Study of $B \to K\pi\pi\gamma$ Decays

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representing the BABAR collaboration

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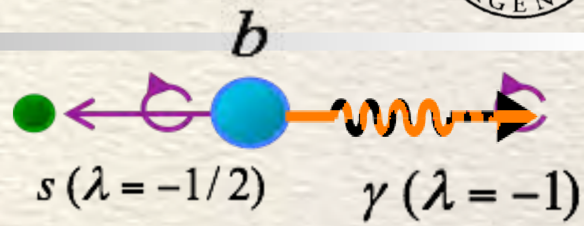
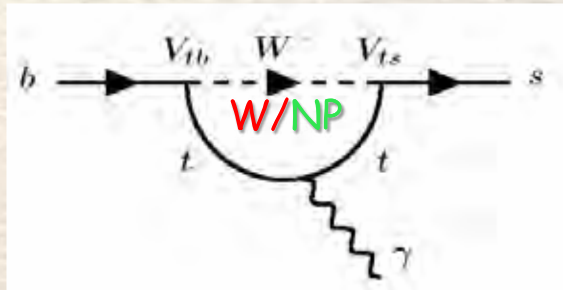
Introduction

- The V-A structure of the SM weak interaction yields predominantly left-handed photons in $b \to s\gamma$ decays



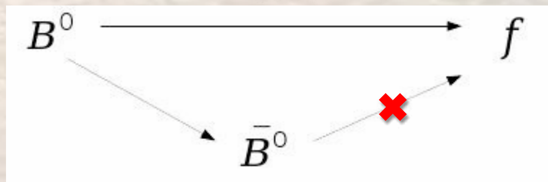
- Thus, a B meson decays predominantly to a right-handed γ while a $\bar{B}$ meson decays predominantly to a left-handed γ (apart from m_s/m_b effects) Atwood *et al.*, PRL79, 185 (1987)



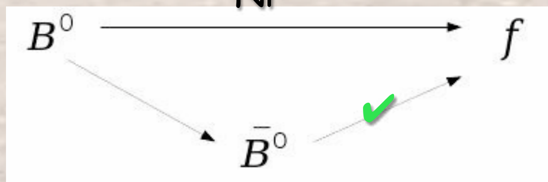
- The mixing-induced CP asymmetry in $B \to f_{CP}\gamma$ decays is expected to be small in the Standard Model

- New physics processes in which opposite-helicity photons are involved may alter the SM prediction
Fujikawa *et al.*, PRD 49, 5890 (1994) Babu *et al.*, PL B333, 196 (1994)
Cho *et al.*, PRD 49, 5894 (1994)

SM



NP



- Measurement of $\mathcal{B}(B \to X_s\gamma) = (3.40 \pm 0.21) \times 10^{-4}$ agrees with the SM prediction of $\mathcal{B}(B \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$, the study of $B \to K\pi\pi\gamma$ provides an independent measurement
PDG., Chin.Phys. C38, 090001 (2014) Misiak *et al.*, PRL98, 022002 (2007)

- In addition, we explore the resonance structure of the $K\pi\pi$ system

- Data samples: 426 fb^{-1} at $\Upsilon(4S)$ (471×10^6 $B\bar{B}$) & 44.5 fb^{-1} 40 MeV below $\Upsilon(4S)$ peak

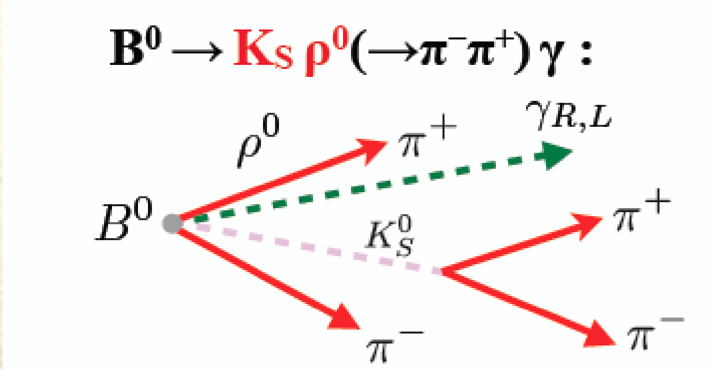
Analysis Methodology

- Goal is to measure mixing-induced *CP* asymmetry parameters $\mathcal{S}_{K0\pi\pi\gamma}$ & $\mathcal{C}_{K0\pi\pi\gamma}$ in $B^0_d \to K^0_s\pi\pi\gamma$ and $\mathcal{S}_{K0\rho\gamma}$ in $B^0_d \to K^0_s\,\rho_0\,\gamma$

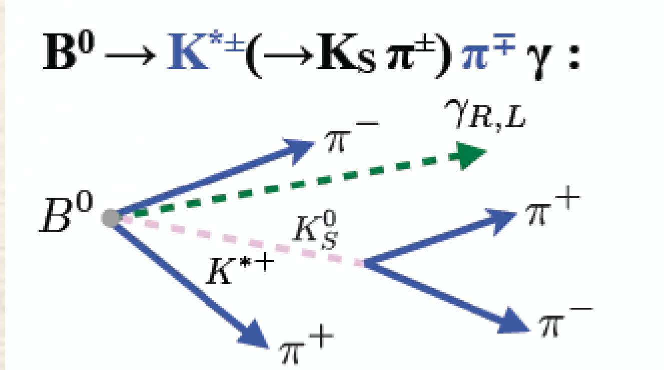
- Due to background from $B^0_d \to K^{*+}(K^0_s\pi^+)\pi^-\gamma$, $\mathcal{S}_{K0\rho\gamma}$ is diluted by

Hebinger *et al.*, LAL 15-75 (2015)

Signal



Background



$$\mathcal{D}_{K^0_S\rho\gamma} = \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{S}_{K^0_S\rho\gamma}} = \frac{\int\left[\left|A_{\rho K^0_S}\right|^2 - \left|A_{K^{*+}\pi^-}\right|^2 - \left|A_{(K\pi)^+_0\pi^-}\right|^2 + 2\Re\left(A^*_{\rho K^0_S}A_{K^{*+}\pi^-}\right) + 2\Re\left(A^*_{\rho K^0_S}A_{(K\pi)^+_0\pi^-}\right)\right]dm^2}{\int\left[\left|A_{\rho K^0_S}\right|^2 + \left|A_{K^{*+}\pi^-}\right|^2 + \left|A_{(K\pi)^+_0\pi^-}\right|^2 + 2\Re\left(A^*_{\rho K^0_S}A_{K^{*+}\pi^-}\right) + 2\Re\left(A^*_{\rho K^0_S}A_{(K\pi)^+_0\pi^-}\right)\right]dm^2}$$

A_f: amplitude for state f

➔ Need amplitude analysis to determine dilution factor $D_{K0\rho\gamma}$

- Use $B^+ \to K^+\pi^+\pi^-\gamma$ decay to determine $D_{K0\rho\gamma}$ since a larger number of signal events is available and it is simply related to $B^0_d \to K^0_s\pi^+\pi^-\gamma$ by isospin

