$B \to \pi\pi\ell\nu$ at Large Dipion Masses

(Ph. Böer, TF, D. van Dyk)

Thorsten Feldmann (TF)

QFET Meeting

— U Siegen, 8. December 2014 —

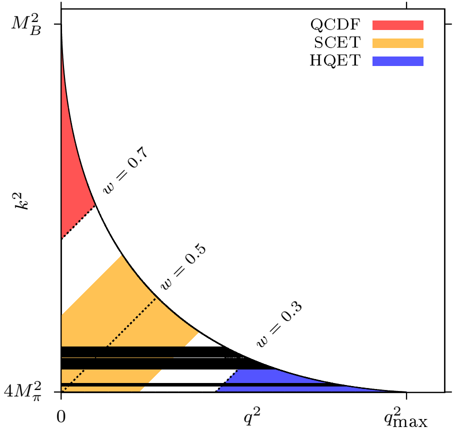
Theor. Physik 1

UNIVERSITÄT SIEGEN

DFG FOR 1873

$B \to \pi\pi\ell\nu$: 2-pion phase space

[arxiv:1310.6660]



Part of Cross-over Project:

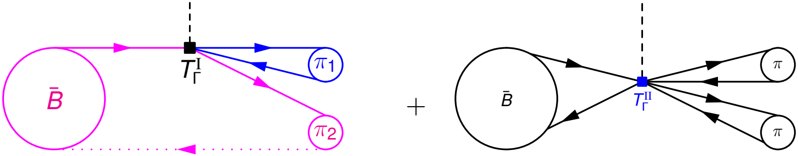
- QCDF region: → **this talk** (P5)
- SCET region: → LCSR (P3 talk)
- HQET region: → P1

- Test of QCD-Factorization approach with tunable q^2
- Independent information on hadronic input parameters ($B \to \pi$ form factors, B and π distribution amplitudes)

QCD Factorization Theorem ?

Similar to BBNS Approach for non-leptonic $B \to \pi\pi$ decays:

$$
\begin{aligned}
&\langle \pi(k_1)\pi(k_2)|\bar{u}\,\Gamma\, b|\bar{B}(p)\rangle \\
&= \xi_\pi(E_2) \int_0^1 du\, \phi_\pi(u)\, T_\Gamma^{\rm I}(u,\ldots) \\
&\quad + \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega}\, \phi_\pi(u)\, \phi_\pi(v)\, \phi_B(\omega)\, T_\Gamma^{\rm II}(u,v,\omega,\ldots) \\
&\quad + \text{power corrections}
\end{aligned}
$$



QCD Factorization Theorem ?

Similar to BBNS Approach for non-leptonic $B \to \pi\pi$ decays:

$$\langle \pi(k_1)\pi(k_2)|\bar{u}\,\Gamma\, b|\bar{B}(p)\rangle$$
$$= \xi_\pi(E_2) \int_0^1 du\, \phi_\pi(u)\, T_\Gamma^{\rm I}(u,\ldots)$$
$$+ \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega}\, \phi_\pi(u)\, \phi_\pi(v)\, \phi_B(\omega)\, T_\Gamma^{\rm II}(u,v,\omega,\ldots)$$
$$+ \text{power corrections}$$

- universal $B \to \pi$ form factor $\xi_\pi(E)$ for $E \sim m_b/2$

$$\langle \pi(p' = En)|\bar{u}\,\Gamma\, b|B(v)\rangle \;\to\; 2E\, \xi_\pi(E)\, {\rm tr}\left[\frac{\not{\bar{n}}\not{n}}{4}\,\Gamma\,\frac{1+\not{v}}{2}\right]$$

- universal LCDAs $\phi_\pi(u)$, $\phi_B(\omega)$ for light and heavy mesons
- perturbatively calculable short-distance kernels $T_\Gamma^{\rm I,II}$

Plan: Diagrammatic Analysis

- Verify factorization theorem to leading non-trivial order.
  (i.e. IR-sensitive contributions can be absorbed into $\phi_{\pi,B}$ or $\xi_\pi(E_2)$.)

