4th QFET Workshop

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Testing models of lepton flavour via mixing sum rules

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Symmetry approach to flavour

Fermion mixing

- mismatch of flavour (weak) and mass eigenstates

$$\Psi_{\text{flavour}} \;=\; V^\dagger\,\Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} \;=\; V_L^u\,V_L^{d\dagger} \;\approx\; \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} \;=\; V_L^e\,V_L^{\nu\dagger} \;\approx\; \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.68 \end{pmatrix}$$

www.nu-fit.org (2014)

neutrino mixing pattern suggestive of non-Abelian family symmetry

Simple mixing patterns – tri-bimaximal

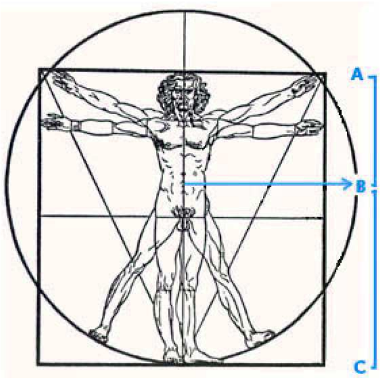
  

Harrison Perkins Scott

$$U_{\mathrm{PMNS}} \approx U_{\mathrm{TB}} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 35.3^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

Simple mixing patterns – golden ratio



$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

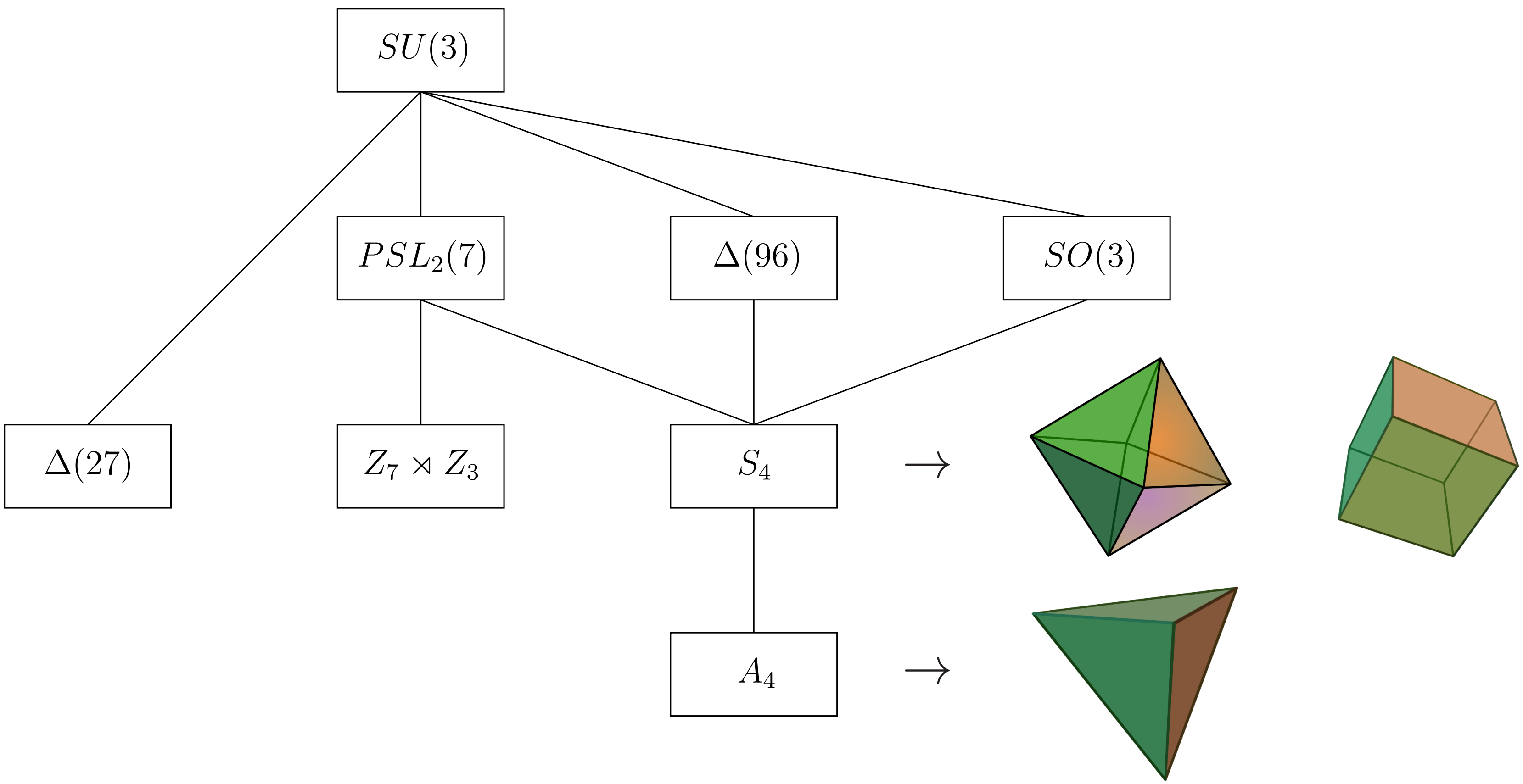
$$\tan\theta_{12} = \frac{1}{\varphi}$$

$$U_{\mathrm{PMNS}} \approx U_{\mathrm{GR}} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} \approx 31.7^\circ \qquad \theta_{23} = 45^\circ \qquad \theta_{13} = 0^\circ$$

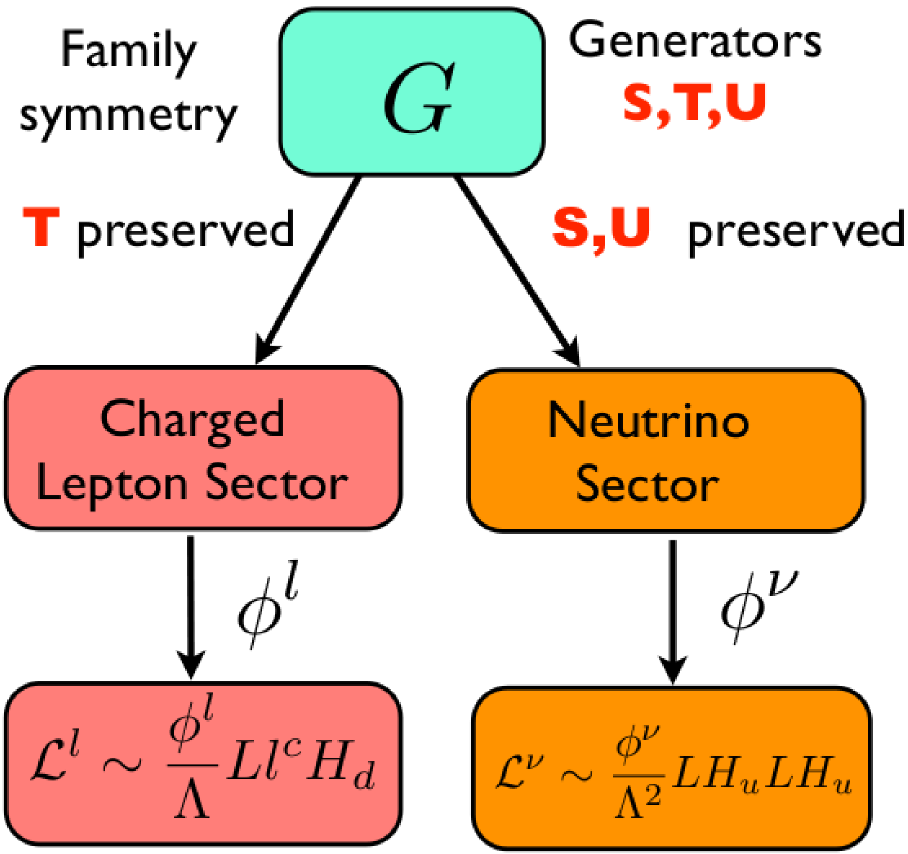
Candidate symmetry groups G

- non-Abelian to unify families (G should have triplet representations)
- discrete to facilitate obtaining simple mixing patterns

