Matching the Nagy-Soper parton shower at next-to-leading order

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Outline

Motivation

Nagy-Soper parton shower

Parton shower matching

Results: $pp \to t\bar{t}j + X$ @ LHC

Summary

Motivation

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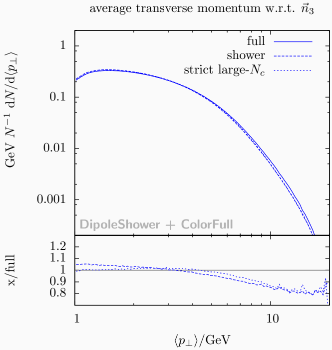
Monte Carlo Event generators

- Provide realistic description of hadron collisions
- Present in all experimental analyses
- Need to be improved as data become more precise



[*Platzer,Sjodahl '12*]

Improving the accuracy of the predictions

- Fixed-order: NLO QCD + EW, NNLO QCD
- Matching parton shower to fixed-order calculations
- Merging several matched calculations for different jet multiplicities
- Current parton showers have only leading colour and leading log accuracy
- Improve parton showers by including subleading effects

Nagy-Soper parton shower

Nagy-Soper parton shower

- Cross section for an observable F: [Nagy,Soper '07'08'12'14]

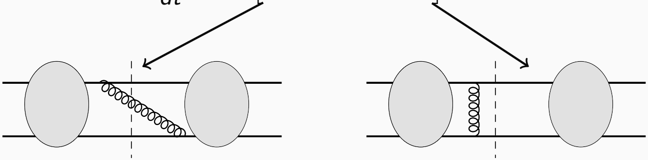
$$\sigma[F] = \sum_m \frac{1}{m!} \int [d\{p,f\}_m] \langle \mathcal{M}(\{p,f\}_m) | F(\{p,f\}_m) | \mathcal{M}(\{p,f\}_m) \rangle \frac{f_a(\eta_a, \mu_F^2) f_b(\eta_b, \mu_F^2)}{4 n_c(a) n_c(b) \times flux}$$

- Quantum density matrix:

$$\rho(\{p,f\}_m) \sim |\mathcal{M}(\{p,f\}_m)\rangle \langle \mathcal{M}(\{p,f\}_m)|$$

- Parton shower evolves ρ with a unitary operator $U(t_F, t_0)$

$$\frac{dU(t,t_0)}{dt} = \left[\mathcal{H}_I(t) - \mathcal{V}(t) \right] U(t,t_0)$$



- Unitarity: $(1|U(t_F,t_0)|\rho) = (1|\rho) \Rightarrow (1|\left[\mathcal{H}_I(t) - \mathcal{V}(t)\right] = 0$

Nagy-Soper parton shower

- Evolution operator:

$$U(t_F, t_0) = N(t_F, t_0) + \int_{t_0}^{t_F} d\tau \, [\mathcal{H}_I(\tau) - \mathcal{V}_S(\tau)] \, U(\tau, t_0)$$

- Sudakov form factor:

$$N(t_F, t_0) = \exp\left(-\int_{t_0}^{t_F} d\tau \; \mathcal{V}_E(\tau)\right) \; , \qquad \mathcal{V}(t) = \mathcal{V}_E(t) + \mathcal{V}_S(t)$$

Exponentiation of full colour is non-trivial [*Platzer,Sjodahl '12*]

- Exponentiate colour diagonal part $\mathcal{V}_E(t)$ to all orders
- Treat colour off-diagonal part $\mathcal{V}_S(t)$ as perturbation

