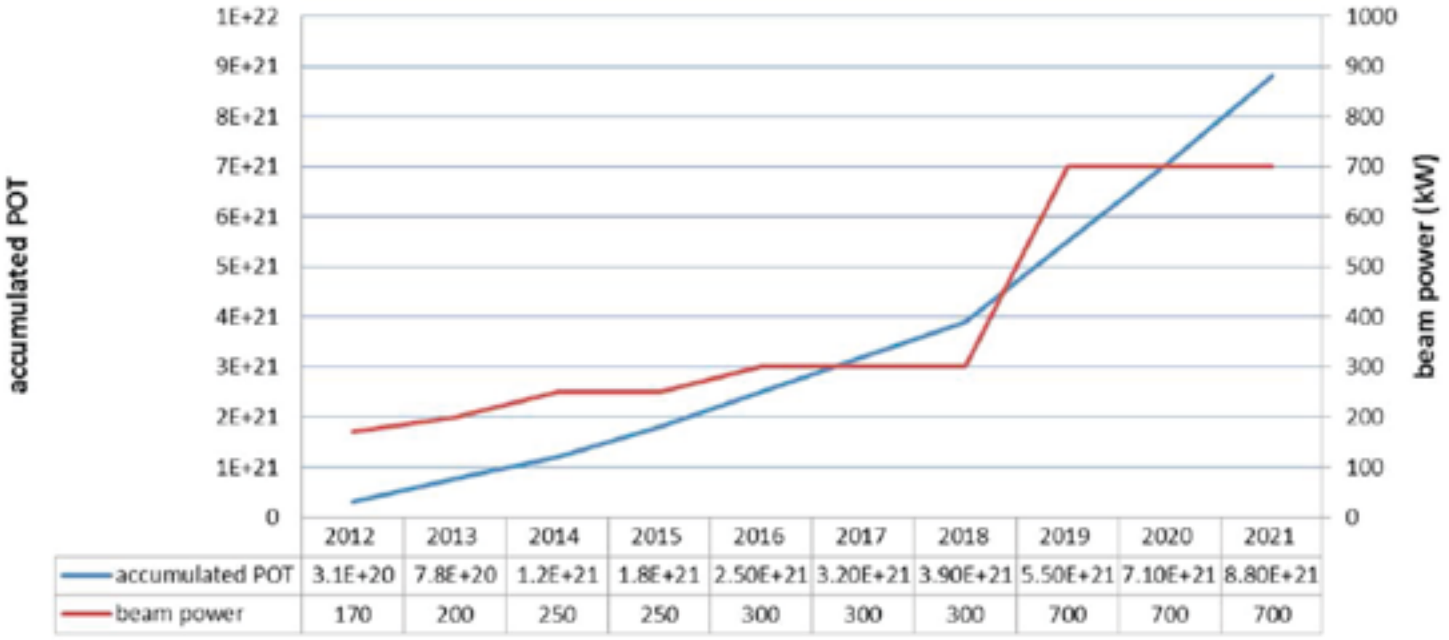
Take linear combinations





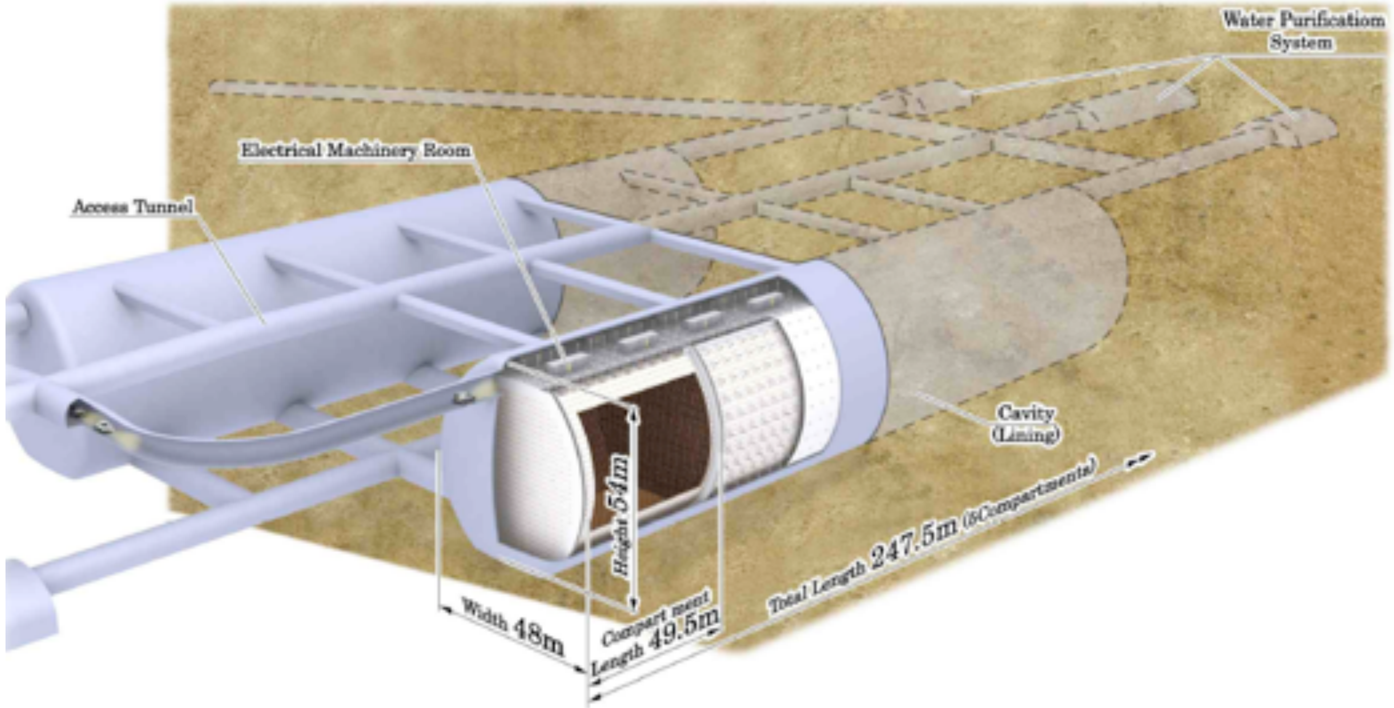
Increasing the statistics

- Accelerator upgrades
  - beam power: 250kW→700kW
  - by increased rep. rate:
    - every 3sec→1sec
    - keeping the similar instantaneous rate