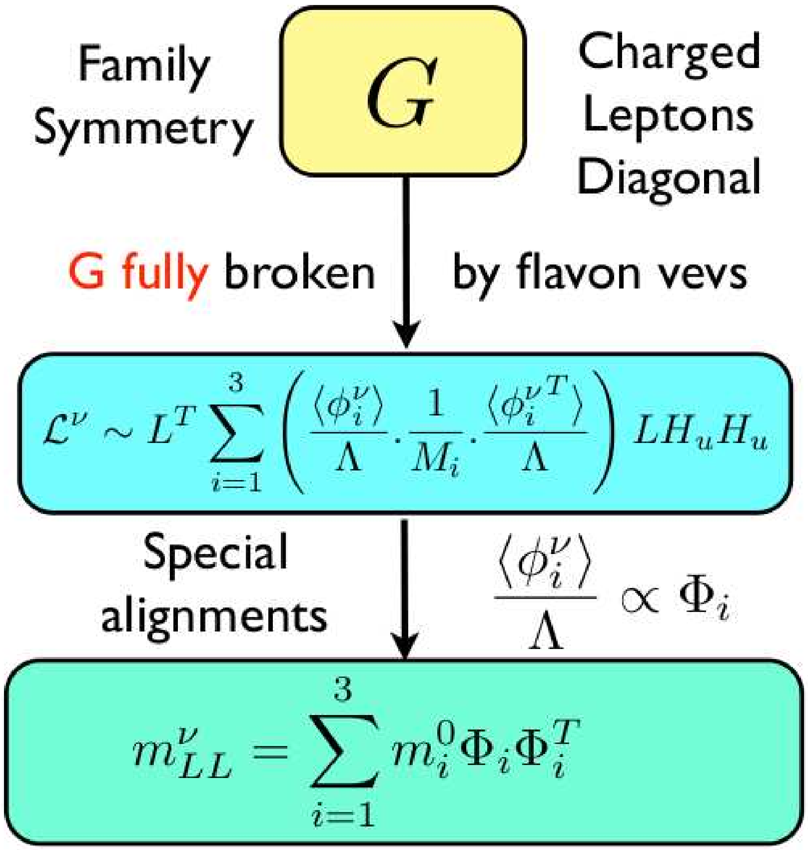


Two model building strategies

direct models
(mixing from residual symmetries)



indirect models
(mixing from flavon alignments Φ)



King, Luhn (JHEP 0910, 2009)

Tri-bimaximal mixing from S_4

- three generators S, U and T
- identify VEV configurations for family symmetry breaking fields ϕ

$$S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle \qquad T\langle\phi^\ell\rangle = \langle\phi^\ell\rangle \qquad \text{flavon VEVs}$$

S_4	S	U	T	$\langle\phi^\nu\rangle$	$\langle\phi^\ell\rangle$
$\mathbf{1},\mathbf{1}'$	1	±1	1	$\mathbf{1}$	$\mathbf{1},\mathbf{1}'$
$\mathbf{2}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}0&1\\1&0\end{pmatrix}$	$\begin{pmatrix}\omega&0\\0&\omega^2\end{pmatrix}$	$\mathbf{2}\propto\begin{pmatrix}1\\1\end{pmatrix}$	$-$
$\mathbf{3},\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix}$	$\mp\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{pmatrix}$	$\mathbf{3}'\propto\begin{pmatrix}1\\1\\1\end{pmatrix}$	$\mathbf{3},\mathbf{3}'\propto\begin{pmatrix}1\\0\\0\end{pmatrix}$

- control coupling of flavons to fermions by extra Z_N or $U(1)$ symmetry

$$\frac{\phi^\nu}{\Lambda^2}LH_uLH_u \quad\rightarrow\quad M_\nu = S^TM_\nu S = U^TM_\nu U \quad\rightarrow\quad \text{tri-bimaximal } M_\nu$$