- Expectation value of observable F including shower effects:

$$\sigma[F] = (F|\rho(t_F)) = (F|U(t_F, t_0)|\rho(t_0))$$

Features of Nagy-Soper shower

- Splitting functions are different from Altarelli-Parisi
- Massive initial state charm and bottom quarks
- Designed to include spin correlations [*Nagy,Soper '08*]
- ... and full colour in the evolution [*Nagy,Soper '12*]
- Ordering parameter Λ_l [*Nagy,Soper '14*]

$$e^{-t_l} = \frac{\Lambda_l^2}{Q^2}\,, \qquad \Lambda_l^2 = \frac{|(\hat{p}_l \pm \hat{p}_{m+1})^2 - m^2(l)|}{2p_l \cdot Q} Q^2$$

- PDFs evolved according to PS splitting functions [*Nagy,Soper '14*]
- Global momentum mapping

Public code: Deductor

- LC+ approximation [*Nagy,Soper '12*]
  - Full colour for collinear and soft-collinear limits
  - Shower can start from subleading colour configurations
- Spin-averaged evolution

Parton shower matching

- Quantum density at next-to-leading order:

$$|\rho) = \underbrace{|\rho_m^{(0)})}_{\text{Born, } \mathcal{O}(1)} + \underbrace{|\rho_m^{(1)})}_{\text{Virtual, } \mathcal{O}(\alpha_s)} + \underbrace{|\rho_{m+1}^{(0)})}_{\text{Real, } \mathcal{O}(\alpha_s)} + \mathcal{O}(\alpha_s^2)$$

✗ Double counting:

$$(F|U(t_F, t_0)|\rho) \approx (F|\rho) + \int_{t_0}^{t_F} d\tau \; (F| \left[\mathcal{H}_I(\tau) - \mathcal{V}(\tau)\right] |\rho_m^{(0)}) + \mathcal{O}(\alpha_s^2)$$

- Modify the Quantum density matrix (Mc@Nlo) *[Frixione, Webber '02]*

$$|\bar{\rho}) \equiv |\rho) - \int_{t_0}^{t_F} d\tau \left[\mathcal{H}_I(\tau) - \mathcal{V}(\tau)\right] |\rho_m^{(0)}) + \mathcal{O}(\alpha_s^2)$$

- Matched differential cross section:

$$\bar{\sigma}[F] = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m)(\Phi_m| \left[|\rho_m^{(0)}) + |\rho_m^{(1)}) + \int_{t_0}^{t_F} d\tau \mathcal{V}(\tau)|\rho_m^{(0)}) \right]$$
$$+ \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1})(\Phi_{m+1}| \left[|\rho_{m+1}^{(0)}) - \int_{t_0}^{t_F} d\tau \mathcal{H}_I(\tau)|\rho_m^{(0)}) \right]$$

Matching fully inclusive processes

- Shower kernels define subtraction scheme ($t_F \to \infty$)

$$\int_{t_0}^{\infty} d\tau \mathcal{H}_I(\tau) = \sum_I \mathbf{S}_I \int_{t_0}^{\infty} d\tau \delta(\tau - t_I)\Theta(\tau - t_0) \sum_I \mathbf{S}_I \Theta(t_I - t_0)$$

$$\int_{t_0}^{\infty} d\tau \mathcal{V}(\tau) = \sum_I \int d\Gamma_I \mathbf{S}_I \Theta(t_I - t_0) \equiv \mathbf{I}(t_0) + \mathbf{K}(t_0)$$

- Matched cross section including shower effects:

$$\bar{\sigma}[F] = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m)(\Phi_m|S) + \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1})(\Phi_{m+1}|H)$$

$$(\Phi_m|S) \equiv (\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + (\Phi_m|\mathbf{I}(t_0) + \mathbf{K}(t_0) + \mathbf{P}|\rho_m^{[0]})$$

$$(\Phi_{m+1}|H) \equiv (\Phi_{m+1}|\rho_{m+1}^{(0)}) - \sum_I (\Phi_{m+1}|\mathbf{S}_I|\rho_m^{(0)})\Theta(t_I - t_0)$$

  - Matching in two steps: generation of $(\Phi_m|S)$ and $(\Phi_{m+1}|H)$
  - Application of $U(t_F, t_0)$