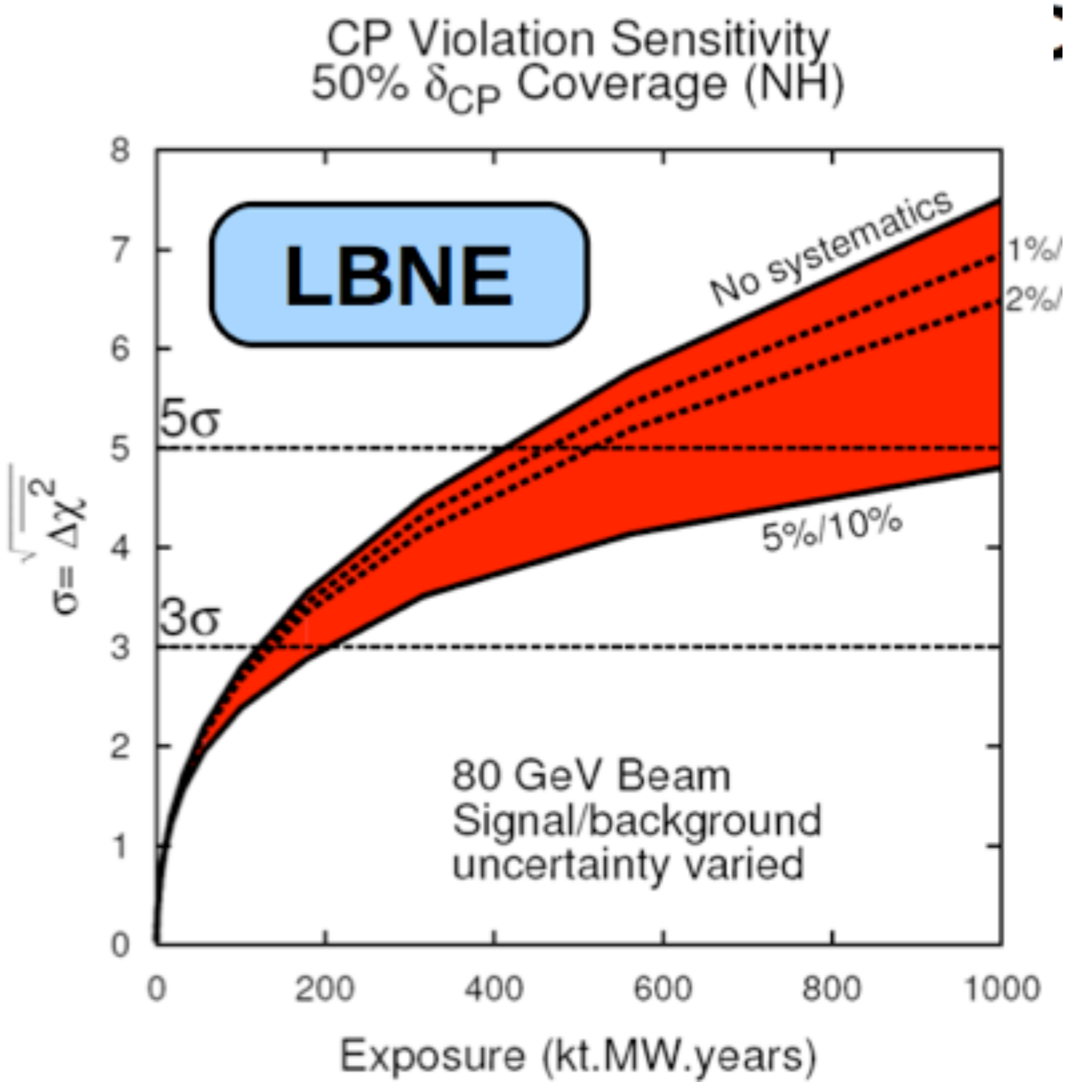
	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
accumulated POT	3.1E+20	7.8E+20	1.2E+21	1.8E+21	2.50E+21	3.20E+21	3.90E+21	5.50E+21	7.10E+21	8.80E+21
beam power	170	200	250	250	300	300	300	700	700	700

