# QUENCHING OF $g_{\mathrm{A}}$ AND ITS IMPACT IN DOUBLE BETA DECAY 

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## INTRODUCTION

Unanswered questions in neutrino physics (2015):

- What is the absolute mass scale of neutrinos?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?

An answer to these questions can be obtained from neutrinoless double-beta decay (DBD)

$$
{ }_{Z}^{A} X_{N} \rightarrow{ }_{Z+2}^{A} Y_{N-2}+2 e^{-}
$$

## DOUBLE BETA DECAY



Half-life for processes not allowed

$$
{ }_{34}^{76} 5 e_{42}
$$ by the standard model:

$$
\left[\tau_{1 / 2}^{0 \nu \beta \beta}\left(0^{+} \rightarrow 0^{+}\right)\right]^{-1}=G_{0 \nu}\left|M_{0 \nu}\right|^{2}\left|f\left(m_{i}, U_{e i}\right)\right|^{2}
$$

Beyond the standard model (Particle physics)

Phase-space factor (Atomic physics) PSF

Matrix elements
(Nuclear physics)
NME

For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form


A special case is 0 vECEC , which is forbidden by energy and momentum conservation, but can occur under resonance conditions. In this case the inverse half-life is given by

$$
\xrightarrow{\left[\tau_{1 / 2}^{0 \nu}\right]^{-1}=G_{0 v}\left|M_{0 v}\right|^{2}\left|f\left(m_{i} U_{e i}\right)\right|^{2} \frac{\left(m_{e} c^{2}\right) \Gamma}{\Delta^{2}+\left(\Gamma^{2} / 4\right)}}
$$

Prefactor
(Atomic Physics) PF

Matrix elements Beyond the SM
(Nuclear Physics) (Particle Physics) NME

Degeneracy parameter (Atomic and Nuclear Physics)


For all processes and to extract physics beyond the standard model one needs to calculate the phase space factors (PSF) and the nuclear matrix elements (NME).

## NUCLEAR MATRIX ELEMENTS (NME)

NME can be written as:

$$
\begin{aligned}
& M_{0 v}=g_{A}^{2} M^{(0 v)} \\
& M^{(0 v)} \equiv M_{G T}^{(0 \nu)}-\left(\frac{g_{V}}{g_{A}}\right)^{2} M_{F}^{(0 \nu)}+M_{T}^{(0 v)}
\end{aligned}
$$

Several methods have been used to evaluate $\mathrm{M}_{0 \mathrm{v}}$ : QRPA (Quasiparticle Random Phase Approximation) ISM (Shell Model)
IBM-2 (Interacting Boson Model)
DFT (Density Functional Theory)

Calculations of NME in IBM-2 for all processes have been completed (2015) and are available upon request. A list of references is given in Appendix A.

For $0 v$ processes two scenarios have been considered:
(i) Emission and re-absorption of a light $\left(\mathrm{m}_{\text {light }} \ll 1 \mathrm{MeV}\right)$ neutrino.
(ii) Emission and re-absorption of a heavy ( $\mathrm{m}_{\text {heavy }} \gg 1 \mathrm{GeV}$ ) neutrino.



## Scenario 1: LIGHT NEUTRINO EXCHANGE

Dependence on the average neutrino mass

$$
f=\frac{\left\langle m_{v}\right\rangle}{m_{e}}
$$

$$
\left\langle m_{v}\right\rangle=\sum_{k=\text { ligth }}\left(U_{e k}\right)^{2} m_{k}
$$

Fourier transform of the neutrino "potential"

$$
v(p)=\frac{2}{\pi} \frac{1}{p(p+\tilde{A})}
$$

$$
\tilde{\mathrm{A}}=\text { closure energy }=1.12 \mathrm{~A}^{1 / 2}(\mathrm{MeV})
$$

In the last few years atmospheric, solar, reactor and accelerator neutrino oscillation experiments have provided information on light neutrino mass differences and their mixings. Two possibilities, normal and inverted hierarchy, are consistent with experiment.