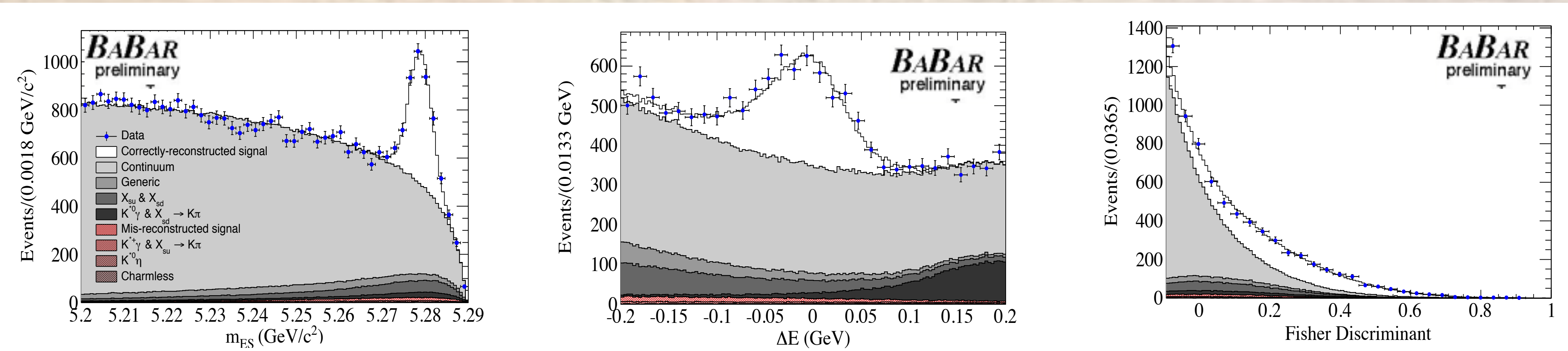
- $K^+\pi^+\pi^-\gamma$ final state is produced by kaonic resonances that decay via intermediate $K^{*0}(892)\pi^+$ or $K^+\rho^0$ states ➔ determine first 3-body resonance content of $m_{K\pi\pi}$ spectrum

Event Selection

- We combine high-energy photon ($1.5 < E_\gamma < 3.5$ GeV) with 2 oppositely charged pions and a charged K

- We use the beam-energy-constrained mass $m_{ES} = \sqrt{\left(\frac{E_{CM}^2}{4} - p_B^{*2}\right)}$
  the energy difference $\Delta E = E_B^* - \frac{E_{CM}}{2}$
  and a Fisher discriminant based on 6 discriminating event shape variables to separate signal from $e^+e^- \to q\bar{q}$: Fisher, Annals Eugen. 7, 179 (1936)
  θ_B, $\theta_{Bthrust}$, θ_{B-roe}, L_0, L_2/L_0, R_2/R_0

- Use likelihood ratio to discriminate against π^0 and η:
  $\mathcal{L}_{\pi 0} < 0.86$ ➔ retains 93% signal, removes 83% $q\bar{q}$ and 63% $B\bar{B}$ backgrounds
  $\mathcal{L}_\eta < 0.957$ ➔ retains 95% signal, removes 87% $q\bar{q}$ and 10% $B\bar{B}$ backgrounds

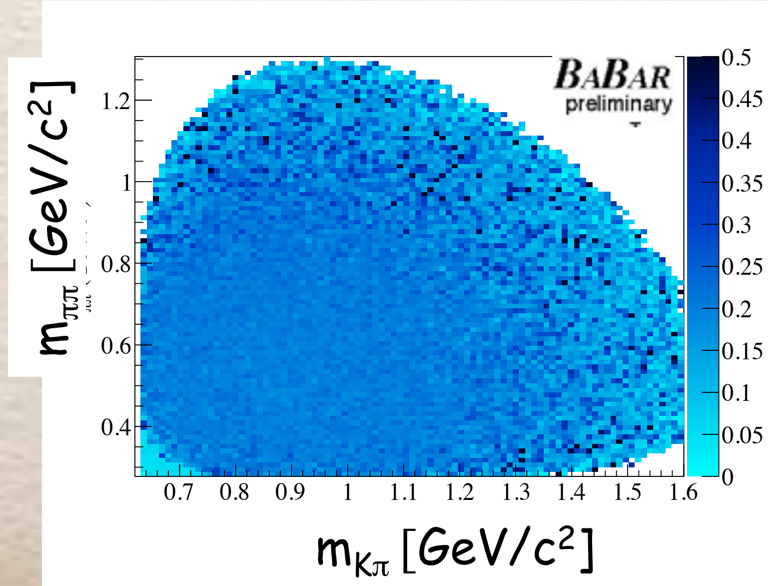


Fit to $m_{K\pi\pi}$ Spectrum

- Do unbinned extended maximum likelihood fit to m_{ES}, ΔE and $\mathcal{F}$ discriminant to extract $B^+ \to K^+\pi^+\pi^-\gamma$ signal yield ➔ extract signal $m_{K\pi\pi}$, $m_{K\pi}$ & $m_{\pi\pi}$ spectra using ${}_s\mathcal{P}$lot technique Pivk *et al.*, NIM A555, 356 (2005)
- Model $m_{K\pi\pi}$ distribution as a coherent sum of 5 resonances [$K_1(1270)+K_1(1400)$, $K^*(1410)+K^*(1680)$, $K^*_2(1430)$] ➔ Model by relativistic Breit-Wigner line shapes
- Fit with 8 free parameters: magnitudes of $K_1(1400)$, $K^*(1410)$, $K^*(1680)$ & $K^*_2(1430)$; 2 relative phases, width of $K_1(1270)$ & $K^*(1680)$
- Extract fit fractions F_R for each resonance & interference fit fractions with a binned ML fit to $m_{K\pi\pi}$ spectrum (80 bins)
- Total branching fraction is given by

$$\mathcal{B}\left(B^+ \to K^+\pi^+\pi^-\gamma\right) = \frac{N_{sig}}{\bar{\varepsilon} N_{B^\pm}}$$

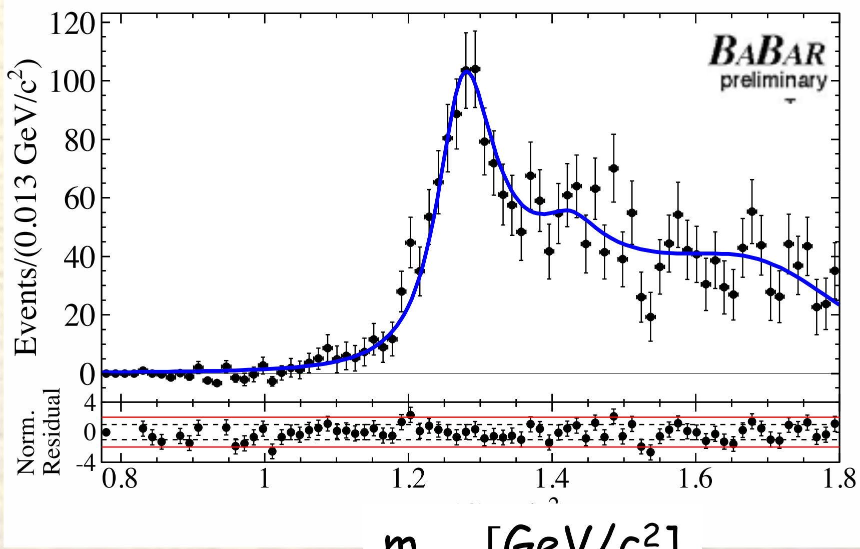
where $N_{B\pm}=483.2\pm6.3\times10^6$ and $\bar{\varepsilon}$ is average efficiency (~0.2) weighted over all 5 resonances