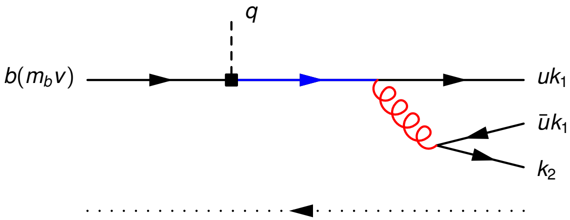
- Calculate short-distance kernels:

$$T_\Gamma^{\mathrm{I}} = \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2)\,, \qquad T_\Gamma^{\mathrm{II}} = \mathcal{O}(\alpha_s^2)$$

- Analyze power-suppressed (non-factorizable) terms a la BBNS.

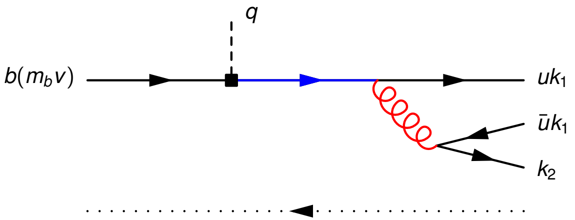
1-Gluon–Exchange – Leading Power

(1a) Radiation off *u*-quark: (Feynman gauge)



$$\propto \frac{g_s^2}{q_2^2 + i\epsilon} \left[\bar{u}(k_{1a})\, \gamma_\alpha\, T^a \, \frac{\not{q}_1}{q_1^2 + i\epsilon} \, \Gamma \, u(m_b v) \right] [\bar{u}(k_{2a})\, \gamma^\alpha \, T^a \, v(k_{1b})]$$

(1a) Radiation off u-quark: (Feynman gauge)



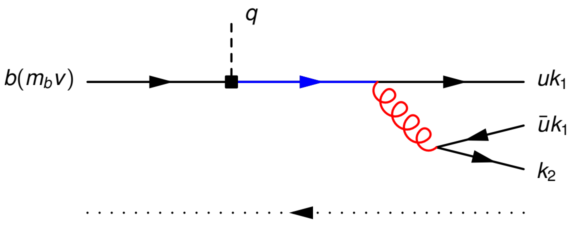
$$\propto \frac{g_s^2}{q_2^2 + i\epsilon} \left[\bar{u}(k_{1a})\, \gamma_\alpha\, T^a\, \frac{\not{q}_1}{q_1^2 + i\epsilon}\, \Gamma\, u(m_b v) \right] \left[\bar{u}(k_{2a})\, \gamma^\alpha\, T^a\, v(k_{1b}) \right]$$

Hard Propagators: $q_1 \simeq k_1 + k_2\,,$ $q_2 \simeq \bar{u}\, k_1 + k_2$

$q_1^2 \simeq k^2\,,$ $q_2^2 \simeq \bar{u}\, k^2$

1-Gluon–Exchange – Leading Power

(1a) Radiation off u-quark: (Feynman gauge)

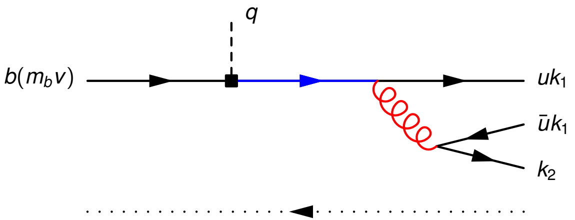


$$\propto \frac{C_F}{N_c} \frac{g_s^2}{q_2^2} \left[\bar{u}^i(k_{1a})\,\gamma_\alpha\,\frac{\not{k}}{k^2}\,\Gamma\,u^j(m_b v)\right]\left[\bar{u}^j(k_{2a})\,\gamma^\alpha\,v^i(k_{1b})\right]$$