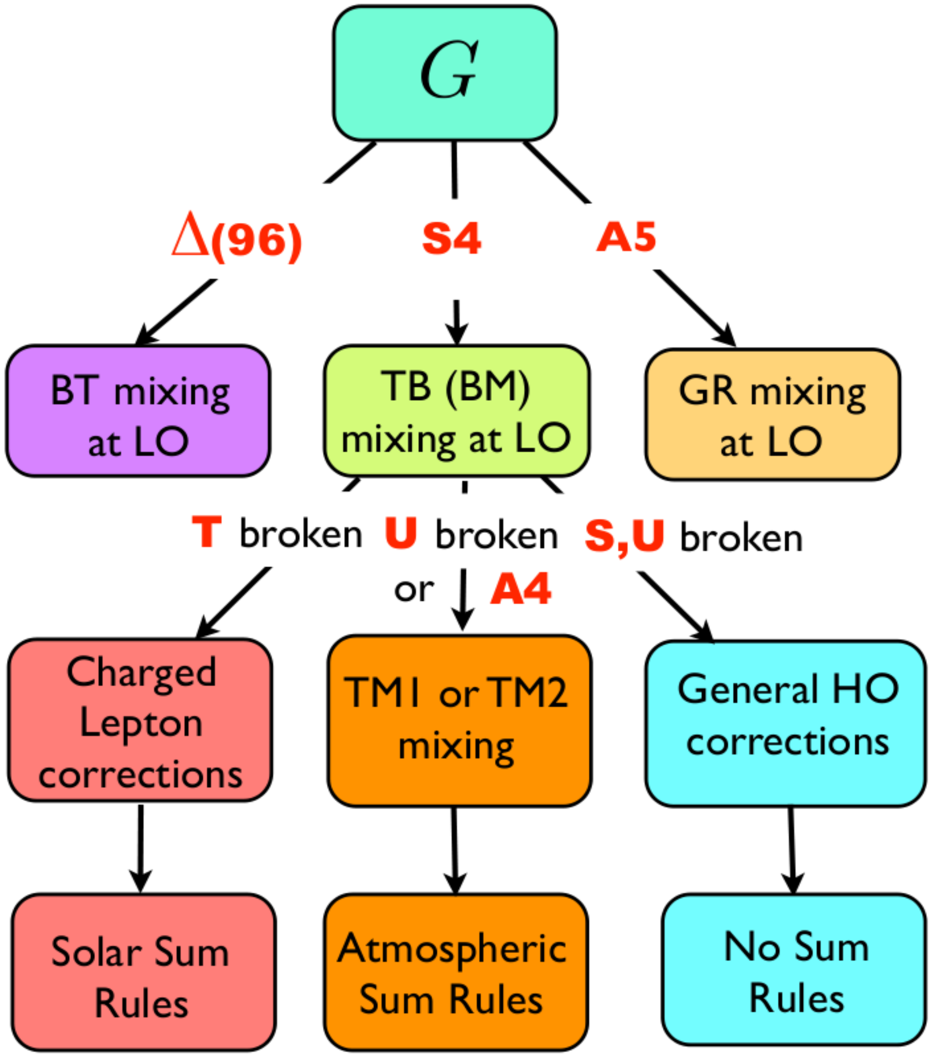
$$\frac{\phi^\ell}{\Lambda}L\,\ell^cH_d \quad\rightarrow\quad \left[M_\ell M_\ell^\dagger\right] = T^T\left[M_\ell M_\ell^\dagger\right]T^* \quad\rightarrow\quad \text{diagonal } \left[M_\ell M_\ell^\dagger\right]$$

Accommodating $\theta_{13} \sim 9^\circ$

King, Luhn (Rept. Prog. Phys. 76, 2013)

King et al. (New J. Phys. 16, 2014)

Direct models after 2012



mixing patterns:

	θ_{13}	θ_{23}	θ_{12}
TB	0°	45°	35.3°
BM	0°	45°	45°
GR	0°	45°	31.7°
BT	12.2°	36.2°	36.2°
TM	$\neq 0^\circ$	$\neq 45^\circ$	35.3°

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

Perturbation I – solar mixing sum rule

- T symmetry of charged lepton sector "slightly" broken (e.g. GUTs)
- $U_{\rm PMNS} = V_{\ell_L} V^\dagger_{\nu_L}$ and $V^\dagger_{\nu_L} = U_{\rm TB}$

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}^*_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}^*_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}^*_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12}\, e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left(e^{i\delta^\nu_{12}} - \theta^\ell_{12}\, e^{i\delta^\ell_{12}} + \theta^\ell_{13}\, e^{i(\delta^\ell_{13} - \delta^\nu_{23})} \right)$$

$$s_{23}\, e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left(e^{i\delta^\nu_{23}} - \theta^\ell_{23}\, e^{i\delta^\ell_{23}} \right)$$

$$s_{13}\, e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left(-\theta^\ell_{12}\, e^{i(\delta^\ell_{12} + \delta^\nu_{23})} - \theta^\ell_{13}\, e^{i\delta^\ell_{13}} \right)$$