INVERTED


The average light neutrino mass can be written as

$$
\begin{aligned}
& \left\langle m_{v}\right\rangle=\left|c_{13}^{2} c_{12}^{2} m_{1}+c_{13}^{2} s_{12}^{2} m_{2} e^{i \varphi_{2}}+s_{13}^{2} m_{3} e^{i \varphi_{3}}\right| \\
& c_{i j}=\cos \theta_{i j}, s_{i j}=\sin \theta_{i j}, \varphi_{2,3}=[0,2 \pi] \\
& \quad\left(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}\right)=\frac{m_{1}^{2}+m_{2}^{2}}{2}+\left(-\frac{\delta m^{2}}{2},+\frac{\delta m^{2}}{2}, \pm \Delta m^{2}\right)
\end{aligned}
$$

A fit to oscillation experiments gives ${ }^{\S}$

$$
\begin{aligned}
& \sin ^{2} \theta_{12}=0.312, \sin ^{2} \theta_{13}=0.016, \sin ^{2} \theta_{23}=0.466 \\
& \delta m^{2}=7.67 \times 10^{-5} \mathrm{eV}^{2}, \Delta m^{2}=2.39 \times 10^{-3} \mathrm{eV}^{2}
\end{aligned}
$$

§ G.L. Fogli et al., Phys. Rev. D75, 053001(2007); D78, 033010 (2008).
[A recent result from Daya Bay, Phys. Rev. Lett. 108, 171803 (2012) gives $\sin ^{2} \theta_{13}=0.024 \pm 0.005$, which slightly modifies the fit.]

Variation of the phases $\varphi_{2}$ and $\varphi_{3}$ from 0 to $2 \pi$ gives the values of $\left\langle\mathrm{m}_{v}\right\rangle$ consistent with oscillation experiments (constraints on the neutrino masses)


Vissani-Strumia plot ${ }^{\text {a }}$

- F. Vissani,
J. High Energy

Phys. 06, 022
(1999)

## Scenario 2: HEAVY NEUTRINO EXCHANGE

Dependence on the average neutrino mass

$$
f=m_{p}\left\langle\frac{1}{m_{v_{n}}}\right\rangle
$$

$$
\left\langle m_{v_{n}}^{-1}\right\rangle=\sum_{k=\text { heary }}\left(U_{e_{k n}}\right)^{2} \frac{1}{m_{k_{k}}}
$$

Fourier transform of the neutrino "potential"

$$
v(p)=\frac{2}{\pi} \frac{1}{m_{p} m_{e}}
$$

Constraints on the average inverse heavy neutrino mass are model dependent. V. Tello et al. ${ }^{〔}$ have recently (2011) worked out constraints from lepton flavor violating processes and (potentially LHC experiments). In this model

$$
\begin{aligned}
f & \equiv \eta=\frac{M_{W}^{4}}{M_{W R}^{4}} \sum_{k=\text { heary }}\left(V_{e k_{n}}\right)^{2} \frac{m_{p}}{m_{k_{n}}} \equiv \frac{M_{W}^{4}}{M_{W R}^{4}} \frac{m_{p}}{\left\langle m_{v_{h}}\right\rangle} \\
M_{W} & =80.41 \pm 0.10 \mathrm{GeV} ; M_{W R}=3.5 \mathrm{TeV}
\end{aligned}
$$

$\eta=$ lepton violating parameter.
Constraints on $\eta$ can then be converted into constraints on the average heavy neutrino mass as

$$
\left\langle m_{v_{h}}\right\rangle=m_{p}\left(\frac{M_{W}}{M_{W R}}\right)^{4} \frac{1}{\eta}
$$

${ }^{〔}$ V. Tello, M. Nemevšek, F. Nesti, G. Senjanović, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

## Most recent (2015) results for $0 v \beta^{-} \beta^{-}$(light neutrino exchange)



IBM-2 *: J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 91, 034304 (2015).
QRPA-Tu *: F. Simkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87, 045501 (2013).