Results of the $m_{K\pi\pi}$ Spectrum Fit

- The weighted efficiency is $\bar{\varepsilon}$=0.1857±0.0007
- For $m_{K\pi\pi}$< 1.8 GeV, the unbinned ML fit to data yields $2441 \pm 91^{+41}_{-54}$ $B^+ \to K^+\pi^+\pi^-\gamma$ *events*
- We measure a branching fraction of

$$\mathcal{B}\left(B^+ \to K^+\pi^+\pi^-\gamma\right) = \left(27.2 \pm 1.0 \pm 1.2\right) \times 10^{-6}$$



$m_{K\pi\pi}$ [GeV/c²]

- We extract fit fractions and branching fractions for the individual kaonic resonances
- We make the first measurements of $B^+\to K^+_1(1400)\gamma$, $B^+\to K^{*+}(1410)\gamma$ & $B^+\to K^{*+}(1680)\gamma$ modes

Mode	$\mathcal{B}(B^+ \to \text{Mode})\times$ $\mathcal{B}(K_{\text{res}} \to K^+\pi^+\pi^-) \times 10^{-6}$	$\mathcal{B}(B^+ \to \text{Mode}) \times 10^{-6}$	Previous world average [17] ($\times 10^{-6}$)
$B^+ \to K^+\pi^+\pi^-\gamma$	$\cdots$	$27.2 \pm 1.0 \pm 1.2$	27.6 ± 2.2
$K_1(1270)^+\gamma$	$14.5^{+2.0}_{-1.3}\pm 1.2$	$44.0^{+6.0+3.5}_{-4.0-3.6} \pm 4.6$	43 ± 13
$K_1(1400)^+\gamma$	$4.1^{+1.9+1.3}_{-1.2-0.8}$	$9.6^{+4.6+3.0}_{-2.9-1.8} \pm 0.6$	< 15 at 90% CL
$K^*(1410)^+\gamma$	$10.5^{+2.1+2.1}_{-1.9-0.9}$	$25.8^{+5.2+5.1}_{-4.6-2.2} \pm 2.6$	—
$K_2^*(1430)^+\gamma$	$1.2^{+1.2+0.9}_{-1.0-1.2}$	$8.7^{+8.7+6.2}_{-7.0-8.5} \pm 0.4$	14 ± 4
$K^*(1680)^+\gamma$	$16.6^{+1.7+3.6}_{-1.4-2.7}$	$70.0^{+7.2+15}_{-5.7-11} \pm 5.7$	< 1900 at 90% CL

PDG., Chin.Phys. C38, 090001 (2014)

Fit to $m_{K\pi}$ Spectrum

- We do a binned ML fit (90 bins) of $m_{K\pi}$ spectrum after $\bar{\varepsilon}$ correction, i.e. by dividing ($m_{K\pi}$, $m_{\pi\pi}$) ${}_s\mathcal{P}$lot by combined efficiency map and then integrating over $m_{\pi\pi}$ dimension

- Branching fractions of intermediate resonances are obtained from

$$\mathcal{B}\left(B^+ \to \begin{matrix}(K\pi)_R\pi\gamma \\ (\pi\pi)_R K\gamma\end{matrix}\right) = F_R \frac{N_{sig}}{\bar{\varepsilon} N_{B^\pm}}$$

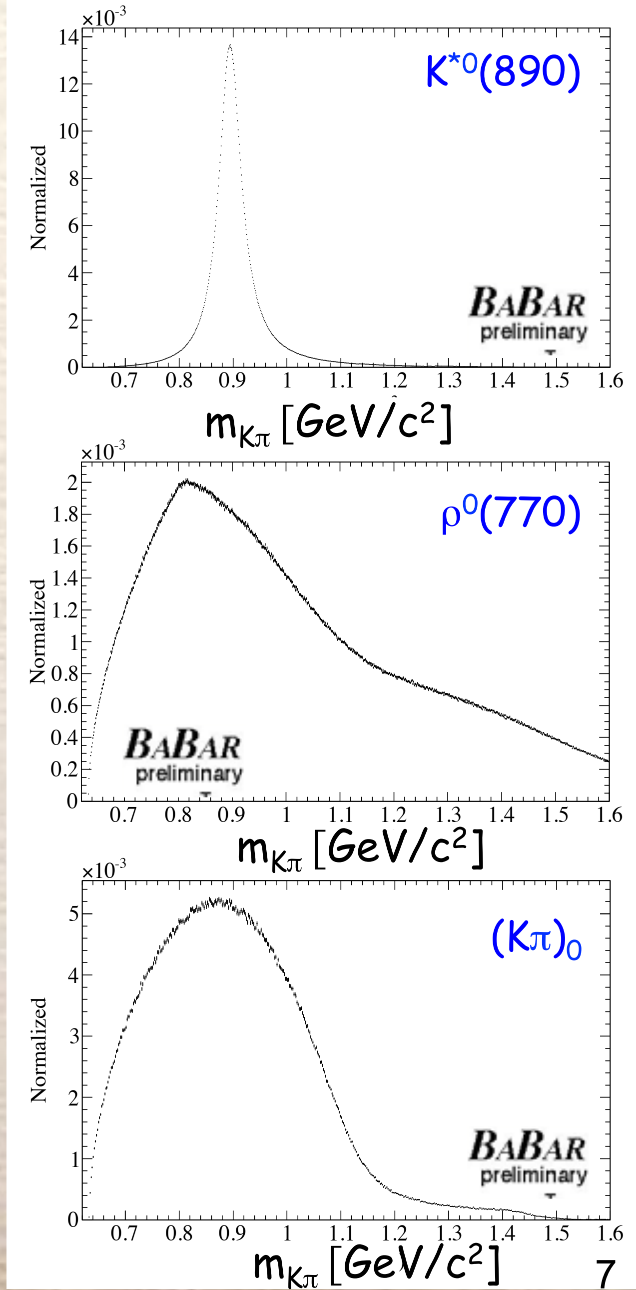
- For $(K\pi)_R$, we include $K^{*0}(892)$ (P-wave) and 0^+ (S-wave) components, for $(\pi\pi)_R$ we include $\rho^0(770)$ (P-wave)

- Model $K^{*0}(892)$ by relativistic BW, $\rho^0(770)$ by Gounaris-Sakurai lines shapes and 0^+ by LASS parameterization
  Gounaris *et al.*, PRL 21, 244 (1968) LASS, Nucl.Phys B296, 493 (1988)

- For each resonance, we account for line shape distortions above the pole mass caused by low-mass $K\pi\pi$ resonances

- We include interference between $K\pi$ and $\pi\pi$ P-wave and between $K\pi$ S-wave and $\pi\pi$ P-wave

PDFs for $m_{K\pi}$ mass fit



Results for $B^+ \to K^+\pi^+\pi^-\gamma$

- We extract efficiency-corrected $m_{K\pi}$ ${}_s\mathcal{P}$lot distribution from unbinned ML fit to m_{ES}, ΔE and $\mathcal{F}$ discriminant
- We extract fit fractions and branching fractions from binned ML fit to $m_{K\pi}$ ${}_s\mathcal{P}$lot
- We separate $K^*_0(1430)$ contribution from from $(K\pi)_0$ component
- This is the first observation of $K^+\rho^0\gamma$ and $(K\pi)^{*0}_0\pi^+\gamma$ NR-contribution
- From $(K\pi)^{*0}_0\pi^+\gamma$, we extract