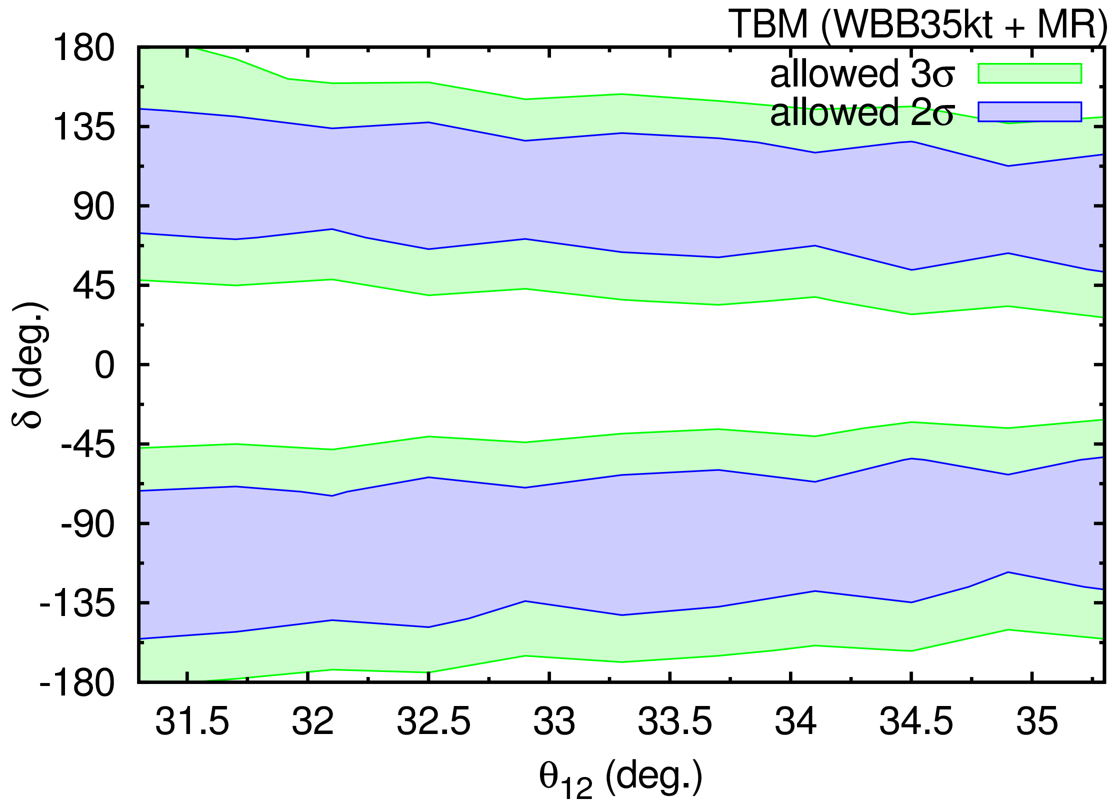
$$c_{ij} = \cos\theta_{ij}$$

$$\hat{s}_{ij} = \sin\theta_{ij}\, e^{-i\delta_{ij}}$$

- $\theta^\ell_{12} \sim \theta_C \sim 0.22 \quad \rightarrow \quad \theta_{13} \sim 9^\circ$
- first order relation $\boxed{\theta_{12} \approx 35.3^\circ + \theta_{13} \cos\delta}$

Testing the solar sum rule

- JUNO will measure θ_{12} with high precision
- wide-band superbeam (LBNO/LBNE/LBNF) could access Dirac phase δ
- expected sensitivity for ruling out solar sum rule



Ballett et al.
(arXiv:1410.7573)