ISM: J. Menendez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. A 818, 139 (2009).

* With isospin restoration and Argonne SRC

Most recent (2015) results for $0 \vee \beta \beta$ - (heavy neutrino exchange)


* With isospin restoration and Argonne SRC


## PHASE SPACE FACTORS (PSF)

PSF were calculated in the 1980's by Doi et al. *. Also, a calculation of phase-space factors is reported in the book of Boehm and Vogel ${ }^{\text {8 }}$. These calculations use an approximate expression for the electron wave functions at the nucleus.

PSF have been recently recalculated ${ }^{* *}$ with exact Dirac electron wave functions and including screening by the electron cloud.

These new PSF are available from jenni.kotila@yale.edu and are on the webpage nucleartheory.yale.edu

[^0]
## QUENCHING OF $g_{A}$

Results in the previous slides are obtained with $g_{A}=1.269$.
It is well-known from single $\beta$-decay/EC ${ }^{\boldsymbol{q}}$ and from $2 \nu \beta \beta$ that $\mathrm{g}_{\mathrm{A}}$ is renormalized in models of nuclei. Two reasons:
(i) Limited model space
(ii) Omission of non-nucleonic degrees of freedom ( $\Delta, \ldots$ )
${ }^{\top}$ I J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965).
D.H. Wilkinson, Nucl. Phys. A225, 365 (1974).

## ORIGIN OF QUENCHING OF $g_{A}$ IN DBD


$\mathrm{p} \leftarrow$ Quenching factor $q_{\Delta} \cong 0.7$
( $\Delta$ means excited states of the nucleon)


Quenching factor $q_{N^{E X}} \cong 0.7$ (nuclear model dependent)
( $\mathrm{N}^{E X}$ means excited states of the nucleus not included explicitly)

Maximal quenching:

$$
Q=q_{\Delta} q_{N^{E X}} \cong 0.5
$$

For each model (ISM/QRPA/IBM-2) one can define an effective $g_{A, e f f}$ by writing

$$
\begin{aligned}
& M_{2 v}^{e f f}=\left(\frac{g_{A, e f f}}{g_{A}}\right)^{2} M_{2 v} \\
& M_{\beta / E C}^{e \text { eff }}=\left(\frac{g_{A, \text { eff }}}{g_{A}}\right) M_{\beta / E C}
\end{aligned}
$$

The value of $g_{A, e f f}$ in each nucleus can then be obtained by comparing the calculated and measured half-lives for $\beta / E C$ and for $2 v \beta \beta$.

## Values of $\left|\mathrm{M}_{2 \mathrm{v}}{ }^{\text {eff }}\right|$ obtained from experimental half-lives


${ }^{\text {§ }}$ From a compilation by A.S. Barabash, Phys. Rev. C 81, 035501 (2010). For ${ }^{136} \mathrm{Xe}$, N. Ackerman et al. (EXO Collaboration), Phys. Rev. Lett. 107, 212501 (2011).

Effective axial vector coupling constant in nuclei from $2 \nu \beta \beta$ ब


One obtains $\mathrm{g}_{\mathrm{A}, \mathrm{eff}} \mathrm{IBM}-2 \sim 0.6-0.5$.
The extracted values can be parametrized as

$$
g_{A, e f f}^{I B M 2}=1.269 A^{-0.18}
$$

A similar analysis can be done for the ISM for which $g_{A, e f f}{ }^{\text {ISM }} \sim 0.8-0.7$.

