

A critical examination of the $SU(3)$ framework in the hadronic decays of D.

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ERC Ideas: NPFlavour

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the importance of $\eta - \eta'$ mixing

- ✓ Complete set of measurements of branching fraction available.
- ✓ The mixing angle is well measured (with a grain of salt).
- ✓ Previously left unconsidered in analyses based on the complete SU(3) framework.
- ✓ The singlet-octet mixing is a consequence of broken SU(3)
- ✓ There are convincing theoretical arguments and experimental hints that the states have not only quark content but also a gluonic component.

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\cos\theta & +\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

C. Di Donato et al., Phys. Rev. D **85**, 013016 (2012).

S. V. Donskov, V. N. Kolosov, A. A. Lednev, Yu. V. Mikhailov, V. A. Polyakov, V. D.

Samoylenko, G. V. Khaustov, IHEP 2012.22, arXiv:1301.6987

the problem with not considering $\eta - \eta'$ mixing

- ✓ The transfer matrix between the amplitudes and the reduced matrix elements are square.
- ✓ Hence there are as many complex amplitudes as reduced matrix elements.
- ✓ Not considering all PP channels makes the transfer matrix non-square
- ✓ This leads to un-physical combinations of the reduced matrix elements when the number of free parameters are reduced by Gaussian reduction
- ✓ While $SU(3)$ breaking can possibly be inferred from a judicious combination of these transformed reduced matrix elements, they lack the full information that could be carried by taking all channels into consideration.

Caveat: Considering $\eta - \eta'$ mixing increases the number of parameters because then one has to distinguish between the singlet and octet reduced matrix elements. However, the increase in the number of branching fractions is greater than the increase in the number of parameters.

the analysis

1. Choose you favourite set of singlet/octet final states and triplet initial state.
2. Construct your Hamiltonian from the triplet of quarks.

$$\begin{pmatrix} \mathcal{A}(D_s^+ \rightarrow \pi^+ \pi^0) \\ \mathcal{A}(D^+ \rightarrow \pi^+ \bar{K}^0) \\ \mathcal{A}(D^0 \rightarrow \pi^+ K^-) \\ \mathcal{A}(D_s^+ \rightarrow \pi^+ \eta_8) \\ \mathcal{A}(D^0 \rightarrow \pi^0 \bar{K}^0) \\ \mathcal{A}(D_s^+ \rightarrow K^+ \bar{K}^0) \\ \mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta_8) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 & -\frac{\sqrt{5}}{6} & -\frac{1}{2\sqrt{3}} & \frac{\sqrt{5}}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \frac{1}{6\sqrt{5}} & \frac{1}{2\sqrt{3}} & \frac{2}{3\sqrt{5}} & -\frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \sqrt{\frac{2}{15}} & 0 & \sqrt{\frac{2}{15}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{3}} \\ -\sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \frac{2}{3\sqrt{5}} & 0 & \frac{2}{3\sqrt{5}} & \frac{\sqrt{2}}{3} & 0 \\ -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & 2\sqrt{\frac{2}{15}} & 0 & \frac{1}{\sqrt{30}} & -\frac{1}{\sqrt{3}} & 0 \end{pmatrix} \begin{pmatrix} \langle 8 | \bar{6}_1 | \bar{3} \rangle \\ \langle 8 | 15_1 | \bar{3} \rangle \\ \langle 27 | \bar{24}_1 | \bar{3} \rangle \\ \langle 27 | \bar{24}_2 | \bar{3} \rangle \\ \langle 27 | 15_1 | \bar{3} \rangle \\ \langle 27 | 42_1 | \bar{3} \rangle \\ \langle 27 | 42_2 | \bar{3} \rangle \end{pmatrix}$$

decay amplitudes
Clebsch-Gordon coefficients
reduced matrix elements

$$\begin{pmatrix} \mathcal{A}(D_s^+ \rightarrow \pi^+ \eta_1) \\ \mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta_1) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \langle 8' | \bar{6}_1 | \bar{3} \rangle \\ \langle 8' | 15_1 | \bar{3} \rangle \end{pmatrix}$$

singlet-octet final state different from octet-octet final state

onward to the Hamiltonian...