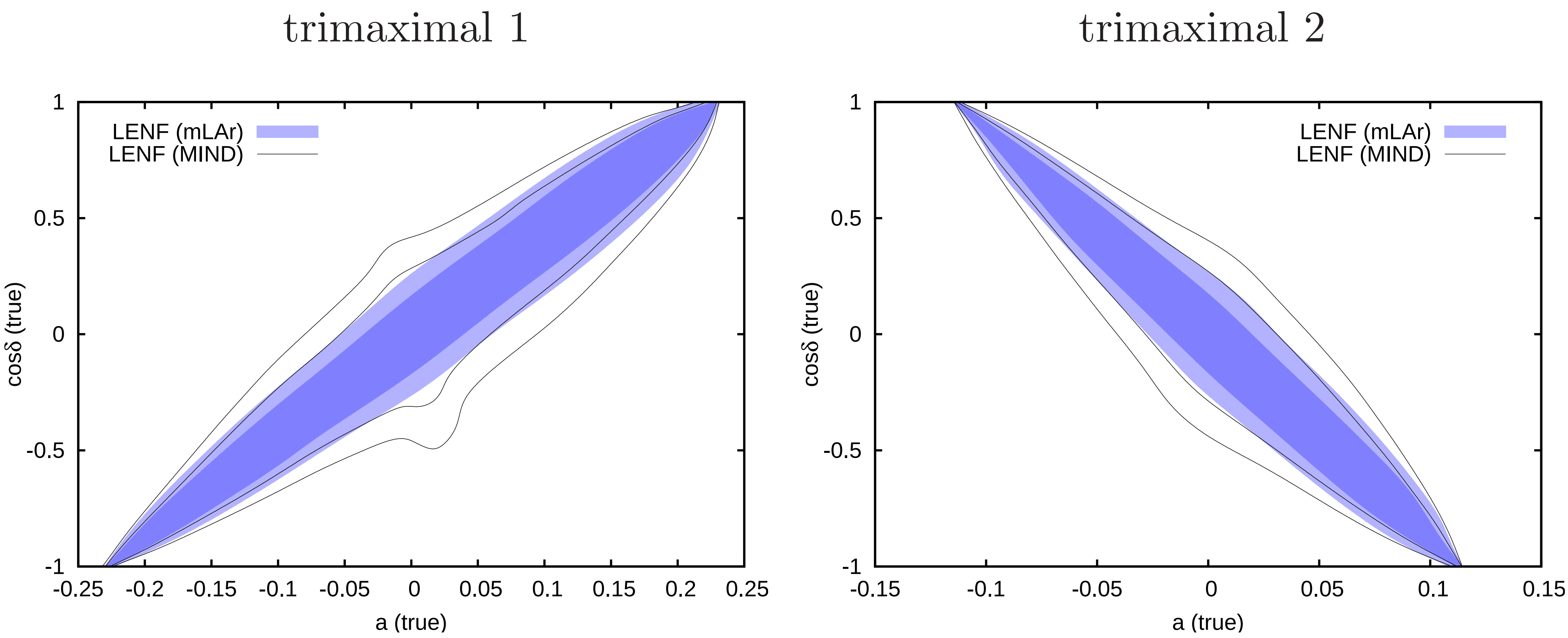
Perturbation II – atmospheric mixing sum rule

- U symmetry of neutrino sector "slightly" broken $\rightarrow$ $U^{13}_{\mathrm{PMNS}} \neq 0$
- conserve a Z_2 subgroup of neutrino symmetry $Z_2^S \times Z_2^U$

	trimaximal 1 (TM_1)	trimaximal 2 (TM_2)
unbroken Z_2	$SU = -\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}$	$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$
PMNS mixing	$\frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \cdot & \cdot \\ -1 & \cdot & \cdot \\ -1 & \cdot & \cdot \end{pmatrix}$	$\frac{1}{\sqrt{3}}\begin{pmatrix} \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \\ \cdot & 1 & \cdot \end{pmatrix}$
solar angle	$\theta_{12} \approx 34.2^\circ$	$\theta_{12} \approx 35.8^\circ$
first order relation	$\boxed{\theta_{23} \approx 45^\circ + \sqrt{2}\,\theta_{13}\cos\delta}$	$\boxed{\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta}$

Testing the atmospheric sum rule

- low energy neutrino factory could measure θ_{23} and δ to high precision
- expected sensitivity for ruling out atmospheric sum rule



Ballett et al. (Phys. Rev. D89, 2014)

