Error reduction using the covariant approximation averaging

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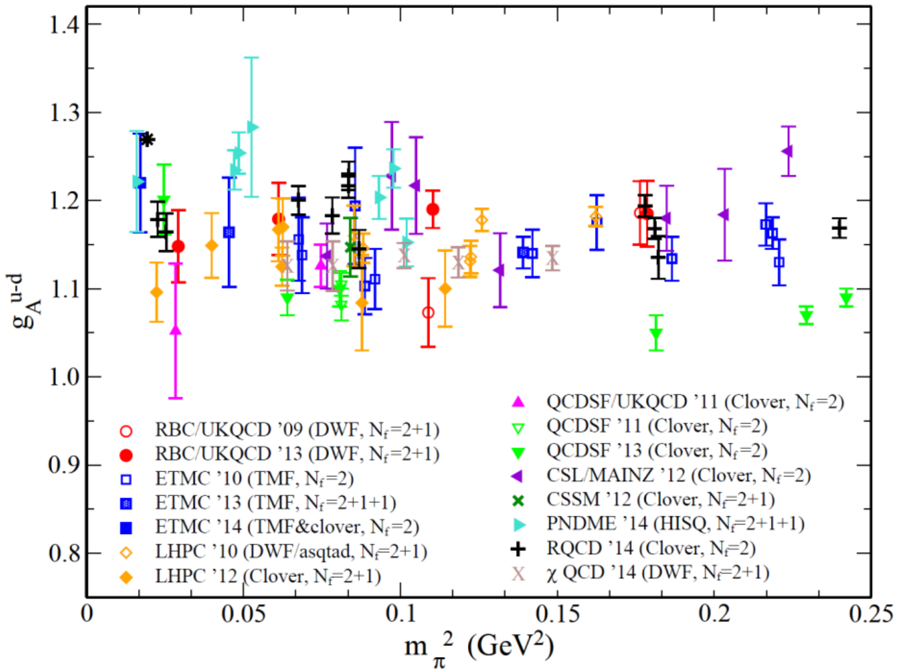
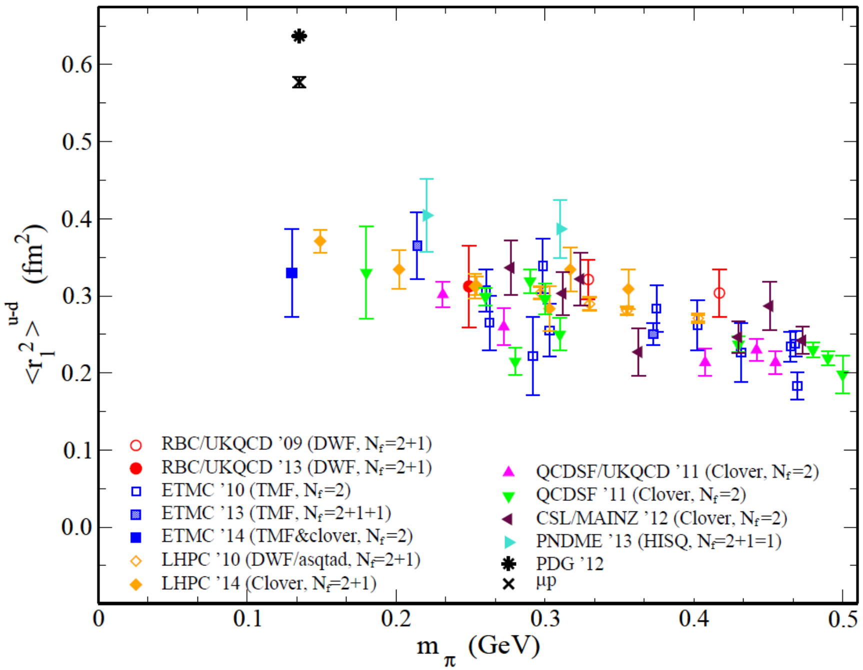
OUTLINE

- Introduction
- Error reduction technique
- Application
  - Nucleon mass
  - Axial charge, Scalar and tensor charge
  - Isovector form factor and Charge radius
- Summary

"Puzzle" of nucleon form factor in LQCD

Constantinou, lattice2014





- There is slight tension from experiment, even in different group
  $\Delta g_A \sim 5$ -- 10%, $\Delta r_E^2 \sim 10$ -- 20%
- Large statistical error of Monte-Carlo simulation is serious issue.
- Careful estimate of systematic uncertainty should be carried out.

1. Introduction

Lattice computation of matrix element

- 2pt, 3pt function

$$\langle 0|\mathcal{N}(t)\mathcal{N}^\dagger(0)|0\rangle = |\langle 0|\mathcal{N}|N\rangle|^2 e^{-m_N t} + \underbrace{|\langle 0|\mathcal{N}|N'\rangle|^2 e^{-m'_N t}}_{\text{First excited state contamination}} + \cdots$$

$$\langle 0|T\{\mathcal{N}(t_s,0)J_\mu(t,q)\mathcal{N}^\dagger(0,p)|0\rangle$$
$$= \langle 0|\mathcal{N}|N\rangle\langle N|J_\mu|N\rangle\langle N|\mathcal{N}^\dagger|0\rangle e^{-E_N t - m_N(t_s-t)} + \langle 0|\mathcal{N}|N'\rangle\langle N'|J_\mu|N'\rangle\langle N'|\mathcal{N}^\dagger|0\rangle e^{-E'_N t - m'_N(t_s-t)} + \cdots$$
$$\simeq Z_N(0)Z_N(p)e^{-E_N t - m_N(t_{\rm sep}-t)} \times \left[\underbrace{\{G_X, g_A\}}_{\text{Matrix element of ground state}} + \underbrace{c_1 e^{-\Delta(t_{\rm sep}-t)} + c_2 e^{-\Delta' t}}_{\text{First excited state contamination}}\right]$$

$\Delta = m'_N - m_N > 0, \Delta' = E'_N - E_N > 0$

- Signal-to-noise ratio of nucleon correlation function

$$\mathrm{S/N} \sim \sqrt{N}\exp[-(m_N - 3m_\pi/2)t]$$

Our strategy:

- To much reduce statistical error, the all-mode-averaging (AMA) is applied.
- Systematic study of excited state contamination is performed in light pion mass and large volume, $m_\pi L > 4$.

All-mode-averaging

Blum, Izubuchi, ES (2013)

- Effective technique to reduce statistical error of correlation function without additional computational cost, by using covariant symmetry
  - Master formula

Bias correction

$$O^{(\mathrm{imp})} = O - O^{(\mathrm{appx})} + \frac{1}{N_G} \sum_{g \in G} O^{(\mathrm{appx}),g}$$

High precision

Low precision

- $O^{(appx)}$ should be good approximation to O.
- Computation cost of $O^{(appx)}$ is much small.
- Covariant symmetry of $O^{(appx)}$ guarantees no bias.