Matching exclusive processes

✗ The problem:

- Missmatch with the naive approach:

$$\bar{\sigma}[F] = \frac{1}{m!}\int [d\Phi_m](F|U(t_F,t_0)|\Phi_m)(\Phi_m|S)F_I(\{\hat{p},\hat{f}\}_m) + \frac{1}{(m+1)!}\int [d\Phi_{m+1}](F|U(t_F,t_0)|\Phi_{m+1})(\Phi_{m+1}|H)F_I(\{p,f\}_{m+1})$$

- Expanding the evolution operator:

$$\bar{\sigma}[F]^{PS} \approx \frac{1}{m!}\int [d\Phi_m](F|\Phi_m)(\Phi_m|\left[|\rho_m^{(0)}) + |\rho_m^{(1)}) + \mathbf{P}|\rho_m^{(0)})\right] F_I(\{\hat{p},\hat{f}\}_m)$$
$$+ \frac{1}{(m+1)!}\int [d\Phi_{m+1}](F|U(t_F,t_0)|\Phi_{m+1})(\Phi_{m+1}|\rho_{m+1}^{(0)})F_I(\{p,f\}_{m+1})$$
$$+ \int \frac{[d\Phi_m]}{m!}\frac{[d\Phi_{m+1}]}{(m+1)!}\int_{t_0}^{t_F} d\tau (F|\Phi_{m+1})(\Phi_{m+1}|\mathcal{H}_I(\tau)|\Phi_m)$$
$$\times (\Phi_m|\rho_m^{(0)})\left[F_I(\{\hat{p},\hat{f}\}_m) - F_I(\{p,f\}_{m+1})\right] + \mathcal{O}(\alpha_s^2)$$

✓ The solution:

- Modifying the subtraction terms:

$$(\Phi_{m+1}|H) \to (\Phi_{m+1}|\tilde{H}) \equiv (\Phi_{m+1}|\rho^{(0)}_{(m+1)}) - \sum_l (\Phi_{m+1}|\mathbf{S}_l|\rho^{(0)}_m)\Theta(t_l - t_0) F_I(Q_l(\{p,f\}_{m+1}))$$

where $F_I(Q_l(\{p,f\}_{m+1})) = F_I(\{\hat{p},\hat{f}\}_m)$ and Q_l is inverse momentum mapping.

- Expanding the shower again:

$$\bar{\sigma}[F]^{PS} \approx \sigma^{NLO} + \int \frac{[d\Phi_m]}{m!} \frac{[d\Phi_{m+1}]}{(m+1)!} \int_{t_0}^{t_F} d\tau (F|\Phi_{m+1})(\Phi_{m+1}|\mathcal{H}_I(\tau)|\Phi_m) \times (\Phi_m|\rho^{(0)}_m) \left[1 - F_I(\{p,f\}_{m+1})\right] F_I(\{\hat{p},\hat{f}\}_m) + \mathcal{O}(\alpha_s^2)$$

- Double counting is removied if $[1 - F_I(\{p,f\}_{m+1})]\, F(\{p,f\}_{m+1}) = 0$

$$F_I(\{p,f\}_{m+1}) = 1 \text{ for } F_I(\{p,f\}_{m+1}) \neq 0$$

Generation cuts more inclusive than the cuts on the final observable

Results: $pp \to t\bar{t}j + X$ @ LHC

$pp \to t\bar{t}j + X$ @ LHC

NLO calculations: [Dittmaier, Uwer, Weinzierl '07; Melnikov, Schulze '10]
NLO+PS using POWHEG method [Kardos,Papadopoulos, Trocsanyi '11]
[Alioli, Moch, Uwer '11]