Color Fierz Identity: $(T^a)_{ij}(T^a)_{kl} = \underbrace{\frac{C_F}{N_c}}\,\delta_{il}\,\delta_{kj} - \frac{1}{N_c}\,(T^a)_{il}(T^a)_{kj}$

1-Gluon–Exchange – Leading Power

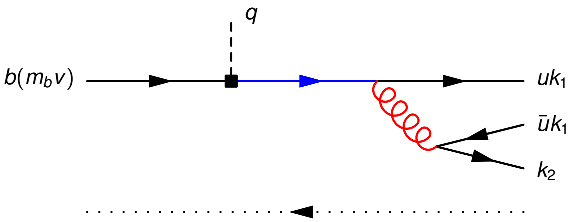
(1a) Radiation off u-quark: (Feynman gauge)



$$\propto \frac{C_F}{N_c}\,\frac{g_s^2}{q_2^2}\,\frac{1}{4\,k^2}\left[\bar{u}(k_{1a})\,\overbrace{\gamma_\alpha\,\not{k}\,\Gamma^A\gamma^\alpha}\,v(k_{1b})\right][\bar{u}(k_2)\,\Gamma_A\,\Gamma\,u(m_b v)]$$

Dirac Fierz Identity: $\Gamma\,u(m_b v)\,\bar{u}(k_2) \quad = \frac{1}{4}\sum_A \Gamma_A\left[\bar{u}(k_2)\,\Gamma^A\,\Gamma\,u(m_b v)\right],$

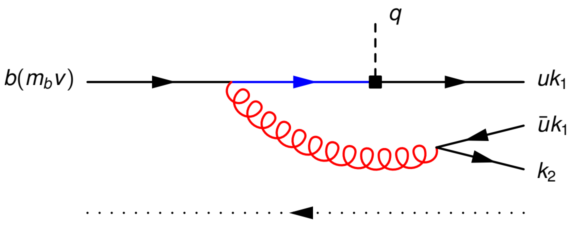
(1a) Radiation off u-quark: (Feynman gauge)



$$\propto \frac{C_F}{N_c}\,\frac{g_s^2}{q_2^2}\,\frac{1}{4\,k^2}\left[\bar u(k_{1a})\,\overbrace{\gamma_\alpha\,\not k\,\Gamma^A\gamma^\alpha}\,v(k_{1b})\right][\bar u(k_2)\,\Gamma_A\,\Gamma\,u(m_b v)]$$

Twist-2 Projection: $\frac{1}{4}\,\mathrm{tr}\left[\not k_1\gamma_5\,\gamma_\alpha\not k\Gamma^A\gamma^\alpha\right] \to \begin{cases} -k^2 & \Gamma^A=\gamma_5\,, \\ 2\,\epsilon_{\rho\sigma\mu\nu}\,k_1^\rho k_2^\sigma\,, & \Gamma^A=\sigma_{\mu\nu}\,, \\ 0 & \text{otherwise} \end{cases}$

(1b) Radiation off b-quark: (Feynman gauge)



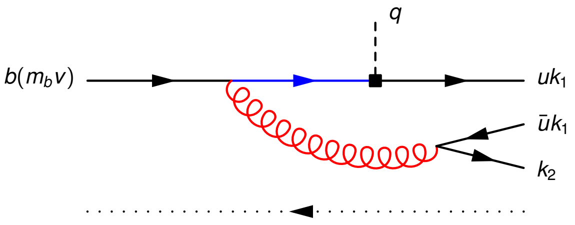
$$\propto \frac{C_F}{N_c}\,\frac{g_s^2}{q_2^2}\left[\bar u^i(k_{1a})\,\Gamma\,\frac{\not q_1 + m_b}{q_1^2 - m_b^2}\,\gamma_\alpha\,u^j(m_b v)\right]\left[\bar u^j(k_2)\,\gamma^\alpha\,v^i(k_{1b})\right]$$

