



Improved prediction for the mass  
of the  $W$  boson in the SM, the  
MSSM and the NMSSM

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EPS-HEP 2015,  
Vienna, 07 / 2015

# Outline

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Introduction

What is experimentally measured?

$M_W$  prediction in the Standard Model

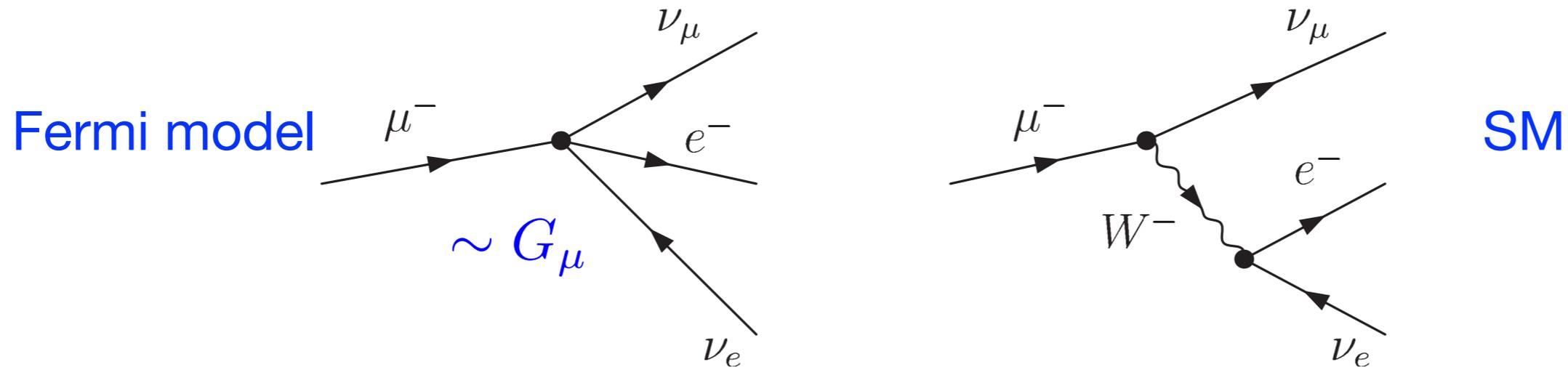
$M_W$  prediction in the MSSM

$M_W$  prediction in the NMSSM

Conclusions

Based on collaboration with:  
S. Heinemeyer, W. Hollik, O. Stål, L. Zeune

# Introduction: Prediction for the W-boson mass from muon decay: relation between $M_W$ , $M_Z$ , $\alpha$ , $G_\mu$



$M_W$ : Comparison of prediction for muon decay with experiment (Fermi constant  $G_\mu$ ); QED corrections in Fermi model incl. in def. of  $G_\mu$

$$\Rightarrow M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} (1 + \Delta r),$$

↕  
**loop corrections**

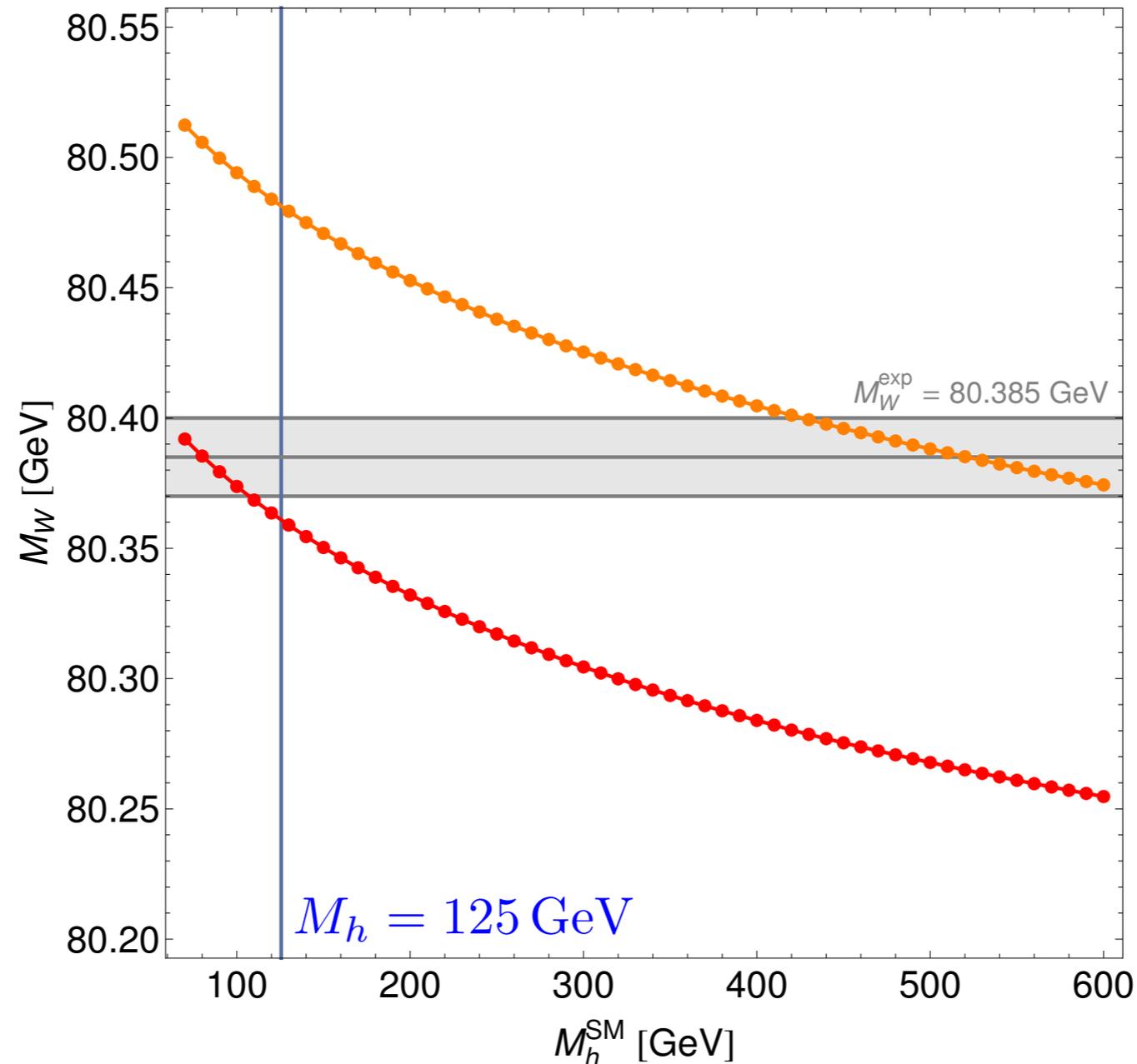
$\Rightarrow$  Theo. prediction for  $M_W$  in terms of  $M_Z$ ,  $\alpha$ ,  $G_\mu$ ,  $\Delta r(m_t, m_{\tilde{t}}, \dots)$

Tree-level prediction:  $M_W^{\text{tree}} = 80.939 \text{ GeV}$ ,  $M_W^{\text{exp}} = 80.385 \pm 0.015 \text{ GeV}$   
 $\Rightarrow$  off by many  $\sigma$  (accuracy of  $2 \times 10^{-4}$ )

# W-mass prediction within the SM:

## one-loop result vs. state-of-the-art prediction

[L. Zeune, G. W. '14]



⇒ Pure one-loop result would imply preference for heavy Higgs,  $M_h > 400$  GeV  
Corrections beyond one-loop order are crucial for reliable prediction of  $M_W$

# Sources of theoretical uncertainties

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- From experimental errors of the input parameters

$$\delta m_t = 0.9 \text{ GeV} \Rightarrow \Delta M_W^{\text{para}} \approx 5.4 \text{ MeV}, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx 2.8 \times 10^{-5}$$

$$\delta(\Delta\alpha_{\text{had}}) = 0.00014 \Rightarrow \Delta M_W^{\text{para}} \approx 2.5 \text{ MeV}, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx 4.8 \times 10^{-5}$$

- From unknown higher-order corrections (“intrinsic”)

**SM:** Complete 2-loop result + leading higher-order corrections known for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$

⇒ Remaining uncertainties:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03, '04*]

[*M. Awramik, M. Czakon, A. Freitas '06*]

$$\Delta M_W^{\text{intr}} \approx 4 \text{ MeV}, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{intr}} \approx 5 \times 10^{-5}$$

# The role of the $W$ -boson mass as a precision observable

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- Very accurately known both experimentally and theoretically
- Highly sensitive to quantum corrections of new physics
- Global fits in the Standard Model: dominated by the two observables  $M_W$  and  $\sin^2\theta_{\text{eff}}$

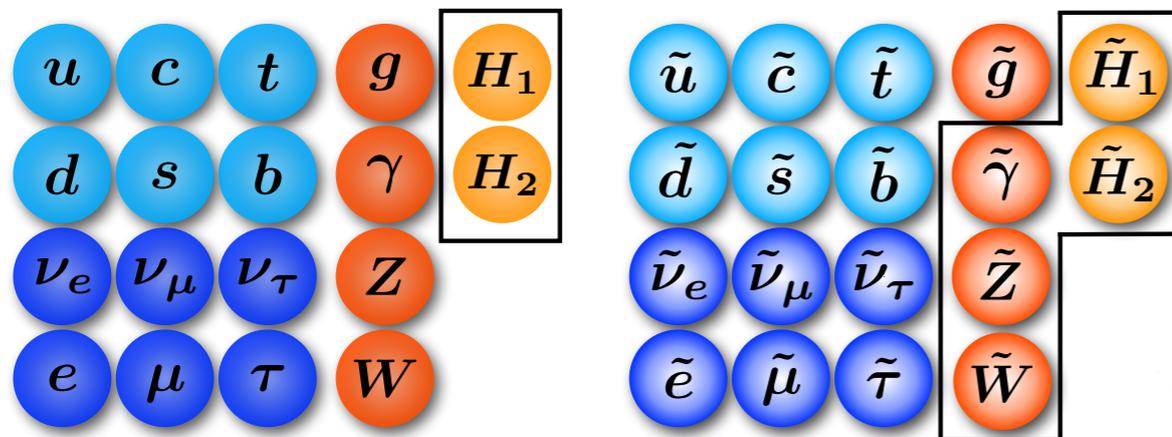
## Note:

- Prospects for further experimental improvements of  $M_W$  from analysis of Tevatron data, LHC, future  $e^+e^-$  collider
- Interpretation of constraints from  $\sin^2\theta_{\text{eff}}$  is complicated by the fact that the two most precise individual measurements differ from each other by more than  $3\sigma$

# Theoretical framework

- Every theoretical calculation that is expressed in terms of the Fermi constant  $G_\mu$  requires a prediction for  $\Delta r$  in the considered model and to the accuracy of that calculation
- Coherent theoretical framework for comparison between different models: SM, MSSM, NMSSM, ...