Conclusion

- non-Abelian discrete symmetries
  - unify three families of chiral fermions
  - still attractive despite $\theta_{13} \sim 9^\circ$
- perturbations of simple mixing patterns
  - correlations among mixing parameters
  - solar/atmospheric mixing sum rules
- testing models of lepton flavour
  - measurement of CP phase (e.g. superbeam/neutrino factory)
  - precision measurement of mixing angles (e.g. JUNO)
- other aspects
  - gain predictivity by imposing CP symmetry
  - connection to quarks via grand unified theories

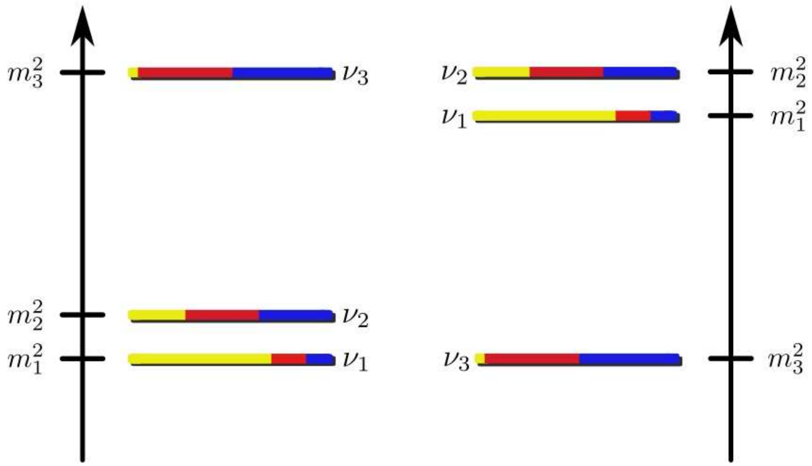
Thank you

Three neutrino flavour mixing

(in diagonal charged lepton basis)

flavour PMNS mixing mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



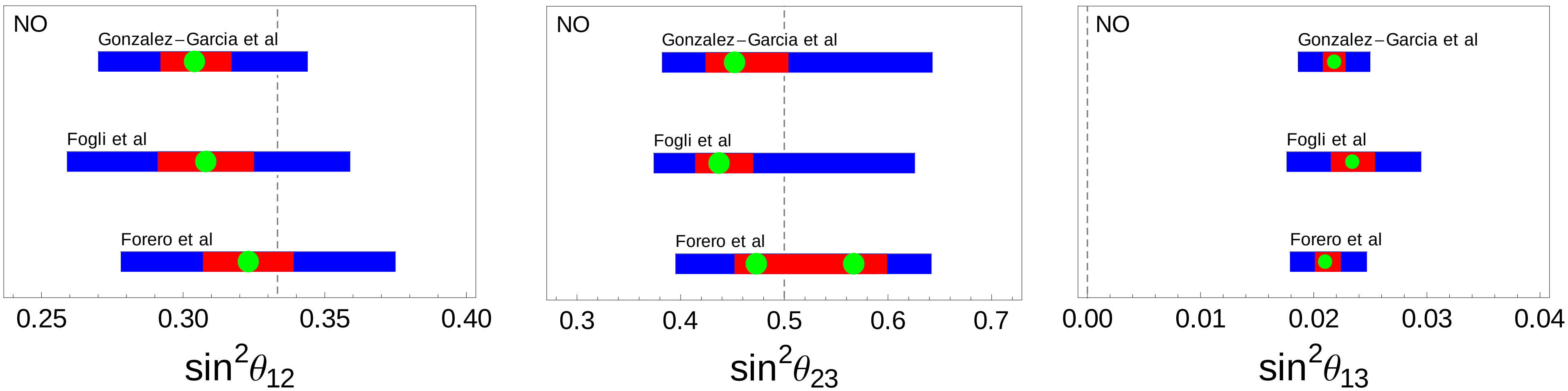
atmospheric reactor + Dirac solar Majorana