$$
g_{A, e f f}^{I S M}=1.269 A^{-0.12}
$$

『 J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).
$g_{A, \text { eff }}$, has been extracted also from single $\beta / E C$ in QRPA, very recently by Suhonen and Civitarese (QRPA-Jy), $\mathrm{g}_{\mathrm{A}, \mathrm{eff}}$ QRPA $\sim 0.8-$ $0.4 \S$, and a few years ago by Faessler et al. (QRPA-Tü) ~0.7 *
[In some earlier (1989) QRPA papers ${ }^{『}$, it is claimed that no renormalization of $g_{A}$ is needed. However, this claim is based on results where the renormalization of $g_{A}$ is transferred to a renormalization of the free parameter $g_{p p}$ used in the calculation and adjusted to the experimental $2 v \beta \beta$ half-life.]
§ J. Suhonen and O. Civitarese, Phys. Lett. B 725, 153 (2013).

* A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A.M. Rotunno, and F. Šimkovic, J. Phys. G: Nucl. Part. Phys. 35, 075104 (2008).
${ }^{〔}$ K. Muto, E. Bender, H.V. Klapdor, Z. Phys. A334, 177 (1989); 187 (1989).


## IMPACT OF THE RENORMALIZATION

The axial vector coupling constant, $\mathrm{g}_{\mathrm{A}}$, appears to the second power in the NME

$$
\begin{aligned}
& M_{0 v}=g_{A}^{2} M^{(0 v)} \\
& M^{(0 v)}=M_{G T}^{(0 v)}-\left(\frac{g_{V}}{g_{A}}\right)^{2} M_{F}^{(0 v)}+M_{T}^{(0 v)}
\end{aligned}
$$

and hence to the fourth power in the half-life!

Therefore, the results of the previous slides should be multiplied by 6-34 to have realistic estimates of expected half-lives. [See also, H. Robertson 『, and S. Dell’Oro, S. Marcocci, F. Vissani\#.]
${ }^{\text {® }}$ R.G.H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).
\# S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

The question of whether or not $\mathrm{g}_{\mathrm{A}}$ in $0 v \beta \beta$ is renormalized as much as in $2 \nu \beta \beta$ is of much debate. In $2 \nu \beta \beta$ only the $1^{+}$(GT) multipole contributes. In $0 v \beta \beta$ all multipoles $1^{+}, 2^{-}, \ldots ; 0^{+}, 1^{-} \ldots$ contribute. Some of these could be unquenched. However, even in $0 v \beta \beta, 1^{+}$ intermediate states dominate. Hence, our current understanding is that $\mathrm{g}_{\mathrm{A}}$ is renormalized in $0 \vee \beta \beta$ as much as in $2 v \beta \beta$.

This problem is currently being addressed from various sides. Experimentally by measuring the matrix elements to and from the intermediate odd-odd nucleus in $2 v \beta \beta$ decay $\$$. Theoretically, by using effective field theory (EFT) to estimate the effect of nonnucleonic degrees of freedom (two-body currents) ${ }^{\uparrow}$.

[^1]Another question is whether or not the vector coupling constant, $\mathrm{g}_{\mathrm{V}}$, is renormalized in nuclei.
Because of CVC, the mechanism (ii) omission of nonnucleonic degrees of freedom cannot contribute. However, the mechanism (i), limited model space, can contribute, and, if so, the ratio $g_{\mathrm{V}} / \mathrm{g}_{\mathrm{A}}$ may remain the same as the non-renormalized ratio $1 / 1.269$.
No experimental information is available, but is could be obtained by measuring with $\left({ }^{3} \mathrm{He}, \mathrm{t}\right)$ and $\left(\mathrm{d},{ }^{2} \mathrm{He}\right)$ reactions the F matrix elements to and from the intermediate odd-odd nucleus.

## CONCLUSIONS

Major progress has been made in the last few years to narrow down predictions of $0 \vee \beta \beta$ decay to realistic values in all nuclei of interest.
Current (2015) limits on the neutrino mass from $0 v \beta-\beta^{-}$(light neutrino exchange) with $g_{A}=1.269$, IBM-2 NME, and KI PSF:

x H.V. KlapdorKleingrothaus et al., Phys. Lett. B586, 198 (2004).