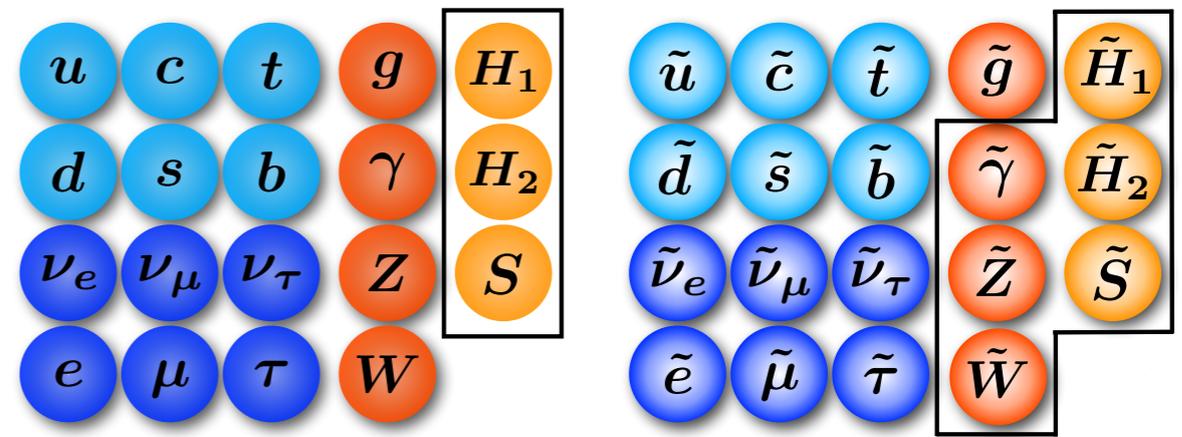
Minimal supersymmetric Standard Model (MSSM)



5 physical Higgs bosons:  
 $h, H, A, H^\pm$

neutral components:  $\tilde{\chi}_{1,2,3,4}^0$   
charged components:  $\tilde{\chi}_{1,2}^\pm$

Next-to-minimal supersymmetric Standard Model (NMSSM)



7 physical Higgs bosons:  
 $h_1, h_2, h_3, a_1, a_2, H^\pm$

neutral components:  $\tilde{\chi}_{1,2,3,4,5}^0$   
charged components:  $\tilde{\chi}_{1,2}^\pm$

- Solves the  $\mu$ -problem of the MSSM:  
 $\mu$  parameter generated dynamically from the vacuum expectation value of the singlet

$$W^{\text{MSSM}} = \dots + \mu H_2 H_1$$

$$W^{\text{NMSSM}} = \dots + \lambda S H_2 H_1 + \dots$$

# What is experimentally measured?

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- LEP:  $e^+e^- \rightarrow W^+W^-$  in the continuum and at threshold (small amount of data); impact of fully hadronic final state suffered from uncertainties due to BE correlations, colour reconnections
- Tevatron, LHC (under study): transverse mass distribution

How is the measured parameter (Monte Carlo mass) related to the theoretically well-defined quantity  $M_W$ ?

Similar question as for top-quark mass, where the latter is conceptually much more difficult (coloured object, renormalon ambiguities, ...), but here we are aiming for a two orders of magnitude higher accuracy

# What is the mass of an unstable particle?

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Particle masses are **not** directly physical observables

Can only measure cross sections, branching ratios, kinematical distributions, ...

⇒ masses are “pseudo-observables”

Need to **define** what is meant by  $M_Z$ ,  $M_W$ ,  $m_t$ , ... :

$\overline{\text{MS}}$  mass, pole mass (real pole, real part of complex pole, Breit–Wigner shape with running or constant width), ...

⇒ Determination of  $M_Z$ ,  $M_W$ ,  $m_t$ , ... involves deconvolution procedure (unfolding)

Mass obtained from comparison data – Monte Carlo

⇒  $M_Z$ ,  $M_W$ ,  $m_t$ , ... are not strictly model-independent

# Expansion around the complex pole (example: $M_Z$ )

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Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \rightarrow f\bar{f}) = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2) S' + \dots$$

$$\mathcal{M}_Z^2 = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z$$

Expanding up to  $\mathcal{O}(\alpha^2)$  using  $\mathcal{O}(\overline{\Gamma}_Z/\overline{M}_Z) = \mathcal{O}(\alpha)$

From 2-loop order on:

real part of complex pole,  $\overline{M}_Z \neq$  pole of real part,  $\widetilde{M}_Z^2$

$$\delta\overline{M}_{(2)}^2 = \delta\widetilde{M}_{(2)}^2 + \underbrace{\text{Im} \{ \Sigma'_{T,(1)}(M^2) \} \text{Im} \{ \Sigma_{T,(1)}(M^2) \}}_{\text{gauge-parameter dependent!}}$$

**gauge-parameter dependent!**

# Physical mass of unstable particles: real part of complex pole

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⇒ Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with **constant width**

For historical reasons, the experimental values of  $M_Z$ ,  $M_W$  are defined according to a Breit–Wigner shape with **running width**

⇒ Need to correct for the difference in definition when comparing theory with experiment

# Deconvolution and residual model dependence

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LEP legacy: the experimental value quoted for the Z-boson mass actually depends (slightly) on the Higgs-boson mass of the Standard Model!

$\delta M_Z^{\text{exp}} \approx \pm 0.2 \text{ MeV}$  for  $100 \text{ GeV} < M_H < 1 \text{ TeV}$ , corresponds to about 10% of the experimental error

⇒ Careful assessment of this kind of effects will be crucial for the upcoming analyses at the Tevatron, the LHC and future colliders

# $M_W$ prediction in the Standard Model

Contributions beyond one-loop order:

$$\begin{aligned} \Delta r^{\text{SM(h.o.)}} = & \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r_{\text{ferm}}^{(\alpha^2)} + \Delta r_{\text{bos}}^{(\alpha^2)} \\ & + \Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)} + \Delta r^{(G_\mu m_t^2 \alpha_s^3)} \end{aligned}$$

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas,  
Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong,

...

Impact of different contributions to  $\Delta r$  ( $\times 10^4$ ) for fixed  
 $M_W = 80.385$  GeV and  $M_H^{\text{SM}} = 125.09$  GeV:

[O. Stål, G. W., L. Zeune '15]

$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r_{\text{ferm}}^{(\alpha^2)} + \Delta r_{\text{bos}}^{(\alpha^2)}$	$\Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}$	$\Delta r^{(G_\mu m_t^2 \alpha_s^3)}$
297.17	36.28	7.03	29.14	-1.60	1.23

# Methods for estimating theoretical uncertainties from unknown higher-order corrections

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- Parametric factors, e.g.  $\alpha$ ,  $\alpha_s$ ,  $N_c$ ,  $N_f$ , ...
- Geometric progression, e.g.  $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalisation scale dependence: affects only part of the higher-order corrections; often underestimates theoretical uncertainties
- Renormalisation scheme dependence
- ...

SM result

$$M_W^{\text{SM}}(m_t = 173.34 \text{ GeV}, M_H^{\text{SM}} = 125.09 \text{ GeV}) = 80.358 \text{ GeV}$$

Differs from the measurement by  $1.8 \sigma$

# Renormalisation scheme dependence

[A. Freitas '15]

- a) Uncertainty of  $\mathcal{O}(\alpha^2)$  corrections beyond leading  $\alpha^2 m_t^4$  and  $\alpha^2 m_t^2$  from comparison of  $\overline{\text{MS}}$  and OS schemes: Degrassi, Gambino, Sirlin '96

$$\delta M_W \sim 2 \text{ MeV} \quad (\text{for } M_H \sim 100 \text{ GeV})$$

Actual remaining  $\mathcal{O}(\alpha^2)$  corrections: Freitas, Hollik, Walter, Weiglein '00

$$\delta M_W \sim 3 \text{ MeV} \quad (\text{for } M_H \sim 100 \text{ GeV})$$

- b) Estimate of missing  $\mathcal{O}(\alpha^3)$  corrections from comparison of  $\overline{\text{MS}}$  and OS results: Awramik, Czakon, Freitas, Weiglein '03  
Degrassi, Gambino, Giardino '14

$$\delta M_W \sim 4 \dots 5 \text{ MeV} \quad (\text{after accounting for } \mathcal{O}(\alpha_t \alpha_s^3) \text{ corrections})$$

→ Saturates previous  $\delta M_W$  estimate!