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

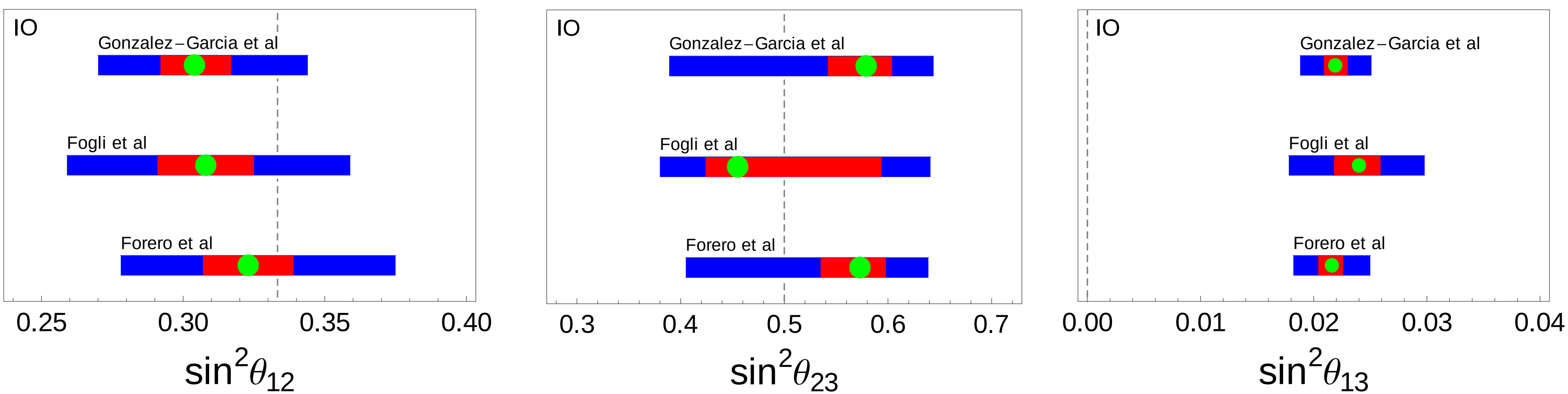
$\theta_{23} \approx 45^\circ$ $\theta_{13} \approx 9^\circ$ $\theta_{12} \approx 33^\circ$

Global neutrino fits

normal mass ordering

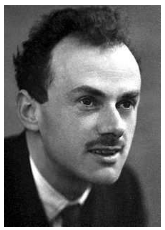


inverted mass ordering



Symmetries of the mass matrices (in flavour basis)

charged leptons



Dirac

$$M_\ell = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

symmetric under diagonal phase transformation h

$$\boxed{M_\ell = h^T M_\ell\, h^*}$$

e.g. $h = \mathrm{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$

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neutrinos

Majorana

$$M_\nu = U_{\mathrm{PMNS}}\,\mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,U^T_{\mathrm{PMNS}}$$

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu = k^T M_\nu\, k}$$

$k = U^*_{\mathrm{PMNS}}\,\mathrm{diag}\,(+1, -1, -1)\,U^T_{\mathrm{PMNS}}$

four different k $\rightarrow$ generate $Z_2 \times Z_2$ symmetry group

Klein symmetry: $K = \{1, k_1, k_2, k_3\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

Origin of the Klein symmetry K

- direct models
  - $\boxed{\text{Klein symmetry } K \ \subset \ \text{family symmetry } G}$
  - flavons ϕ are multiplets of G
  - their VEVs $\langle\phi\rangle$ break G down to K in neutrino sector
  - for TB mixing (k_1, k_2, h) generate permutation group S_4
- indirect models
  - $\boxed{\text{Klein symmetry } K \ \not\subset \ \text{family symmetry } G}$
  - G responsible for generating particular flavon VEV configurations $\langle\phi\rangle$
  - for TB mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\ H\,H$$

Essentials of finite group theory – S_4 example

e.g. Ramond, Group theory: a physicist's survey (2010)

- finite number of group elements g $\rightarrow$ each g has finite order ($g^n = 1$)
- construct all elements from a small number of generators
- presentation of S_4: generators S, T, U which satisfy

$$S^2 = T^3 = U^2 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$$

- matrix representations for S, T, U $\rightarrow$ irreps $\mathbf{1}$ $\mathbf{1}'$ $\mathbf{2}$ $\mathbf{3}$ $\mathbf{3}'$
- in physics we are mainly interested in multiplication rules
- Kronecker products: e.g. $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3}'$
- Clebsch-Gordan coefficients: e.g. $\mathbf{3} \otimes \mathbf{3} \rightarrow \mathbf{1}$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \rightarrow \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

basis dependent !!