Softened Gravity & the Extension of the SM up to Infinite Energy

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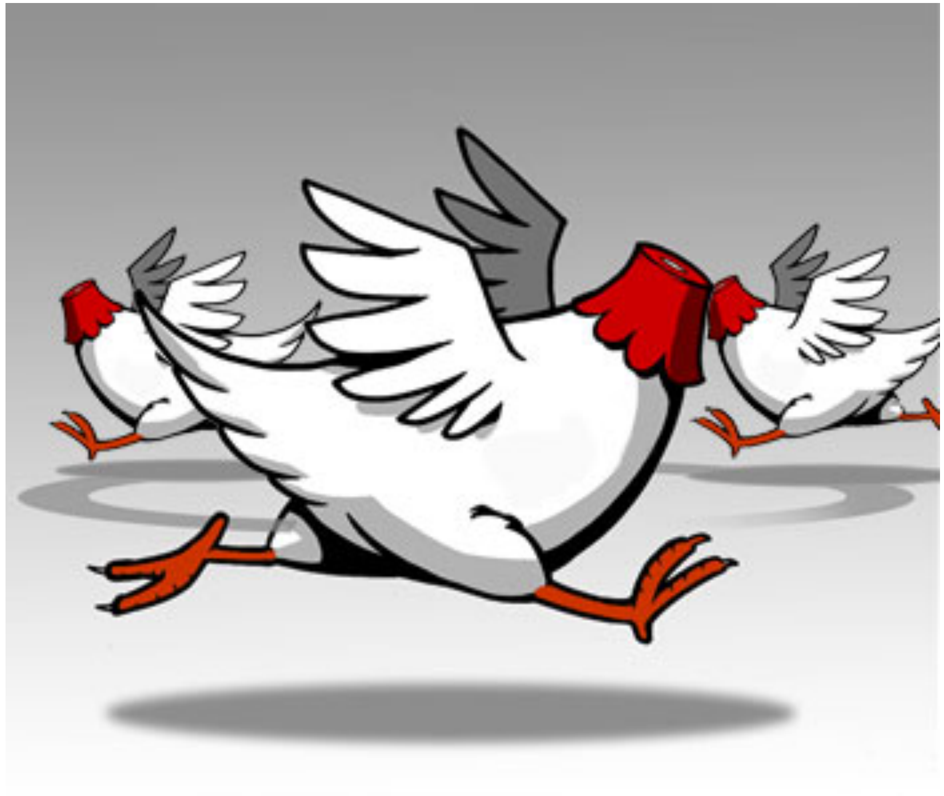
arXiv:1412.2769 with G. Isidori, A. Salvio, A. Strumia

Higgs naturalness is the central issue



Desperate times call for desperate measures

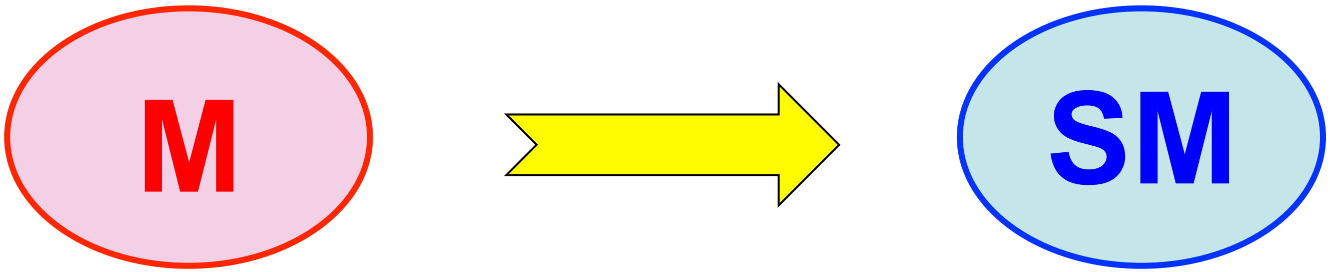
Single mass scale ⇒ no hierarchy

(goes back to ADD extra dim)

Many reasons against:

- quantum gravity
- gauge unification
- inflation
- neutrino mass
- baryogenesis
- axion, ...