**Note:** Differences in (implicitly) resummed higher-order contributions

# $M_W$ prediction in the MSSM

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$\Delta r$  in the MSSM and the NMSSM, treatment of higher-order contributions:

full one-loop + higher orders (SM) + higher orders (SUSY)

$$\Delta r^{(N)\text{MSSM}} = \Delta r^{(N)\text{MSSM}(\alpha)} + \Delta r^{(N)\text{MSSM}(\text{h.o.})}$$

$$\Delta r^{(N)\text{MSSM}(\text{h.o.})} = \Delta r^{\text{SM}(\text{h.o.})} + \Delta r^{\text{SUSY}(\text{h.o.})}$$

⇒ State-of-the art SM prediction recovered in decoupling limit, all available higher-order corrections of SUSY-type included

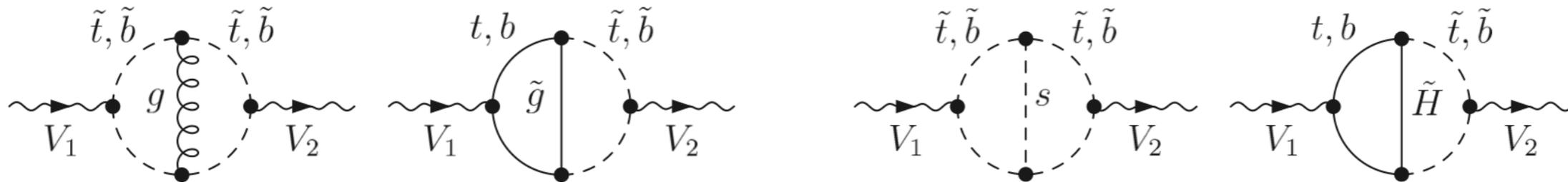
For light SUSY particles: additional theoretical uncertainty from higher-order SUSY-loop corrections

# SUSY higher-order contributions

leading reducible 2-loop corrections, gluon/gluino 2-loop corrections, higgsino 2-loop corrections

Djouadi, Haestier, Heinemeyer, Stoeckinger, Weiglein, Consoli, Hollik, Jegenlehner, ...

$$\Delta r^{\text{SUSY(h.o.)}} = \Delta r_{\text{red}}^{\text{SUSY}(\alpha^2)} - \frac{c_W^2}{s_W^2} \Delta \rho^{\text{SUSY},(\alpha\alpha_s)} - \frac{c_W^2}{s_W^2} \Delta \rho^{\text{SUSY},(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2)}$$

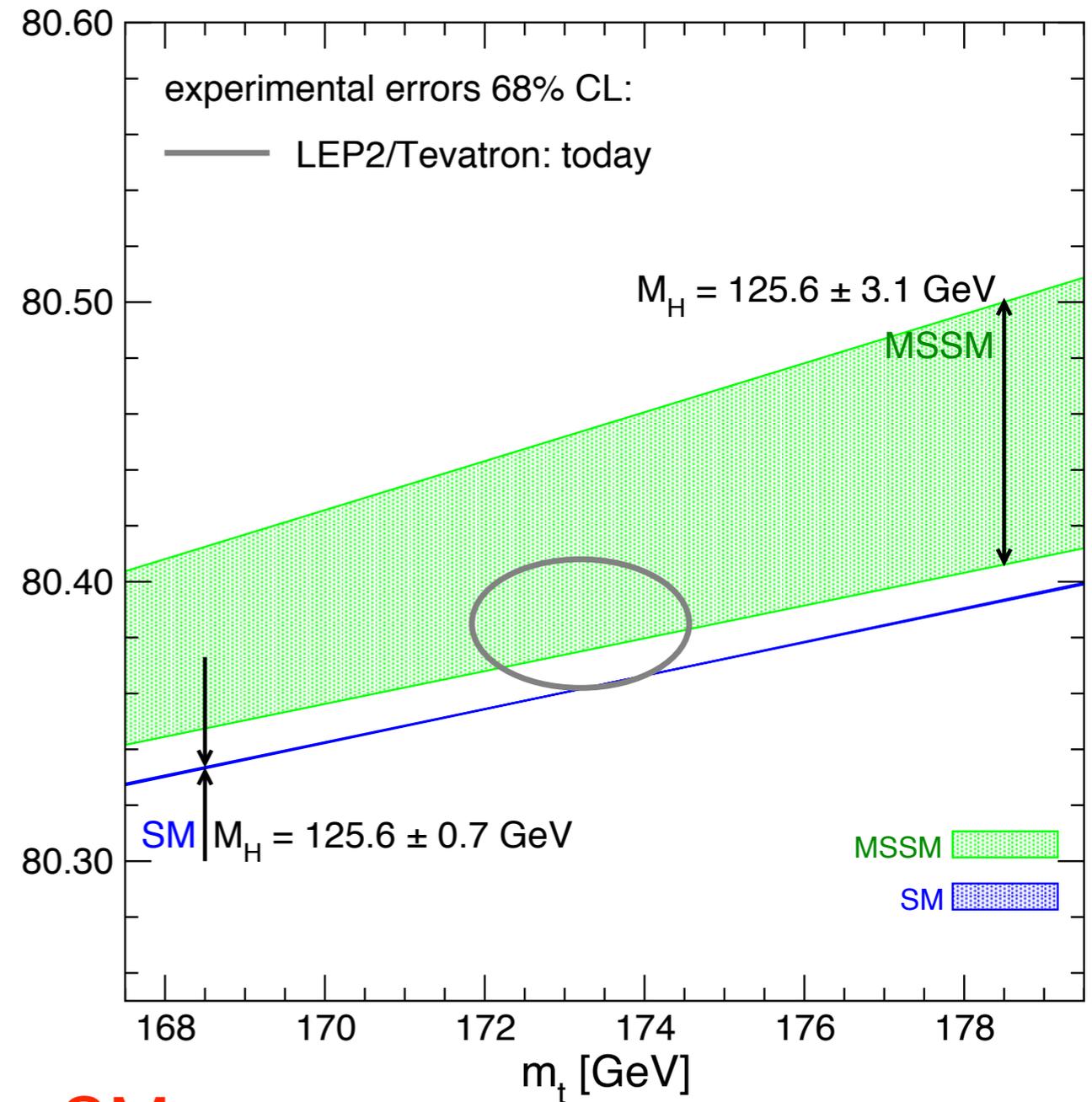
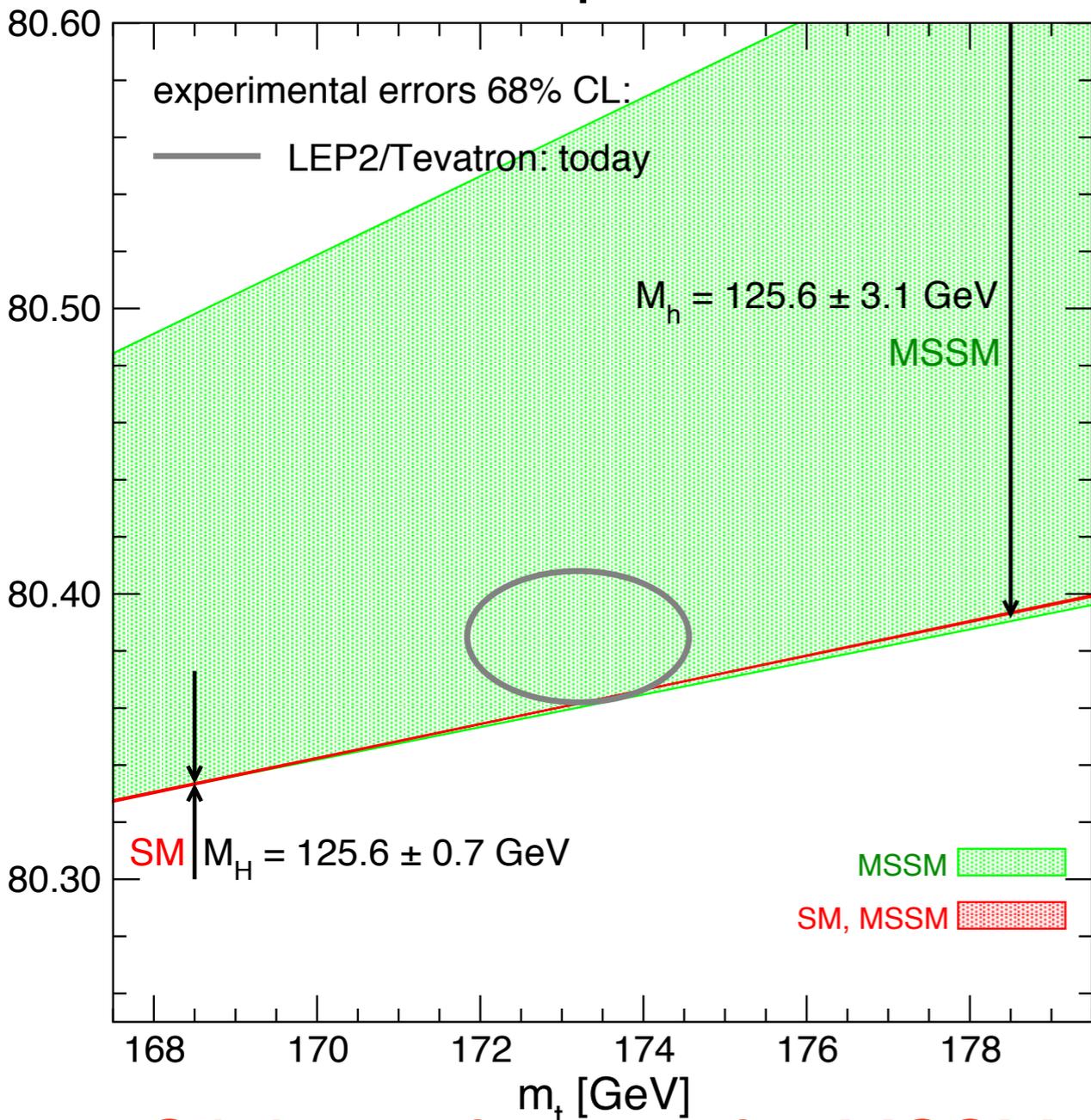