in the limit of SU(3) conservation

$$\mathcal{H} = \frac{V_{cd}^* V_{us}}{\sqrt{2}} \mathcal{H}_0^6 + \frac{(V_{cd}^* V_{ud} - V_{cs}^* V_{us})}{2} \mathcal{H}_{1/2}^6 - \frac{V_{cs}^* V_{ud}}{\sqrt{2}} \mathcal{H}_1^6 - \frac{(V_{cd}^* V_{ud} - 3V_{cs}^* V_{us})}{2\sqrt{6}} \mathcal{H}_{1/2}^{15} \\ + \frac{(V_{cd}^* V_{us} + V_{cs}^* V_{ud})}{\sqrt{2}} \mathcal{H}_1^{15} + \frac{V_{cd}^* V_{ud}}{\sqrt{3}} \mathcal{H}_{3/2}^{15}$$

Grinstein-Lebed Relations

$$\frac{\langle f || 6_{I=1/2} || i \rangle}{\langle f || 6_{I=0} || i \rangle} = \frac{V_{cd}^* V_{ud} - V_{cs}^* V_{us}}{\sqrt{2} V_{cd}^* V_{us}}, \quad \frac{\langle f || 6_{I=1} || i \rangle}{\langle f || 6_{I=0} || i \rangle} = -\frac{V_{cs}^* V_{ud}}{V_{cd}^* V_{us}} \\ \frac{\langle f || 6_{I=1} || i \rangle}{\langle f || 6_{I=1/2} || i \rangle} = -\frac{\sqrt{2} V_{cs}^* V_{ud}}{V_{cd}^* V_{ud} - V_{cs}^* V_{us}}, \quad \frac{\langle f || 15_{I=1} || i \rangle}{\langle f || 15_{I=1/2} || i \rangle} = -\frac{2\sqrt{3}(V_{cs}^* V_{ud} + V_{cd}^* V_{us})}{V_{cd}^* V_{ud} - 3V_{cs}^* V_{us}} \\ \frac{\langle f || 15_{I=3/2} || i \rangle}{\langle f || 15_{I=1/2} || i \rangle} = -\frac{2\sqrt{2} V_{cd}^* V_{ud}}{V_{cd}^* V_{ud} - 3V_{cs}^* V_{us}}, \quad \frac{\langle f || 15_{I=3/2} || i \rangle}{\langle f || 15_{I=1} || i \rangle} = \sqrt{\frac{2}{3}} \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{ud} + V_{cd}^* V_{us}}$$

“The ratios of pieces of the Hamiltonian of the same representation are independent of isospin and depend only on the Clebsch-Gordon coefficients and CKM elements for a fixed pair of initial and final state representations.”