2. Error reduction technique

All-mode-averaging

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  - Error reduction formula

$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'}}$$

$$r = \frac{\langle \Delta O \Delta O^{(\mathrm{appx})}\rangle}{\sigma\sigma^{(\mathrm{appx})}} \qquad r_{gg'} = \frac{\langle \Delta O^{(\mathrm{appx}),g} \Delta O^{(\mathrm{appx}),g'}\rangle}{\sigma^{(\mathrm{appx}),g}\sigma^{(\mathrm{appx}),g'}}$$

r: correlation between O and $O^{(\mathrm{appx})}$

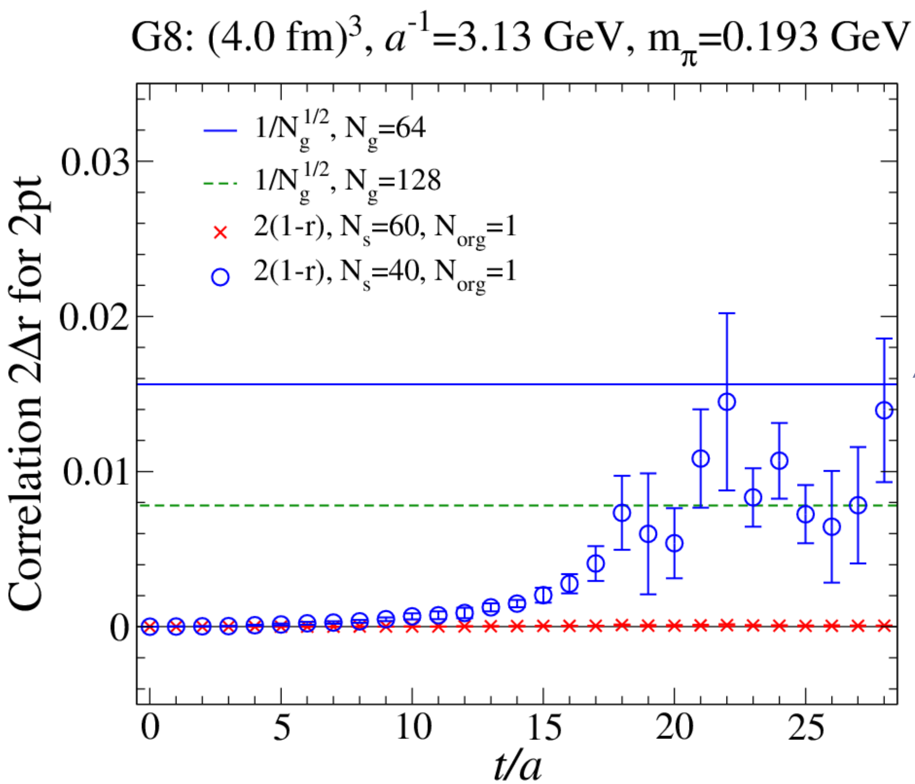
$r_{gg'}$: correlation between $O^{(\mathrm{appx}),g}$ and $O^{(\mathrm{appx}),g'}$

2. Error reduction technique
Tuning and Correlation

- Approximation

Luscher, 2004

  - Relaxed GCR+Deflation field for preconditioning in solver algorithm.
  - Deflation space N_s is related to quality and cost of approximation.



Expected error reduction in AMA:

$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'}}$$

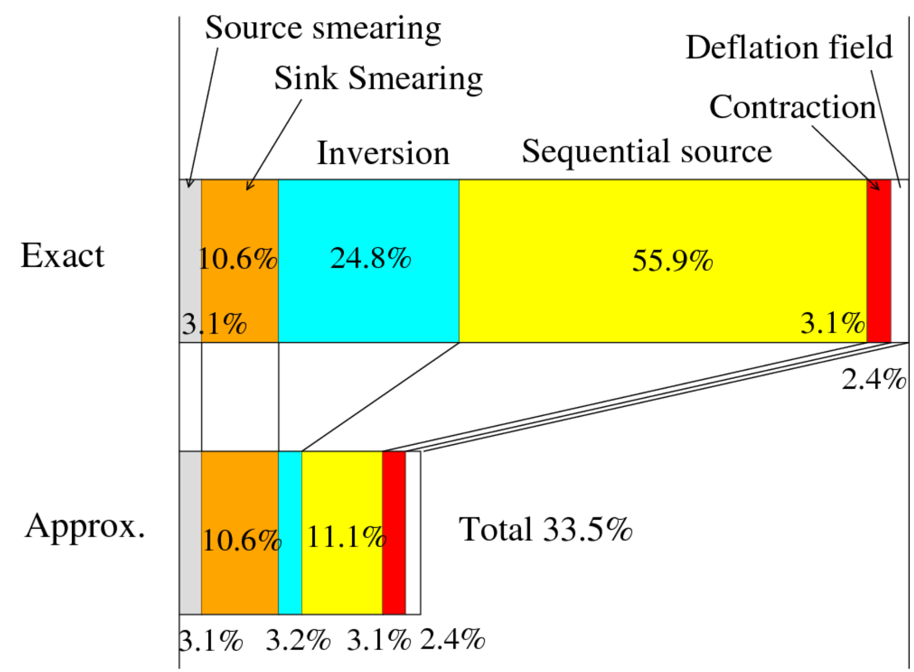
$1/N_G = 1/64$

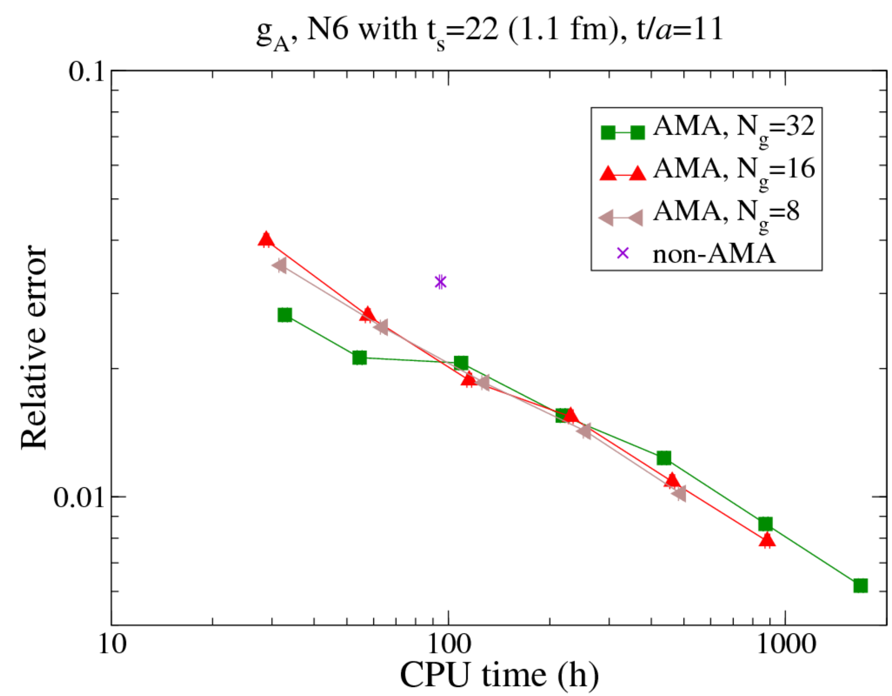
$1/N_G = 1/128$

- At t ~ 24, size of correlation is similar to $1/N_G$, ⇒maximum point to reduce error