But naturalness can work with a weaker constraint than single mass scale...



$$\delta m_H^2 \approx L\, M^2 \qquad L = \frac{g^{2\ell}}{(4\pi)^{2\ell}}$$

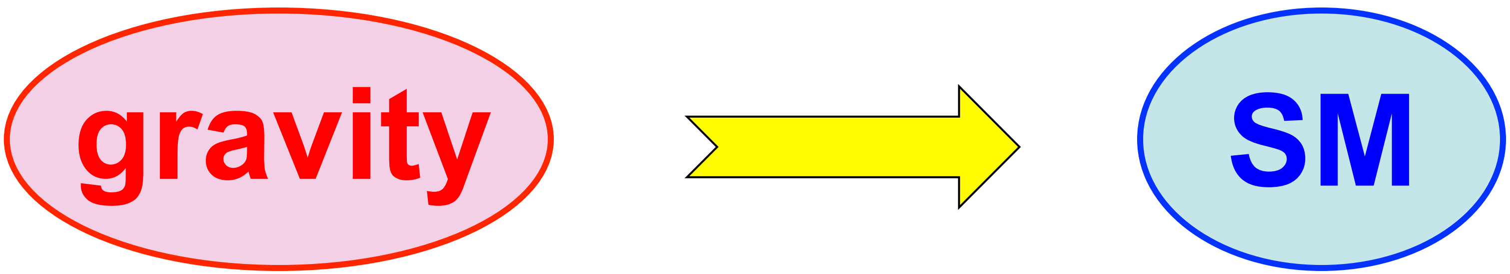
$$g < 4\pi \frac{m_H}{M} \approx 10^{-7} \left(\frac{10^{10}\,\mathrm{GeV}}{M} \right) \qquad \ell = 1$$

Naturalness OK if heavy sector is very-weakly coupled to SM

Gravity is special

- Dimensionful $G_N \Rightarrow$ is it a coupling constant or a mass?
- Derivatively coupled $\partial \Rightarrow$ strength grows with energy

Shutting off short distances at Λ_G



$$\delta m_H^2 \approx \ell\, G_N \Lambda_G^4 \qquad \ell = \frac{1}{(4\pi)^4}$$

$$\Lambda_G < 4\pi\sqrt{m_H M_{Pl}} \approx 10^{11}\,\mathrm{GeV}$$

Requirements for naturalness

- premature UV softening ($\Lambda_G << M_{Pl}$)
- gravitational interactions with SM remain weak above Λ_G

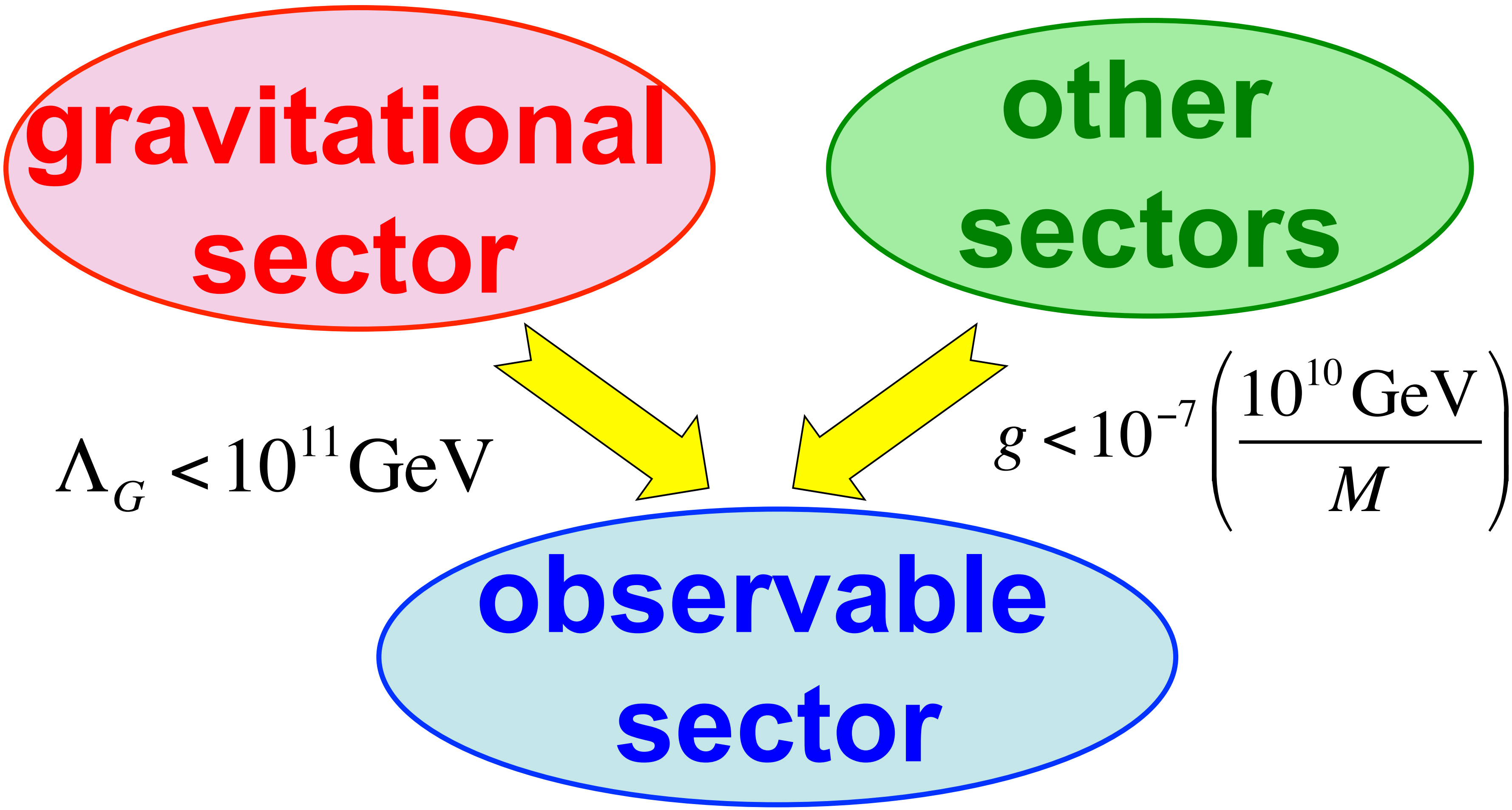
Even without knowing the theory,
I can infer an important result

