NNLO penguin amplitudes in QCDF

(Project 5)

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Related publications

- G. Bell, TH
  "Master integrals for the two-loop penguin contribution in non-leptonic B-decays"
  QFET-2014-18, arXiv:1410.2804, accepted by JHEP

- G. Bell, M. Beneke, X.-Q. Li, TH
  "in preparation"

Penguin amplitudes in QCDF

$$\langle\pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = A_{\pi\pi}\,\left\{\lambda_u\left[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\right]+\lambda_c\,\alpha_4^c(\pi\pi)\right\}$$
$$-\,\langle\pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = A_{\pi\pi}\,\left\{\lambda_u\left[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\right]-\lambda_c\,\alpha_4^c(\pi\pi)\right\}$$

$$\langle\pi^-\bar{K}^0|\,\mathcal{H}_{eff}\,|B^-\rangle = A_{\pi\bar{K}}\,\left[\lambda_u^{(s)}\,\alpha_4^u+\lambda_c^{(s)}\,\alpha_4^c\right]$$
$$\langle\pi^+K^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = A_{\pi\bar{K}}\left[\lambda_u^{(s)}\,(\alpha_1+\alpha_4^u)+\lambda_c^{(s)}\,\alpha_4^c\right]$$

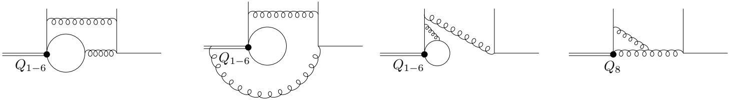
- Main motivation: direct CP asymmetries at NLO
- Penguin amplitudes to NLO

$$\alpha_4^u(\pi\pi) = -0.029-[0.002+0.001i]_V+[0.003-0.013i]_P+[??\;+\;??\;i]_{\mathcal{O}(\alpha_s^2)}$$
$$+\left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO}+[0.001+0.000i]_{HV+HP}+[0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002}+(-0.012^{+0.003}_{-0.002})i$$

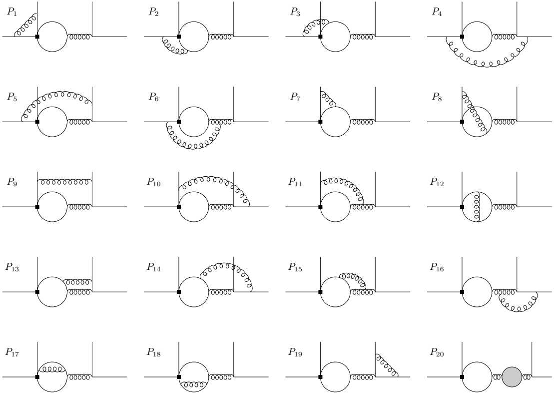
$$\alpha_4^c(\pi\pi) = -0.029-[0.002+0.001i]_V-[0.001+0.007i]_P+[??\;+\;??\;i]_{\mathcal{O}(\alpha_s^2)}$$
$$+\left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO}+[0.001+0.001i]_{HV+HP}+[0.001]_{\rm tw3}\right\} = -0.028^{+0.005}_{-0.003}+(-0.006^{+0.003}_{-0.002})i$$

Diagrams

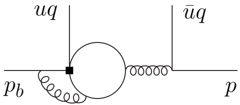
- $\mathcal{O}(70)$ diagrams at NNLO.



- Example: “Genuine” penguin diagrams



- Kinematics:



$$p^2 = q^2 = 0$$
$$p_b^2 = m_b^2$$

- Fermion loop with either $m = 0$ or $m = m_c$.
- Genuine two-scale problem: $\bar{u}$, m_c^2/m_b^2
- Threshold at $\bar{u} = 4m_c^2/m_b^2$
- Choice of suitable kinematic variables crucial

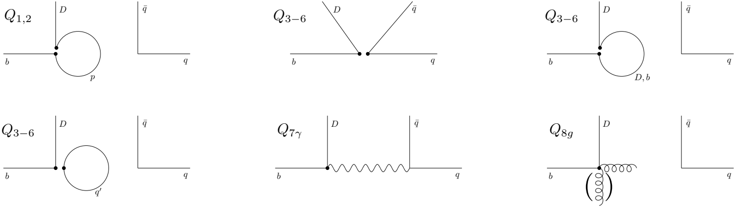
$$s = \sqrt{1 - 4z_c/\bar{u}}\ ,\ r = \sqrt{1 - 4z_c} \quad \longleftrightarrow \quad \bar{u}\ ,\ z_c = \frac{m_c^2}{m_b^2} \quad \longleftrightarrow \quad s_1 = \sqrt{1 - 4/\bar{u}}\ ,\ r$$

$$\updownarrow$$

$$p = \frac{1 - \sqrt{u^2 + 4\bar{u}z_c}}{\bar{u}}\ ,\ r$$

Insertions

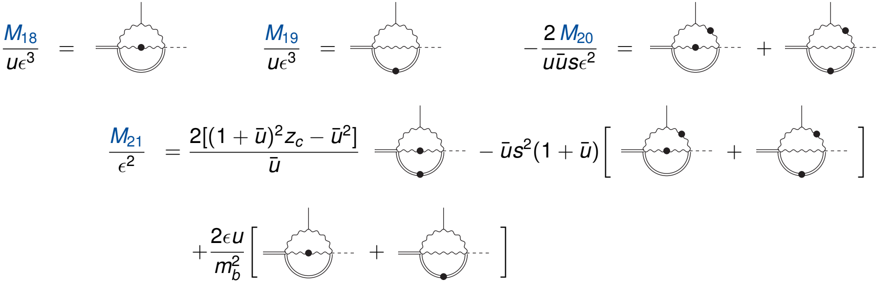
- Quite some book-keeping due to various insertions