Performance test of AMA

- Reduction of computational cost





- Cost of computing quark propagator is reduced to 1/5 and less.
- Total speed-up is about factor 2 and more. (depending on lattice size and pion mass)

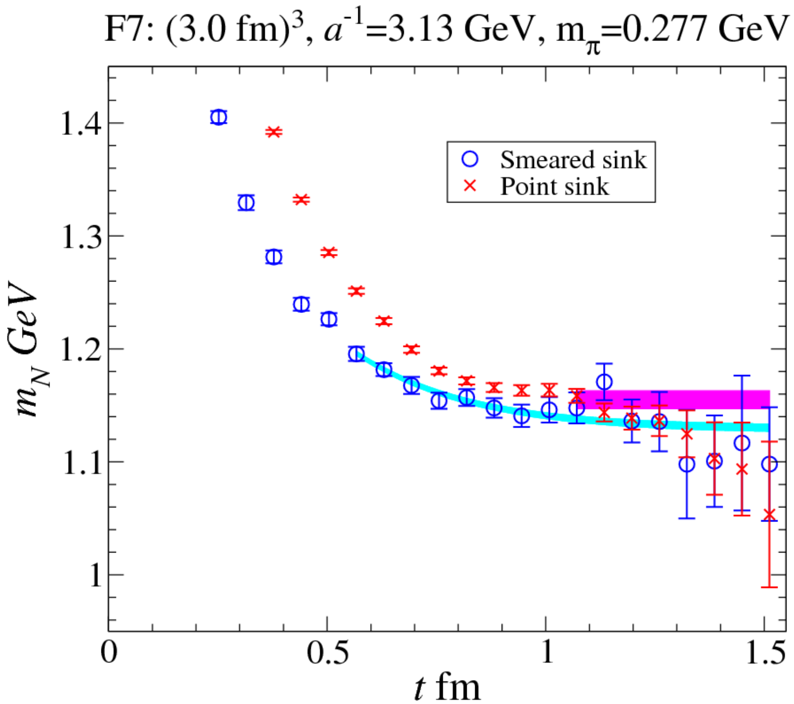
CLS config, N_f = 2 Wilson-clover fermion

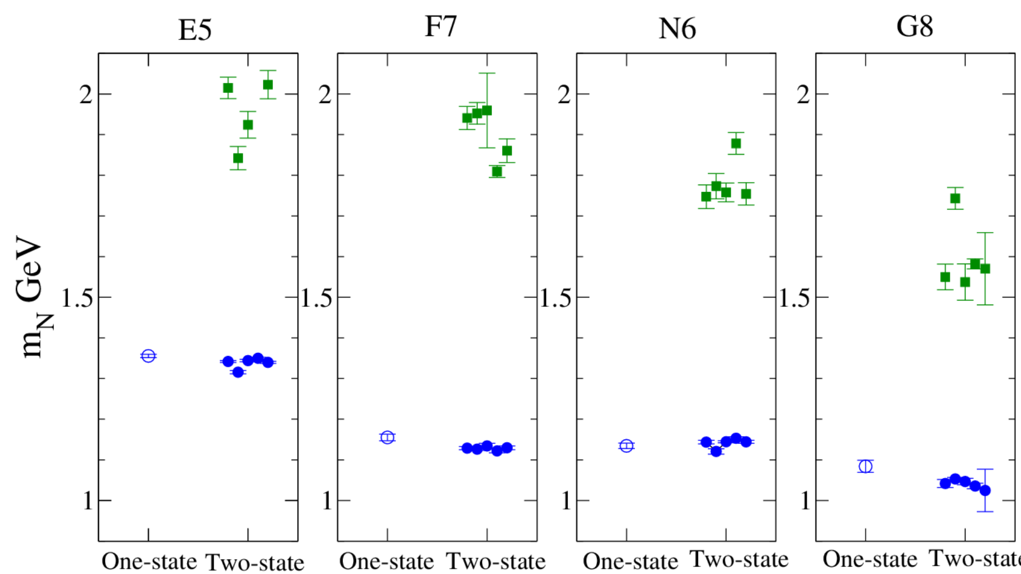
	Lattice	a (fm)	m_π (GeV)	N_G	t_s (fm)	#conf	#meas(*)
E5	64×32^3	0.063	0.456	64	0.82, 0.95, 1.13	~480	~30,000
	$(2.0\ \text{fm})^3$		($m_\pi L$=4.7)		1.32	994	63,616
					1.51	1605	102,720
F7	96×48^3	0.063	0.277	64	0.82, 0.95, 1.07	250	16,000
	$(3.0\ \text{fm})^3$		($m_\pi L$=4.2)	128	1.20, 1.32	250	32,000
				192	1.51	250	64,000
N6	96×48^3	0.05	0.332	32	0.9	110	3,520
	$(2.4\ \text{fm})^3$		($m_\pi L$=4.1)	32	1.1,1.3	888	28,416
				32	1.5, 1.7	936	30,272
G8	128×64^3	0.063	0.193	80	0.88	184	14,720
	$(4.0\ \text{fm})^3$		($m_\pi L$=4.0)	112	1.07	170	19,040
				160	1.26	178	28,480
				160	1.51	179	28,640

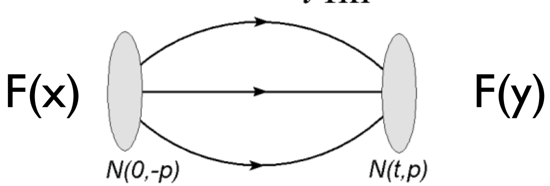
* Effective statistics : #mes = $N_G \times$ #conf