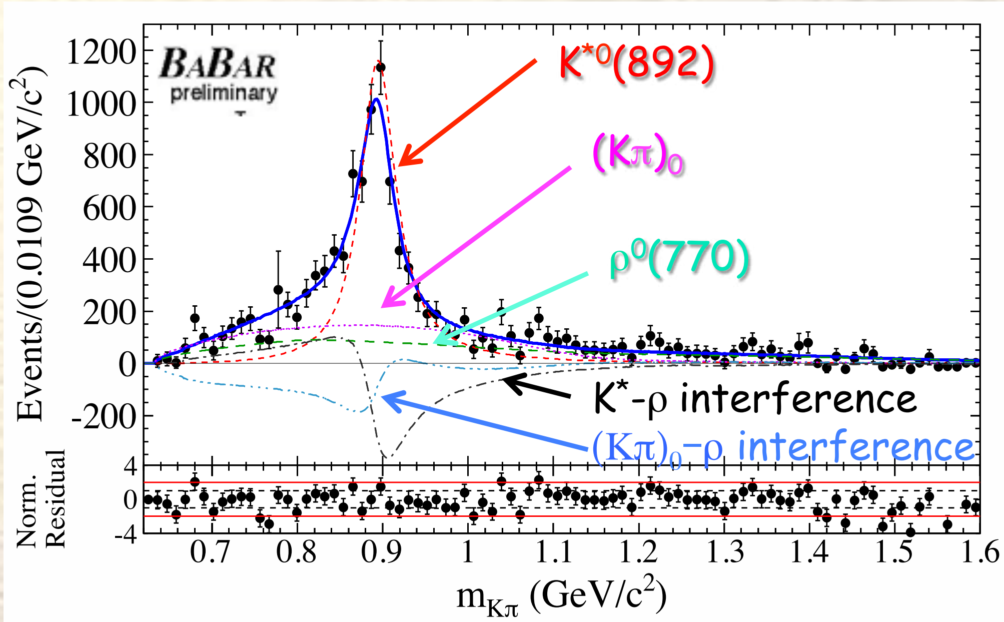
$$B\left(K_1(1270) \to K^*_0(1430)\pi\right) = 3.34^{+0.62+0.64}_{-0.54-0.82}\,\%$$

➔ is in good agreement with Belle

Belle, PRD 83, 032005 (2011)

- We measure dilution factor for $0.6<m_{\pi\pi}<0.9$ GeV, $m_{K\pi}<0.845$ or $m_{K\pi}>0.945$ GeV and $m_{K\pi\pi}<1.8$ GeV

$$D_{K^0_S\rho\gamma} = -0.79^{+0.18}_{-0.17}$$



Mode	$\mathcal{B}(B^+ \to \text{Mode})\times \mathcal{B}(R \to hh) \times 10^{-6}$	$\mathcal{B}(B^+ \to \text{Mode}) \times 10^{-6}$	World average [16] $(\times 10^{-6})$
$B^+ \to K^+\pi^+\pi^-\gamma$	...	$27.2 \pm 1.0^{+1.1}_{-1.3}$	27.6 ± 2.2
$K^*(892)^0\pi^+\gamma$	$17.3 \pm 0.9^{+1.2}_{-1.1}$	$26.0^{+1.4}_{-1.3} \pm 1.8$	20^{+7}_{-6}
$K^+\rho(770)^0\gamma$	$9.1^{+0.8}_{-0.7} \pm 1.3$	$9.2^{+0.8}_{-0.7} \pm 1.3 \pm 0.02$	< 20 CL$=90\%$
$(K\pi)^{*0}_0\pi^+\gamma$	$11.3 \pm 1.5^{+2.0}_{-2.6}$	...	—
$(K\pi)^0_0\pi^+\gamma$ (NR)	...	$10.8^{+1.4+1.9}_{-1.5-2.5}$	< 9.2 CL$=90\%$
$K^*_0(1430)^0\pi^+\gamma$	$0.89 \pm 0.12^{+0.16}_{-0.21}$	$1.44 \pm 0.19^{+0.26}_{-0.34} \pm 0.14$	—

PDG., Chin.Phys. C38, 090001 (2014)

Selection of $B^0 \to K^0_S \pi^+\pi^-\gamma$ Events

- Selection of $B^0_d \to K^0_S\, \pi^+\pi^-\gamma$ events is identical to that of the $B^+ \to K^+\pi^+\pi^-\gamma$ mode except for replacement of K^+ with K^0_S ($|m_{\pi\pi} - m_{K0s}| < 11$ MeV; $\sigma_{\tau(K0s)} > 5$,

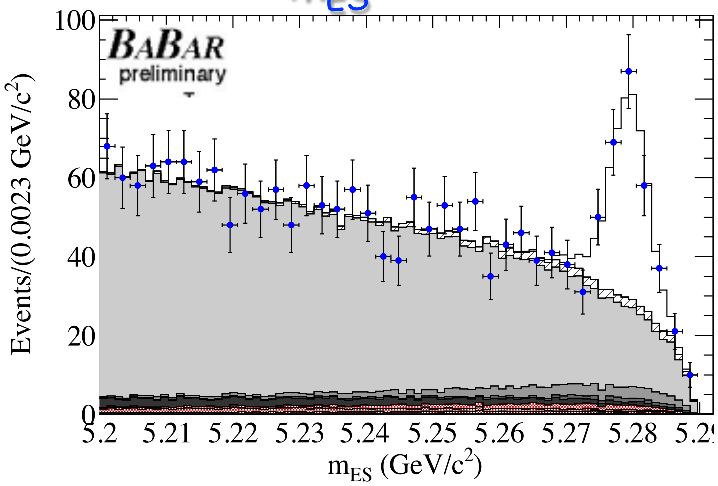
$$\vec{p}_{K^0_S} \bullet \vec{\ell}^{\,flight}_{K^0_S} \Big/ \left| \vec{p}_{K^0_S} \bullet \vec{\ell}^{\,flight}_{K^0_S} \right| > 0.995$$

- Apply same $m_{\pi\pi}$, $m_{K\pi}$, and $m_{KS\pi\pi}$ mass selections

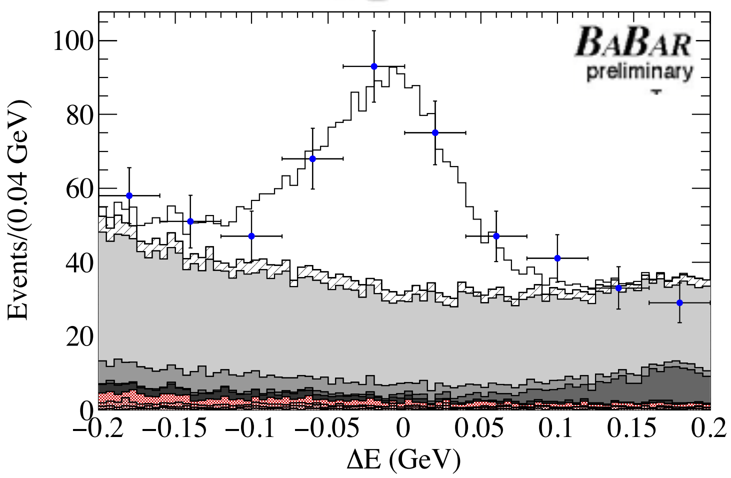
- For most backgrounds, we use Argus PDF for m_{ES}, Chebychev polymomial for ΔE and Gaussian/exponential for $\mathcal{F}$ ARGUS, Z. Phys. C48, 543 (1990)

- We optimize the selection on $\mathcal{F}$ to minimize statistical errors of *CP* parameters