# Prediction for $M_W$ (parameter scan): SM vs. MSSM

Signal interpreted as light (left) / heavy (right) CP-even Higgs

Exp. result for  $m_t$  interpreted (perturb.) as pole mass

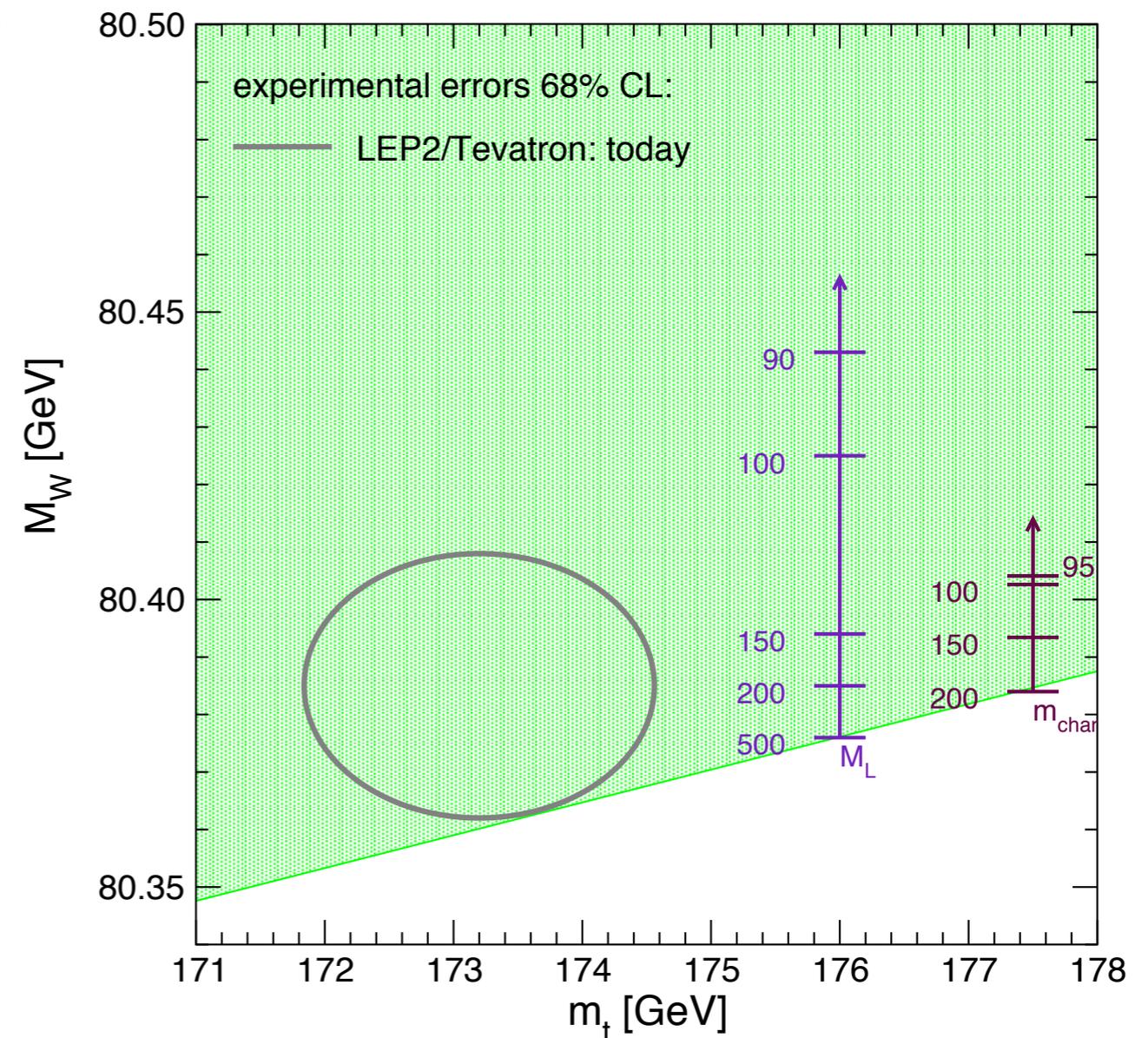
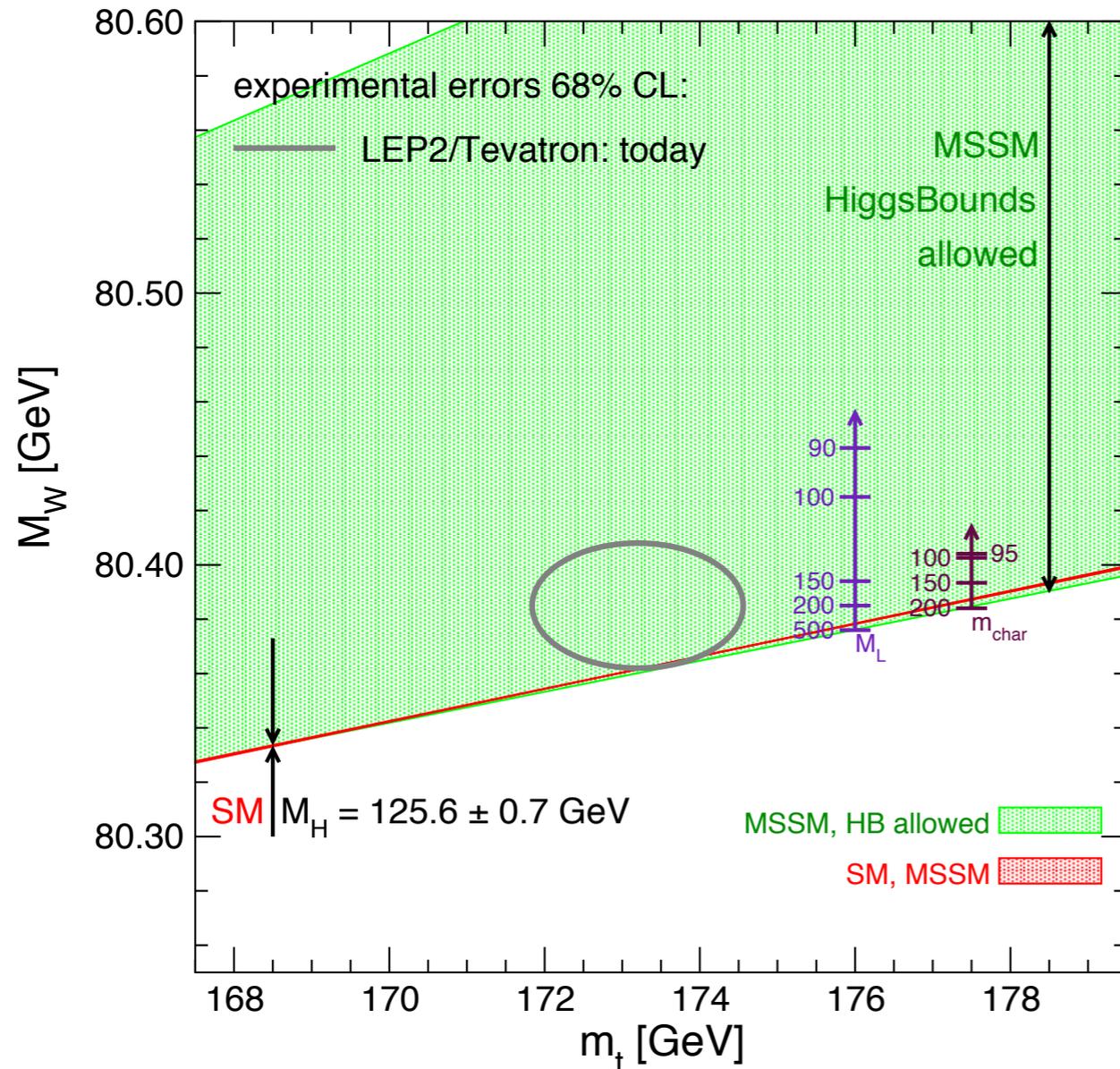
**MSSM:** SUSY parameters varied [S. Heinemeyer, W. Hollik, G. W., L. Zeune '14]



⇒ Slight preference for MSSM over SM

# Impact of different SUSY contributions

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '14]



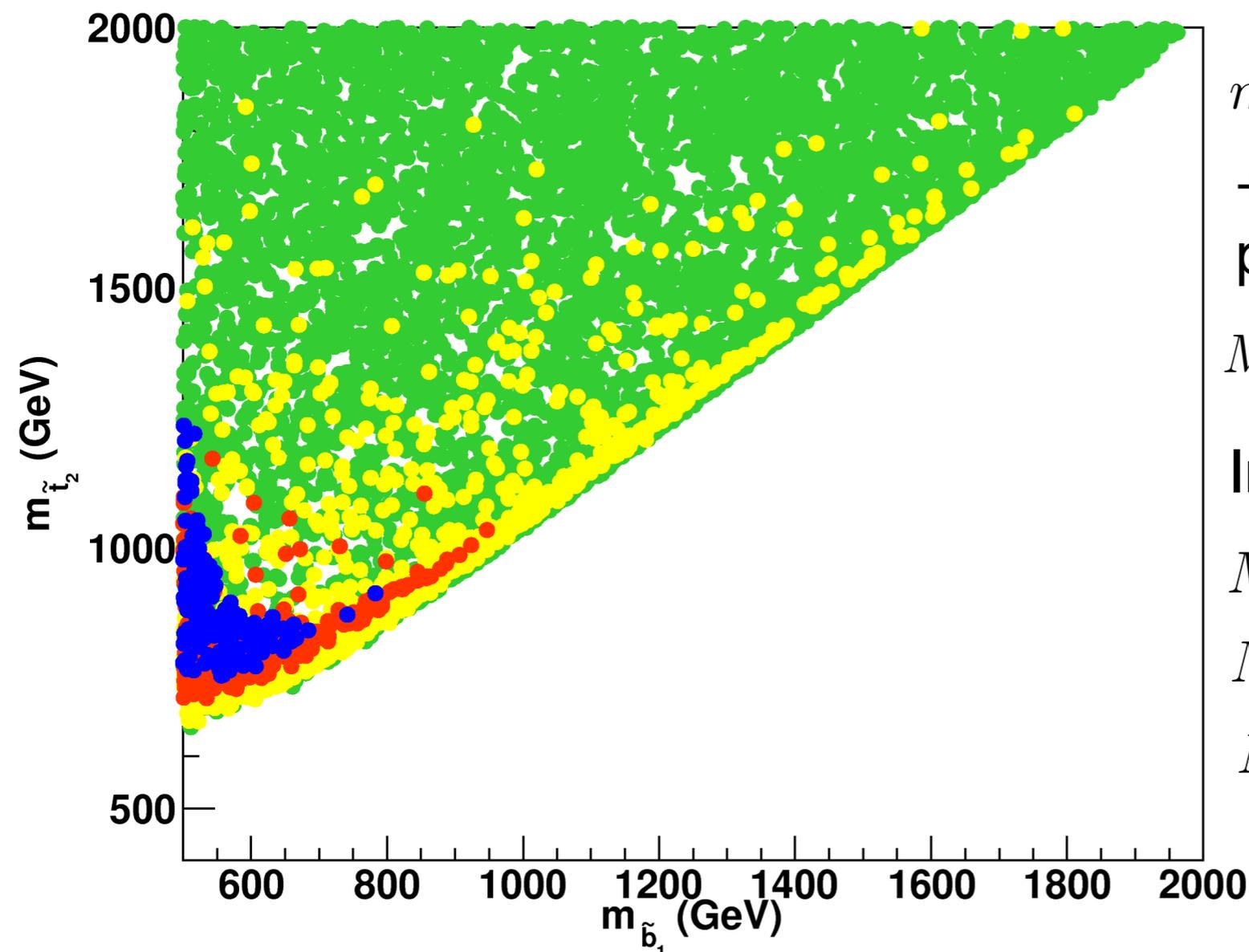
⇒ Dominant contributions from superpartners of t, b  
 But sizable SUSY contributions also possible for heavy squarks