3. Application

Nucleon mass and its excited state







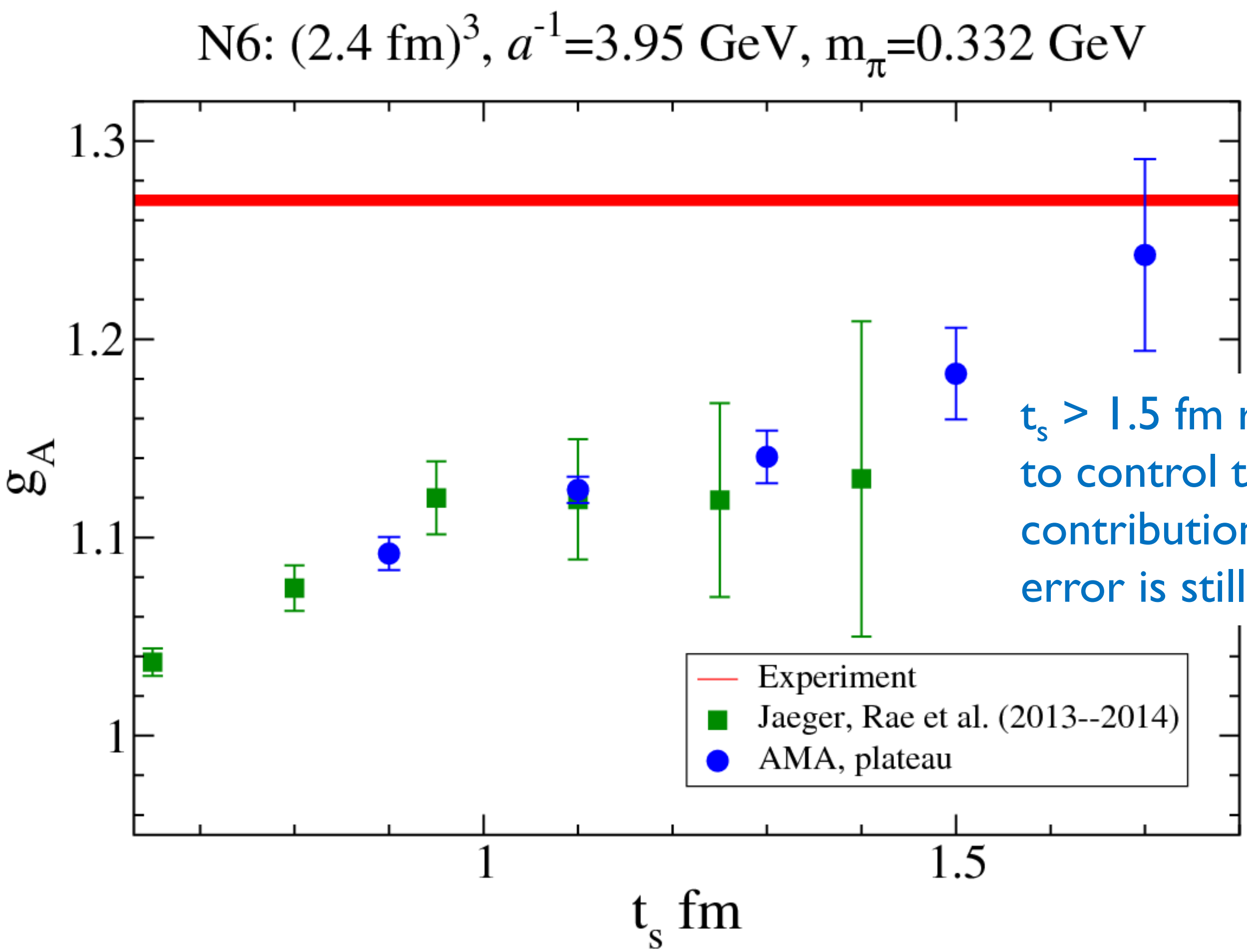
F(x): Jacobian function with APE smearing link.

- The ground-state dominant, t = 1--1.5 fm.
- Including the excited state, t = 0.5 -- 1.5 fm
- Fitting function
  One-state : Ze^{-mt},
  Two-state : $Z e^{-m t} + Z' e^{-m' t}$
- almost comparable with two fitting results

3. Application

Axial charge

- AMA results at $t_s > 1.5$ fm

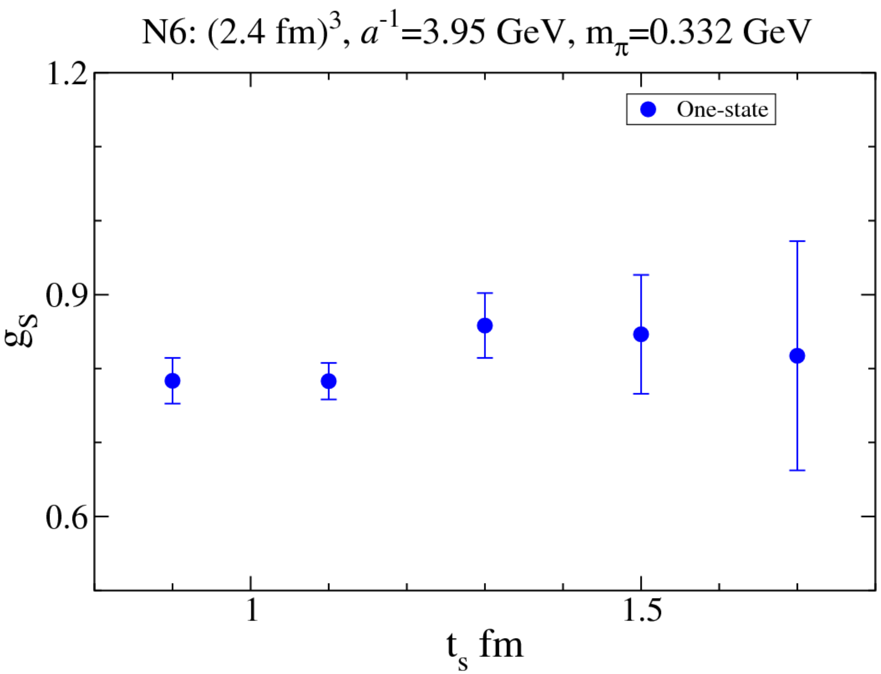


$t_s > 1.5$ fm region is much better to control the excited state contribution, although statistical error is still large.

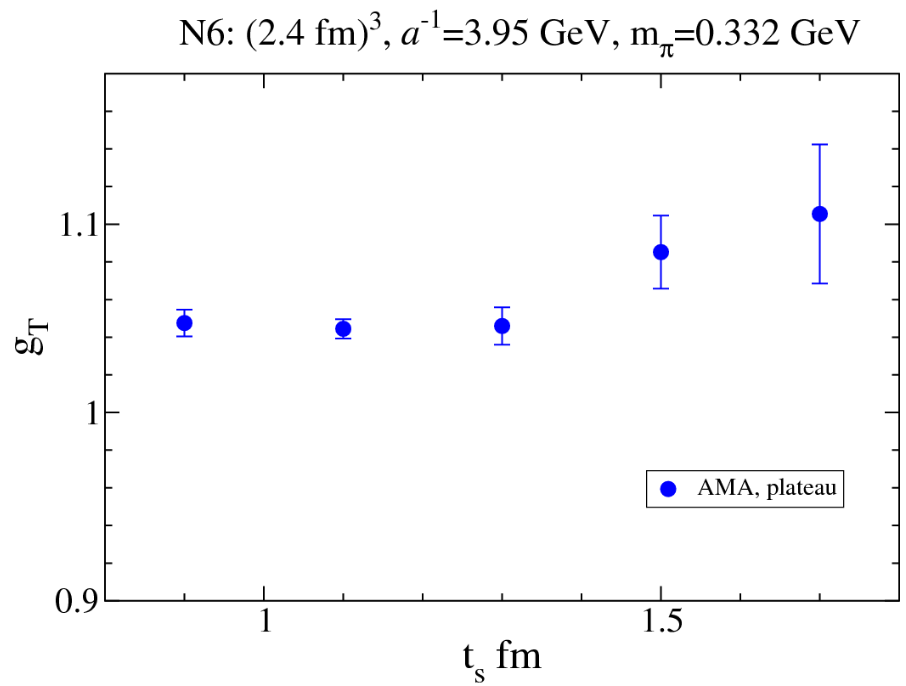
3. Application

Scalar and tensor charge

Scalar (lattice)