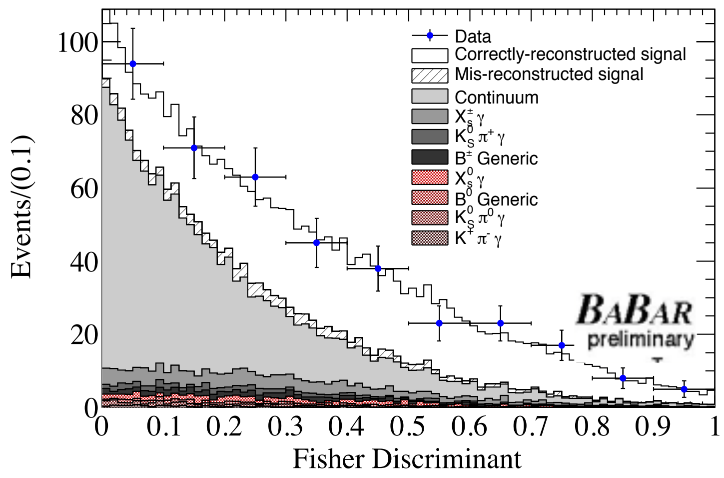
m_{ES}



ΔE



$\mathcal{F}$



Time-Dependent Analysis of $B^0 \to K^0_S \pi^+\pi^-\gamma$

- We perform an unbinned extended ML fit to extract $B^0 \to K^0_S \pi^+\pi^-\gamma$ event yield along with time-dependent *CP* asymmetry parameters $\mathcal{S}$ and $\mathcal{C}$

- The Likelihood function for event i is a sum over class j (signal & backgrounds) in which each PDF depends on m_{ES}, ΔE, $\mathcal{F}$ and Δt ➔ factorize for most classes

$$\mathcal{P}^i_j\left(m_{ES},\Delta E,\mathcal{F},\Delta t,\sigma_{\Delta t};q_{tag},c\right) = \mathcal{P}^i_j\left(m_{ES}\right)\mathcal{P}^i_j\left(\Delta E\right)\mathcal{P}^i_j\left(\mathcal{F}\right)\mathcal{P}^i_j\left(\Delta t,\sigma_{\Delta t};q_{tag},c\right)$$

q_{tag}=+1(-1) for $B_{tag}=B^0$ ($\bar{B}^0$); c is tagging category (use 6 mutually exclusive tags)

- The proper time distribution for $B^0_d \to K^0_S\rho^0\gamma$ events for tagging category c is

$$P^i_{sig}\left(\Delta t,\sigma_{\Delta t};q_{tag},c\right) = \frac{exp\left[-\left|\Delta t\right|/\tau_{B^0}\right]}{4\tau_{B^0}}\left[1+q_{tag}\frac{\Delta D_c}{2}+q_{tag}\left\langle D\right\rangle_c\left(\mathcal{S}\sin(\Delta m_{B_d}\Delta t)-\mathcal{C}\cos(\Delta m_{B_d}\Delta t)\right)\right]\otimes R^c_{sig}(\Delta t,\sigma_{\Delta t})$$

$\langle D\rangle_c$: B^0-$\bar{B}^0$ averaged tagging dilution for c, ΔD_c: difference in D_c between B^0 & $\bar{B}^0$ tags

- We use tagging algorithm and the $\langle D\rangle_c$, ΔD_c values of 6 tagging categories from $B^0_d \to (c\bar{c})K^{(*)0}$ analysis; use untagged events as 7th category (➔ direct *CP*)

BABAR, PRL 99, 171803 (2007); BABAR, PRL 94, 161803 (2005) Gardner et al., PRD 69, 034011 (2004)

- We add backgrounds Δt PDFs: charged *B* decays, B^0 decays to flavor eigenstates, B^0 decays to *CP* eigenstates and those for $q\bar{q}$

Results for $B^0 \to K^0_S\pi^+\pi^-\gamma$

- We require $|\Delta t|$>20 ps and $\sigma_{\Delta t}$<2.5 ps
- The ML fit yields $246 \pm 24^{+14}_{-16}$ signal events
- Resulting in a branching fraction of

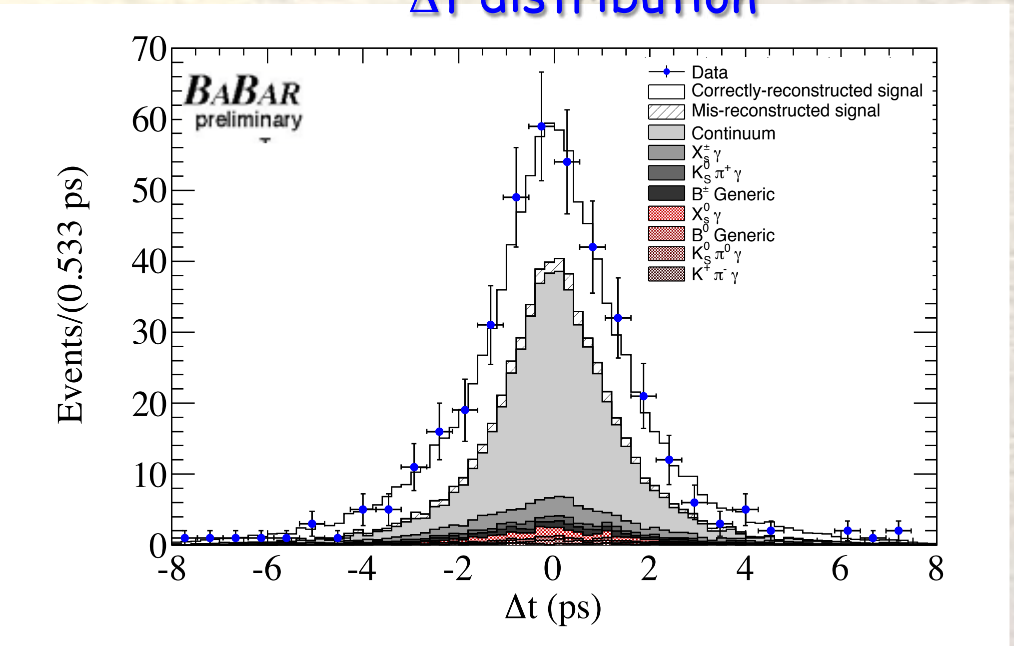
$$\mathcal{B}\left(B^0 \to K^0\pi^+\pi^-\gamma\right) = \left(24.0 \pm 2.4^{+1.7}_{-1.8}\right) \times 10^{-6}$$

- The time-dependent *CP* parameters are measured to be

$$S_{K^0_S\pi^+\pi^-\gamma} = 0.14 \pm 0.25^{+0.04}_{-0.03}$$

$$C_{K^0_S\pi^+\pi^-\gamma} = -0.39 \pm 0.20^{+0.05}_{-0.05}$$

Δt distribution



Continuum background dominates

- For $B^0 \to K^0_S\rho^0\gamma$, we measure after correcting for the dilution factor $\mathcal{D}_{K^0_S\rho^0\gamma}$

$$S_{K^0_S\rho^0\gamma} = -0.17 \pm 0.32^{+0.07}_{-0.06}$$

- Our result is consistent with the Belle result

Belle *et al.*, PRL 101, 251601 (2008)