# High-precision measurements of $m_t$ and $M_W$

Upper bounds on the heavier stop mass and the lighter sbottom mass in a hypothetical future scenario where the LHC has detected the lighter stop

Parameter scan:

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '14]



$$m_{\tilde{t}_1} = 400 \pm 40 \text{ GeV}$$

+ lower limits on other SUSY particles

$$M_h = 125.6 \pm 3.1 \text{ GeV}$$

Improved  $M_W$  precision:

$$M_W = 80.375 \pm 0.005 \text{ GeV} \quad \text{(yellow)}$$

$$M_W = 80.385 \pm 0.005 \text{ GeV} \quad \text{(red)}$$

$$M_W = 80.395 \pm 0.005 \text{ GeV} \quad \text{(blue)}$$

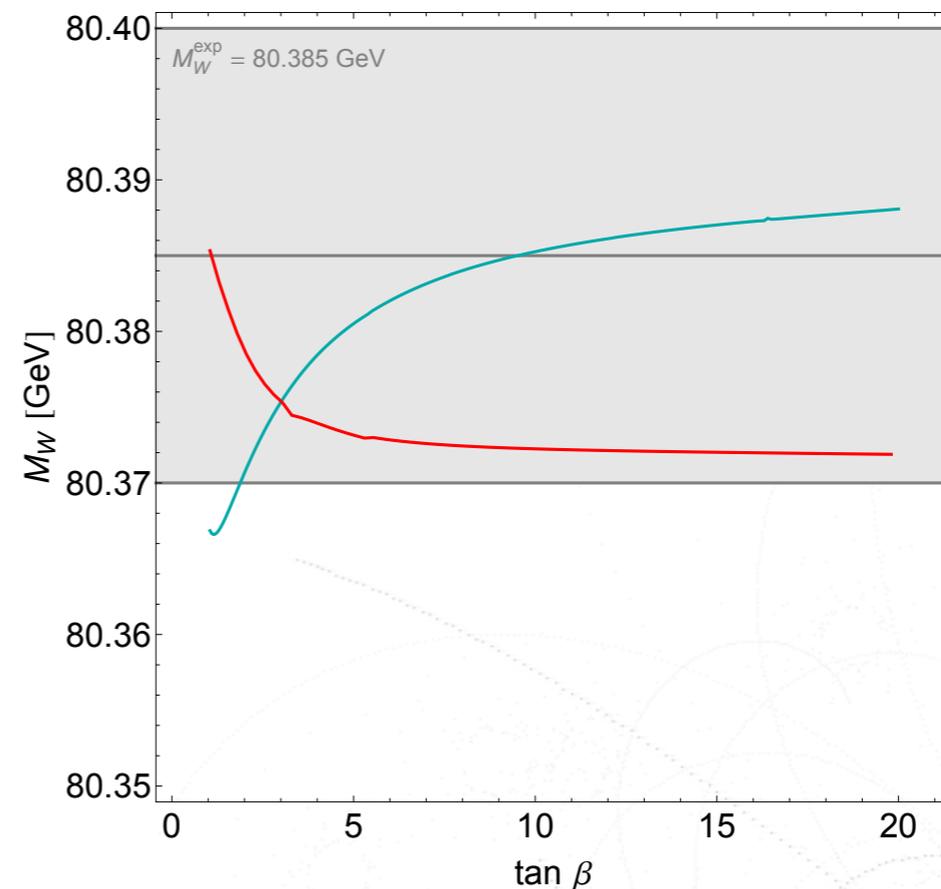
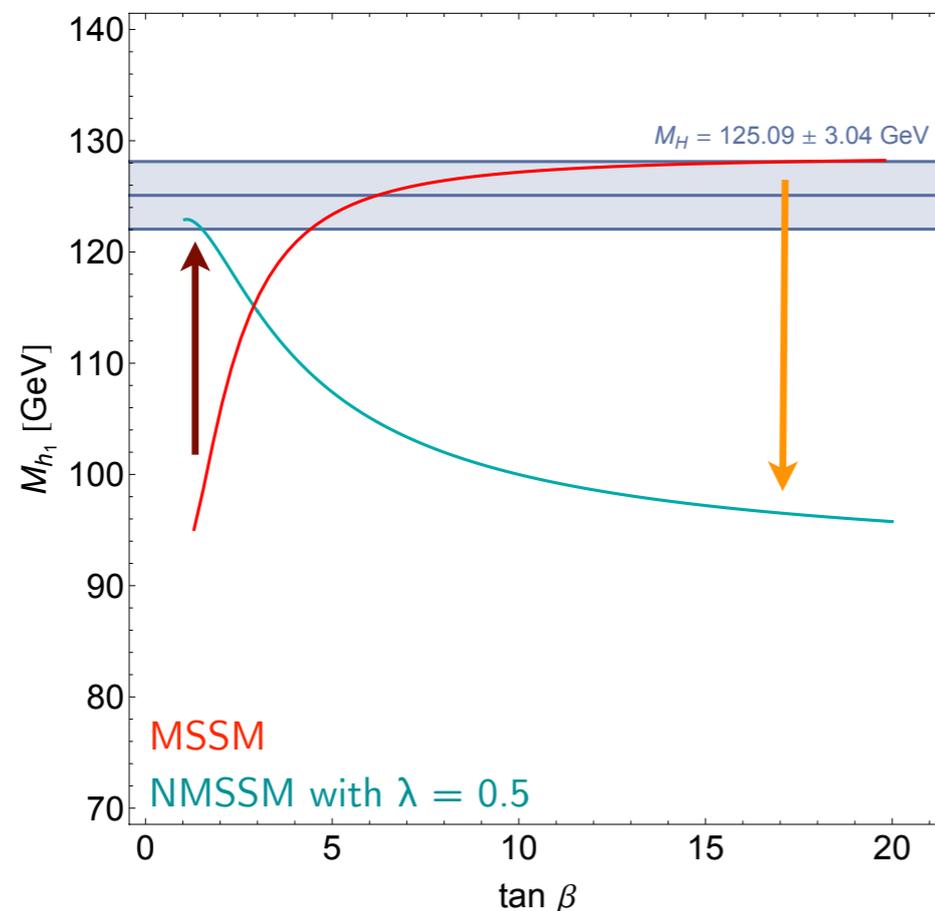
⇒ Precision observables provide constraints on undetected particles