Scale invariance is broken at the quantum level $\Rightarrow$ new mass scale

No naturalness problem $\Rightarrow$
no new masses above EW in SM sector $\Rightarrow$
SM interactions must be asymptotically free

Gravity cannot cure the problem $\Rightarrow$
always weaker than $G_N \Lambda_G^2 \approx 10^{-16}$

Softened Gravity



- gravity is modified prematurely ($\Lambda_G << M_{Pl}$)
- new physics modifies SM at the EW scale

Result # 1

In the context of theories with no dynamical protection of the Higgs mass, naturalness requires that any safe modification of gravity must occur at scales no larger than

$$\Lambda_{\mathrm{G}} \approx 4\pi(m_H\, M_{Pl})^{1/2} \approx 10^{11}\ \mathrm{GeV}$$

Result # 2

Theories intended to deal with naturalness without new dynamics in the TeV range actually need a large number of new particles around the TeV scale.

- These results are based on common intuition derived from EFT and DA

- They could be wrong in setups that defy this intuition (anthropic arguments in the multiverse?)

How to construct a TAF theory

Gauge coupling

(one loop)

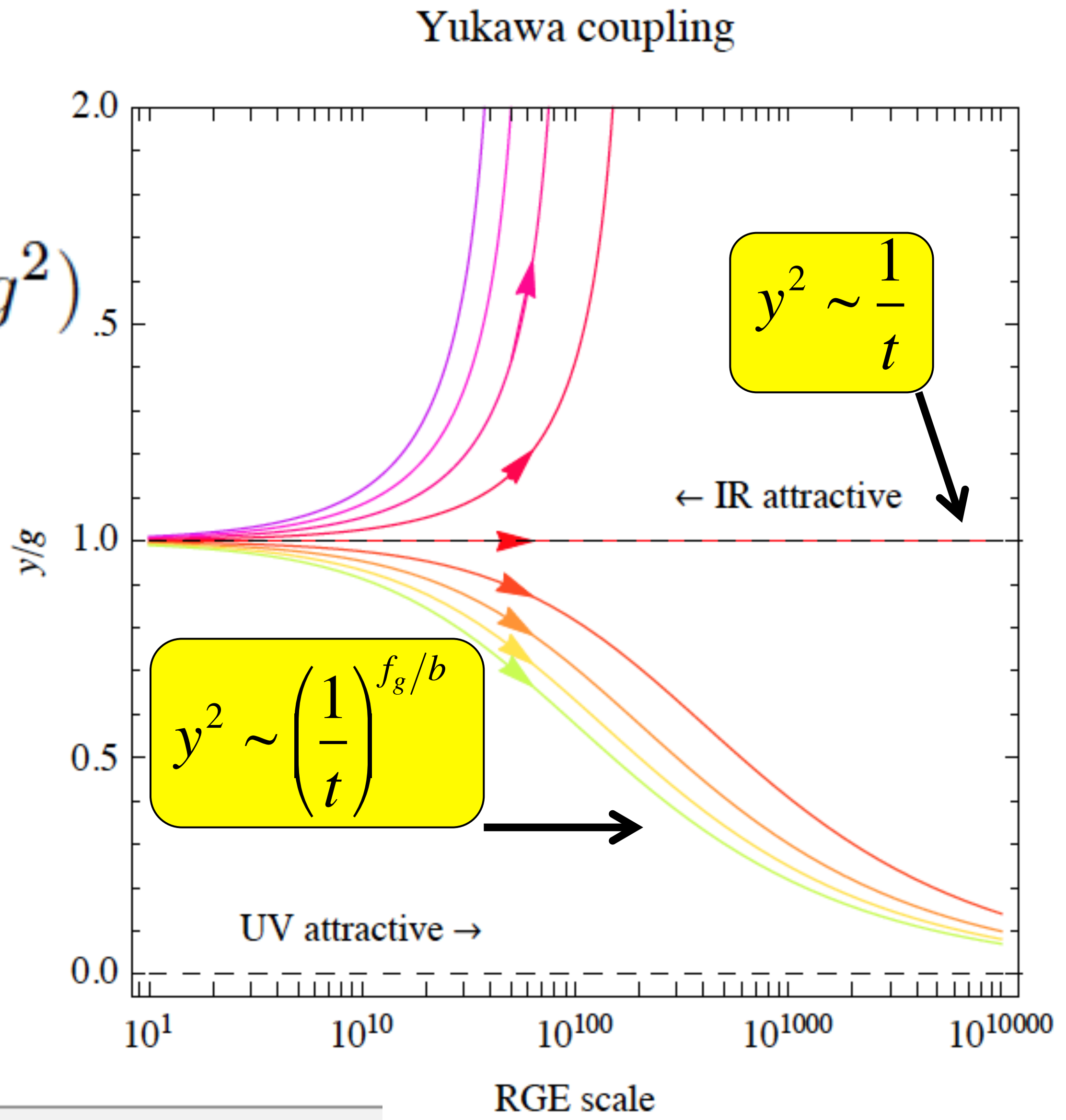
$$\frac{d}{dt} g^2 = -b g^4 \qquad t = \ln(\mu^2/\mu_0^2)/(4\pi)^2$$