Setup

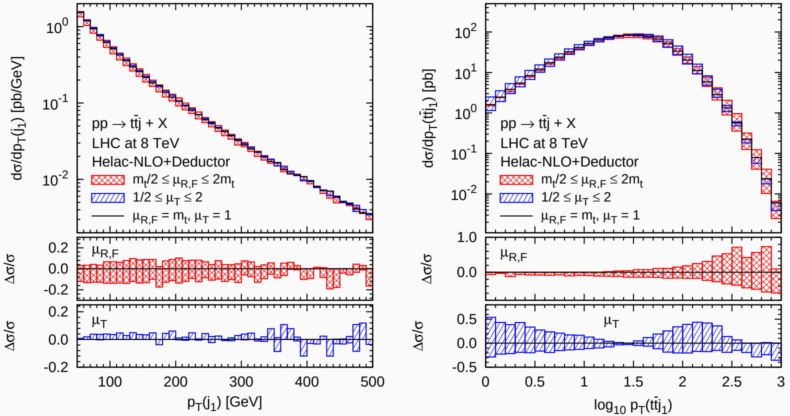
- $\sqrt{s} = 8$ TeV, $m_t = 173.5$ GeV, in PS: $m_b = 4.75$ GeV, $m_c = 1.4$ GeV
- MSTW2008NLO PDF sets, in PS: provided at $\mu_F = 1$ GeV

$$p_T^{gen}(j) > 30 \text{ GeV}, \; p_T(j) > 50 \text{ GeV}, \; |y(j)| < 5, \; \mu_R = \mu_F = m_t$$

- anti-k_T jet algorithm with $\Delta R = 1$
- LC and spin-averaged shower evolution, full correlations in the subtraction
- Top decays, hadronization and multiple interactions are not included
- Shower initial conditions:

$$e^{-t_0} = \min_{i \neq j} \left\{ \frac{2p_i \cdot p_j}{\mu_T^2 Q^2} \right\} \qquad \mu_T = 1 \text{ for central prediction}$$

Perturbative uncertainties

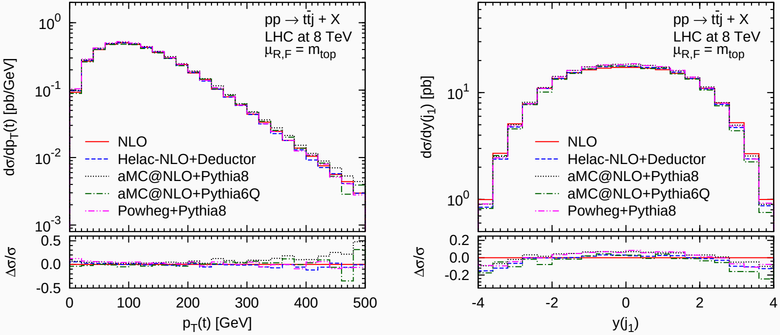


✓ Scales: $m_t/2 < \mu_{R,F} < 2m_t$

✓ PS initial conditions: $1/2 < \mu_T < 2$

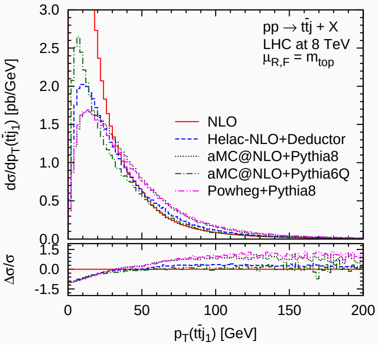
✗ Non-perturbative: Hadronization, Multiple interactions...

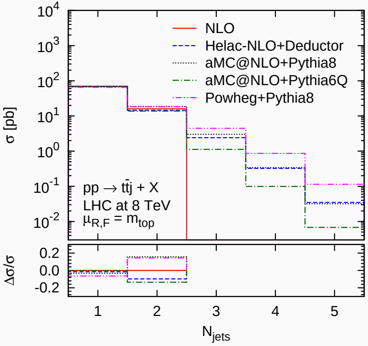
Comparison with other MC generators



- ✓ Comparison of different generators, matching schemes and showers
- ✓ Good agreement for inclusive NLO observables