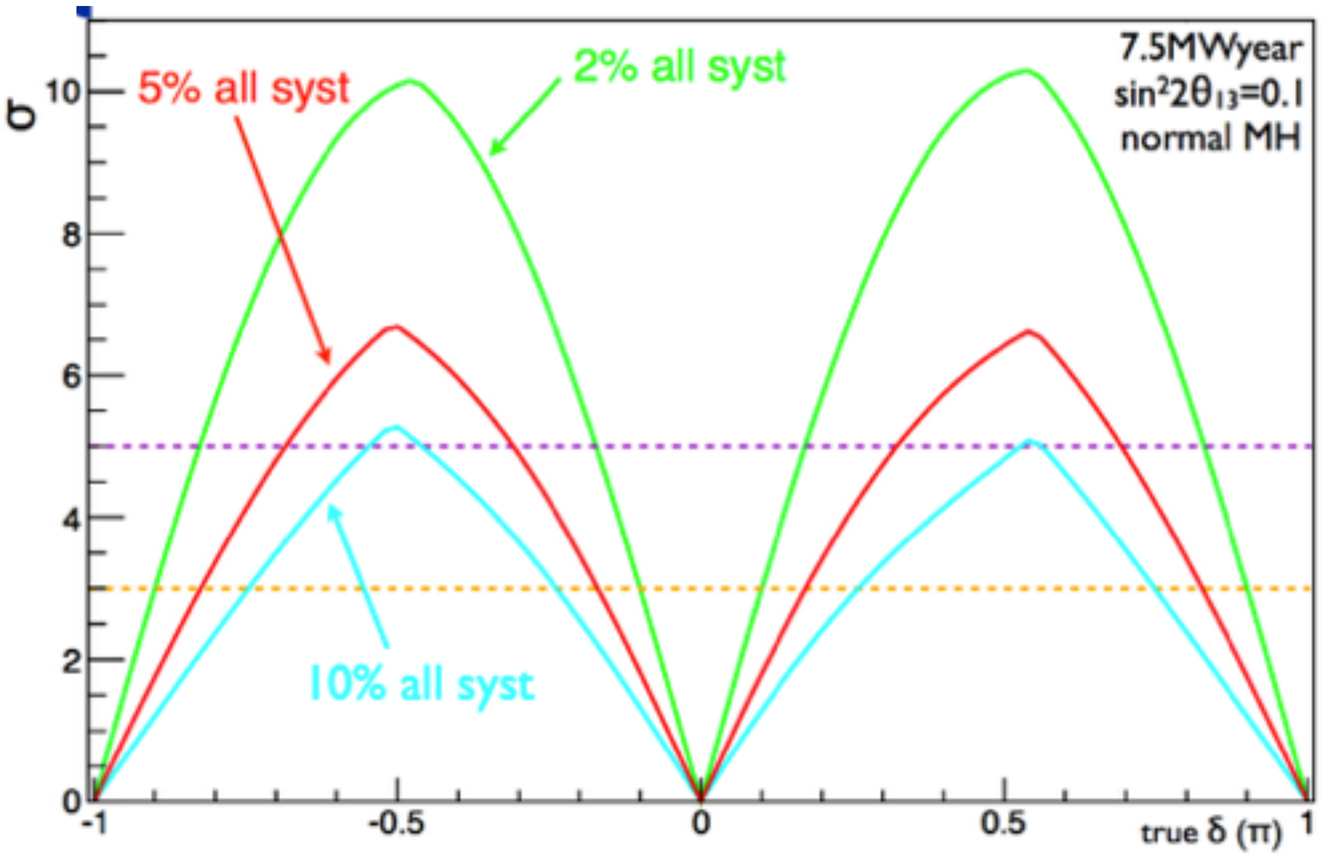
- Far detector upgrade: Hyper-Kamiokande
  - 1Mton water Cerenkov
  - x20 larger fiducial volume than SK