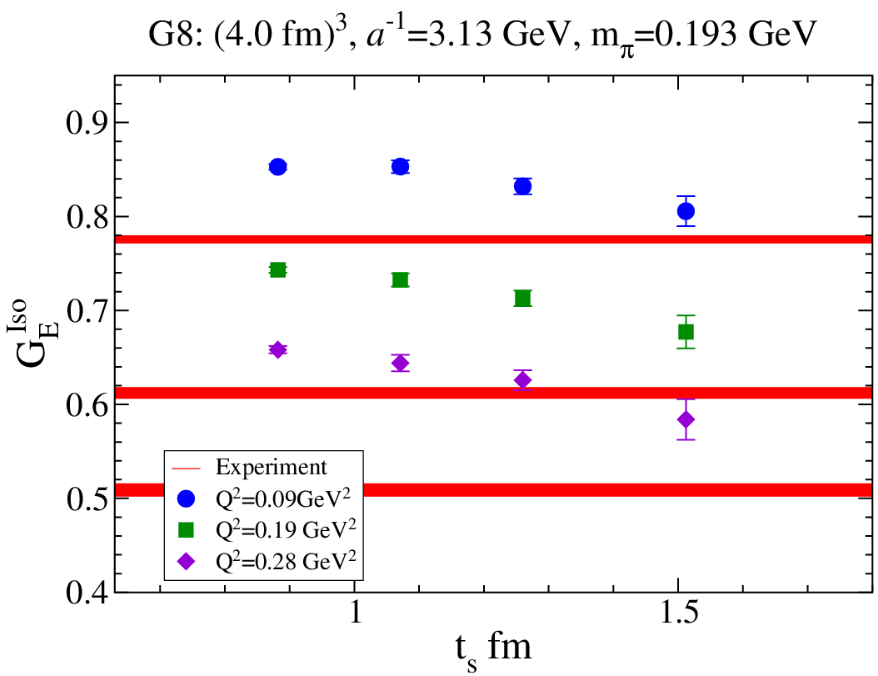
Tensor (lattice)

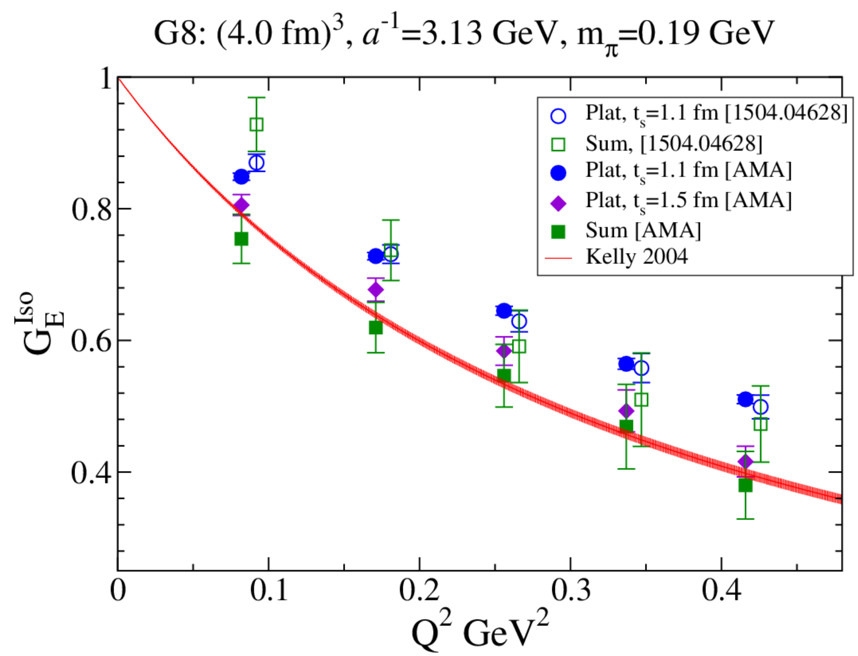


- There does not appear significant effect of excited state.