$$g^2 = \frac{1}{bt} \quad \text{for } t \to \infty$$

TAF condition: $b > 0$

Yukawa coupling

$$\frac{d}{dt}y^2 = y^2(f_y y^2 - f_g g^2)$$

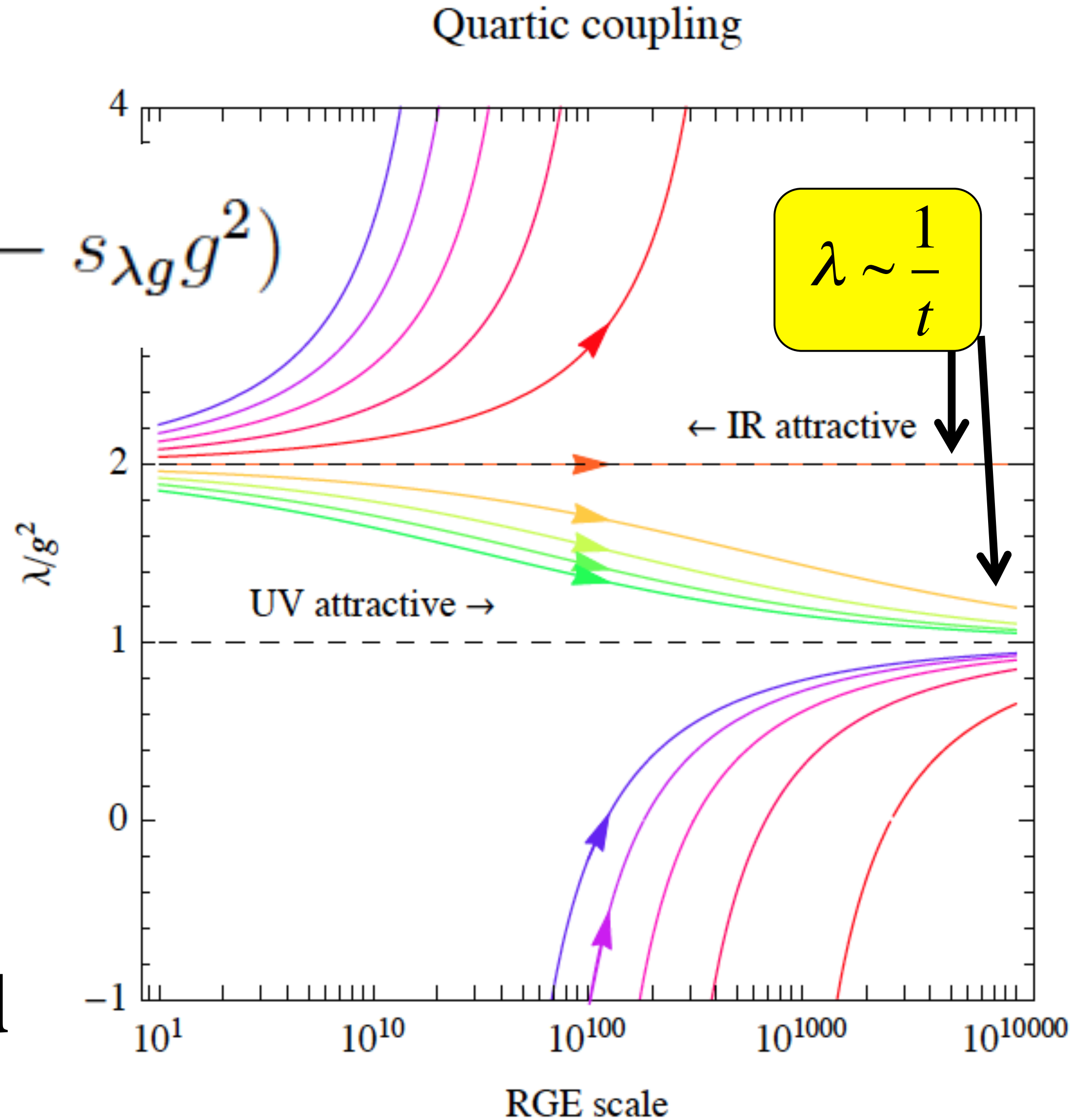


TAF condition:

