$B^0 - \bar{B}^0$ mixing beyond factorization: three loop QCD SR analysis

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Gauge interaction of $U_i = (u, c, t)$ and $D_j = (d, s, b)$ with W-bosons includes CKM matrix V_{ij}

$$V_{ij} \cdot \bar{U}_i \gamma_\mu D_j W^\mu$$

Nonvanishing off-diagonal elements of V_{ij} initiate transitions between flavors

$$b \to c, \quad b \leftarrow s, \quad s \to u, \quad c \leftrightarrow d, \quad \ldots$$

$\Delta F = 2$ transitions: mixing of different flavor mesons

$$sd: \; K^0 - \bar{K}^0; \quad cu: \; D^0 - \bar{D}^0; \quad bd, bs: \; B^0 - \bar{B}^0$$

is important for CP violation studies

$B^0 - \bar{B}^0$ mixing phenomenology

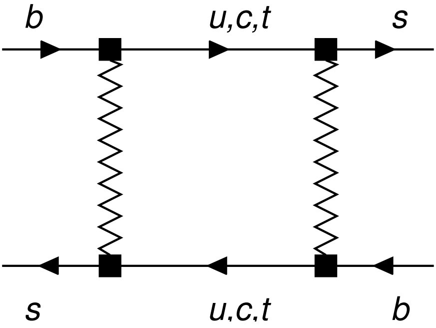
Evolution of $(B^0, \bar{B}^0)$ system is described

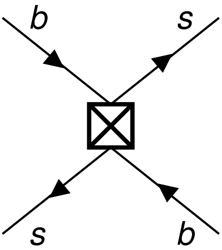
$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = H_{eff}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

with H_{eff} being a 2×2 matrix

$$H_{eff} = (M - i\Gamma/2)_{ij}\,, \quad i,j = 1,2$$

Non-diagonal elements M_{12} and Γ_{12} are effective $\Delta B = 2$ interactions: calculable in SM.
Observables of $B^0 - \bar{B}^0$ system:
mass difference: $\Delta m = M_{heavy} - M_{light} \approx 2\,|M_{12}|$
decay rates difference:
$\Delta\Gamma = \Gamma_L - \Gamma_H \approx -2\,|\Gamma_{12}|\cos\Phi,\ \Phi = \arg(-M_{12}/\Gamma_{12})$





At quark level $\Delta B = 2$ processes go through
a box diagram that becomes a transition operator.

$$M_{12} = \frac{G_F^2 M_W^2}{4\pi^2} \left(V_{tb}{}^* V_{td}\right)^2 \langle \bar{B}^0 | Q(\mu) | B^0 \rangle$$

$Q(\mu) = (\bar{b}_L \gamma_\sigma d_L)(\bar{b}_L \gamma_\sigma d_L)(\mu)$ – local four-quark operator

$$\Delta\Gamma \sim \Gamma_{12} = \langle \bar{B}_s | \mathcal{T} | B_s \rangle / 2M_{B_s}$$

Final states in decays are (c, u) "quarks", $m_b \gg m_c, m_u$ and HQE in $1/m_b$ is used

$$\langle \bar{B}_s | \mathcal{T} | B_s \rangle = \sum_n \frac{C_n}{m_b^n} \langle \bar{B}_s | \mathcal{O}_n^{\Delta B=2} | B_s \rangle$$

C_n are calculable in PT. nonPT physics is contained in ME of local operators $\mathcal{O}_n^{\Delta B=2}$.
At LO in $1/m_b$ two four-quark operators are in $\mathcal{T}$

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \quad Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}$$

At NLO in $1/m_b$ there are more.

Phenomenology: factorization

Since $Q_i \sim J \cdot J$ with $J \sim \bar{s}b$ and $\langle \bar{B}|'' ='' s\bar{b}$ it is prompting "to factorize" (vacuum saturation)

$$\langle \bar{B}|Q_i|B\rangle = \langle \bar{B}|J \cdot J|B\rangle = C_{\mathrm{comb}}\langle \bar{B}|J|0\rangle\langle 0|J|B\rangle$$

For $J \sim \bar{b}_L\gamma_\mu d_L, \qquad \langle 0|\bar{b}_L\gamma_\mu d_L|B^0(\mathbf{p})\rangle = ip_\mu f_B/2.$
Main problem: accuracy of such factorization
Writting

$$\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle = B_i\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle^{fac}$$

one introduces bag parameters B_i – QCD quantities controlling the accuracy of the factorization
normalization $B_i = 1$ in factorization approximation

For relevant operators

$$\begin{aligned}
\langle \bar{B}|Q|B\rangle &= f_B^2 M_B^2 2\left(1+\frac{1}{N_c}\right) B \\
\langle \bar{B}|Q_S|B\rangle &= -f_B^2 M_B^2 \frac{M_B^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right) B_S \\
\langle \bar{B}|R_2|B\rangle &= -f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1-\frac{1}{N_c}\right) B_2 \\
\langle \bar{B}|R_3|B\rangle &= f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1+\frac{1}{2N_c}\right) B_3,
\end{aligned}$$

Main theoretical uncertainties of $B^0-\bar{B}^0$ mixing analysis are related to the ME of the local operators $\mathcal{O}_i \in \{Q, Q_S, R_2, R_3\}$, or the bag parameters B_i.