3. Application

Isovector form factor

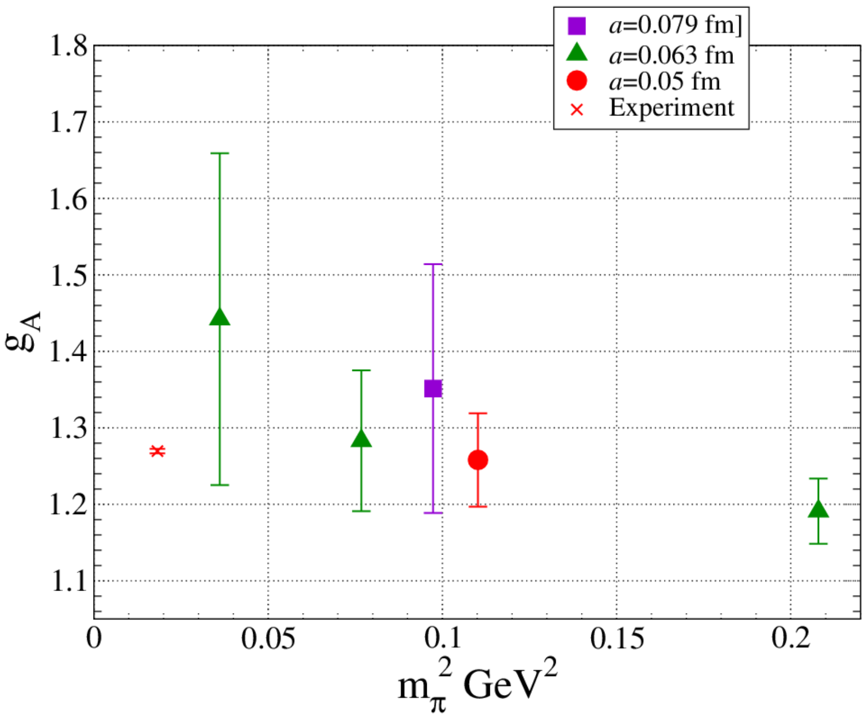


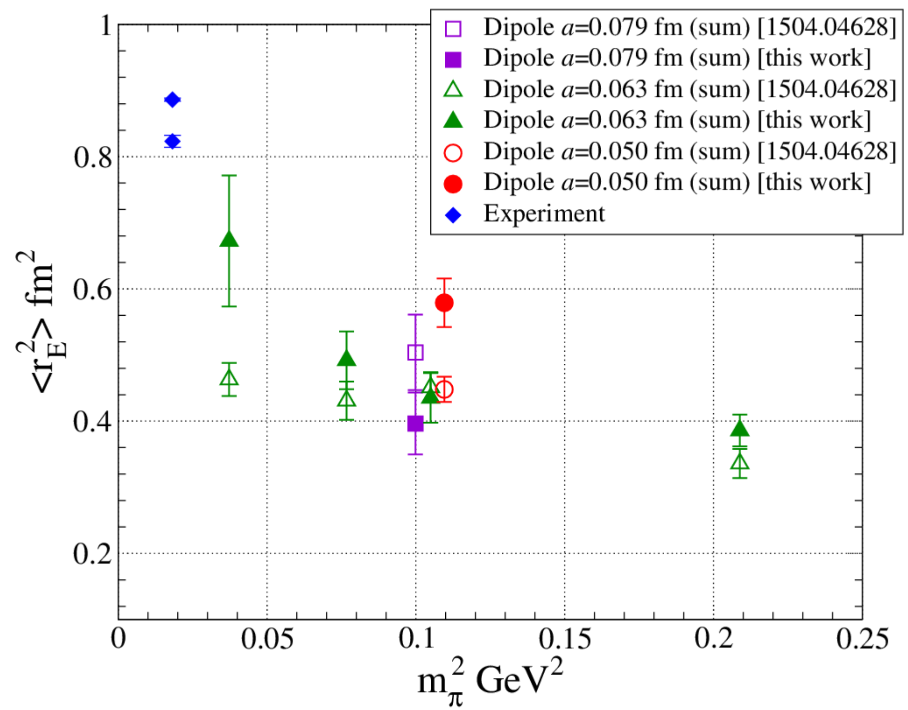


- From t_s > 1 fm, there is still tendency to decrease by ~5%.
- using at t_s > 1.5 fm is compatible with experimental value.

- Comparison with previous work on the same ensemble.
- Large discrepancy between plateau method at t_s = 1.1 fm and 1.5 fm, due to excited state contamination.
- Approaching to experimental value.

Axial charge and charge radius





- Analysis of axial charge and charge radius with large t_s up to 1.7 fm.
- Result has still large statistical error, even though statistics $O(10^5)$ is used.
- In t_s = 1.1 fm, there is still unsuppressed excited state effect, which may be one of the reason for large discrepancy from experiment $\Rightarrow$ need more than 1.5 fm.
- Axial charge may not have strong m_π dependence, but $\langle r_E \rangle$ may have.

Summary

- All-mode-averaging technique is applied for reduction of statistical error in lattice QCD.
- High statistics calculation of nucleon form factor is performed in N_f=2 Wilson-clover at $Lm_\pi > 4$ with m_π = 0.19--0.46 GeV.
- t_s > 1.5 fm is required for small contribution of excited state contamination in axial charge and (iso)vector form factor.
- Axial charge and charge radius are approaching to experimental value.
- Feasible study for application to N_f = 2+1 CLS configurations with open boundary condition.

Thank you for your attention.

Extraction of g_A

- Ground and excited state ansatz
  - Ground state dominance (plateau method)

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \quad \simeq \quad g_A,\ (t_s, t_s - t \gg 1)$$

- Evaluation from constant fitting for t with fixed t_s.
- To suppress the excited state contamination, measurement at large t_s is needed.

  - First excited state (two-state) PNDME(2014), RQCD(2014), …

$$R_A(t,t_s) \quad \simeq \quad g_A + c\left(e^{-\Delta t_s} + e^{-\Delta(t_s - t)}\right)$$

- Δ is mass difference between ground and 1st excited state.

- Summation method Capitani et al. PRD86 (2012)

$$R_A^{\rm sum}(t_s) = \sum_{t=0}^{t_s} R_A(t,t_s) \simeq a_0 + t_s(g_A + O(e^{-\Delta t_s}))$$

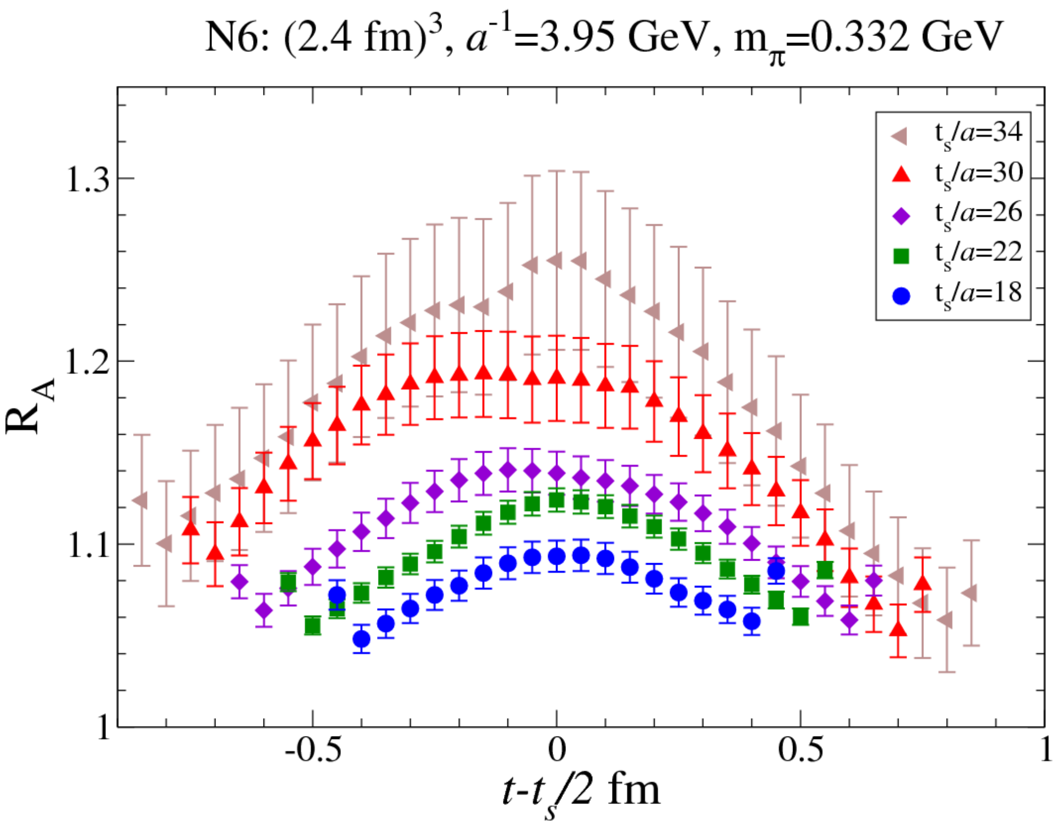
- Using summation in [0,t_s] at fixed t_s , the excited state effect is ~ $O(e^{-\Delta t_s})$
- g_A is given from t_s linear part at t_s >> 1.