- Focus on $Q_1^{u,c}$ and $Q_2^{u,c}$ insertions
- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- Reduction: Integration-by-parts relations, Laporta algorithm
  [Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08; Studerus,von Manteuffel'10,'12]
  - Obtain a set of 29 master integrals

Computing the masters

- Use differential equations in canonical form *[Henn'13]*
  - Found canonical basis for all masters, including boundary conditions ✓
  - First example of canonical basis in case of 2 different internal masses
  - Found analytical solution in terms of iterated integrals ✓ *[Bell,TH'14]*



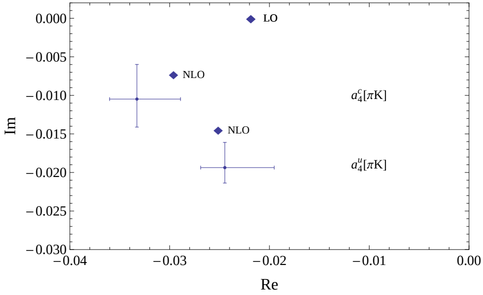
- Benefits of canonical basis
  - No fake higher weights
  - QCD amplitude much simpler
  - Suitable for convolution with LCDA

Results: Penguin Amplitudes

- Preliminary numbers: Only $Q_{1,2}$ contribution. Inputs from *[Beneke,Li,TH'09]*

$$\alpha_4^u(\pi K) = -0.029 + [0.004 - 0.014i]_{\rm NLO} + [-0.003 - 0.007i]_{\rm NNLO}$$
$$+ \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.001]_{\rm LOsp} + [0.001 + 0.002i]_{\rm NLOsp} + [0.001]_{\rm tw3}\right\}$$
$$= (-0.024^{+0.005}_{-0.002}) + (-0.019^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi K) = -0.029 + [-0.001 - 0.008i]_{\rm NLO} + [-0.008 - 0.005i]_{\rm NNLO}$$
$$+ \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.001]_{\rm LOsp} + [0.001 + 0.001i]_{\rm NLOsp} + [0.001]_{\rm tw3}\right\}$$
$$= (-0.033^{+0.004}_{-0.003}) + (-0.010^{+0.004}_{-0.004})i$$



Results: Amplitude ratios

Ratio	NLO	NNLO
$\frac{P_{\pi\pi}}{T_{\pi\pi}}$	$-0.121-0.021i$	$-0.124^{+0.031}_{-0.060}+(-0.026^{+0.045}_{-0.046})i$
$\frac{P_{\rho\rho}}{T_{\rho\rho}}$	$-0.035-0.009i$	$-0.041^{+0.020}_{-0.016}+(-0.014^{+0.019}_{-0.018})i$
$\frac{P_{\pi\rho}}{T_{\pi\rho}}$	$-0.038-0.005i$	$-0.040^{+0.016}_{-0.030}+(-0.009^{+0.026}_{-0.026})i$
$\frac{P_{\rho\pi}}{T_{\rho\pi}}$	$0.040+0.002i$	$0.036^{+0.042}_{-0.023}+(-0.001^{+0.033}_{-0.033})i$
$\frac{C_{\pi\pi}}{T_{\pi\pi}}$	$0.317-0.040i$	$0.320^{+0.255}_{-0.142}+(-0.030^{+0.150}_{-0.091})i$
$\frac{C_{\rho\rho}}{T_{\rho\rho}}$	$0.165-0.064i$	$0.176^{+0.187}_{-0.133}+(-0.054^{+0.142}_{-0.104})i$
$\frac{C_{\pi\rho}}{T_{\pi\rho}}$	$0.219-0.064i$	$0.212^{+0.197}_{-0.112}+(-0.062^{+0.114}_{-0.079})i$
$\frac{C_{\rho\pi}}{T_{\rho\pi}}$	$0.092-0.080i$	$0.112^{+0.189}_{-0.144}+(-0.065^{+0.152}_{-0.115})i$
$\frac{T_{\rho\pi}}{T_{\pi\rho}}$	$0.821+0.016i$	$0.810^{+0.262}_{-0.200}+(\ 0.010^{+0.062}_{-0.062})i$
$\frac{\alpha_4^c(\pi K)}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}$	$-0.085-0.019i$	$-0.087^{+0.022}_{-0.036}+(-0.021^{+0.029}_{-0.029})i$
$\frac{\alpha_4^c(\pi K^*)}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}$	$-0.029-0.005i$	$-0.030^{+0.015}_{-0.026}+(-0.007^{+0.023}_{-0.023})i$
$\frac{\alpha_4^c(\rho K)}{\alpha_1(\rho\rho)+\alpha_2(\rho\rho)}$	$0.037+0.004i$	$0.034^{+0.039}_{-0.021}+(\ 0.001^{+0.030}_{-0.030})i$
$\frac{\alpha_4^c(\rho K^*)}{\alpha_1(\rho\rho)+\alpha_2(\rho\rho)}$	$-0.023-0.010i$	$-0.027^{+0.027}_{-0.016}+(-0.012^{+0.024}_{-0.023})i$