$$f_g > b \quad \text{and} \quad \frac{y_0^2}{g_0^2} \leq \frac{f_g - b}{f_y}$$

Scalar coupling

$$\frac{d}{dt}\lambda = \lambda(s_\lambda \lambda + s_{\lambda y} y^2 - s_{\lambda g} g^2) - s_y y^4 + s_g g^4$$



TAF condition (more easily satisfied if *y* on QFP):

$$s_{\lambda g} - b \geq 2\sqrt{s_\lambda s_g} \quad \text{and} \quad \frac{\lambda_0}{g_0^2} \leq \frac{(s_{\lambda g} - b) + \sqrt{(s_{\lambda g} - b)^2 - 4 s_\lambda s_g}}{2 s_\lambda}$$

General procedure

Define

$$g_i^2(t) = \frac{\tilde{g}_i^2(t)}{t}, \qquad y_a^2(t) = \frac{\tilde{y}_a^2(t)}{t}, \qquad \lambda_m(t) = \frac{\tilde{\lambda}_m(t)}{t}$$

1-loop RGE

$$\frac{d\tilde{g}_i}{d\ln t} = \frac{\tilde{g}_i}{2} + \beta_{g_i}(\tilde{g}), \qquad \frac{d\tilde{y}_a}{d\ln t} = \frac{\tilde{y}_a}{2} + \beta_{y_a}(\tilde{g},\tilde{y}), \qquad \frac{d\tilde{\lambda}_m}{d\ln t} = \tilde{\lambda}_m + \beta_{\lambda_m}(\tilde{g},\tilde{y},\tilde{\lambda})$$

$$\frac{dx_I}{d\ln t} = V_I(x)\,, \qquad x_I = \{\tilde{g}_i, \tilde{y}_a, \tilde{\lambda}_m\}$$

vector flow

(does not depend explicitly on t)

1) Determine FF (solving an algebraic eq.)

$$V_I(x_\infty) = 0 \qquad x_\infty = \{\tilde{g}_{i\infty}, \tilde{y}_{a\infty}, \tilde{\lambda}_{m\infty}\}$$

2) Determine nature of FF

$$V_I(x) \simeq \sum_J M_{IJ}(x_J - x_{J\infty}) \quad \text{where} \quad M_{IJ} = \left.\frac{\partial V_I}{\partial x_J}\right|_{x=x_\infty}$$

FF is UV-attractive (IR-repulsive)

if all eigenvalues of $M(x_\infty)$ are –

FF is UV-repulsive (IR-attractive)

if all eigenvalues of $M(x_\infty)$ are +

$$\frac{d\Delta_I}{d\ln t} = \sum_J M_{IJ}\Delta_J \qquad \Delta_I \equiv x_I - x_{I\infty}$$

Each + eigenvalue ⇒
- TAF determines IR physical quantity (tuning or prediction?)
- Reduced dim. of basin of attraction

Example: SM

Gauge couplings

$$\frac{dg_1^2}{dt} = \frac{41}{10} g_1^4, \qquad \frac{dg_2^2}{dt} = -\frac{19}{6} g_2^4, \qquad \frac{dg_3^2}{dt} = -7 g_3^4$$

	$\tilde{g}_{1\infty}^2$	$\tilde{g}_{2\infty}^2$	$\tilde{g}_{3\infty}^2$	$M(x_\infty)$ eigenvalues
Solution 1	0	6/19	1/7	$+--$
Solution 2	0	6/19	0	$+-+$
Solution 3	0	0	1/7	$++-$
Solution 4	0	0	0	$+++$

IR prediction ⇒ $g_1 = 0$ (unphysical)

Yukawa couplings

$$\frac{dy_t^2}{dt} = y_t^2\left(-\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + y_\tau^2 + y_\nu^2\right)$$

$$\frac{dy_b^2}{dt} = y_b^2\left(-\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{3}{2}y_t^2 + \frac{9}{2}y_b^2 + y_\tau^2 + y_\nu^2\right),$$

$$\frac{dy_\tau^2}{dt} = y_\tau^2\left(-\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + 3y_t^2 + 3y_b^2 + \frac{5}{2}y_\tau^2 - \frac{1}{2}y_\nu^2\right),$$

$$\frac{dy_\nu^2}{dt} = y_\nu^2\left(-\frac{9}{20}g_1^2 - \frac{9}{4}g_2^2 + 3y_t^2 + 3y_b^2 - \frac{1}{2}y_\tau^2 + \frac{5}{2}y_\nu^2\right).$$