3. Lattice results (preliminary)

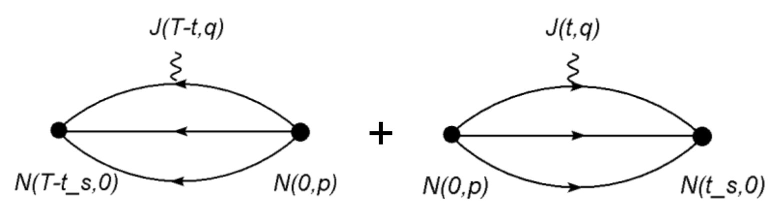
Axial charge

- Single ratio of 2pt and 3pt with fixed t_s

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \simeq g_A + c_1 e^{-\Delta t_s} + c_2 e^{-\Delta'(t_s-t)}$$



- Computation of 3pt and 2pt function at zero momentum with spin projection P.
- Signal is regarded as plateau.
- There is significant size of excited state (2nd and 3rd terms) → fitting including 1st excited state
- Forward and backward averaging



Isovector form factor

- Ratio with momentum transition

$$R_G(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_1)J_\mu(t,q)\mathcal{N}^\dagger(0,p_0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_0)\mathcal{N}^\dagger(0,p_0)|0\rangle}K(p_1,p_0) \simeq G_X + d_1 e^{-\Delta t_s} + d_2 e^{-\Delta'(t_s-t)}$$

$$K(p_1,p_0) = \sqrt{\frac{C^{\rm lc}_{\rm 2pt}(p_1,t_s-t)C^{\rm sm}_{\rm 2pt}(p_0,t)C^{\rm lc}_{\rm 2pt}(p_0,t_s)}{C^{\rm lc}_{\rm 2pt}(p_0,t_s-t)C^{\rm sm}_{\rm 2pt}(p_1,t)C^{\rm lc}_{\rm 2pt}(p_1,t_s)}},$$

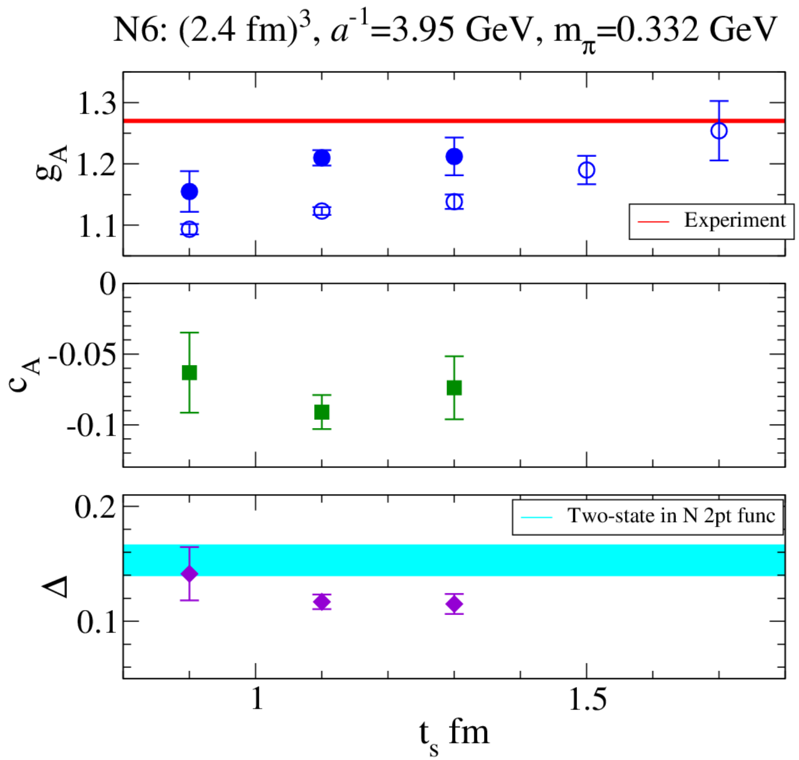
- The ratio consists of 3pt and 2pt, with combination of local "lc" and smeared "sm" sink.
- Matrix element with Sachs form factor

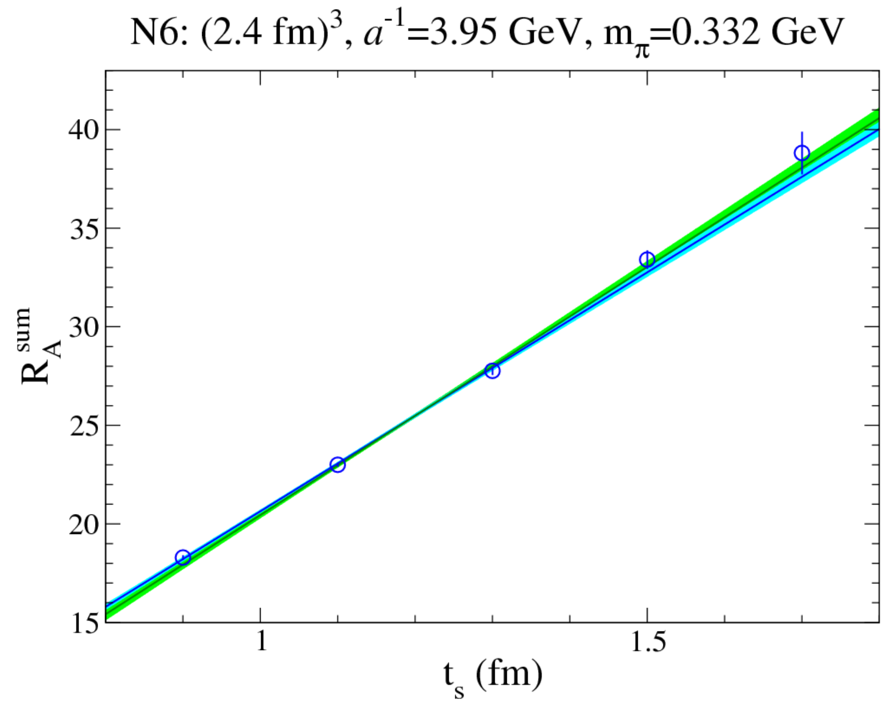
$$\langle N(\vec{p}_1)|J_\mu|N(\vec{p}_0)\rangle = \bar{u}(p_1)\left[F_1^v(q^2)\gamma_\mu + F_2 q_\nu \sigma_{\mu\nu}/2m_N\right]u_N(p_0)$$

$$G_E = F_1 - \frac{q^2}{4m_N^2}F_2,\ G_M = F_1 + F_2$$

- Form factor G_X as a function of q^2, $q = p_1 - p_0$, in which $p_1 = (0,m_N)$ $p_0 = (p,E)$ are used.
- Systematic study of excited state contamination with plateau and summation method is necessary.