With $\mathrm{g}_{\mathrm{A}}=1.269$ :

For light neutrino exchange, only the degenerate region can be tested in the immediate future. The current best limit (with $\mathrm{g}_{\mathrm{A}}=1.269$ ) is from EXO/KamLAND-Zen, $\mathrm{m}_{\mathrm{v}}<0.20 \mathrm{eV}$. Exploration of the inverted region $>1$ ton Exploration of the normal region $\gg 1$ ton

For heavy neutrino exchange, the limit is model dependent. In the model of Tello et al. 9, the current best limit from $\mathrm{EXO} / \mathrm{KamLAND}-\mathrm{Zen}$ is $\mathrm{m}_{\mathrm{vh}}>257 \mathrm{GeV}\left(3.5 / \mathrm{M}_{\mathrm{WR}}\right)^{4}$.

『 V. Tello, M. Nemevšek, F. Nesti, O. Senjanovic, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

The major remaining question is the value of $g_{A}$.
Three scenarios arell/s :

$$
\begin{array}{lll}
g_{A}=1.269 & \longleftarrow & \text { Free value } \\
g_{A}=1 & \longleftarrow & \text { Quark value } \\
g_{A}=1.269 A^{-0.18} & \longleftarrow & \text { Maximal quenching }
\end{array}
$$

© J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 87, 014315 (2013).
§ S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

If $g_{A}$ is renormalized to $\sim 0.8-0.5$, all estimates for half-lives should be increased by a factor of $\sim 6-34$ and limits on the average neutrino mass should be increased by a factor $\sim 2.5-6$, making it impossible to reach in the foreseeable future even the inverted region.


If $g_{A}$ is renomalized to $\sim 0.8-0.5$, even the exploration of the inverted hierarchy will require a multiton large neutrino infrastructure.

Possibilities to escape this negative conclusion are:
(1) Neutrino masses are degenerate and large.


$$
\begin{array}{ll}
\sum_{i} m_{i} \leq 0.6 \mathrm{eV} & (2008) \\
\sum_{i} m_{i} \leq 0.230 \mathrm{eV} & (2015) \text { Planck } \\
& 68 \% \text { confidence level }
\end{array}
$$

${ }^{\text {II }}$ S. Matarrese for the Planck collaboration, Proc. XVI
Int. Workshop NEUTEL 2015, in press.
(2) Both mechanisms, light and heavy exchange, contribute simultaneously, are of the same order of magnitude, and interfere constructively.

$$
\left[\tau_{1 / 2}^{0 \vee \beta \beta}\left(0^{+} \rightarrow 0^{+}\right)\right]^{-1}=G_{0 v}\left|M_{0 v, \text { light }} \frac{\left\langle m_{v}\right\rangle}{m_{e}}+M_{0 v, \text { heavy }} \frac{m_{p}}{\left\langle m_{v_{h}}\right\rangle}\right|^{2}
$$

This possibility requires a fine tuning which is quite unlikely.
(3) Other scenarios (Majoron emission, ...) and/or new mechanisms (sterile neutrinos, ...) must be considered.

3

$\mathrm{n} \quad \mathrm{n}$


Majoron means a massless neutral boson

Sterile means no standard model interactions ${ }^{\text {8 }}$
§B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984

## Scenario 3: MAJORON EMISSION

The inverse half-life for this scenario ( $0 v \beta \beta \varphi$ decay) is given by

$$
\left[\tau_{1 / 2}^{0 v \beta \beta \varphi}\left(0^{+} \rightarrow 0^{+}\right)\right]^{-1}=G_{o v \varphi}\left|M_{0 \nu}\right|^{2}\langle g\rangle^{2}
$$

effective Majoron coupling constant
NME are the same as for scenario 1 and 2.
PSF have been recalculated recently.
Best limit ${ }^{〔}$ with IBM-2 NME, KBI PSF and $g_{A}=1.269$ from EXO/KamLAND-Zen

$$
\left\langle g^{2}\right\rangle<6.2 \times 10^{-5}
$$

© J. Kotila, J. Barea and F. Iachello, in preparation (2015).
This scenario was suggested by H.M. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