	$\tilde{y}^2_{t\infty}$	$\tilde{y}^2_{b\infty}$	$\tilde{y}^2_{\tau\infty}$	$\tilde{y}^2_{\nu\infty}$	$M(x_\infty)$ eigenvalues
Solution 1	$227/1197$	0	0	0	$+-++$
Solution 2	0	$227/1197$	0	0	$-+++$
Solution 3	$227/1596$	$227/1596$	0	0	$++++$
Solution 4	0	0	0	0	$--++$

3 IR predictions $\Rightarrow$
- $m_t = 186$ GeV
- $m_\tau = 0$
- $m_\nu = 0$

Higgs quartic coupling

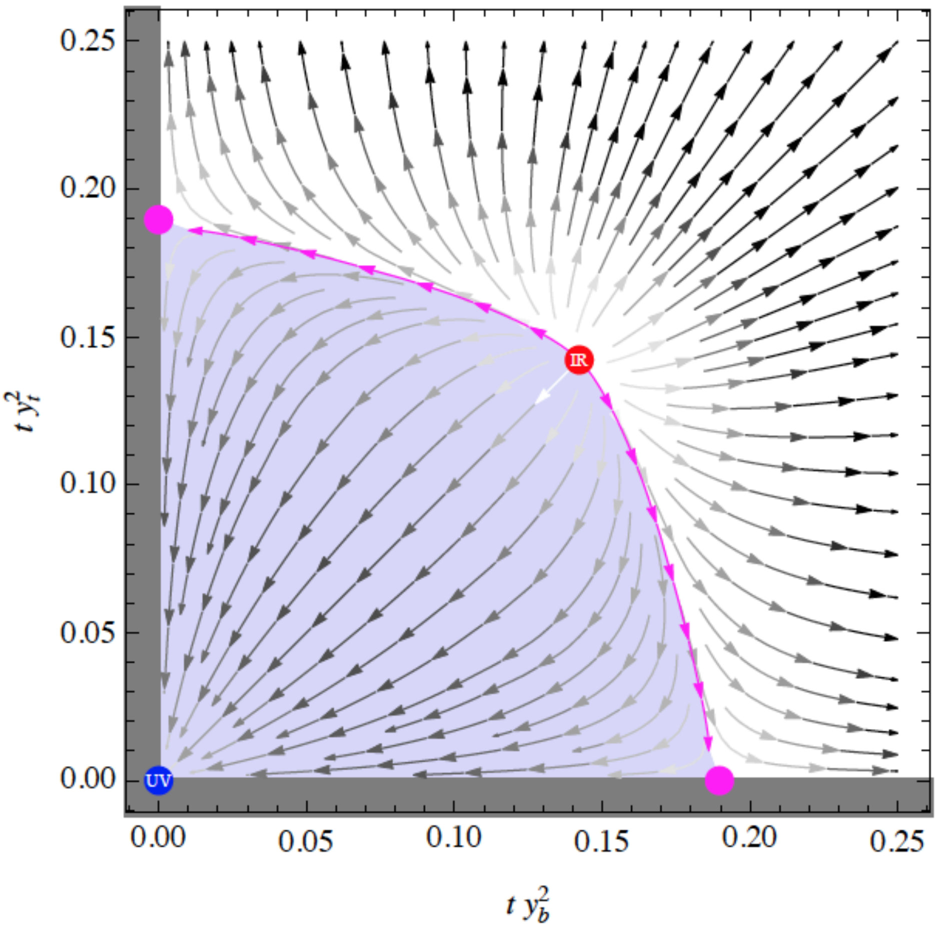
$$\frac{d\lambda}{dt} = 12\lambda^2 + \lambda\left(6y_t^2 + 6y_b^2 + 2y_\tau^2 + 2y_\nu^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2\right) - 3y_t^4 - 3y_b^4 - y_\tau^4 - y_\nu^4 + \frac{9}{16}g_2^4 + \frac{27}{400}g_1^4 + \frac{9}{40}g_2^2 g_1^2$$