# $M_W$ prediction in the NMSSM

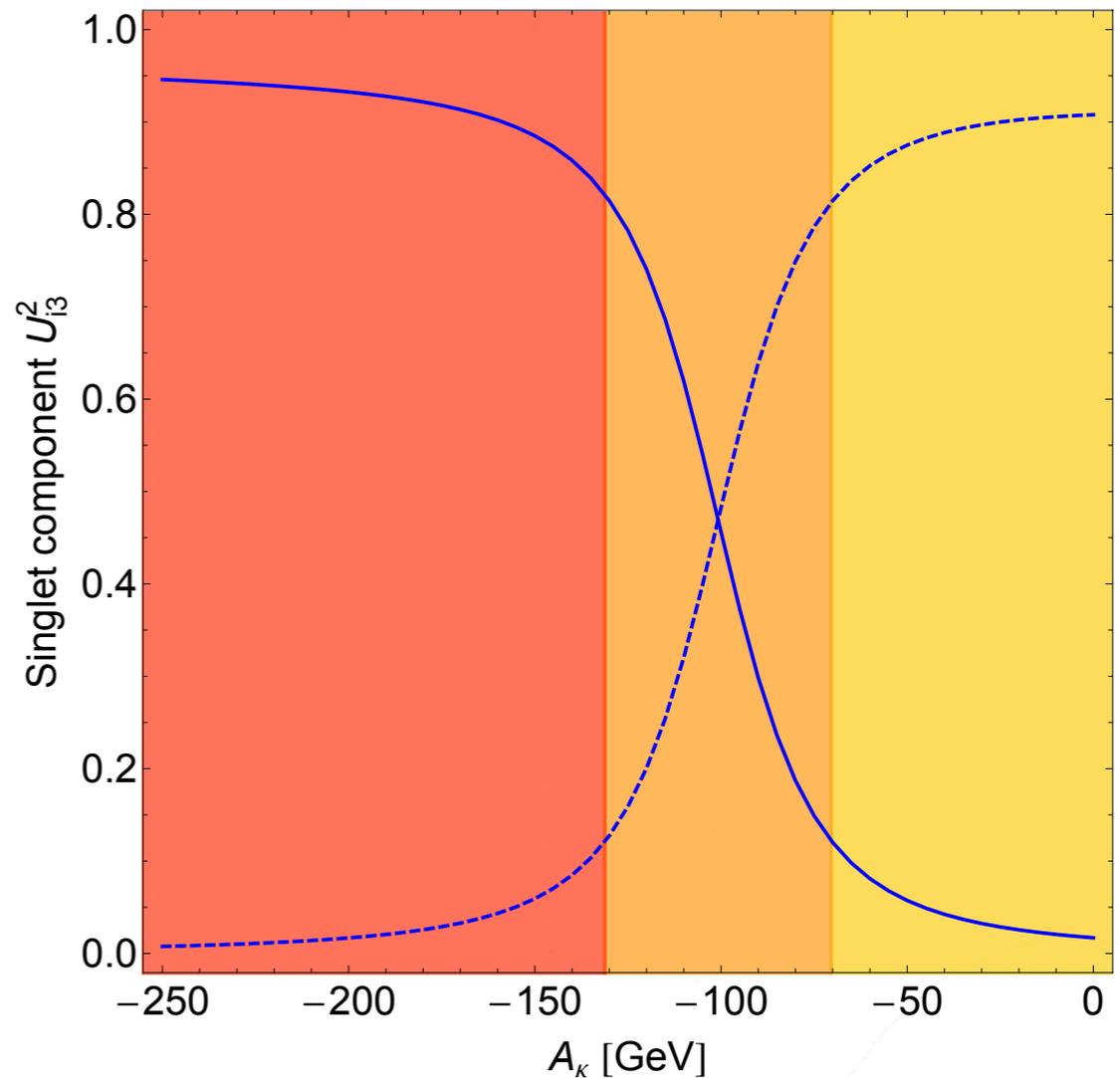
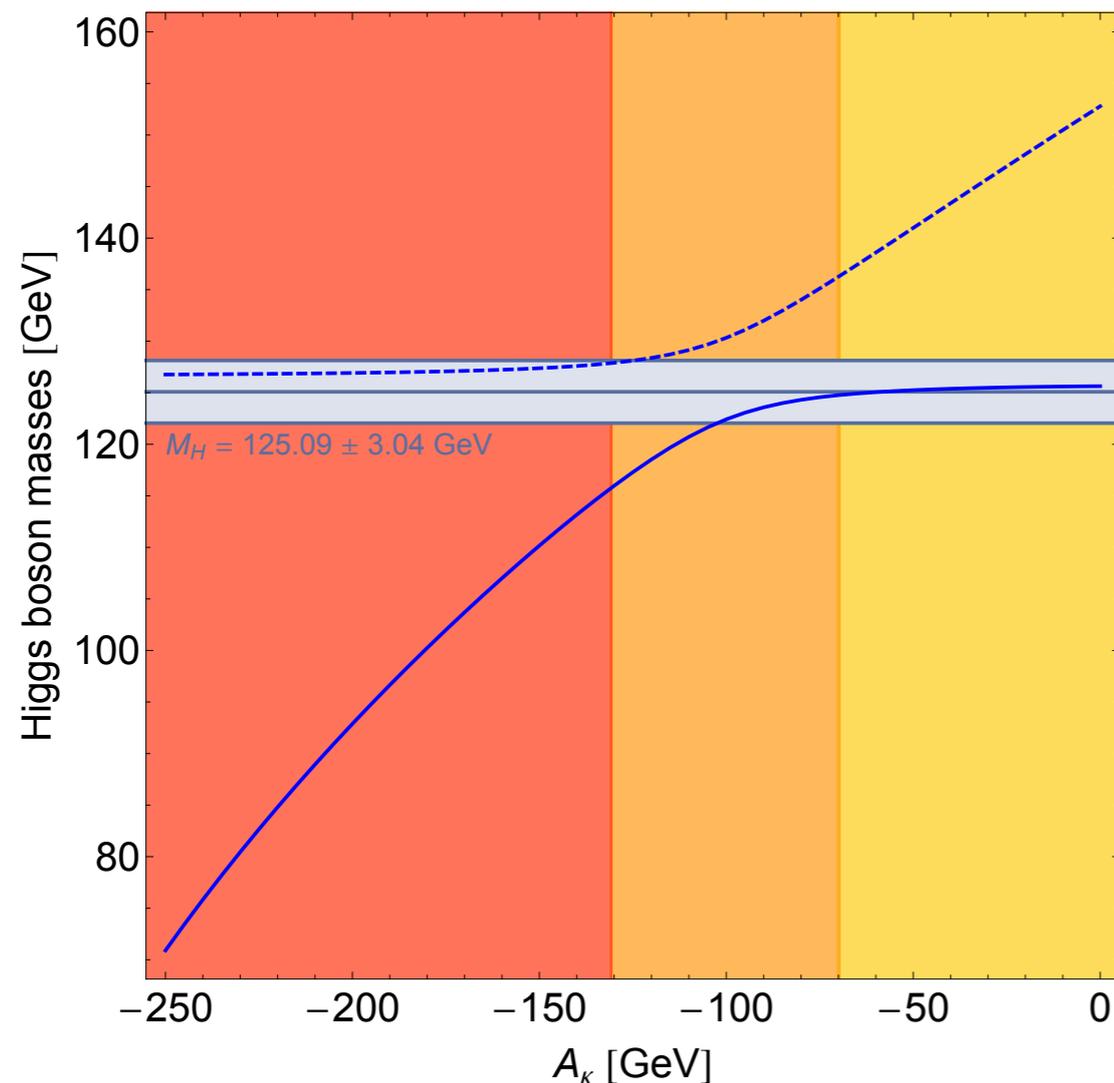
[O. Stål, G. W., L. Zeune '15]

## Higgs sector:

- In the NMSSM: Additional tree-level Higgs mass contribution  
Can reduce the size of the radiative corrections needed to 'push' the lightest Higgs mass up to the experimental value
- Here the NMSSM Higgs sector contribution to  $M_W$  is predominately SM like with  $M_{h_{SM}} = M_{h_1^{NMSSM}}$



# Singlet-doublet mixing in the NMSSM



- Higgs signal can be interpreted as heavy NMSSM Higgs

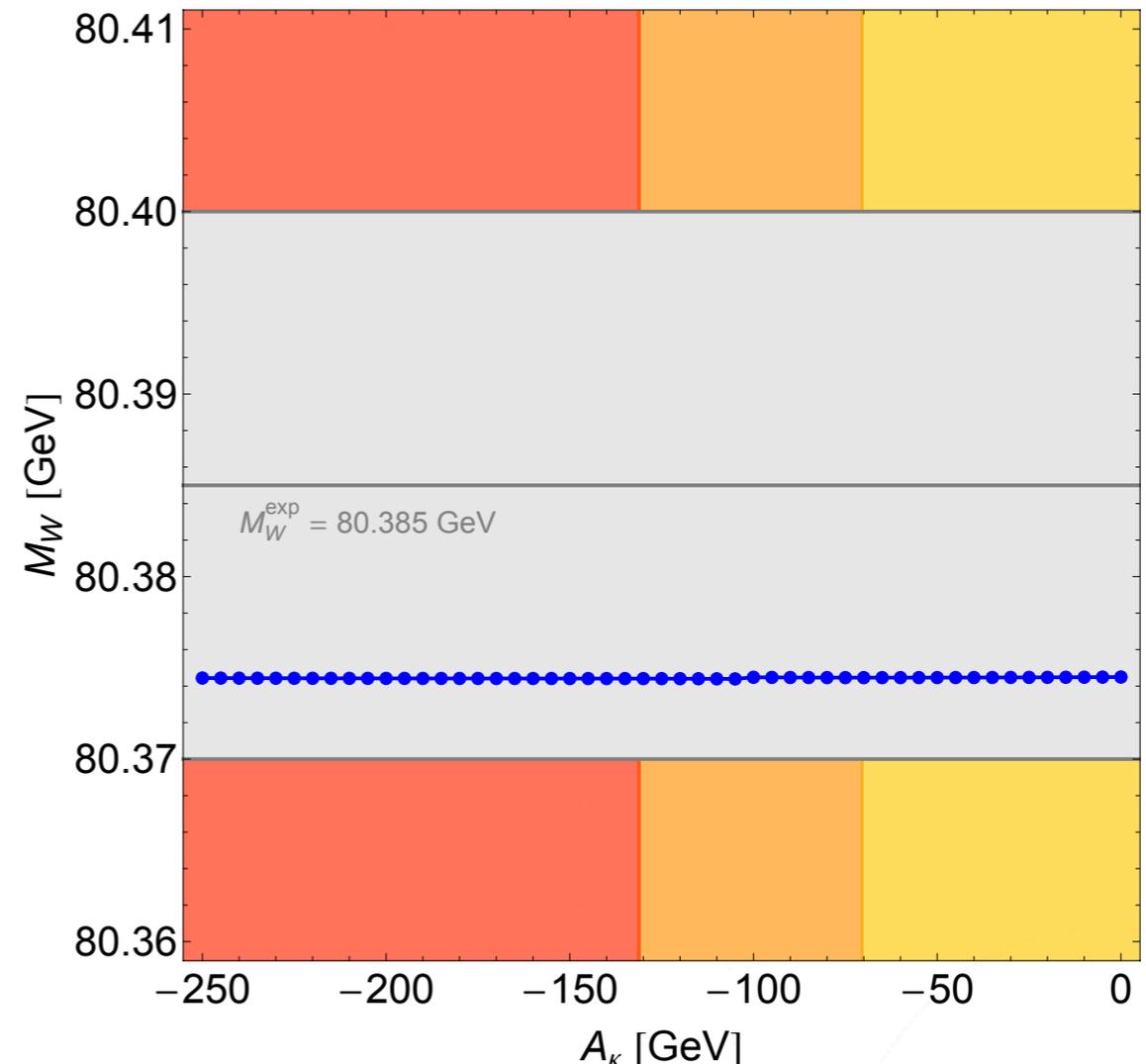
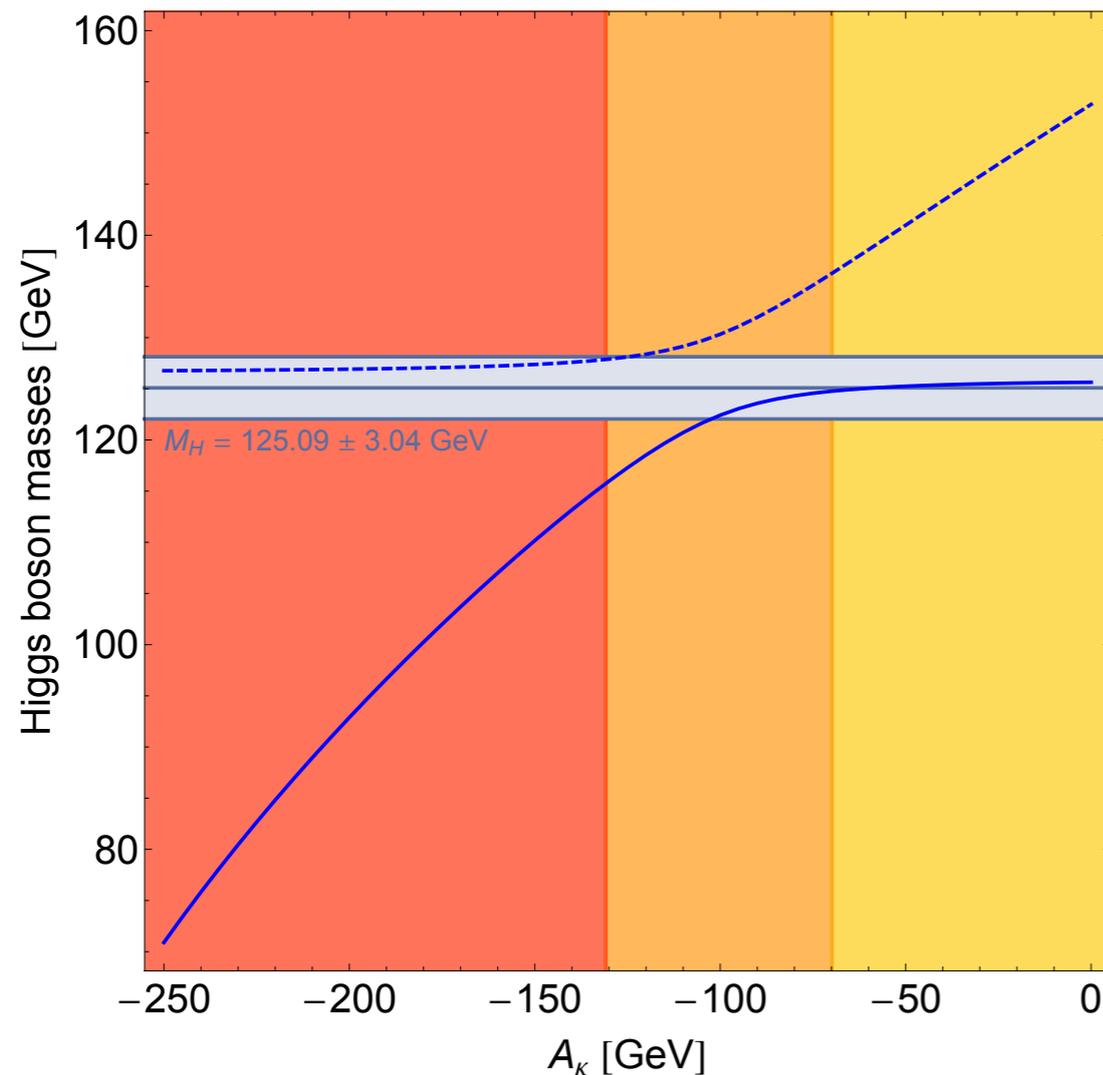
[O. Stål, G. W., L. Zeune '15]