Two state and summation method



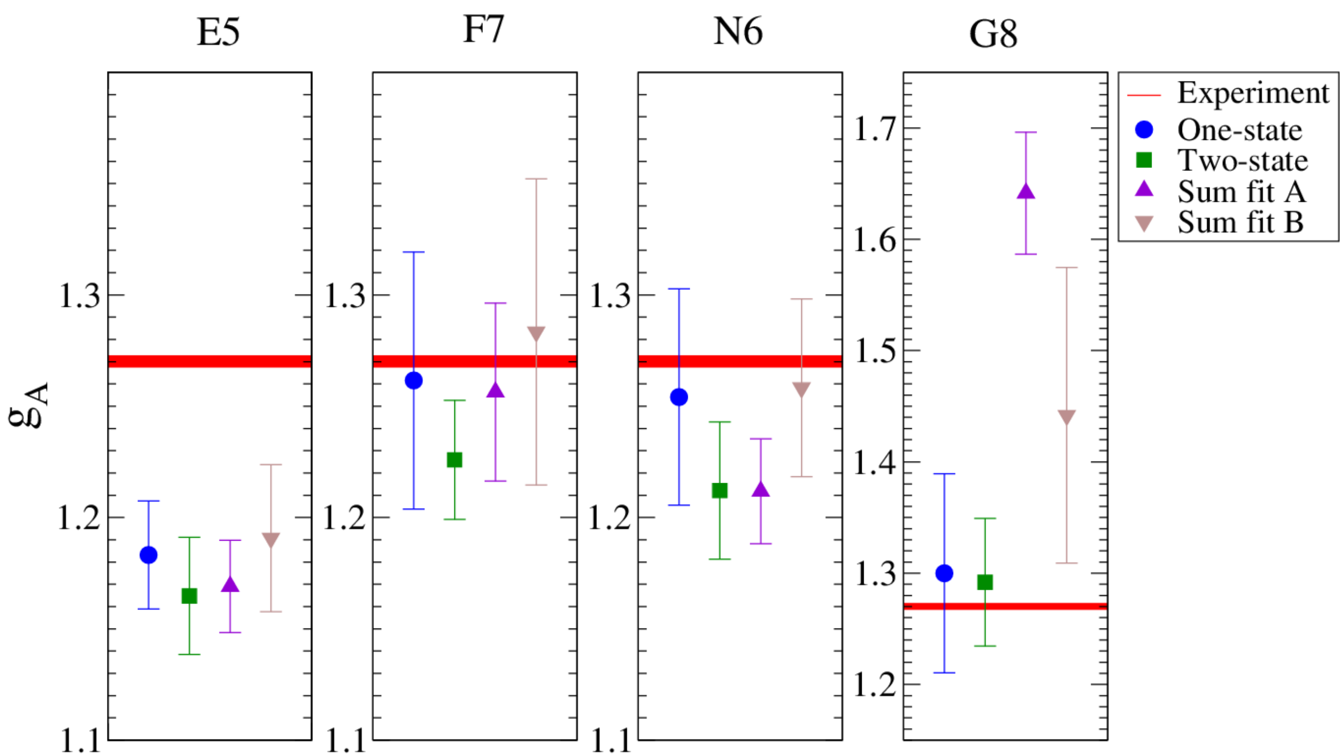


- After correction to excited state, g_A increases, and in agreement with plateau method in t_s > 1.5 fm.
- Mass difference Δ is compatible with two state fit of 2pt function.

- Linear behavior which is consistent with linear ansatz as expected.
- Comparison between two fitting range: t_s = (fit A)[0.9, 1.7], (fit B)[1.1, 1.7] ⇒ estimate of systematic uncertainty

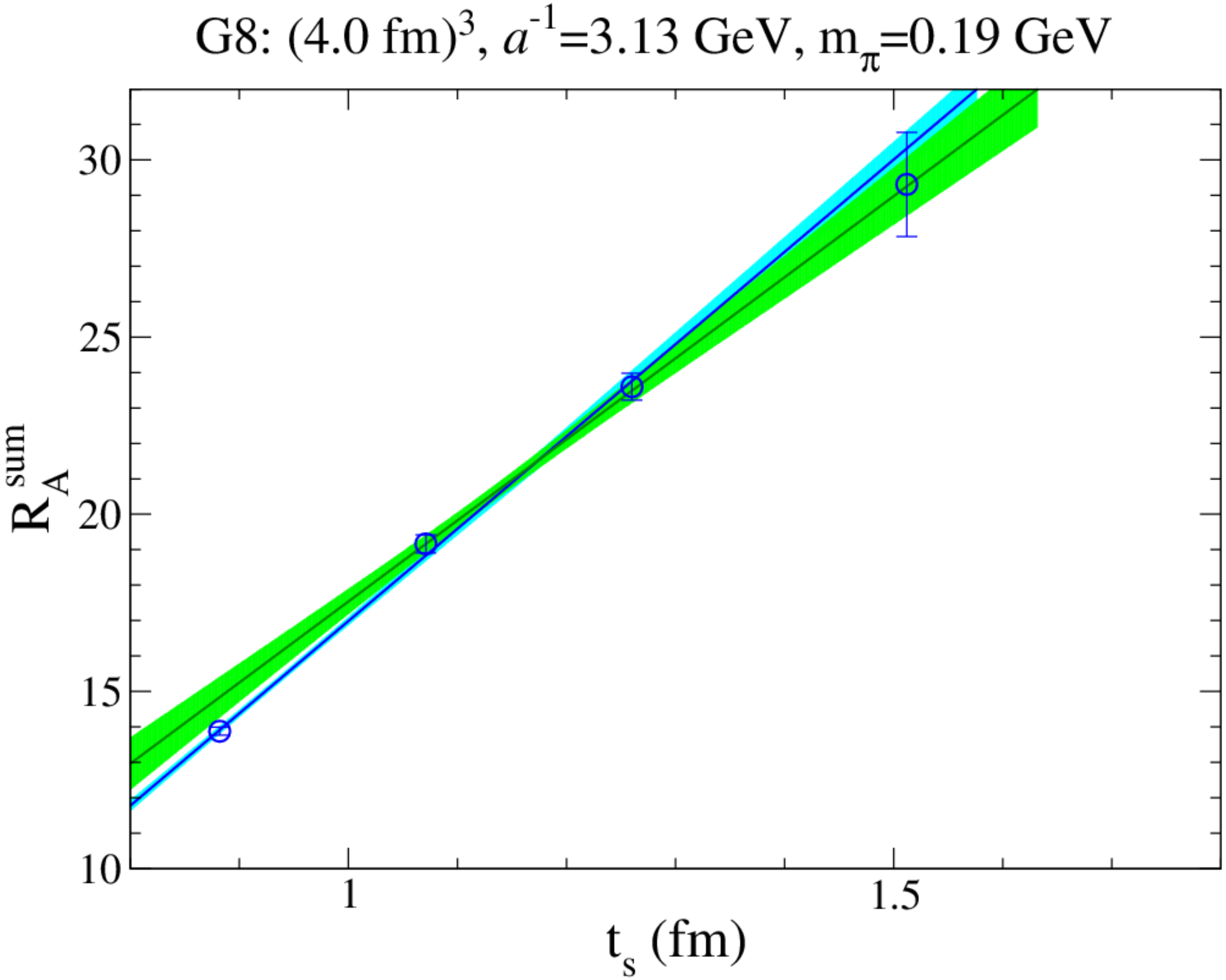
3. Lattice results (preliminary): axial charge

Comparison



- Four methods provide comparable result except for G8 ensemble at m_π = 0.19 GeV .
- On G8 summation method with fit A (including short t_s) is discrepancy from others → expect systematic uncertainty in linear fit function.
- Finite pion mass effect of g_A is rather mild.

Summation method on G8



t dependence of G_E