- All *CP* asymmetries are consistent with zero and thus agree with the SM

Conclusions and Outlook

- We have studied of $B^+ \to K^+ \pi^+\pi^-\gamma$ and $B^0 \to K^0_S \pi^+\pi^-\gamma$ decays, measuring branching fractions of

$$B(B^+ \to K^+\pi^+\pi^-\gamma) = (27.2 \pm 1.0 \pm 1.2) \times 10^{-6}$$

$$B(B^0 \to K^0\pi^+\pi^-\gamma) = (24.0 \pm 2.4^{+1.7}_{-1.8}) \times 10^{-6}$$

- We used the charged B decay to measure the dilution of $K^{*0}(892)\pi\gamma$ in $K\rho\gamma$ decay, yielding

$$D_{K^0_S\rho\gamma} = -0.79^{+0.18}_{-0.17}$$

- We measured branching fractions of 5 exclusive radiative decays $B^+ \to (K^+\pi^+\pi^-)_{resonance}\gamma$ (first measurements for $K^+_1(1400)\gamma$, $K^{*+}(1410)\gamma$, $K^{*+}(1400)\gamma$)

- We measured branching fractions of $B^+\to K^{*+}(892)\pi\gamma$, $B^+\to K^+\rho^0\gamma$, $B^+\to(K^+\pi^-)\pi^+\gamma$ (first measurements of $B^+\to K^+\rho^0\gamma$ and $B^+\to(K^+\pi^-)\pi^+\gamma$; measure $K_1(1270) \to K^{*0}(1430)\pi$

- We measured time-dependent CP asymmetries that are consistent with the SM expectation

$$S_{K^0_S\pi^+\pi^-\gamma} = 0.14 \pm 0.25^{+0.04}_{-0.03}$$

$$C_{K^0_S\pi^+\pi^-\gamma} = -0.39 \pm 0.20^{+0.05}_{-0.05}$$

$$S_{K^0_S\rho^0\gamma} = -0.17 \pm 0.32^{+0.07}_{-0.06}$$

Backup Slides

Fit Model

- We model the $m_{K\pi\pi}$ distribution as a sum of 5 resonances, each described by a relativistic Breit-Wigner line shape

$$\left|A(m;c_k)\right|^2 = \sum_J \left|\sum_k c_k R_k^J(m)\right|^2 \Bigg|_{m=m_{K\pi\pi}}$$

with $c_k = \alpha_k e^{i\phi_k}$

- The fit fractions are obtained from

$$FF(k)\frac{\left|c_k\right|^2 \left\langle R_k R_k^* \right\rangle}{\sum_{\mu\nu} (c_\mu c_\nu^*) \left\langle R_\mu R_\nu^* \right\rangle}$$

$$FF(k,l) = \frac{2\Re e\left\{\left(c_k c_l^*\right)\left\langle R_k R_l^* \right\rangle\right\}}{\sum_{\mu\nu} \left(c_\mu c_\nu^*\right)\left\langle R_\mu R_\nu^* \right\rangle}$$

- where $\left\langle R_\mu R_\nu^* \right\rangle = \int R_\mu R_\nu^* \, dm$

- The average efficiency is obtained from

$$\bar{\varepsilon} = \sum_i \varepsilon_i \frac{FF_i}{\sum_j FF_j}$$

PDF Shapes

- Crystal Ball PDF

$$CB(x;\mu,\sigma,\alpha,n)=\begin{cases}\left(\frac{n}{\alpha}\right)^{n}\frac{exp(-\alpha^{2}/2)}{((\mu-x)/\sigma+n/\alpha-\alpha)^{n}} & x\le\mu-\alpha\sigma\\ exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}\right] & x>\mu-\alpha\sigma\end{cases}$$

Skwarnicki, DESY-F13-86-01 (1986)
Oreglia, SLAC-0236 (1980)
Gaiser, SLAC-R-0225 (1982)

- Asymmetric Gaussian with powertail

$$\tilde{G}(x;\mu,\sigma_{l},\sigma_{r},\alpha_{l},\alpha_{r})=\exp\left[\frac{(x-\mu)^{2}}{2\sigma_{i}^{2}+\alpha_{i}(x-\mu)^{2}}\right]\begin{cases}x-\mu<0: i=l\\ x-\mu\ge 0: i=r\end{cases}$$

- Relativistic Breit-Wigner
  X(|q|r) is Blatt Weisskopf barrier

$$R_{j}(m)=\frac{1}{(m_{0}^{2}-m^{2})-im_{0}\Gamma(m)}$$

$$\Gamma(m)=\Gamma_{0}\left(\frac{|\vec{q}|}{|\vec{q}|_{0}}\right)^{2J+1}\left(\frac{m_{0}}{m}\right)\frac{X_{J}^{2}(|\vec{q}|r)}{X_{J}^{2}(|\vec{q}|_{0}r)}$$

- Gounaris-Sakurai

$$GS_{j}(m)=\frac{1+d\bullet\Gamma_{0}/m_{0}}{(m_{0}^{2}-m^{2})-f(m)-im_{0}\Gamma(m)}$$

Gounaris *et al.*, PRL 21, 244 (1968)

$$f(m)=\Gamma_{0}\frac{m_{0}^{2}}{q_{0}^{3}}\left[q^{2}\left(h(m)-h(m_{0})\right)+\left(m_{0}^{2}-m^{2}\right)q_{0}^{2}\left.\frac{dh}{dm^{2}}\right|_{m=m_{0}}\right]$$

$$d=\frac{3}{\pi}\frac{m_{\pi}^{2}}{q^{2}}\ln\left(\frac{m_{0}+2q_{0}}{2m_{\pi}}\right)+\frac{m_{0}}{2\pi q_{0}}-\frac{m_{\pi}^{2}m_{0}}{\pi q_{0}^{3}}$$

$$h(m)=\frac{2}{\pi}\frac{q}{m}\ln\left(\frac{m+2q}{2m_{\pi}}\right)$$

- LASS

$$L_{j}(m)=\frac{m_{K\pi}}{q\cot\delta_{B}-iq}+e^{2i\delta_{B}}\frac{m_{0}\Gamma_{0}\frac{m_{0}}{q_{0}}}{\left(m_{0}^{2}-m_{K\pi}^{2}\right)-im_{0}\Gamma_{0}\frac{q}{m_{K\pi}}\frac{m_{0}}{q_{0}}}$$

$$\cot\delta_{B}=\frac{1}{aq}+\frac{1}{2}rq$$

LASS, Nucl.Phys B296, 493 (1988)

Background Categories for $B^+ \to K^+\pi^+\pi^-\gamma$

- We sort the B backgrounds into 6 classes
- For each class, we define a specific PDF for m_{ES}, ΔE and $\mathcal{F}$ listed in the table

Class		PDFs		Varied	Number of events
	m_{ES}	ΔE	$\mathcal{F}$		
$B^0 \to X_{sd}(\nrightarrow K\pi)\gamma$ $B^+ \to X_{su}(\nrightarrow K\pi)\gamma$	$\tilde{G}$ + ARGUS	Exp.	Gaussian	no	2872 ± 242
$B^0 \to K^{*0}(\to K\pi)\gamma$ $B^0 \to X_{sd}(\to K\pi)\gamma$	Two-dimensional non parametric		$\tilde{G}$	yes	1529 ± 116
$B^+ \to K^{*+}(\to K\pi)\gamma$ $B^+ \to X_{su}(\to K\pi)\gamma$	Linear + ARGUS	Exp.	$\tilde{G}$	no	442 ± 50
$B^0 \to K^{*0}\eta$	$\tilde{G}$ + ARGUS	Gaussian + Constant	$\tilde{G}$	no	56 ± 21
$B^+ \to a_1^+(\to \rho^0\pi^+)\pi^0\gamma$ $B^+ \to K^{*0}(\to K\pi)\pi^+\pi^0\gamma$	CB	Asymmetric Gaussian	Asymmetric Gaussian	no	17 ± 9
$B \to$ {charged and neutral generic decays}	ARGUS	Exp.	Gaussian	yes	3270 ± 385