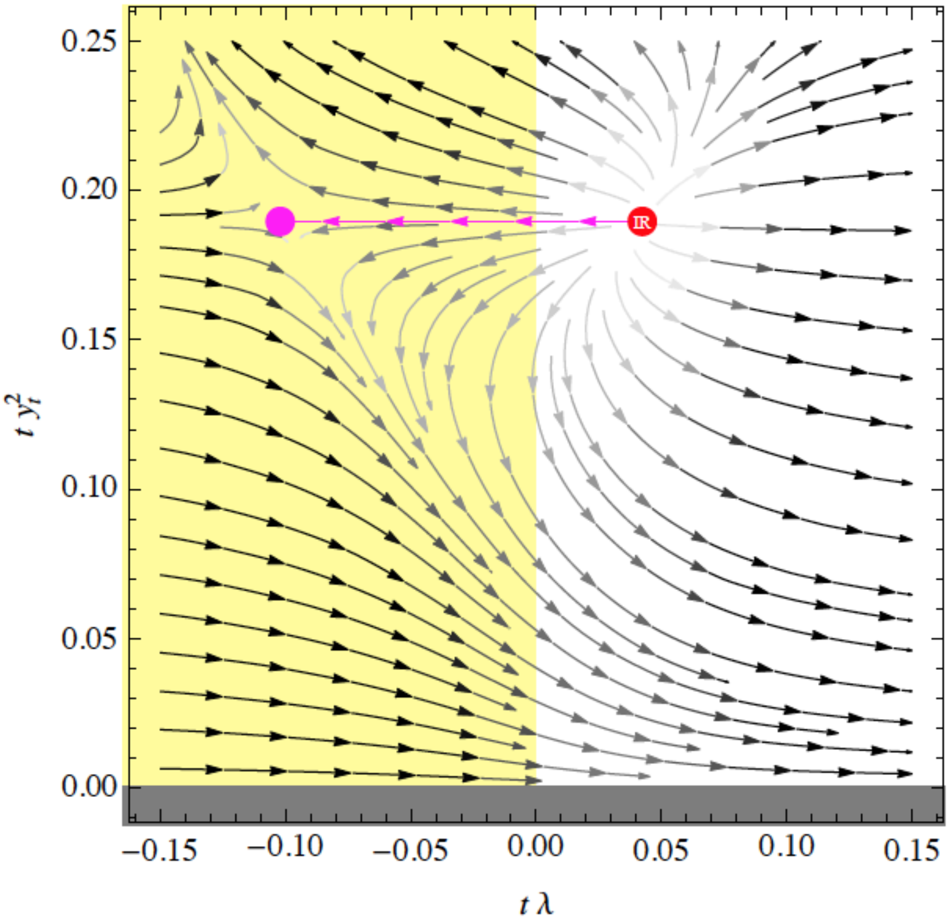
Landau poles unless Yukawa is on its UV-repulsive FF

	$\tilde{\lambda}_\infty$	M-eigenvalue	potential
Solution 1	$\frac{-143+\sqrt{119402}}{4788} \approx +0.0423$	$+$	stable
Solution 2	$\frac{-143-\sqrt{119402}}{4788} \approx -0.1020$	$-$	unstable

IR prediction $\Rightarrow$ $m_H = 163$ GeV

Basins of attraction





Conclusions from the exercise:

- SM is not asymptotically free
- The closest TAF approximation to physical reality is

$\tilde{g}^2_{1\infty}$	$\tilde{g}^2_{2\infty}$	$\tilde{g}^2_{3\infty}$	$\tilde{y}^2_{t\infty}$	$\tilde{y}^2_{b\infty}$	$\tilde{y}^2_{\tau\infty}$	$\tilde{y}^2_{\nu\infty}$	$\tilde{\lambda}_\infty$
0	$\frac{6}{19}$	$\frac{1}{7}$	$\frac{227}{1197}$	0	0	0	$\frac{-143+\sqrt{119402}}{4788}$
$+$	$-$	$-$	$+$	$-$	$+$	$+$	$+$

5 IR predictions $\Rightarrow$

- $g_1 = 0$
- $m_t = 186$ GeV (7% off)
- $m_H = 163$ GeV (30% off)
- $m_\tau = 0$
- $m_\nu = 0$

Extending the SM into a TAF theory at the EW scale

Embed Y into a non-abelian group

$$Y = T_{3R} + \frac{B-L}{2}$$

Embed in:
$SU(2)_R$

Embed in:
$SU(4)$ Pati-Salam
$SU(3)_L \times SU(3)_R$ trinification

Charge quantization

$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$

	Fields	spin	generations	$SU(2)_L$	$SU(2)_R$	$SU(4)_{PS}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	1/2	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	1/2	2	1	1	10
	Q_R	1/2	2	1	1	$\overline{10}$
	Σ	0	1	1	1	15

Extra fields required to avoid $Y_E = Y_D$ and $Y_N = Y_U$ (quark-lepton unification) and to obtain TAF

$SU(3)_c \times SU(3)_L \times SU(3)_R$

Matter fields	spin	$SU(3)_L$	$SU(3)_R$	$SU(3)_c$
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	1	3	$\bar{3}$
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^1 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	$1/2$	$\bar{3}$	1	3
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	$\bar{3}$	1
H_1, H_2	0	3	$\bar{3}$	1

Extra fields needed to obtain TAF

(no model found with realistic flavour)