Not necessary also a light charged Higgs like in the MSSM

- Strong singlet-doublet mixing, two Higgs bosons close in mass

- Higgs signal can be interpreted as lightest NMSSM Higgs

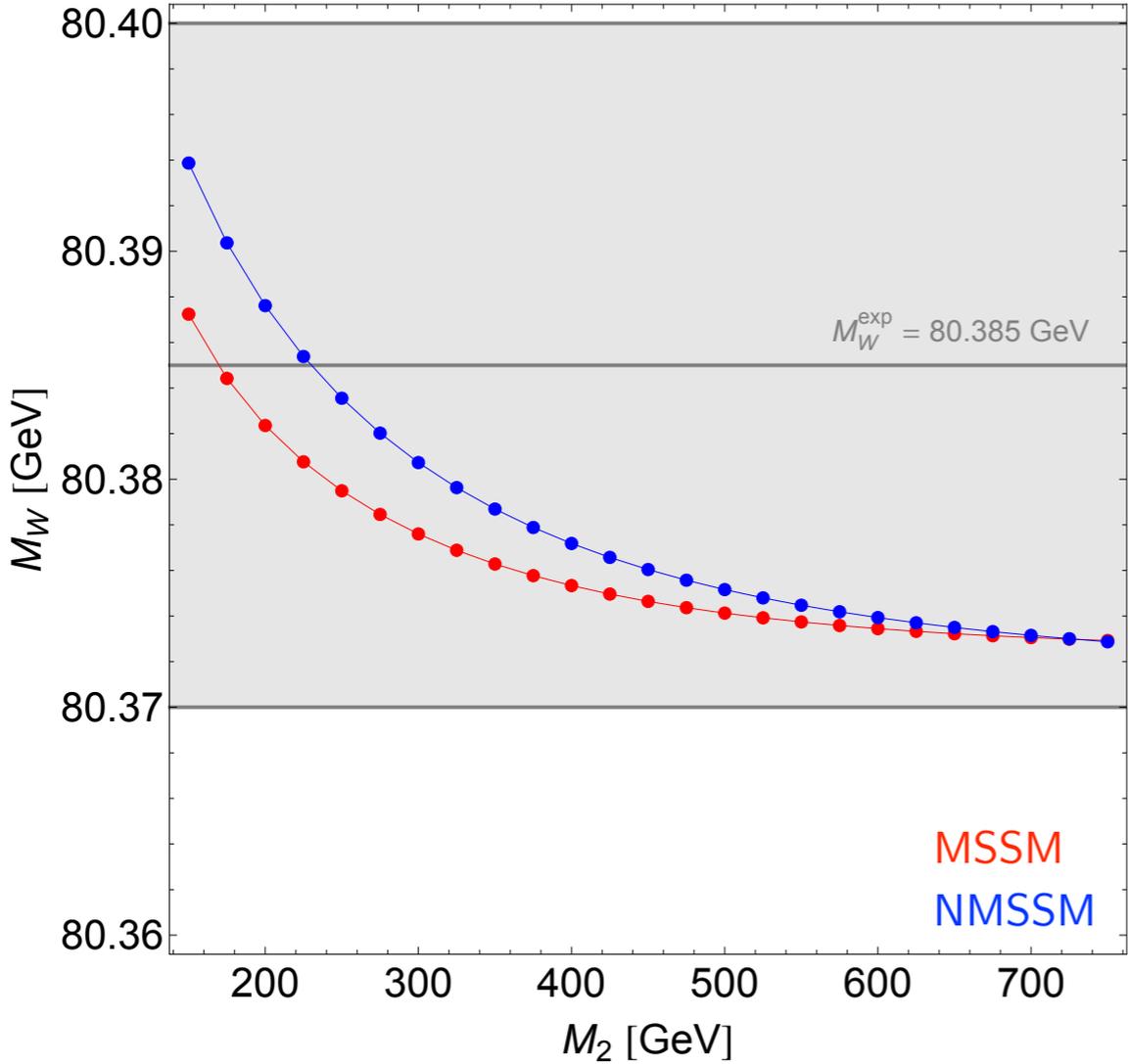
# Singlet-doublet mixing in the NMSSM



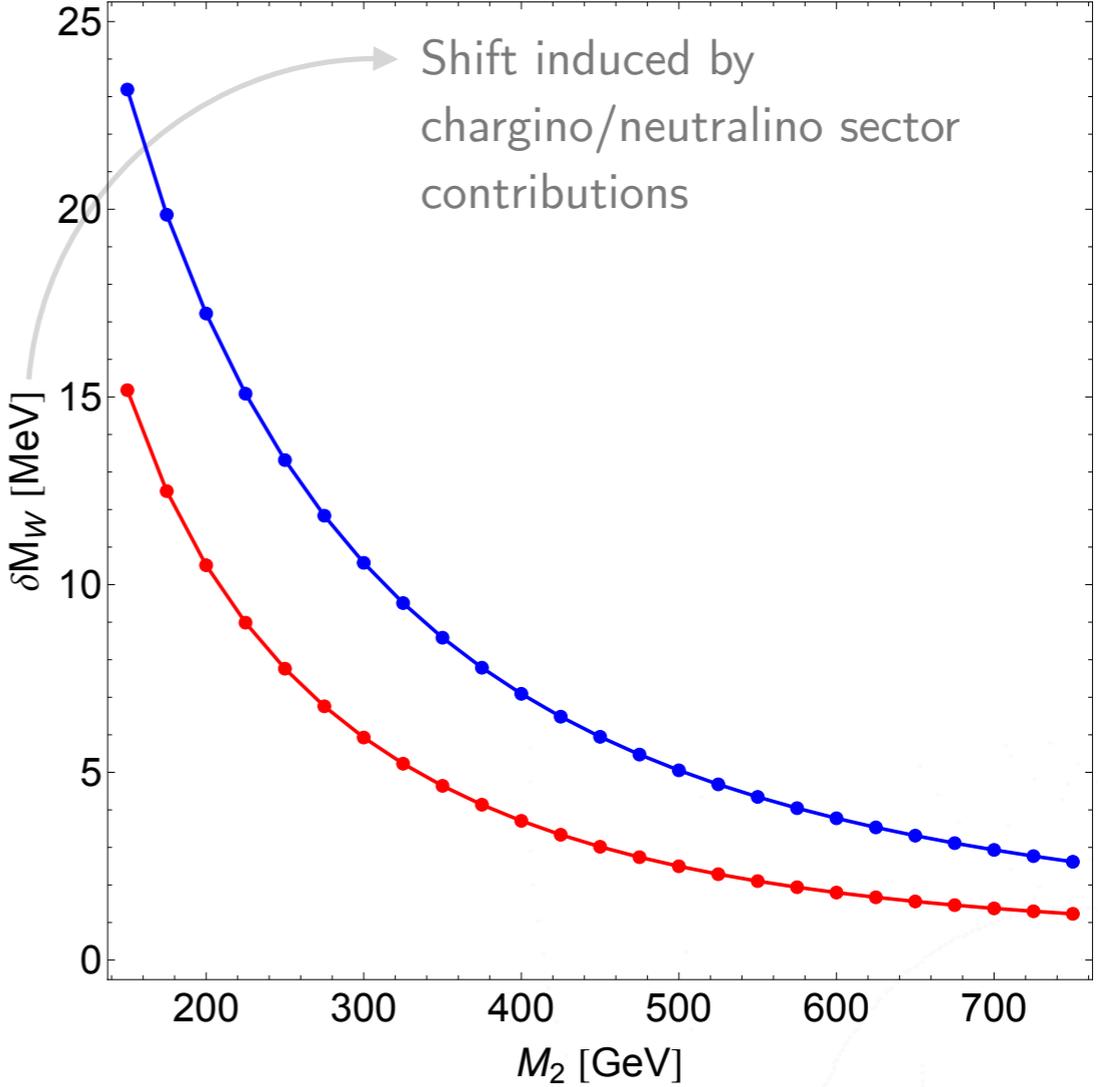
- Sizeable doublet–singlet mixing has only a minor effect on the  $M_W$  prediction
- Both NMSSM Higgs signal interpretations lead to the same  $M_W$  prediction which is well compatible with the  $M_W$  measurement