with SU(3) breaking

$$\mathcal{H} = \mathcal{H} + \epsilon \Delta \mathcal{H} \longleftarrow$$

$$+$$

Grinstein-Lebed Relations

$$\begin{aligned} \Delta \mathcal{H} = & \frac{1}{40\sqrt{3}} \left((-5 + 25\sqrt{2} + 3\sqrt{5}) V_{cd}^* V_{ud} - (5 + 5\sqrt{2} + 9\sqrt{5}) V_{cs}^* V_{us} \right) \mathcal{H}_{1/2}^3 \\ & + \frac{1}{40} \left((20 - 5\sqrt{2} + 2\sqrt{5} + \sqrt{10}) V_{cd}^* V_{ud} - (-20 + 5\sqrt{2} + 2\sqrt{5} + 3\sqrt{10}) V_{cs}^* V_{us} \right) \mathcal{H}_{1/2}^6 \\ & + \left(\frac{1}{3\sqrt{61}} (7\sqrt{3} - 6) V_{cd}^* V_{us} + \left(\frac{1}{\sqrt{61}} - \frac{7}{2\sqrt{183}} - \frac{1}{\sqrt{2}} \right) V_{cs}^* V_{ud} \right) \mathcal{H}_1^{15} \\ & + \frac{1}{1464} \left((-183\sqrt{3} + 549\sqrt{6} - 8\sqrt{61} + 11\sqrt{183}) V_{cd}^* V_{ud} \right) \mathcal{H}_{1/2}^{15} \\ & + \frac{1}{488} \left((-61\sqrt{3} + 61\sqrt{6} + 8\sqrt{61} - 11\sqrt{183}) V_{cs}^* V_{us} \right) \mathcal{H}_{1/2}^{15} \\ & + \left(\frac{1}{\sqrt{3}} V_{cd}^* V_{us} + \sqrt{\frac{2}{15}} (1 - \sqrt{3}) V_{cs}^* V_{ud} \right) \mathcal{H}_1^{24} \\ & + \frac{1}{\sqrt{10}} \left((\sqrt{3} - \frac{1}{3}) V_{cd}^* V_{ud} - (\sqrt{3} - 1) V_{cs}^* V_{us} \right) \mathcal{H}_{1/2}^{24} \\ & + \frac{1}{\sqrt{3}} (V_{cd}^* V_{us} + V_{cs}^* V_{ud}) \mathcal{H}_1^{42} + \left(\sqrt{\frac{2}{5}} V_{cs}^* V_{us} - \frac{\sqrt{2}}{3} V_{cd}^* V_{ud} \right) \mathcal{H}_{1/2}^{42} \\ & + \left(4\sqrt{\frac{2}{183}} V_{cd}^* V_{ud} + \frac{5}{6\sqrt{122}} V_{cd}^* V_{ud} \right) \mathcal{H}_{3/2}^{15} + \frac{1}{3} \sqrt{2} V_{cd}^* V_{ud} \mathcal{H}_{3/2}^{24} + \frac{1}{6} \sqrt{5} V_{cd}^* V_{ud} \mathcal{H}_{3/2}^{42} \\ & + \frac{V_{cd}^* V_{us}}{2\sqrt{2}} \mathcal{H}_{3/2}^{151} + \sqrt{\frac{2}{5}} V_{cd}^* V_{us} \mathcal{H}_0^6 + \sqrt{\frac{3}{5}} V_{ud} V_{us} \mathcal{H}_0^{24} + \frac{(\sqrt{2} - 2) V_{cs}^* V_{ud}}{2\sqrt{5}} \mathcal{H}_1^6 \end{aligned}$$

NOTE 1: ϵ is not a measure of SU(3) breaking, it is merely a numerical tag

NOTE 2: in addition to new reduced matrix elements generated by the breaking the ones previously present in the conserving limit also get corrections

NOTE 3: while ϵ can be reabsorbed into the definition of the matrix elements that appear only in $\Delta \mathcal{H}$ it cannot be completely removed the the full Hamiltonian

after all these rearrangements one ends up with 22 parameters and 30 branching fraction.