## Scenario 4: STERILE NEUTRINOS

Another scenario is currently being discussed, namely the mixing of additional "sterile" neutrinos.
[The question on whether or not "sterile" neutrinos exist is an active areas of research at the present time with experiments planned at FERMILAB and CERN-LHC.]
NME for sterile neutrinos of arbitrary mass can be calculated by using a transition operator as in scenario 1 and 2 but with

$$
f=\frac{m_{v I}}{m_{e}}
$$

$$
v(p)=\frac{2}{\pi} \frac{1}{\sqrt{p^{2}+m_{v I}^{2}}\left(\sqrt{p^{2}+m_{v I}^{2}}+\tilde{A}\right)}
$$

Effective mass of the sterile neutrinos

IBM-2 NME for this scenario have just been calculated (April 2015). PSF are the same as in scenarios 1 and 2.

Several types of sterile neutrinos have been suggested.

## Scenario 4a: HEAVY STERILE NEUTRINOS

Sterile neutrinos with masses $\quad m_{v I} \gg 1 \mathrm{eV}$
Possible values of the sterile neutrino, $4 \mathrm{a}, 5 \mathrm{a}, 6 \mathrm{a}, \ldots$, masses in the keV GeV range have been suggested by T. Asaka and M. Shaposhnikov, Phys. Lett. B620, 17 (2005) and T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys. Lett. B631, 151 (2005).

## Scenario 4b: LIGHT STERILE NEUTRINOS

Sterile neutrinos with masses $\quad m_{\nu I} \sim 1 e V$
Very recently C. Giunti and M. Laveder have suggested sterile neutrinos, $4 \mathrm{~b}, \ldots$, with masses in the eV range to account for the reactor anomaly in oscillation experiments, G. Giunti, XVI International Workshop on Neutrino Telescopes, Venice, Italy, March 4, 2015.

HYPOTHETICAL NEUTRINO SPECTRUM


## CONTRIBUTIONS OF HYPOTHETICAL NEUTRINOS ALL

$$
\begin{gathered}
{\left[\tau_{1 / 2}^{0 \vee \beta \beta}\left(0^{+} \rightarrow 0^{+}\right)\right]^{-1}=G_{0 v} g_{A}^{4} M\left|\frac{m_{p}}{B} \sum_{k=1}^{3} U_{e k}^{2} m_{k}+m_{p} \sum_{i_{b}} \frac{U_{e i_{b}}^{2}}{m_{i_{b}}}+\frac{m_{p}}{B} \sum_{i_{a}} U_{e_{i}}^{2} m_{i_{a}}+m_{p} \sum_{h} \frac{U_{e h}^{2} m_{h}}{B+m_{h}^{2}}\right|^{2}} \\
\text { Known neutrinos } \\
\text { Unknown light sterile }
\end{gathered}
$$

Unknown heavy neutrinos

The values of $M$ and $B$ in IBM-2 have been just calculated ${ }^{\text {® }}$
${ }^{9}$ J. Barea, J. Kotila and F. Iachello, paper in preparation (2015).

No matter what the mechanism of neutrinoless DBD is, its observation will answer the fundamental questions:

- What is the absolute neutrino mass scale?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?


## APPENDIX A: REFERENCES

## PSF

$2 \nu \beta-\beta-/ 0 v \beta-\beta^{-}$
J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).
$2 v \beta^{+} \beta^{+} / 0 v \beta^{+} \beta^{+}$
J. Kotila and F. Iachello, Phys. Rev. C 87, 024313 (2013).