Comparison with other MC generators





- ✗ Strong dependence on initial conditions for LO observables
- ✓ Helac-Nlo+Deductor and aMc@Nlo+Pythia6Q preserve NLO high energy predictions

Summary

Summary

Already done:

- ✓ NLO matching scheme for Nagy-Soper parton shower (Mc@Nlo)
- ✓ Implementation in Helac-Nlo framework (LC & spin-averaged)
- ✓ Study of $t\bar{t}j$ production at LHC using Helac-Nlo+Deductor
- ✓ Comparison with other MC generators

Has to be done:

- In Deductor
  - ✗ Resonance decays
  - ✗ Hadronization model
  - ✗ Full colour evolution and spin correlations
- In Helac-Nlo
  - ✗ Full colour and spin-correlated matching scheme implementation

BACKUP

Total cross sections - $pp \to t\bar{t}j + X$

$$\sigma^{\rm NLO} = 86.04\,^{+5.10\ (+6\%)}_{-11.41\ (-13\%)}\ \text{pb}$$

Helac-Nlo+Deductor

$$\sigma^{\rm NLO+PS} = 86.11\,^{+4.38\ (+5\%)}_{-10.88\ (-13\%)}\ \text{[scales]}\ ^{+0.80\ (+1\%)}_{+2.17\ (+3\%)}\ \text{[PS time] pb}$$

aMc@Nlo+Pythia6Q

$$\sigma^{\rm NLO+PS} = 84.85\,^{+8.95\ (+11\%)}_{-13.75\ (-16\%)}\ \text{[scales] pb}$$

aMc@Nlo+Pythia

$$\sigma^{\rm NLO+PS} = 89.55\,^{+8.44\ (+9\%)}_{-15.41\ (-17\%)}\ \text{[scales] pb}$$

Powheg+Pythia

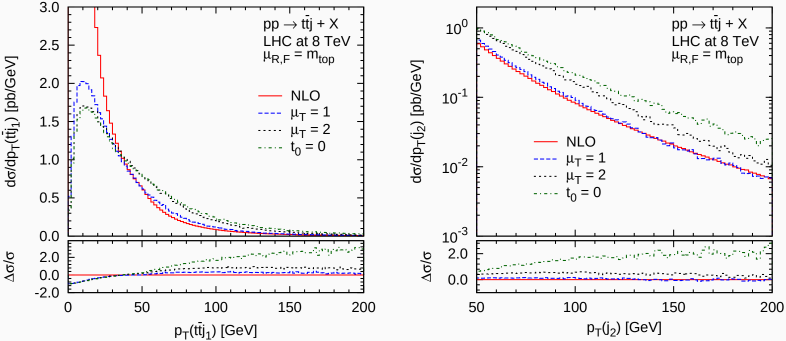
$$\sigma^{\rm NLO+PS} = 89.12\,^{+26.22\ (+29\%)}_{-8.96\ (-10\%)}\ \text{[scales] pb}$$

Generation cut dependence

Generation cut: $p_T(j_1) > p_T^{cut}$
Analysis cut: $p_T(j_1) > 50$ GeV.

p_T^{cut} [GeV]	$\sigma^{\mathrm{NLO+PS}}_{pp \to t\bar{t}j+X}$ [pb]	ϵ [‰]
5	86.51 ± 0.21	2.4
10	86.26 ± 0.17	2.0
15	86.22 ± 0.14	1.6
30	86.11 ± 0.13	1.5
40	86.01 ± 0.08	0.9
50	84.58 ± 0.07	0.8

CHOICE OF t_0



$$e^{-t_0} = \min_{i \neq j} \left\{ \frac{2p_i \cdot p_j}{\mu_T^2 Q^2} \right\}$$

[Plehn, Rainwater, Skands '07]
[Hoeche, Krauss, Schoenherr, Siegert '12]

Choose $\mu_T = 1$ to recover the NLO prediction