the fit

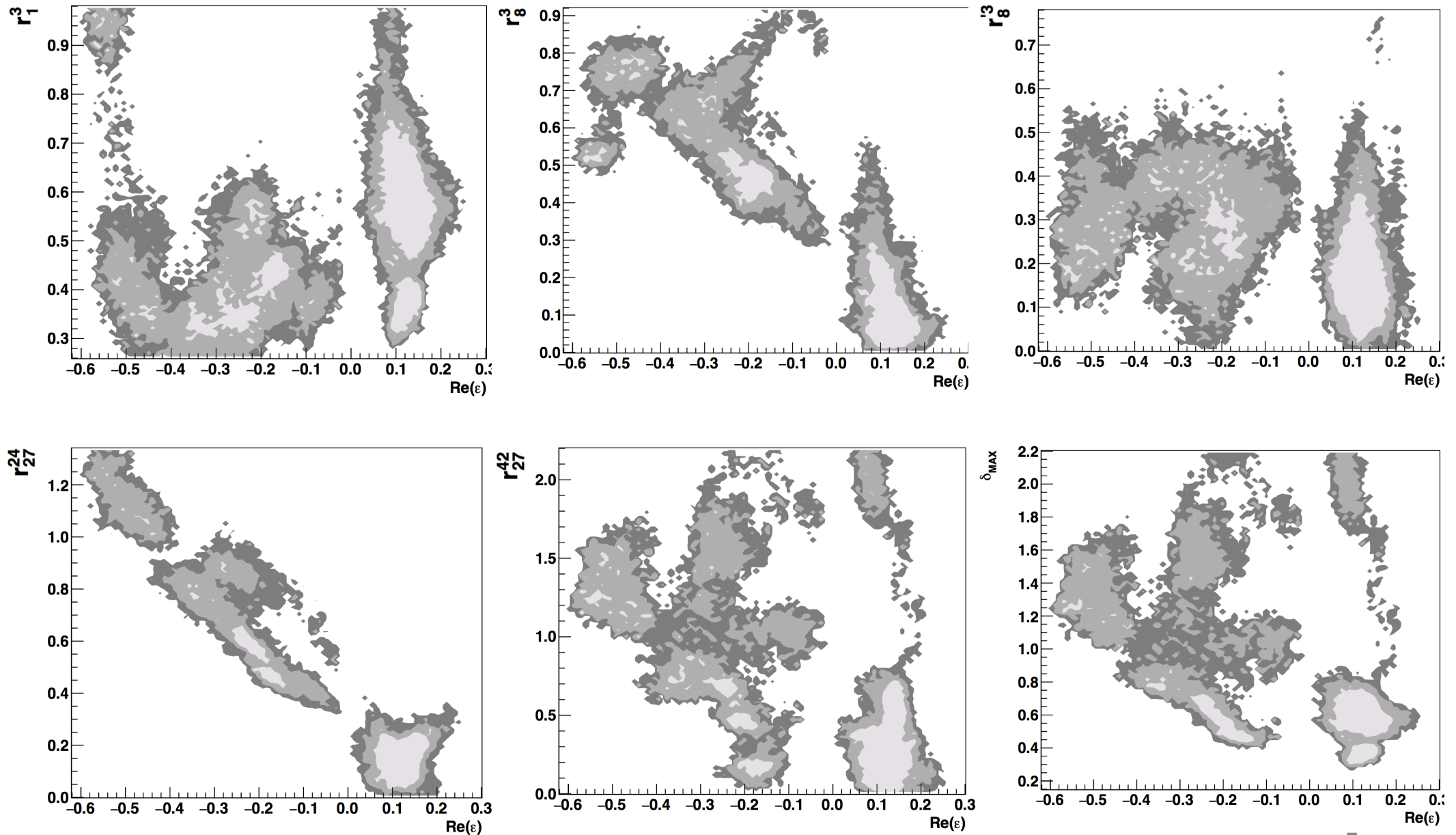
Parameter	Re()	Im()
ϵ	0.13 ± 0.01	
R_1^3	-0.418 ± 0.023	0.156 ± 0.033
R_8^3	0.096 ± 0.020	0.055 ± 0.022
R_8^6	0.114 ± 0.017	0.050 ± 0.016
R_8^{15}	1.164 ± 0.019	
R_{27}^{15}	0.014 ± 0.010	-0.330 ± 0.0097
R_{27}^{24}	-0.113 ± 0.021	-0.138 ± 0.026
R_{27}^{42}	-0.080 ± 0.038	-0.326 ± 0.051
$R_1'^3$	0.279 ± 0.173	-0.625 ± 0.457
$R_8'^3$	0.072 ± 0.022	-0.0038 ± 0.050
$R_8'^6$	0.285 ± 0.042	0.177 ± 0.047
$R_8'^{15}$	-0.374 ± 0.059	0.369 ± 0.094

NOTE: this is one of many possible equivalent solutions!