[O. Stål, G. W., L. Zeune '15]

# Contributions of the neutralino sector: NMSSM / MSSM



= SU(2) gaugino mass parameter



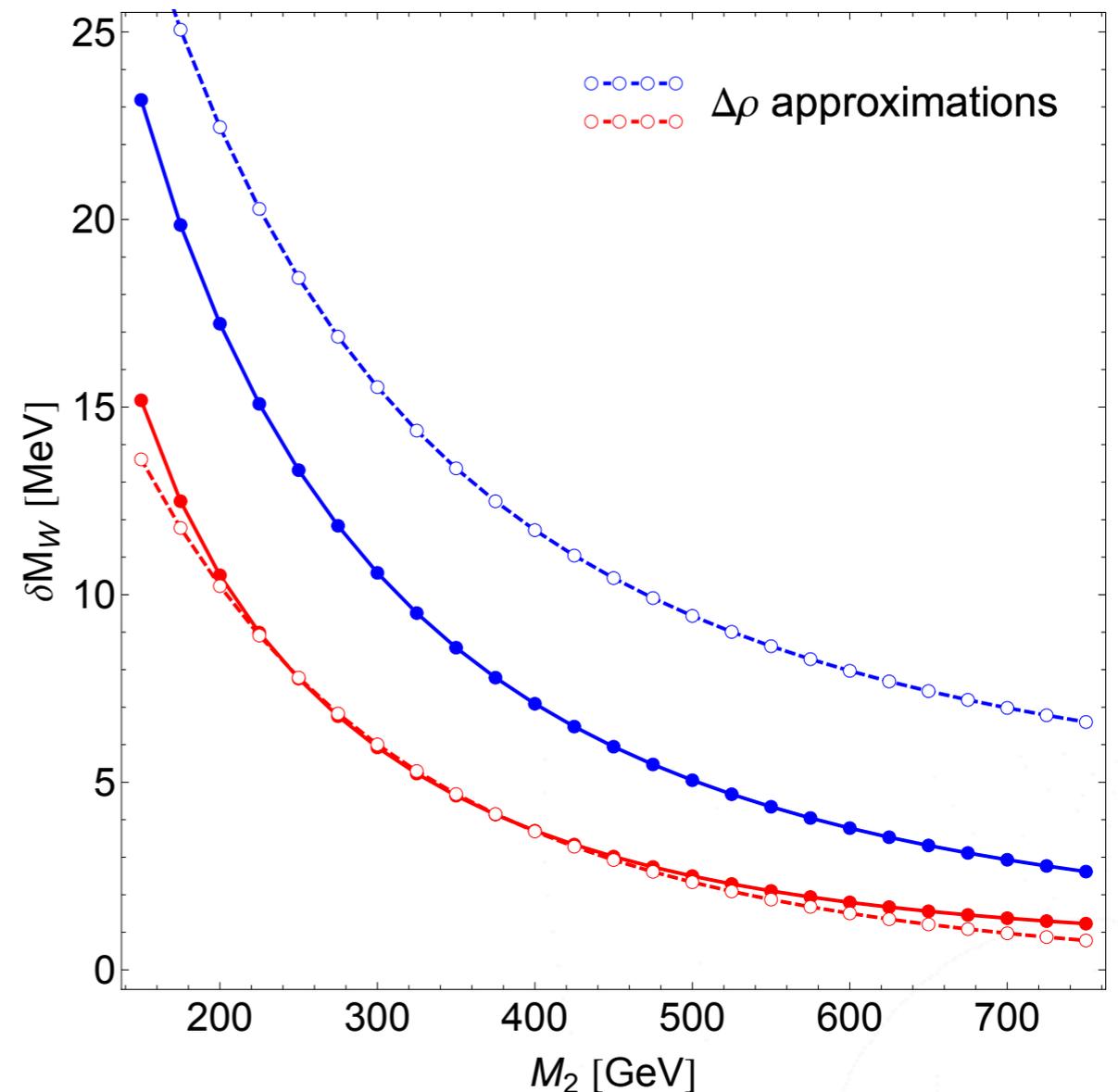
[O. Stål, G. W., L. Zeune '15]

- Modified mass relations lead to lighter (compared to MSSM) NMSSM neutralinos with small singlet component
- Sizeable NMSSM W boson mass contributions from neutralino sector

# Contributions of the neutralino sector in the NMSSM

[O. Stål, G. W., L. Zeune '15]

Sizable difference between the full chargino/neutralino contributions and the approximation by the  $\Delta\rho$  term (T parameter)



# Conclusions

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Relation between  $M_W$ ,  $M_Z$ ,  $\alpha$ ,  $G_\mu$  provides unique sensitivity to quantum effect of new physics; crucial for discriminating between different possible scenarios

Coherent state-of-the-art predictions in the SM, MSSM, NMSSM: facilitates comparison, identification of possible deviations

Impact will be further enhanced by improvements in the experimental determinations and further information on possible SUSY spectra (both from limits and discovery!)

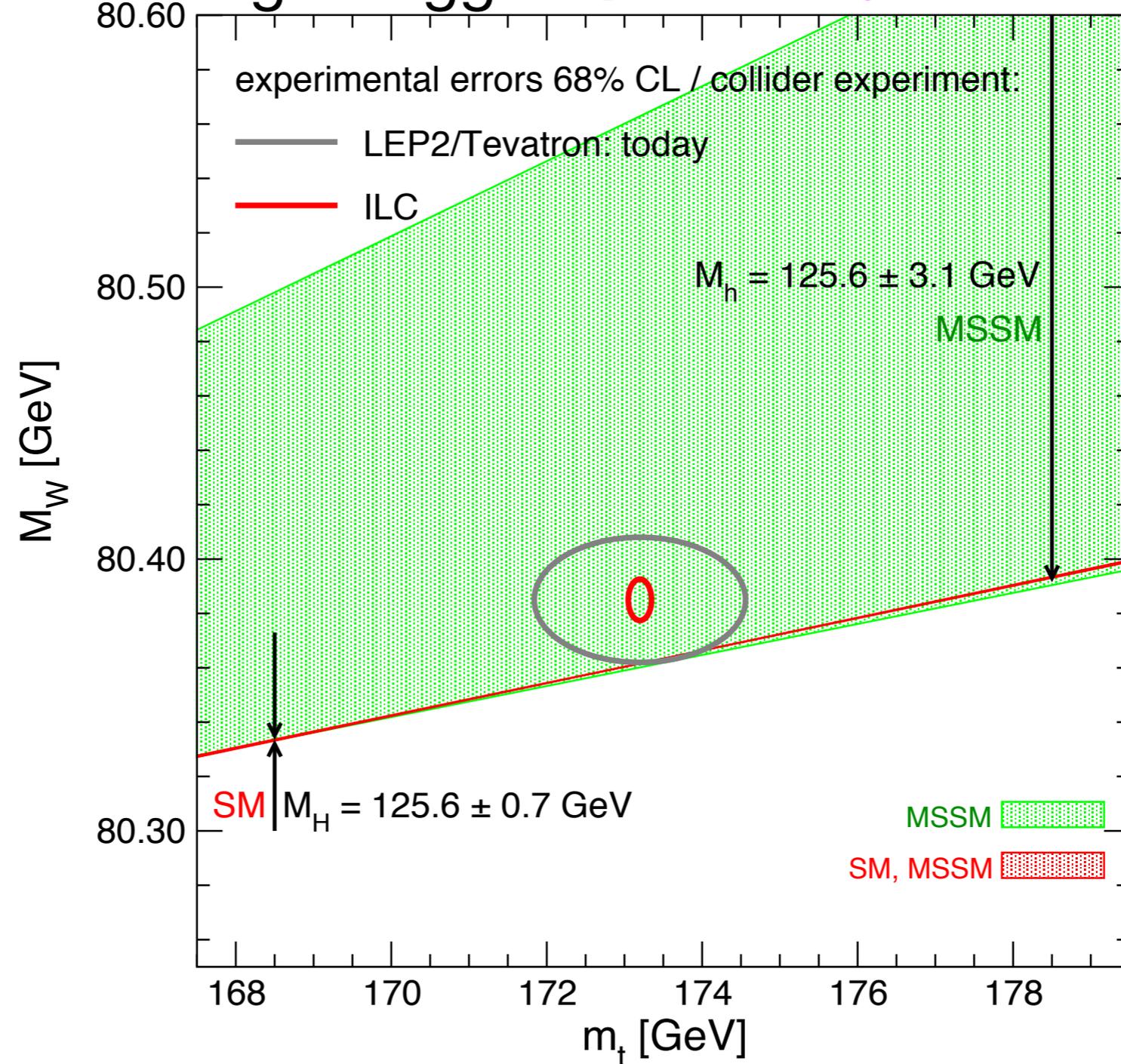
More work will be needed in order to fully exploit the potential of the upcoming experimental results

# Backup

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# Prediction for $M_W$ (parameter scan): SM vs. MSSM

Signal interpreted as light Higgs  $h$  [S. Heinemeyer, W. Hollik, G. W., L. Zeune '14]



⇒ Large improvement at the ILC, high sensitivity to new physics effects