NME
$2 v \beta-\beta-/ 0 v \beta-\beta-$
J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009).
J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).
J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 91, 034304 (2015).
$2 v \beta^{+} \beta^{+} / 0 v \beta^{+} \beta^{+}$
J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87, 057301 (2013). R0vECEC
J. Kotila, J. Barea, and F. Iachello, Phys. Rev. C 89, 064319 (2014).

## APPENDIX B : RECENT IBM-2 RESULTS WITH ERROR FOR $0 v \beta \beta$ (2015)

| Decay | Light neutrino exchange | Heavy neutrino exchange |
| :---: | :---: | :---: |
| ${ }^{48} \mathrm{Ca}$ | $1.75(28)$ | $47(13)$ |
| ${ }^{76} \mathrm{Ge}$ | $4.68(75)$ | $104(29)$ |
| ${ }^{82} \mathrm{Se}$ | $3.73(60)$ | $83(23)$ |
| ${ }^{96} \mathrm{Zr}$ | $2.83(45)$ | $99(28)$ |
| ${ }^{100} \mathrm{Mo}$ | $4.22(68)$ | $164(46)$ |
| ${ }^{110} \mathrm{Pd}$ | $4.05(65)$ | $154(43)$ |
| ${ }^{116} \mathrm{Cd}$ | $3.10(50)$ | $110(31)$ |
| ${ }^{124} \mathrm{Sn}$ | $3.19(51)$ | $79(22)$ |
| ${ }^{128} \mathrm{Te}$ | $4.10(66)$ | $101(28)$ |
| ${ }^{130} \mathrm{Te}$ | $3.70(59)$ | $92(26)$ |
| ${ }^{134} \mathrm{Xe}$ | $4.05(65)$ | $91(26)$ |
| ${ }^{136} \mathrm{Xe}$ | $3.05(59)$ | $73(20)$ |
| ${ }^{148} \mathrm{Nd}$ | $2.31(37)$ | $103(29)$ |
| ${ }^{150} \mathrm{Nd}$ | $2.67(43)$ | $116(32)$ |
| ${ }^{154} \mathrm{Sm}$ | $2.82(45)$ | $113(32)$ |
| ${ }^{160} \mathrm{Gd}$ | $4.08(65)$ | $155(43)$ |
| ${ }^{198} \mathrm{Pt}$ | $2.19(35)$ | $104(29)$ |
| ${ }^{232} \mathrm{Th}$ | $4.04(65)$ | $159(45)$ |
| ${ }^{238} \mathrm{U}$ | $4.81(77)$ | $189(53)$ |

${ }^{76} \mathrm{Ge} \rightarrow{ }^{76} \mathrm{Se}$
$4.68 \pm 0.75$
GERDA
${ }^{130} \mathrm{Te} \rightarrow{ }^{130} \mathrm{Xe}$
$3.70 \pm 0.59$
CUORE
${ }^{136} \mathrm{Xe} \rightarrow{ }^{136} \mathrm{Ba}$
$3.05 \pm 0.59$
EXO
KamLAND-Zen

## APPENDIX C: EXPECTED HALF-LIVES (2015) $0 v \beta-\beta-$



## EXPECTED HALF-LIVES (2015) $0 \nu \beta^{+} \beta^{+} / 0 \nu \beta^{+}$EC



## EXPECTED HALF-LIVES (2015) R0vECEC



APPENDIX D: MATRIX ELEMENTS ALL

$$
\mathrm{g}_{\mathrm{A}}=1.269
$$




[^0]:    * M. Doi, T. Kotani, N. Nishiura, K. Okuda and E. Takasugi, Prog. Theor. Phys. 66 (1981) 1739.
    § F. Bohm and P. Vogel, Physics of massive neutrinos, Cambridge University Press, 1987.
    ** J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

[^1]:    § P. Puppe et al., Phys. Rev. C 86, 044603 (2012).
    『 J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).