- For $q\bar{q}$ backgrounds, we parameterize m_{ES} by an ARGUS function, ΔE by a second-order Chebychev polynomial and $\mathcal{F}$ by an exponential function

Fit Fractions of the $m_{K\pi\pi}$ Spectrum Fit

- We have extracted fit fractions from the unbinned ML fit to $m_{K\pi\pi}$ and $m_{K\pi}$ mass spectra
- Statistical uncertainties of the fit fractions are determined from 10^5 toy experiments

J^P	$K_{\rm res}$	Magnitude α	Phase ϕ (rad.)	Fit fraction
1^+	$K_1(1270)$	1.0 (fixed)	0.0 (fixed)	$0.61^{+0.08+0.05}_{-0.05-0.05}$
	$K_1(1400)$	$0.72\pm0.10^{+0.12}_{-0.08}$	$2.97\pm0.17^{+0.11}_{-0.12}$	$0.17^{+0.08+0.05}_{-0.05-0.03}$
1^-	$K^*(1410)$	$1.31\pm0.16^{+0.21}_{-0.15}$	$3.15\pm0.12^{+0.03}_{-0.04}$	$0.40^{+0.08+0.08}_{-0.07-0.04}$
	$K^*(1680)$	$2.07\pm0.28^{+0.31}_{-0.23}$	0.0 (fixed)	$0.42^{+0.05+0.09}_{-0.04-0.07}$
2^+	$K_2^*(1430)$	$0.29\pm0.09^{+0.08}_{-0.15}$	0.0 (fixed)	$0.05^{+0.04+0.04}_{-0.03-0.06}$
Sum of fit fractions				$1.65^{+0.18+0.14}_{-0.14-0.07}$
interference		$J^P=1^+ : \{K_1(1270)-K_1(1400)\}$		$-0.35^{+0.10+0.05}_{-0.16-0.06}$
		$J^P=1^- : \{K^*(1410)-K^*(1680)\}$		$-0.30^{+0.08+0.06}_{-0.11-0.12}$

Line-shape parameters

$K_{\rm res}$	Mean (GeV/c^2)	Width (GeV/c^2)
$K_1(1270)$	1.272 (fixed)	$0.098\pm0.006\pm0.005$
$K^*(1680)$	1.717 (fixed)	$0.377\pm0.050^{+0.060}_{-0.032}$

	Module α	Phase ϕ (rad.)	Fit Fraction
$K^*(892)^0$	1.0 (fixed)	0.0 (fixed)	$0.635^{+0.011+0.019}_{-0.009-0.014}$
$\rho(770)^0$	$0.721^{+0.015+0.014}_{-0.015-0.019}$	$3.113^{+0.036+0.056}_{-0.035-0.051}$	$0.332^{+0.015+0.031}_{-0.013-0.028}$
$(K\pi)^0_0$	$0.817^{+0.044+0.037}_{-0.050-0.053}$	$3.190^{+0.132+0.109}_{-0.125-0.094}$	$0.423^{+0.039+0.044}_{-0.041-0.072}$
Sum of fit fractions			$1.390^{+0.048+0.099}_{-0.042-0.079}$
Interference	$\{K^*(892)^0$—$\rho(770)^0\}$		$-0.177^{+0.004+0.007}_{-0.006-0.009}$
	$\{(K\pi)^0_0$—$\rho(770)^0\}$		$-0.213^{+0.029+0.032}_{-0.044-0.045}$

Systematic Errors for $B^+ \to K^+\pi^+\pi^-\gamma$

- We vary each of 109 fixed parameters in fit to m_{ES}, ΔE and $\mathcal{F}$ discriminant
- Vary 2-d histograms for m_{ES}-ΔE PDF for $B^0 \to (K^{*0}+X_{sd})\gamma$ bkg
- Account for errors in all $\mathcal{B}$s, binning, mis-reconstructed signal yield
- Vary fixed parameters in line shape
- Vary fit model
- Systematic effects of $_s\mathcal{P}lot$ extraction procedure
- N_{BB}: 0.6%, tracking: 0.24%, PID: 1%, π^0/η vetoes: 1%, γ separation: 2%

J^P	K_{res}	Mass m_k^0 (MeV/c^2)	Width Γ_k^0 (MeV/c^2)
2^-	$K_2(1770)$	1773 ± 8	186 ± 14
	$K_2(1820)$	1816 ± 13	276 ± 35
3^-	$K_3^*(1780)$	1776 ± 7	159 ± 21

Source	+/− signed deviation (%)							
	Magnitude				Phase		Width	
	$K_1(1400)$	$K^*(1410)$	$K_2^*(1430)$	$K^*(1680)$	$K_1(1400)$	$K^*(1410)$	$K_1(1270)$	$K^*(1680)$
Fixed parameters in the fit performed to m_{ES}, ΔE and Fisher	3.8/2.0	4.8/2.0	4.8/8.0	5.4/2.3	0.6/0.8	0.4/0.1	1.6/3.5	3.5/1.1
Fixed line-shape parameters of the kaonic resonances	17/11	14/10	22/42	15/10	3.5/4.0	0.8/0.7	3.6/3.2	12/7.1
Number of bins in the fitted dataset	0.4/0.2	0.4/0.2	0.5/1.9	0.4/0.2	0.1/0.1	0.0/0.0	1.4/3.1	1.5/0.4
$_s\mathcal{P}lot$ procedure	0.4/$\emptyset$	$\emptyset$/1.3	$\emptyset$/2.0	$\emptyset$/2.5	0.1/$\emptyset$	0.0/$\emptyset$	0.9/$\emptyset$	$\emptyset$/0.5
$m_{K\pi\pi}$ fit model (add and remove kaonic resonances)	0.0/0.3	11.6/$\emptyset$	$\emptyset$/20.8	4.8/$\emptyset$	$\emptyset$/0.3	0.1/1.3	$\emptyset$/1.5	14.6/$\emptyset$

Source	+/− signed deviation (%)							
	Fit Fraction							
	$K_1(1270)$	$K_1(1400)$	$K^*(1410)$	$K_2^*(1430)$	$K^*(1680)$	Sum	interference $J^P = 1^+$	interference $J^P = 1^-$
Fixed parameters in the fit performed to m_{ES}, ΔE and Fisher	1.6/1.4	5.2/1.8	5.1/1.4	12/19	2.2/1.2	1.4/0.1	3.2/3.7	1.3/6.8
Fixed line-shape parameters of the kaonic resonances	8.2/8.2	31/18	14/6.1	55/76	21/13	8.5/4.4	14/17	20/40
Number of bins in the fitted dataset	0.1/1.4	4.0/0.6	1.3/1.4	5.0/3.1	1.4/0.1	0.1/0.1	0.6/0.4	0.3/0.3
$_s\mathcal{P}lot$ procedure	1.4/$\emptyset$	3.3/$\emptyset$	$\emptyset$/0.1	$\emptyset$/1.7	$\emptyset$/2.0	$\emptyset$/0.2	$\emptyset$/2.5	1.6/$\emptyset$
$m_{K\pi\pi}$ fit model (add and remove kaonic resonances)	0.0/2.1	0.1/4.2	20/$\emptyset$	$\emptyset$/41	0.2/12	1.0/$\emptyset$	3.2/0.1	$\emptyset$/9.3