Channel	Fit ($\times 10^{-3}$)	Experimental ($\times 10^{-3}$)
CF		
$\text{BR}(D^+ \rightarrow \pi^+ \bar{K}_S)$	14.98 ± 0.44	14.7 ± 0.7
$\text{BR}(D^+ \rightarrow \pi^+ \bar{K}_L)$	14.45 ± 0.39	14.6 ± 0.5
$\text{BR}(D^0 \rightarrow \pi^+ K^-)$	38.86 ± 0.49	38.8 ± 0.5
$\text{BR}(D^0 \rightarrow \pi^0 \bar{K}_S)$	12.37 ± 0.32	11.9 ± 0.4
$\text{BR}(D^0 \rightarrow \pi^0 \bar{K}_L)$	9.35 ± 0.28	10.0 ± 0.7
$\text{BR}(D_s^+ \rightarrow K^+ \bar{K}_S)$	14.6 ± 0.59	14.9 ± 0.6
$\text{BR}(D^0 \rightarrow \bar{K}_S \eta)$	4.81 ± 0.29	4.79 ± 0.3
$\text{BR}(D^0 \rightarrow \bar{K}_S \eta')$	9.35 ± 0.50	9.40 ± 0.5
$\text{BR}(D_s^+ \rightarrow \pi^+ \eta)$	16.51 ± 0.94	16.9 ± 1.0
$\text{BR}(D_s^+ \rightarrow \pi^+ \eta')$	40.17 ± 2.14	39.4 ± 2.5
SCS		
$\text{BR}(D^0 \rightarrow \pi^+ \pi^-)$	1.40 ± 0.02	1.402 ± 0.026
$\text{BR}(D_0^+ \rightarrow \pi^0 \pi^0)$	0.79 ± 0.03	0.82 ± 0.035
$\text{BR}(D^+ \rightarrow \pi^+ \pi^0)$	1.18 ± 0.06	1.19 ± 0.06
$\text{BR}(D^0 \rightarrow K^+ K^-)$	3.89 ± 0.07	3.96 ± 0.08
$\text{BR}(D^0 \rightarrow K_S K_S)$	0.17 ± 0.04	0.17 ± 0.04
$\text{BR}(D^+ \rightarrow K^+ K_S)$	2.91 ± 0.14	2.83 ± 0.16
$\text{BR}(D^0 \rightarrow \eta \eta)$	1.66 ± 0.20	1.67 ± 0.2
$\text{BR}(D^0 \rightarrow \eta \eta')$	1.03 ± 0.23	1.05 ± 0.26
$\text{BR}(D^+ \rightarrow \pi^+ \eta)$	3.53 ± 0.20	3.53 ± 0.21
$\text{BR}(D^+ \rightarrow \pi^+ \eta')$	4.69 ± 0.28	4.67 ± 0.29
$\text{BR}(D^0 \rightarrow \pi^0 \eta)$	0.71 ± 0.06	0.68 ± 0.07
$\text{BR}(D^0 \rightarrow \pi^0 \eta')$	0.93 ± 0.13	0.90 ± 0.14
$\text{BR}(D_s^+ \rightarrow \pi^0 K^+)$	0.72 ± 0.12	0.63 ± 0.21
$\text{BR}(D_s^+ \rightarrow \pi^+ K_S)$	1.22 ± 0.06	1.21 ± 0.06
$\text{BR}(D_s^+ \rightarrow K^+ \eta)$	2.11 ± 0.2	1.76 ± 0.35
$\text{BR}(D_s^+ \rightarrow K^+ \eta')$	2.04 ± 0.3	1.8 ± 0.6
DCS		
$\text{BR}(D^+ \rightarrow \pi^0 K^+)$	0.189 ± 0.02	0.183 ± 0.026
$\text{BR}(D^0 \rightarrow \pi^- K^+)$	0.0140 ± 0.0026	0.138 ± 0.0028
$\text{BR}(D^+ \rightarrow K^+ \eta)$	0.081 ± 0.0087	0.108 ± 0.017
$\text{BR}(D^+ \rightarrow K^+ \eta')$	0.162 ± 0.02	0.176 ± 0.022

the full picture

ratios of SU(3) breaking and the SU(3) conserving part vs. ϵ



what the assumptions are we making?