Hard Propagators:

$$q_1 \simeq u k_1 + q\,,$$

$$q_1^2 - m_b^2 \simeq -\frac{1}{2}\left((2-u)\,m_b^2 + u\,k^2\right)$$

(1b) Radiation off b-quark: (Feynman gauge)



$$\propto \frac{C_F}{N_c}\,\frac{g_s^2}{q_2^2}\,\frac{C_{AB}(k_1,k_2,u)}{q_1^2-m_b^2}\,\left[\bar{u}(k_{1a})\,\not{\bar{n}}\gamma_5\,v(k_{1b})\right]\left[\bar{u}(k_2)\,\Gamma^A\,\Gamma\,\Gamma^B\,u(m_b v)\right]$$

again, using Fierz identities and twist-2 projection ...

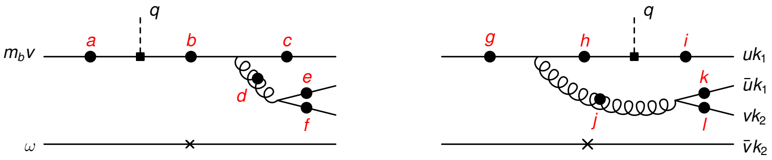
Tentative Conclusions:

- Finite convolution integrals (at leading power).
- LO matching on T_Γ^{I} in factorization theorem works.
- LO expressions for all $B \to \pi\pi$ form factors (QCDF region)
  - Form-factor normalization uncertain, $\propto \alpha_s(\mu)$.
  - Form-factor ratios only depend on kinematics and

$$I_1 \equiv \int_0^1 \frac{\phi_\pi(u)\,du}{1-u}\,, \quad I_3 \equiv \int_0^1 \frac{\phi_\pi(u)\,du}{1+u\,r}\,, \qquad \left(\text{with } r = \tfrac{k^2-m_b^2}{2m_b} \geq -\tfrac{1}{2}\right)$$

NLO Spectator Scattering

Work in Progress



gluon exchange between spectator ($\times$) and any vertex $a - l$

New Features:

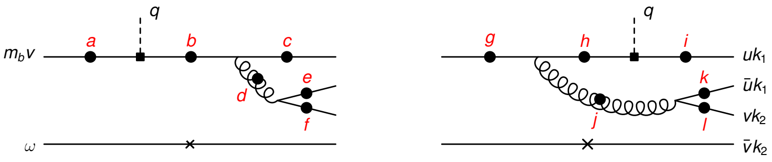
- Exchanged gluon virtuality is "hard-collinear"

$$(k_{\rm soft} - \bar{v}\, E_2 n)^2 \simeq -\bar{v}\, E_2\, (n \cdot k_{\rm soft}) \equiv -\bar{v}\, E_2\, \omega \sim \mathcal{O}(\Lambda_{\rm QCD}\, m_b)$$

- Additional internal propagators may be hard <u>or</u> hard-collinear:
  $\to$ requires careful power counting (!)
- May introduce endpoint divergencies, if $v \to 1$ and/or $\omega \to 0$ (!)
  $\to$ corresponds to kinematic situation of LO diagram ...

NLO Spectator Scattering

Work in Progress



gluon exchange between spectator ($\times$) and any vertex $a - l$

Expectations:

- Diagrams a, f, g and l analogous to $B \to \pi$ FFs: [Beneke/TF '01]
  - endpoint-sensitive contributions absorbed in $B \to \pi$ FFs.
  - factorizable contributions into $T_\Gamma^{\rm II}$.
- Diagrams (c, e) and (i, k) protected by color-transparency. [BBNS '99]

Summary/Outlook

$B \to \pi\pi\ell\nu$ at large dipion mass:

- LO result for T_Γ^{I} in QCD Factorization Theorem $\surd$
- NLO spectator scattering: (work in progress)
  - endpoint-divergent contributions,
    $\to$ to be absorbed into $B \to \pi$ form factor
  - factorizable contributions determine T_Γ^{II}
- Power corrections (work in progress)
- NLO vertex corrections to T_Γ^{I} ($\to$ future work)
- Extrapolation to other phase-space regions ($\to$ future work)