Systematic uncertainties

- Systematic uncertainty of 1-2% is required for the wide range of CP discovery
  - Challenge: ν cross section systematics (multi-nucleon correlation, final state interaction)





T2K systematic uncertainties

- Systematic uncertainties in event rate at SK:

	ν_e app.	**ν_μ disapp.**
flux × cross section	3.1%	2.7%
cross section	5.3%	5.8%
SK detector	2.7%	4.0%
Total	6.8%	7.7%

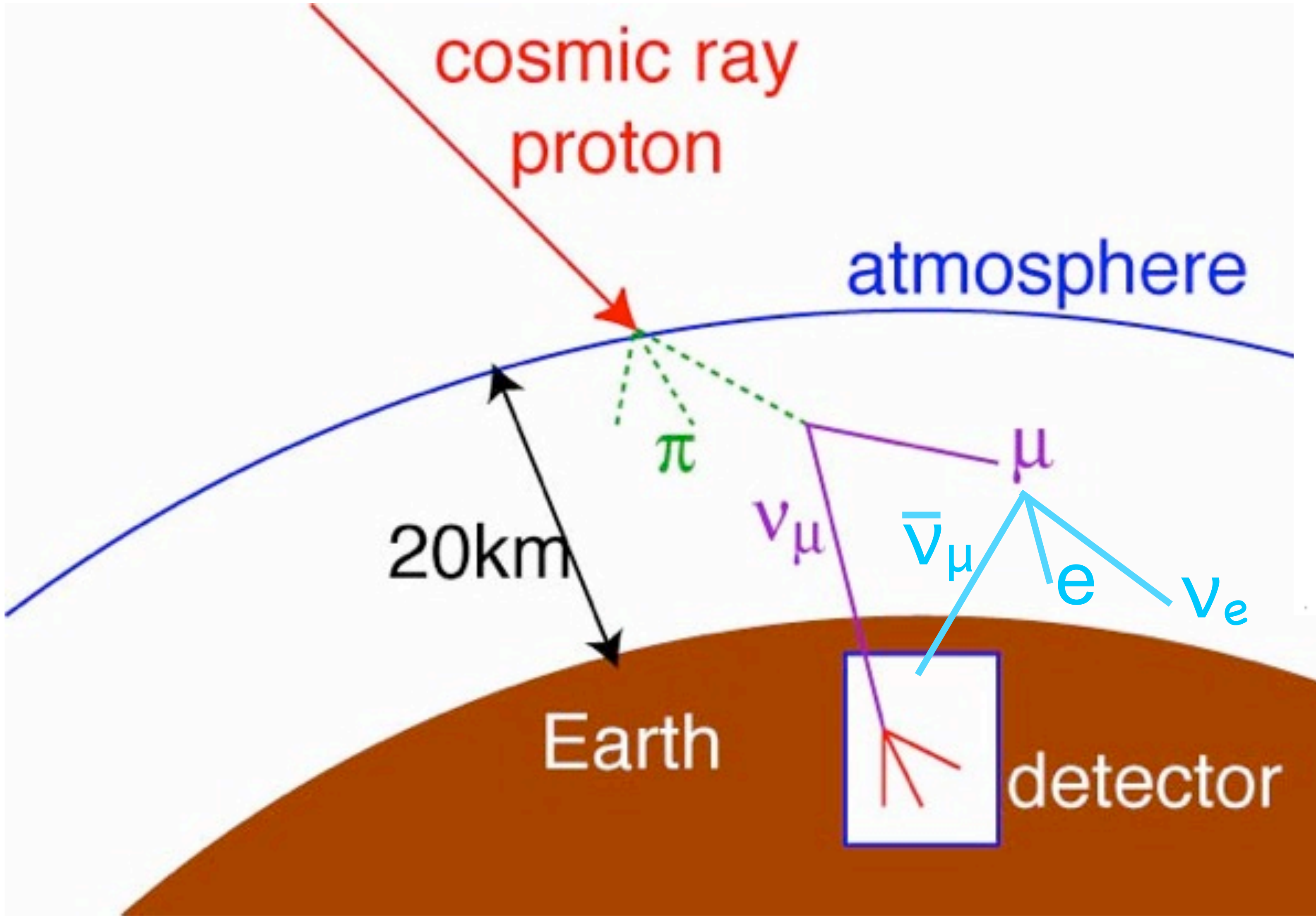
- [Flux × cross section]: ND280 constrains
  - place near detector further away than 280m
- Cross section: Nuclear effects
  - water target with well controlled ν beam
- SK detector: SK detector efficiency
  - water Cherenkov calibration and ν beam test

