Extrapolation and Unitarity Bounds for the $B \to \pi$ Form Factor

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Motivation

- knowledge of the $B \to \pi$ form factor $f^+_{B\pi}(q^2)$ is needed for various interesting phenomenological studies
  - V_{ub} determination (this talk)
  - rare decay $B \to \pi\mu^+\mu^-$ (w/ the tensor form factor $f^T_{B\pi}$)
  - semitauonic decays $B \to \pi\tau\nu$, $B \to \pi\tau\tau$ (w/ the scalar form factor $f^0_{B\pi}$)
- in this work: concentrate on $f^+_{B\pi}$, leave $f^{0,T}_{B\pi}$ for later studies
- apply full statistical (Bayesian) analysis to determine parametric theory uncertainty

$B \to \pi$ form factor in Light-Cone Sum Rules (LCSRs)

$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}(p+q)\rangle$$
$$= f^+_{B\pi}(q^2)\Big[2p_\mu + \big(1 - \frac{m_B^2 - m_\pi^2}{q^2}\big)q_\mu\Big] + f^0_{B\pi}(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q_\mu\,,$$

- calculate FF from LCSRs
  - two-point sum rule for f_B: NLO
  - LCSR: LO for twists 2 to 4; NLO for twist 2,3 (tw-3 only asymptotic)
- parameters $\vec{\theta}$ include
  - quark masses
  - QCD condensate densities
  - coefficients of π distribution amplitudes
  - Borel parameters M^2 and $\overline{M}^2$
  - hadronic thresholds s_0^B and $\bar{s}_0^B$

Applying Bayesian statistics to the correlator(s)

Prior knowledge of input parameters

- prior $P_0(\vec{\theta})$: product of uniform and gaussian probability density functions (PDFs) for all parameters
- most importantly:
  - broad gaussian PDFs for M^2 and $\overline{M}^2$
  - uniform PDFs for s_0^B and $\overline{s}_0^B$

Update knowledge from B-meson mass

- both relevant correlators F_{LCSR} and F_{2ptSR} allow determination of B-meson mass
- schematically

$$\frac{1}{F}\frac{\partial F}{\partial(-1/M^2)} = m_B^2$$

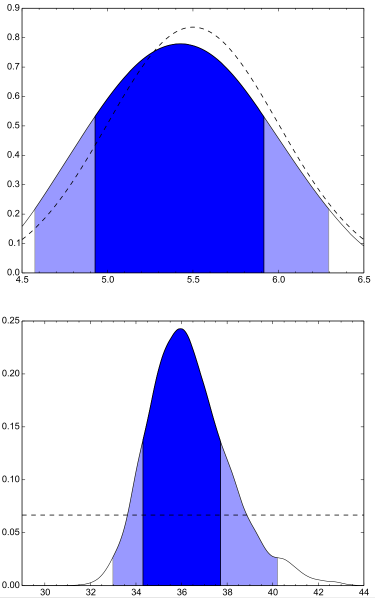
- define likelihood $P(m_B|\vec{\theta})$, with std. deviation $\sigma_B \equiv 1\% \cdot m_B$

Posterior of the parameter space

- compute joint posterior $P(\vec{\theta}|m_B)$
- example: 1D marginal PDFs of
  - 2ptSR Borel parameter (upper)
  - 2ptSR threshold parameter (lower)
- likelihood strongly constrains threshold

prior: dashed lines,

blue: 68% prob., light-blue: 95% prob.



Predictive distribution

compute joint posterior-predictive PDF $P(\vec{F}|m_B)$:

$$P(\vec{F}|m_b) = \int \mathrm{d}\vec{\theta}\delta(\vec{F} - \vec{F}(\vec{\theta}))P(\vec{\theta}|m_B)$$

with

$$\vec{F} \equiv \left(f^+_{B\pi}(0),\, f^{+\prime}_{B\pi}(0),\, f^{+\prime\prime}_{B\pi}(0),\, f^+_{B\pi}(10\,\mathrm{GeV}^2),\, f^{+\prime}_{B\pi}(10\,\mathrm{GeV}^2),\, f^{+\prime\prime}_{B\pi}(10\,\mathrm{GeV}^2)\right)$$

- use $f^+_{B\pi}$ as well as its 1st and 2nd derivative w/r to q^2
- use two q^2 values: $q^2 = 0$ and $q^2 = 10\,\mathrm{GeV}^2$
  - large distance decreases correlations
  - still within LCSR region of applicability
- obtain joint posterior-predictive PDF, $\sim 7\%$ uncertainty on $f^+_{B\pi}$

$$\vec{\mu}^F = \left(0.310,\, 1.55\cdot 10^{-2},\, 1.24\cdot 10^{-3},\, 0.562,\, 4.03\cdot 10^{-2},\, 4.71\cdot 10^{-3}\right)$$
$$\vec{\sigma}^F = \left(0.020,\, 0.10\cdot 10^{-2},\, 0.10\cdot 10^{-3},\, 0.032,\, 0.24\cdot 10^{-2},\, 0.37\cdot 10^{-3}\right)$$

correlation matrix available in the paper

Extrapolation

- use BCL-inspired parametrization

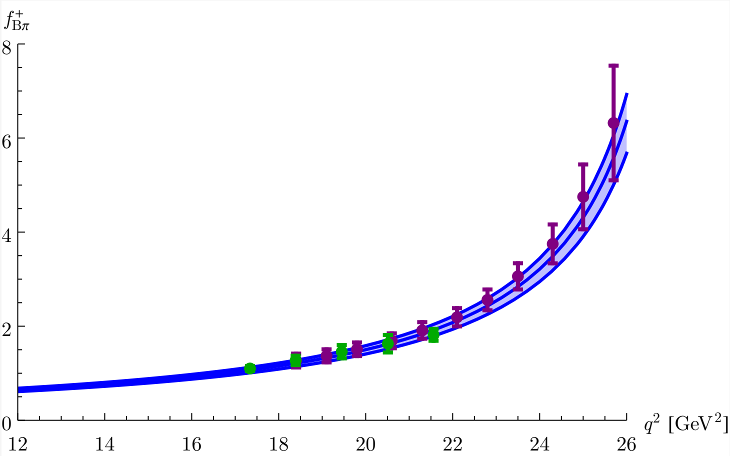
$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B^*}}$$
$$\times \left\{1 + b^+_1 \left[z(q^2, t_0) - z(0, t_0) - \frac{1}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3\right)\right]\right.$$
$$\left. + b^+_2 \left[z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3\right)\right]\right\}.$$