Sum rules

Three-point correlator

$$T(p_1, p_2) = i^2 \int d^4x d^4y e^{ip_1x - ip_2y} \langle Tj(x)\mathcal{O}(0)j(y)\rangle$$

$\mathcal{O} \in \{Q, Q_S, R_2, R_3\}$ is a four-quark operator and j is interpolation current

$$j_5 = \bar{s}i\gamma_5 b$$

The overlap

$$\langle 0|\bar{s}i\gamma_5 b(0)|\bar{B}(p)\rangle = \frac{f_B M_B^2}{m_b + m_s}$$

The dispersion relation

$$T(p_1, p_2) = \int ds_1 ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

determines the spectral density $\rho(s_1, s_2, q^2)$

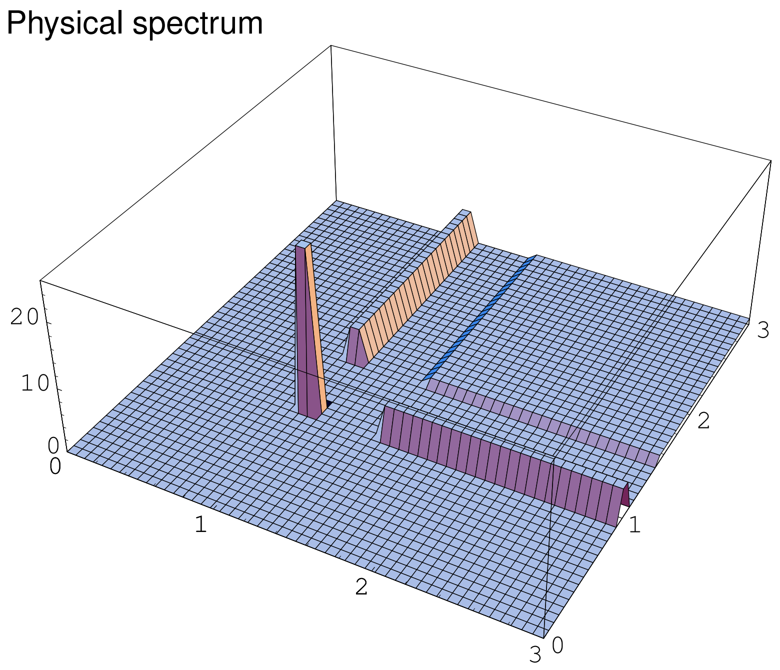
1. Hadronic picture: B-meson pole plus continuum

$$\rho_{\rm AV}^{\rm had}(s_1, s_2) = f_B^2 \delta(s_1 - M_B^2)\delta(s_2 - M_B^2)\langle \bar{B}|\mathcal{O}|B\rangle + \rho_{\rm AV}^{\rm cont}$$

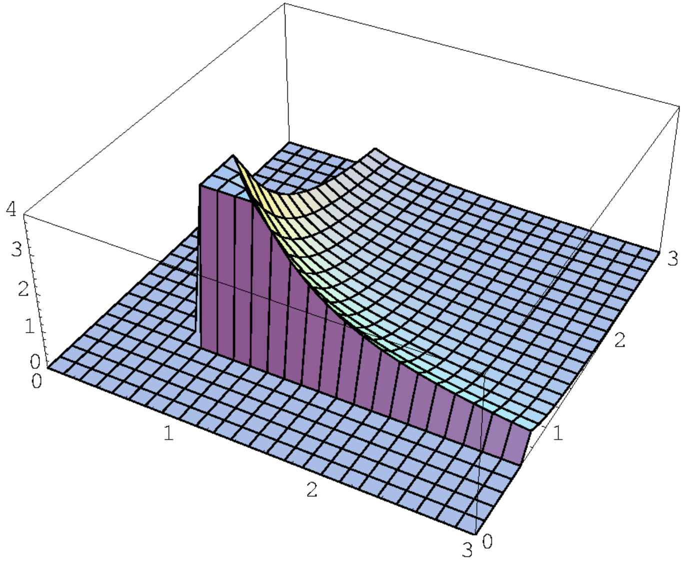
2. With QCD using OPE: $\rho_i^{\rm OPE}$ is a sum of a PT and a nonPT involving condensates.