CP cross section systematics

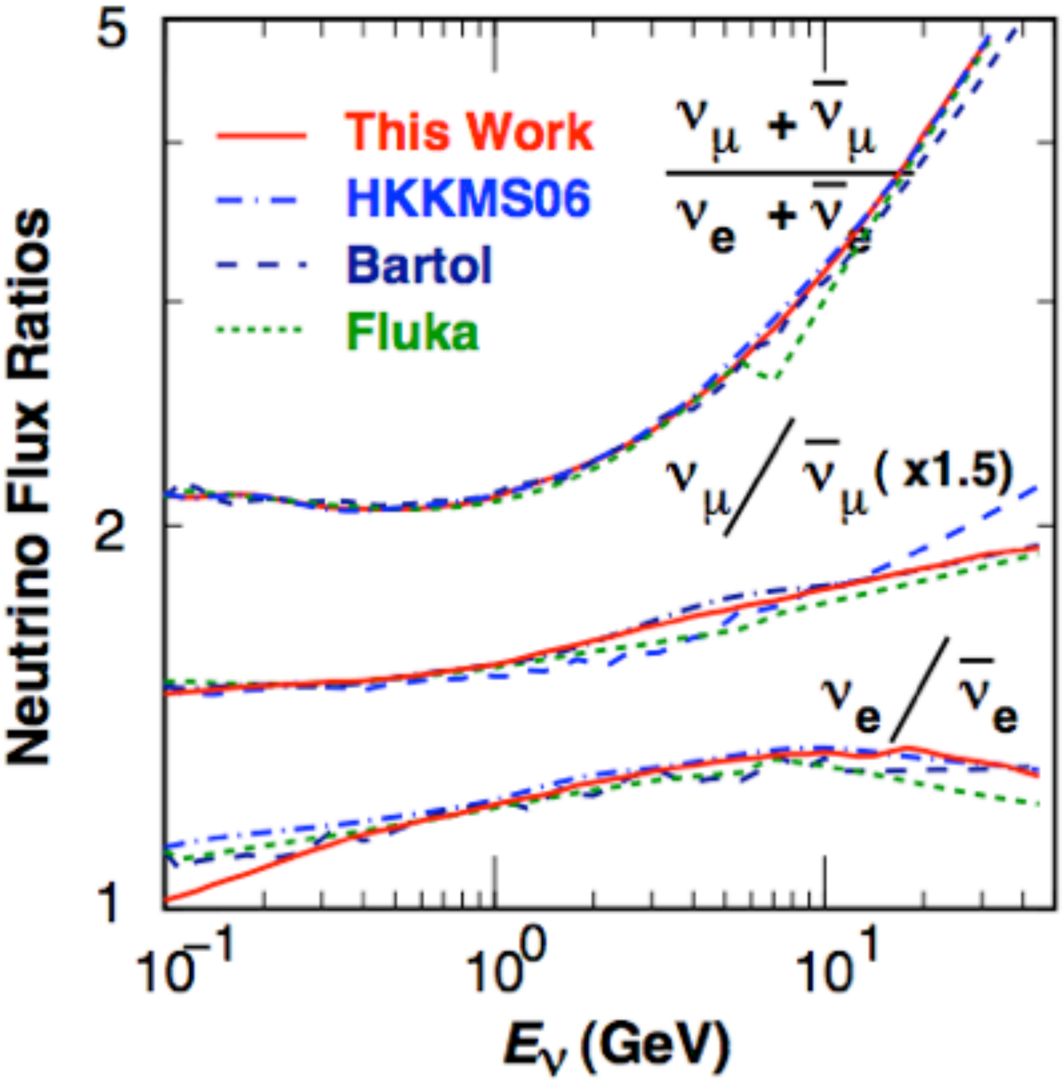
- $\sigma(\nu_e)/\sigma(\nu_\mu)$ ratio
  - For ν_e appearance, the near/far detectors observe ν_e/ν_μ , respectively
  - In the first order, it is the same: kinematical phase spance, radiative correction
- $\sigma(\bar{\nu}_\mu)/\sigma(\nu_\mu)$ and $\sigma(\bar{\nu}_e)/\sigma(\nu_e)$ ratio
  - angular distribution is very different between neutrinos and anti-neutrinos
  - nuclear effect can be different
- nuPRISM to address these
  - nuSTORM (π storage ring) is another proposal

Atmospheric neutrinos

Produced by cosmic ray proton interacting in earth atmosphere:

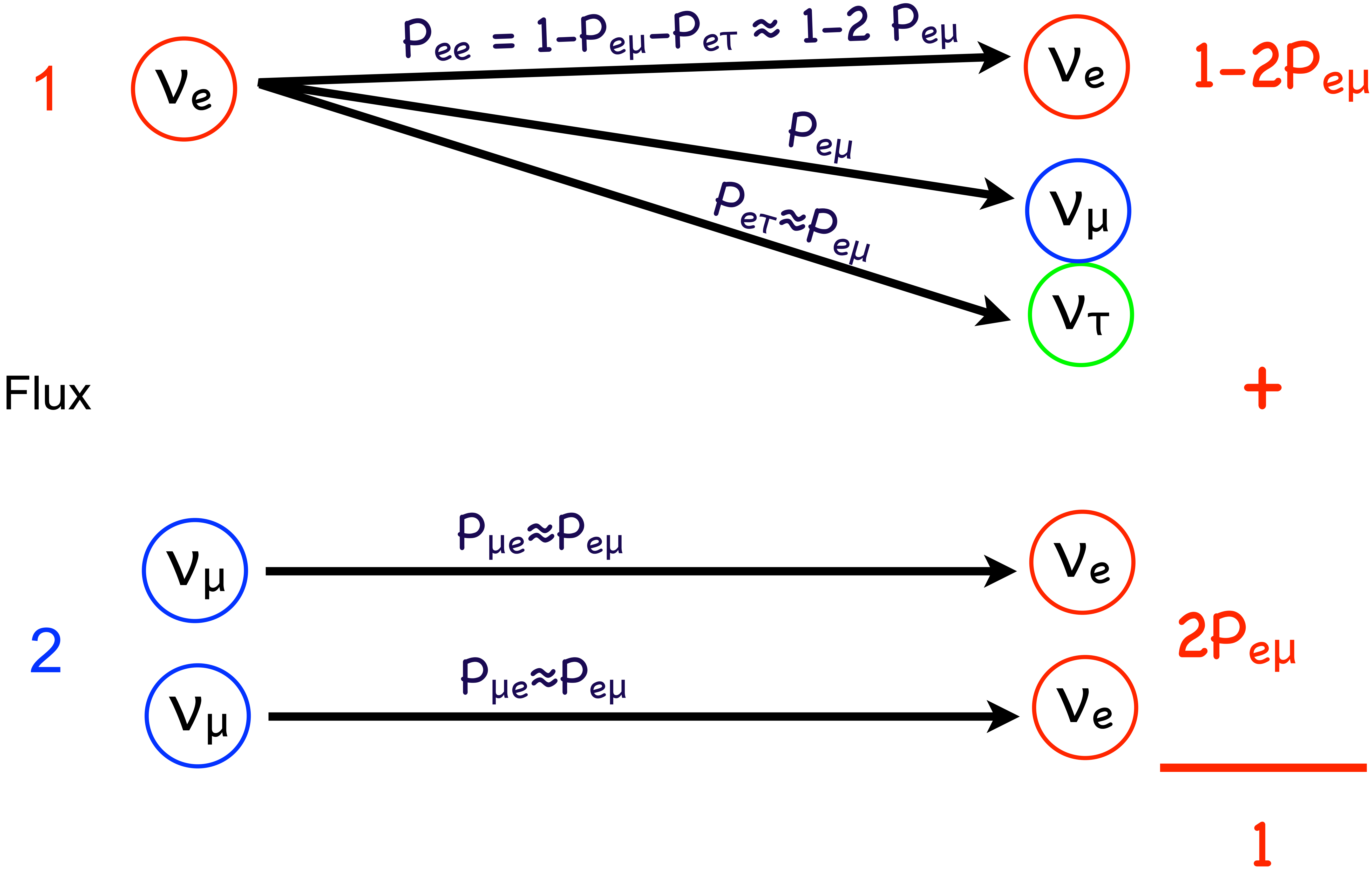


$r=\Phi(\nu_\mu)/\Phi(\nu_e)\sim 2$ for $E_\nu<3\text{GeV}$



arXiv:1102.2688

Cancellation of atm. ν_e appearance



1

ν_e

$P_{ee} = 1-P_{e\mu}-P_{e\tau} \approx 1-2\,P_{e\mu}$

ν_e

$1-2P_{e\mu}$