- z: conformal map from real-valued q^2 to unit disc in the complex plane
- fit parameters $f_{B\pi}(0)$, b^+_1, b^+_2 to LCSR results

$$\vec{\mu} = (0.307, -1.31, -0.904) \qquad \vec{\sigma} = (0.020, 0.42, 0.444)$$
$$\rho = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}$$

Extrapolation

BCL and Lattice:

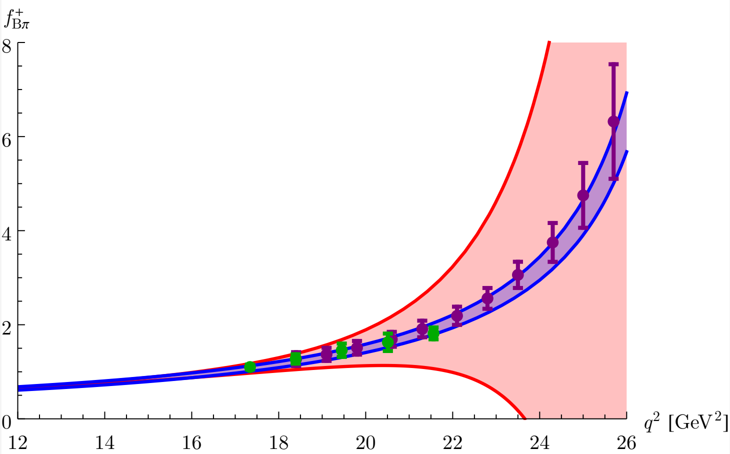


blue: 68% probability envelope and best-fit function,

magenta: Fermilab-MILC, green: HPQCD

Bounds from unitarity/positivity of the correlator

uses FF and 1st/2nd derivatives at $q^2 = 10\,\text{GeV}^2$ as input:

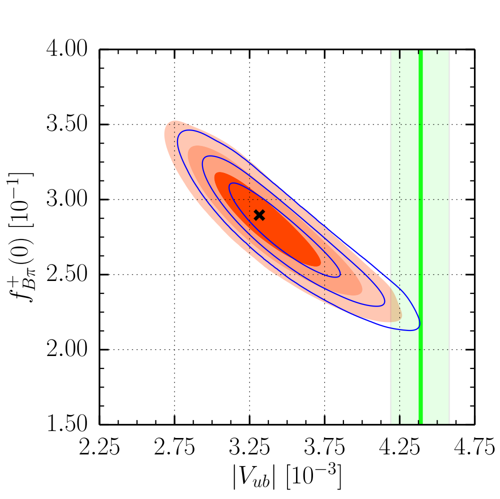


red: 68% envelope from unitarity bounds

blue: 68% probability envelope and best-fit function,

magenta: Fermilab-MILC, green: HPQCD

Determination of $|V_{ub}|$ from $\bar{B}^0 \to \pi^+ \ell \bar{\nu}_\ell$



- 2010 data: Belle+BaBar, 6 bins $q^2 \leq 12\,\text{GeV}^2$
- 2013 data: Belle+BaBar, 6 bins $q^2 \leq 12\,\text{GeV}^2$
- 2010 data vs inclusive: barely compatible @ 99% prob.
- 2013 data increases tension
- 1D marginals:
  - $|V_{ub}|^{2010} = (3.43^{+0.27}_{-0.23}) \cdot 10^{-3}$
  - $|V_{ub}|^{2013} = (3.32^{+0.26}_{-0.22}) \cdot 10^{-3}$

blue lines: 68%, 95%, 99% prob. contours for 2010 data
red area: 68%, 95%, 99% prob. contours for 2013 data
green line/area: central value/68% CL interval for GGOU/HFAG

Summary

- reduction in parametric uncertainty w.r.t. to previous analyses
- first correlation information on $B \to P$ LCSR form factors
- improved semi model-independent bounds, become challenging to lattice data for $16\,\mathrm{GeV}^2 \leq q^2 \leq 20\,\mathrm{GeV}^2$
- find $|V_{ub}|^{2013} = (3.32^{+0.26}_{-0.22}) \cdot 10^{-3}$: 7–8% parametric uncertainty

Outlook

- extend work to other form factors:
  - other currents: scalar (e.g. $B \to \pi\tau\nu$), tensor (e.g. $B \to \pi\ell^+\ell^-$)
  - other transitions: $B \to K$, $B_s \to K$
  - hopefully: full analysis of exclusive $b \to u$ decays using QCD sum rules for f_B, $f_{B\pi}^{+,0}$, $g_{B^*B\pi}$:
    $B \to \pi\mu\nu$, $B \to \pi\tau\nu$, $B \to \tau\nu$
- include systematic theory uncertainties in statistical framework
  - account for $B' \to \pi$ form factor in the LCSR using nuisance parameters
  - ditto: B' decay constant in 2ptSR
  - marginalise