The idea of QCD sum rules is to use duality

$$\int ds_1 ds_2\, \rho_i^{\rm had}(s_1, s_2) = \int ds_1 ds_2\, \rho_i^{\rm OPE}(s_1, s_2).$$


Physical spectrum
20
10
0
0
1
2
3
0
1
2
3

OPE in QCD



OPE diagrams fall into two categories

$$T(p_1, p_2) = T_{\rm fac}(p_1, p_2) + \Delta T(p_1, p_2)$$

The factorized part has the explicit form

$$T_{fac}(p_1, p_2) = \text{combconst} \times \Pi(p_1)\Pi(p_2)$$

with $\Pi(p_i)$ - a 2-point correlator. For V-A operators

$$T^{\rm AV}_{fac}(p_1, p_2) = 2\left(1 + \frac{1}{N_c}\right)\Pi^V(p_1)\Pi^V(p_2)$$

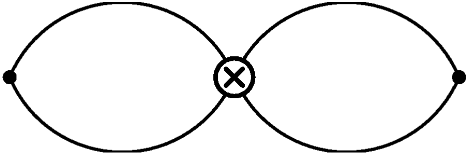
with

$$p^\alpha \Pi^V(p) = i\int dx e^{ipx} \langle Tj(x)\bar{b}\gamma^\alpha(1-\gamma_5)s(0)\rangle.$$

SR for the factorized piece $T_{\rm fac}$ yields $B = 1$.

A sum rule for reads $\Delta B = B - 1$

$$f_B^2 \Delta B e^{-\frac{M_B^2}{M_1^2} - \frac{M_B^2}{M_2^2}} = \int ds_1 ds_2 \Delta\rho^{\rm OPE}_{\rm AV}(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}}$$



At LO in pQCD the three-point function factorizes

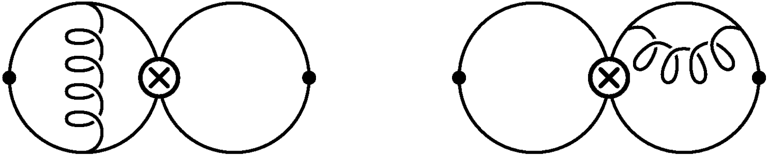
$$T(p_1,p_2)=T_{\rm fac}(p_1,p_2),\quad \Delta T(p_1,p_2)=0$$

and

$$T^{\rm LO}(p_1,p_2)=T^{\rm LO}_{fac}(p_1,p_2)=\mathrm{const}\times\Pi^{\rm LO}(p_1)\Pi^{\rm LO}(p_2)$$

Thus, $B=1$.

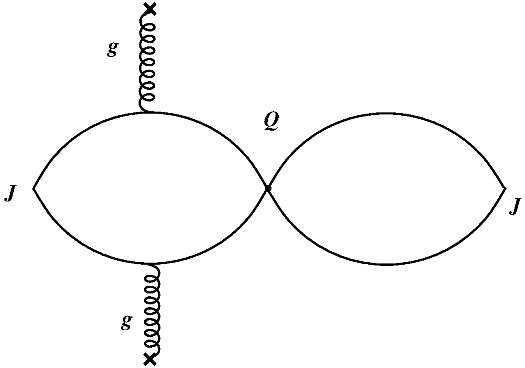
NLO pQCD gives



NLO factorizable contributions are given by the product of two-point correlation functions

$$\Pi^{f}_{\mathrm{NLO}} = \frac{8}{3}(p_1.p_2)\{\Pi_{\mathrm{LO}}(p_1^2)\Pi_{\mathrm{NLO}}(p_2^2) + \mathrm{symm}(p_1, p_2)\}$$

NonPT factorizable contributions: gluon condensate



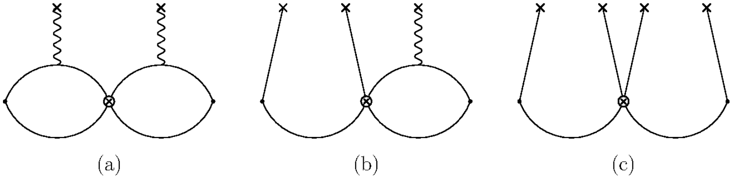
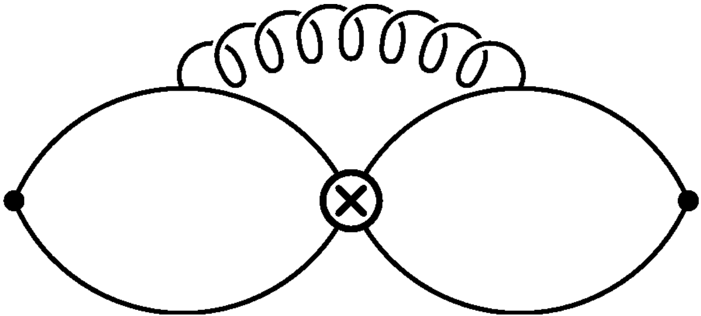


Figure : Non-factorizable condensate contributions

T.Mannel,B.Pecjak, AAP, Eur.Phys.J. C71 (2011) 1607

Non-factorizable contributions. pQCD diagram:



PT non-factorizable contributions at NLO require the calculation of three-loop diagrams.

Status and outlook

Known pieces

- ✓ 1-loop matching of QCD operators to HQET
- ✓ 2-loop anomalous dimensions in HQET
- ✓ Analytical expressions for 3-loop masters *A.Grozin,R.Lee*

We are doing now

- ✓ Computation of 3-loop correlators
- ✓ Reduction to masters (LiteRed package)
- ✓? Spectral densities for disp relations

Nearest future

- Cross-checks, RG analysis of correlators
- Writing down sum rules
- Performing phenomenological analysis (comparison to Lattice...)