$P_{e\mu}$

ν_μ

$P_{e\tau}\approx P_{e\mu}$

ν_τ

Flux

+

ν_μ

$P_{\mu e}\approx P_{e\mu}$

ν_e

2

$2P_{e\mu}$

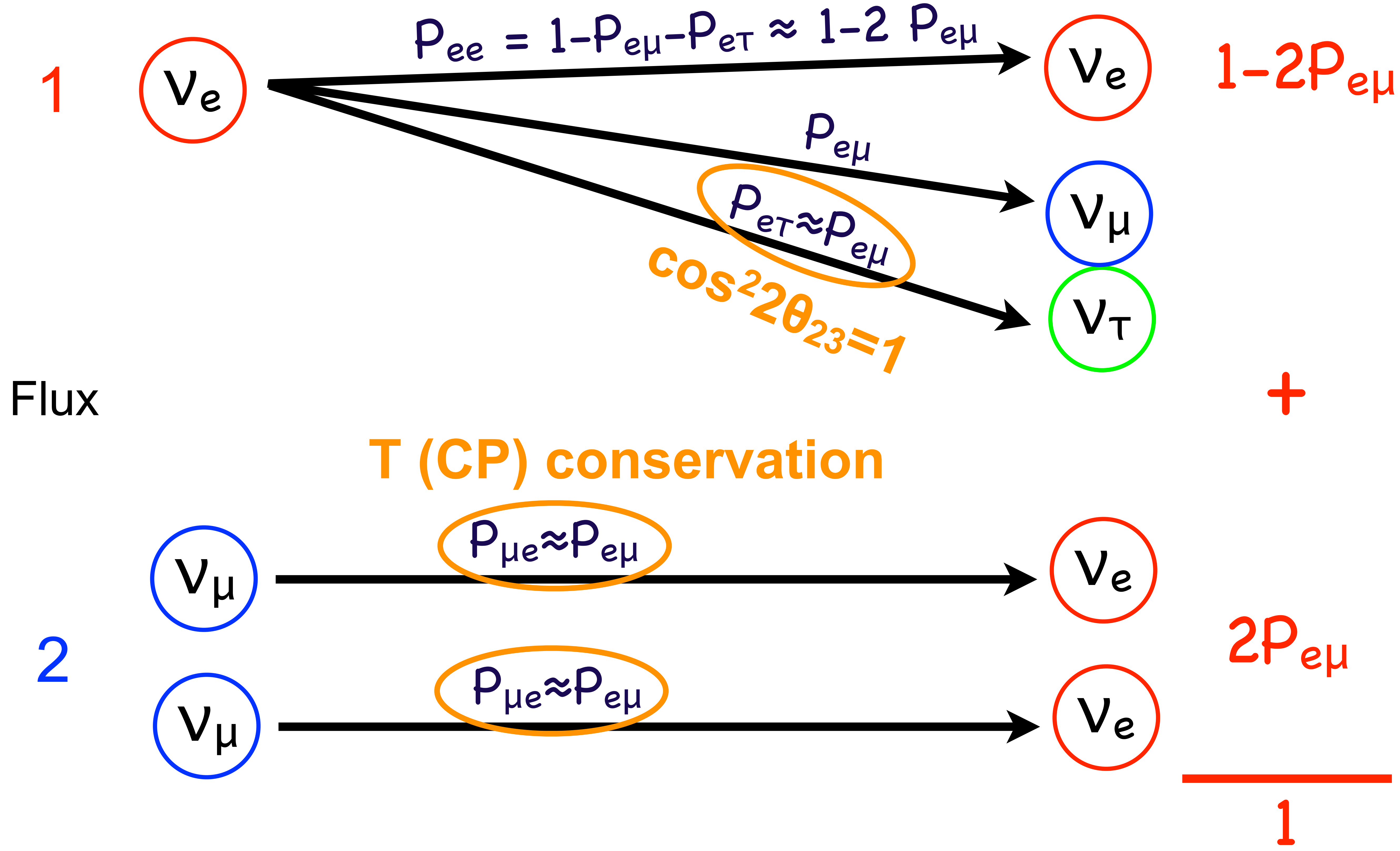
ν_μ

$P_{\mu e}\approx P_{e\mu}$

ν_e

1

atm. ν_e app. sensitive to T(CP) viol.