- Preliminary numbers: Only $Q_{1,2}$ contribution. Inputs from

[Beneke,Li,TH'09]

Results: BR and $A_{\rm CP}$

- Preliminary numbers: Only $Q_{1,2}$ contribution. Inputs from *[Beneke,Li,TH'09]*
- Branching ratios (1st line, in 10^{-6})
- Direct CP asymmetries (2nd line, in 10^{-2}).

	NNLO	NLO	Experiment
$B^- \to \pi^-\pi^0$	$5.43\,^{+2.66}_{-2.14}\,^{+2.05}_{-1.73}\,^{+1.27}_{-0.57}\,^{+0.52}_{-0.50}$	5.33	$5.48\,^{+0.35}_{-0.34}$
	$-0.18\,^{+0.03}_{-0.05}\,^{+0.08}_{-0.07}\,^{+0.03}_{-0.02}\,^{+0.01}_{-0.01}$	-0.09	$2.6\,^{+3.9}_{-3.9}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.47\,^{+3.15}_{-2.61}\,^{+3.36}_{-2.76}\,^{+0.30}_{-0.60}\,^{+1.18}_{-0.66}$	7.30	$5.10\,^{+0.19}_{-0.19}$
	$-9.31\,^{+1.82}_{-2.06}\,^{+3.08}_{-3.39}\,^{+0.40}_{-1.06}\,^{+15.74}_{-15.01}$	-7.67	$-31\,^{+5}_{-5}$
$\bar{B}^0_d \to \pi^0\pi^0$	$0.35\,^{+0.14}_{-0.11}\,^{+0.19}_{-0.11}\,^{+0.33}_{-0.09}\,^{+0.20}_{-0.10}$	0.33	$1.33\,^{+0.46}_{-0.46}$
	$44.2\,^{+7.9}_{-7.3}\,^{+18.3}_{-39.3}\,^{+17.2}_{-25.8}\,^{+47.7}_{-65.8}$	43.7	$43\,^{+24}_{-24}$
$B^- \to \pi^-\bar{K}^0$	$16.03\,^{+0.79}_{-0.77}\,^{+9.66}_{-6.68}\,^{+0.87}_{-1.28}\,^{+13.51}_{-\ \ 5.61}$	14.94	$23.79\,^{+0.75}_{-0.75}$
	$0.67\,^{+0.15}_{-0.16}\,^{+0.25}_{-0.24}\,^{+0.04}_{-0.04}\,^{+0.76}_{-0.64}$	0.62	$-1.5\,^{+1.9}_{-1.9}$
$B^- \to \pi^0K^-$	$9.57\,^{+0.79}_{-0.74}\,^{+5.00}_{-3.50}\,^{+0.18}_{-0.39}\,^{+7.15}_{-3.01}$	8.97	$12.94\,^{+0.52}_{-0.51}$
	$9.58\,^{+2.26}_{-2.25}\,^{+2.18}_{-3.25}\,^{+0.09}_{-0.11}\,^{+10.50}_{-11.61}$	8.86	$4.0\,^{+2.1}_{-2.1}$
$\bar{B}^0_d \to \pi^+K^-$	$14.01\,^{+1.09}_{-1.03}\,^{+8.43}_{-5.76}\,^{+0.12}_{-0.26}\,^{+11.92}_{-\ \ 4.92}$	12.88	$19.57\,^{+0.53}_{-0.52}$
	$7.01\,^{+1.66}_{-1.65}\,^{+2.27}_{-2.59}\,^{+0.36}_{-0.18}\,^{+\ \ 9.91}_{-11.13}$	6.15	$-8.2\,^{+0.6}_{-0.6}$
$\bar{B}^0_d \to \pi^0\bar{K}^0$	$5.82\,^{+0.31}_{-0.31}\,^{+4.05}_{-2.72}\,^{+0.07}_{-0.16}\,^{+5.58}_{-2.26}$	5.31	$9.93\,^{+0.49}_{-0.49}$
	$-4.47\,^{+1.05}_{-0.98}\,^{+3.39}_{-2.08}\,^{+0.23}_{-0.17}\,^{+4.09}_{-3.83}$	-4.47	$-1\,^{+10}_{-10}$

Conclusion

- $Q_{1,2}$-contribution to penguin amplitudes α_4^u and α_4^c at NNLO ready
  - Solution to master integrals in canonical basis enables results which are almost completely analytical and numerically accurate to high precision.
- Preliminary results
  - NNLO shift in amplitudes is rather sizable
  - Shift in amplitude ratios, BRs, CP asymmetries is moderate