Systematic Errors for $B^+ \to K^+\pi^+\pi^-\gamma$

- The systematic uncertainties for the intermediate resonance decays yield

Source	+/− signed deviation (%)									
	Magnitude		Phase		Fit Fraction					
	$\rho(770)^0$	$(K\pi)_0^{*0}$	$\rho(770)^0$	$(K\pi)_0^{*0}$	$K^*(892)^0$	$\rho(770)^0$	$(K\pi)_0^{*0}$	Sum	interference K^{*0}–ρ^0	interference $(K\pi)_0^{*0}$–ρ^0
Fixed parameters in the fit performed to $m_{\rm ES}$, ΔE and the Fisher discriminant	0.7/1.2	2.2/0.6	1.4/0.1	2.3/0.2	0.2/0.4	3.4/2.6	4.8/3.1	1.8/3.3	1.6/0.7	6.0/3.5
Fixed line-shape parameters of the intermediate state resonances	0.2/0.1	0.8/0.2	0.4/0.5	0.6/1.3	0.1/0.6	0.9/3.4	5.1/0.2	0.5/2.8	1.4/0.6	6.9/1.9
Fixed line-shape parameters of the kaonic resonances (in `EvtGen`)	0.1/0.4	1.7/0.1	0.7/1.0	0.8/2.0	∅/0.7	0.1/1.5	1.4/0.1	0.8/1.5	0.9/0.4	3.9/1.5
Number of bins in the PDF	0.0/0.6	2.4/0.0	0.4/0.0	0.4/0.0	0.0/1.0	0.0/0.8	3.6/0.0	0.0/1.6	0.6/0.0	3.5/0.0
Number of bins in the fitted dataset	0.8/0.0	0.0/4.3	0.0/0.3	0.0/0.5	1.8/0.0	4.2/0.0	0.0/7.1	3.8/0.0	0.0/3.3	0.0/9.4
${}_s\mathcal{P}lot$ procedure	∅/2.6	3.7/∅	∅/0.5	∅/1.3	0.2/∅	∅/8.0	10.3/∅	∅/3.5	2.1/∅	6.9/∅
Kaonic resonance weights (taken from a fit to the $m_{K\pi\pi}$ spectrum)	1.5/0.5	1.2/6.0	1.0/1.1	2.6/1.5	2.2/1.2	8.8/2.1	3.1/17.4	6.3/2.2	3.0/4.6	10.6/20.2

Background Categories for $B^0 \to K^0_S \pi^+ \pi^- \gamma$

- We sort B backgrounds into 7 classes

- For each class, we define a specific PDF for m_{ES}, ΔE and $\mathcal{F}$ listed in the table

Mode	PDFs			Varied	Number of events
	m_{ES}	ΔE	$\mathcal{F}$		
$B^+ \to X_{su}(\nrightarrow K\pi)\gamma$	ARGUS	Chebychev (2^{nd} order)	Gaussian	no	94 ± 17
$B^0 \to X_{sd}(\nrightarrow K\pi)\gamma$	ARGUS	Chebychev (2^{nd} order)	Gaussian	no	51 ± 12
$B^+ \to K^{*+}(\to K^0_S\pi^+)\gamma$ $B^+ \to X_{su}(\to K^0_S\pi^+)\gamma$	Two-dimensional non parametric		Gaussian	yes	42 ± 22
$B^0 \to$ {neutral generic decays}	ARGUS	Chebychev (2^{nd} order)	Gaussian	no	35 ± 13
$B^+ \to$ {charged generic decays}	ARGUS	Chebychev (2^{nd} order)	Gaussian	no	34 ± 13
$B^0 \to K^{*0}(\to K^0_S\pi^0)\gamma$ $B^0 \to X_{sd}(\to K^0_S\pi^0)\gamma$	ARGUS	Chebychev (2^{nd} order)	Gaussian	no	30 ± 11
$B^0 \to K^{*0}(\to K^\pm\pi^\mp)\gamma$ $B^0 \to X_{sd}(\to K^\pm\pi^\mp)\gamma$	ARGUS	Chebychev (1^{st} order)	Exp.	no	4 ± 3

- For $q\bar{q}$ backgrounds, we parameterize m_{ES} by an ARGUS function, ΔE by a second-order Chebychev polynomial and $\mathcal{F}$ by an exponential function

Δt PDFs for Backgrounds

- Δt PDF for charged B decays

$$P^i_{B^\pm}\left(\Delta t,\sigma_{\Delta t};q_{tag},c\right)=\frac{\exp\left(-\frac{|\Delta t|}{\tau_j}\right)}{4\tau_j}\left[\left(\frac{1-q_{tag}A_j}{2}\right)\omega_c+\left(\frac{1+q_{tag}A_j}{2}\right)(1-\omega_c)\right]\otimes R^c_{B^0_{FL}}(\Delta t,\sigma_{\Delta t})$$

- Δt PDF for B^0 decays to flavor eigenstates

$$P^i_{B_{Fles}}\left(\Delta t,\sigma_{\Delta t};q_{tag},c\right)=\frac{\exp\left(-\frac{|\Delta t|}{\tau_j}\right)}{4\tau_j}\left[\left(\frac{1-q_{tag}A_j}{2}\right)\omega_c\left(1-\cos(\Delta m_{B_d}\Delta t)\right)+\left(\frac{1+q_{tag}A_j}{2}\right)(1-\omega_c)\left(1+\cos(\Delta m_{B_d}\Delta t)\right)\right]\otimes R^c_{B^0_{FL}}(\Delta t,\sigma_{\Delta t})$$

- Δt PDF for B^0 decays to *CP* eigenstates

$$P^i_{B^0_{CP}}\left(\Delta t,\sigma_{\Delta t};q_{tag},c\right)=\frac{\exp\left(-\frac{|\Delta t|}{\tau_j}\right)}{4\tau_j}\left[1+q_{tag}\frac{\Delta D_c}{2}+q_{tag}\langle D\rangle_c\left(\mathcal{S}\sin(\Delta m_{B_d}\Delta t)-\mathcal{C}\cos(\Delta m_{B_d}\Delta t)\right)\right]\otimes R^c_{B^0_{CP}}(\Delta t,\sigma_{\Delta t})$$

- Δt PDF for $q\bar{q}$ continuum states

$$\mathcal{P}^i_{bg}\left(\Delta t,\sigma_{\Delta t}\right)=\left[f_p\delta(\Delta t'-\Delta t)+(1-f_p)\exp\left(-\frac{|\Delta t|}{\tau_{bg}}\right)\right]$$

Systematic Errors for Time-Dependent Analysis

- We account for:
  - binning effect in fitted $_s\mathcal{P}$lot and binning effect in 2-d histogram ($m_{K\pi}$, $m_{\pi\pi}$)
  - Fixed parameters in fit to m_{ES}, ΔE and $\mathcal{F}$ discriminant
  - Fixed parameters in the fit model
  - Errors on weights for K-resonances
  - We vary distortions of $\rho^0(770)$ & $K^{*0}(892)$
  - We include systematic effects of $_s\mathcal{P}$lot extraction procedure

Parameter	+ signed deviation	− signed deviation
$\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}$	0.041	0.034
$\mathcal{C}_{K^0_S\pi^+\pi^-\gamma}$	0.046	0.049