1

ν_e

$P_{ee} = 1 - P_{e\mu} - P_{e\tau} \approx 1 - 2\ P_{e\mu}$

ν_e

$1-2P_{e\mu}$

$P_{e\mu}$

ν_μ

$P_{e\tau} \approx P_{e\mu}$

$\cos^2 2\theta_{23} = 1$

ν_τ

Flux

+

T (CP) conservation

ν_μ

$P_{\mu e} \approx P_{e\mu}$

ν_e

2

ν_μ

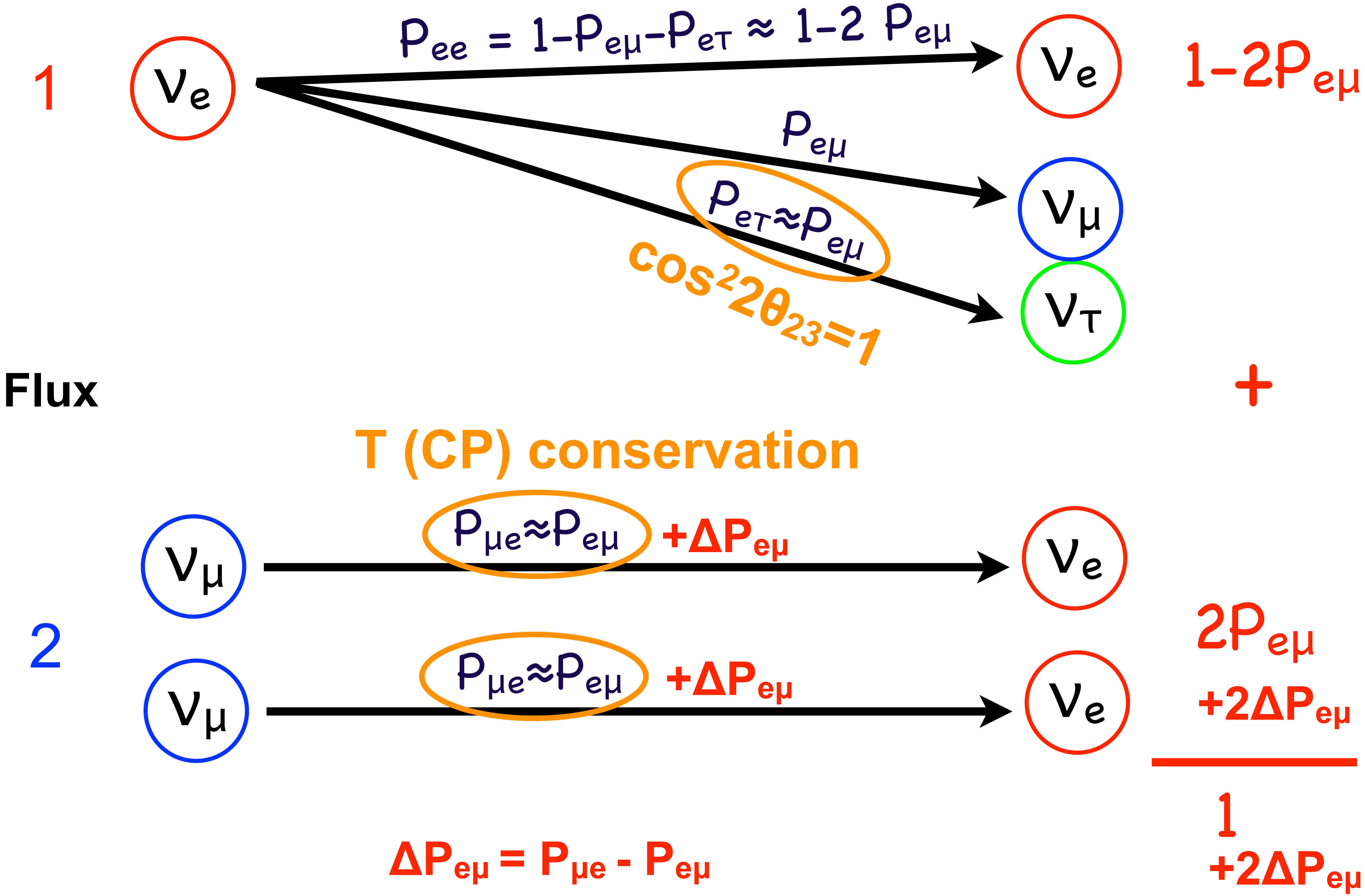
$P_{\mu e} \approx P_{e\mu}$

ν_e

$2P_{e\mu}$

1

atm. ν_e app. sensitive to T(CP) viol.



1 ν_e

$P_{ee} = 1 - P_{e\mu} - P_{e\tau} \approx 1 - 2\,P_{e\mu}$ → ν_e $1-2P_{e\mu}$

$P_{e\mu}$ → ν_μ

$P_{e\tau} \approx P_{e\mu}$ → ν_τ

$\cos^2 2\theta_{23} = 1$

Flux

+

T (CP) conservation

2

ν_μ $P_{\mu e} \approx P_{e\mu}$ $+\Delta P_{e\mu}$ → ν_e

ν_μ $P_{\mu e} \approx P_{e\mu}$ $+\Delta P_{e\mu}$ → ν_e

$2P_{e\mu}$

$+2\Delta P_{e\mu}$

1

$+2\Delta P_{e\mu}$

$\Delta P_{e\mu} = P_{\mu e} - P_{e\mu}$