FCNC from two-Higgs structure

$\phi = (2, \overline{2})$ of $SU(2)_L \times SU(2)_R \Rightarrow$ Two Higgs doublets

$$-\mathscr{L}_Y^q = Y q_L q_R \phi + Y_c q_L q_R \phi^c + \text{h.c.} \qquad \phi^c \equiv \epsilon^T \phi^* \epsilon$$

In terms of h and H ($\langle h \rangle = v$, $\langle H \rangle = 0$):

$$\begin{aligned} -\mathscr{L}_Y^q &= \bar{d}_L \lambda_d d_R h^0 + \bar{d}_L \left(\frac{V^\dagger \lambda_u V_R}{\cos 2\beta} - \tan 2\beta\, \lambda_d \right) d_R H^0 \\ &+ \bar{u}_L \lambda_u u_R h^{0*} + \bar{u}_L \left(\frac{V \lambda_d V_R^\dagger}{\cos 2\beta} - \tan 2\beta\, \lambda_u \right) u_R H^{0*} \end{aligned}$$

For $V_R \approx 1$ $\Rightarrow m_H > 0.75$ TeV

For $V_R \approx$ CKM $\Rightarrow m_H > 19$ TeV

For V_R : $\Rightarrow m_H > 3$ TeV

$$|(V_R)_{us}| \approx |V_{us}| \frac{m_d}{m_s} \approx 10^{-2}, \qquad |(V_R)_{cb}| \approx |V_{cb}| \frac{m_s}{m_b} \approx 10^{-3}, \qquad |(V_R)_{ub}| \approx |V_{ub}| \frac{m_d}{m_b} \approx 10^{-5}$$

Leptoquark vector bosons in SU(4)/SU(3)

Flavour	Experimental constraint	Bound on $M_{W'}$ in TeV
$dd\ e\mu$	$\sigma(\mu\ \text{Ti} \to e\ \text{Ti})/\sigma_0(\mu\ \text{Ti}) < 4.3 \times 10^{-12}$	120
$ss\ e\mu$	$\sigma(\mu\ \text{Ti} \to e\ \text{Ti})/\sigma_0(\mu\ \text{Ti}) < 4.3 \times 10^{-12}$	$12 \times \sqrt{P_{s\bar{s}}/1\%}$
$dd\ e\tau$	$\text{BR}(\tau \to \pi^0 e) < 8.0 \times 10^{-8}$	3.8
$dd\ \mu\tau$	$\text{BR}(\tau \to \pi^0 \mu) < 1.1 \times 10^{-7}$	3.5
$sd\ \mu\mu$	$\text{BR}(K_L \to \bar{\mu}\mu)_{\text{SD}} < 2.5 \times 10^{-9}$	50
$sd\ ee$	$\text{BR}(K_L \to \bar{e}e) = (9.0 \pm 6.0) \times 10^{-12}$	13.4
$bd\ \mu\mu$	$\text{BR}(B_d \to \bar{\mu}\mu) = (3.6 \pm 1.6) \times 10^{-10}$	12.7
$bs\ \mu\mu$	$\text{BR}(B_s \to \bar{\mu}\mu) = (2.9 \pm 0.7) \times 10^{-9}$	10.1
$sd\ e\mu$	$\text{BR}(K_L \to \bar{e}\mu) < 4.7 \times 10^{-12}$	200
$sd\ e\tau$	$\text{BR}(\tau \to K^* e) < 3.2 \times 10^{-8}$	10.3
$sd\ \mu\tau$	$\text{BR}(\tau \to K^* \mu) < 5.9 \times 10^{-8}$	8.8
$bs\ e\mu$	$\text{BR}(B^+ \to K^+ \bar{e}\mu) < 9.1 \times 10^{-8}$	8.3
$bd\ e\mu$	$\text{BR}(B^+ \to \pi^+ \bar{e}\mu) < 1.7 \times 10^{-7}$	7.1
$bd\ \mu\tau$	$\text{BR}(B_d \to \bar{\mu}\tau) < 2.2 \times 10^{-5}$	3.0
$bd\ e\tau$	$\text{BR}(B_d \to \bar{e}\tau) < 2.8 \times 10^{-5}$	2.8

evaded for small mixing

If leptoquarks couples to both L and R currents:

$$\Gamma(\pi \to e\nu)\,/\,\Gamma(\pi \to \mu\nu) \quad \Rightarrow \quad M_{W'} > 250\ \text{TeV}$$

Conclusions

In schemes with no dynamical protection of the hierarchy, for gravity to be compatible with naturalness we need

- Softening of gravity at $\Lambda_G << M_{Pl}$
- Modification of the SM at the EW scale
- Many new particles in the TeV region (introducing the usual difficulties)

Testable at LHC (and 100-TeV collider)
Distinguishable from dynamical mechanisms or anthropic