- ✓ The number of reduced matrix elements are much larger hence one has to make assumptions:

- Isospin universality can be assumed. This leads to a new set of Grinstein-Lebed relationships

this assumption is taken to hold good as there is no way to check it within the $SU(3)$ framework

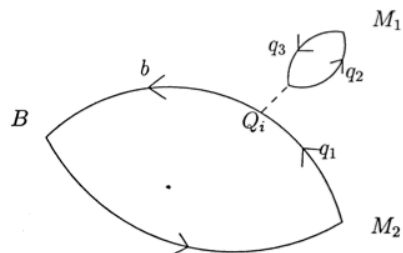
- One can assume the reduced matrix elements from the conserving and breaking part of the Hamiltonian are not independent.

this assumption along with the previous one but without the next one is sufficient to allow a fit if one includes both branching fractions and CP violation data

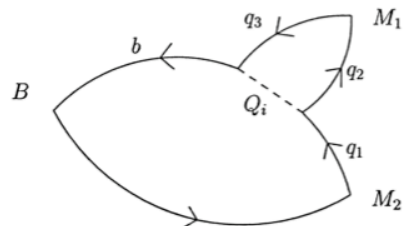
- One can further assume that the breaking is determined by a single parameter representative of the strange quark mass and independent of the representation of the reduced matrix element.

This assumption allows the fitting of all reduced matrix elements with just branching fractions data

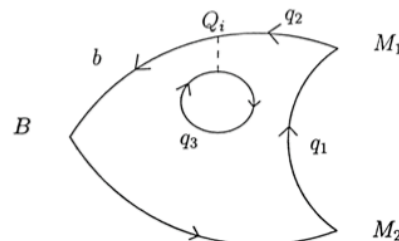
an insight into isospin universality



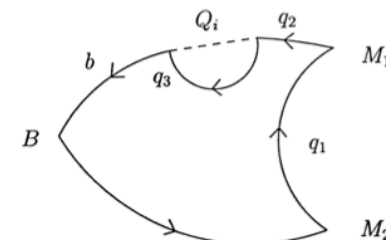
$DE_i(q_3, q_2, q_1; B, M_1, M_2)$



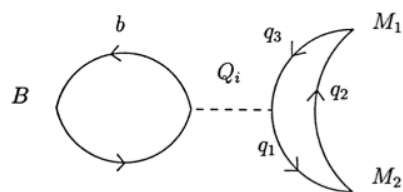
$CE_i(q_3, q_2, q_1; B, M_1, M_2)$



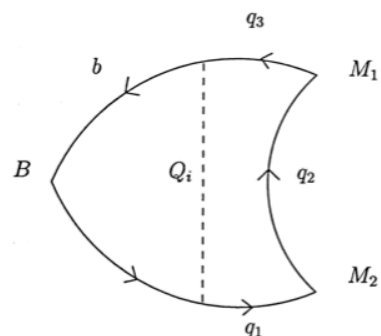
$DP_i(q_3, q_2, q_1; B, M_1, M_2)$



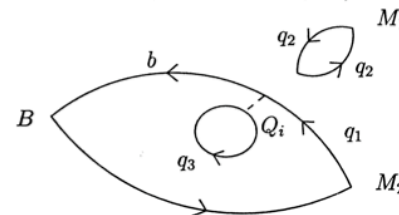
$CP_i(q_3, q_2, q_1; B, M_1, M_2)$



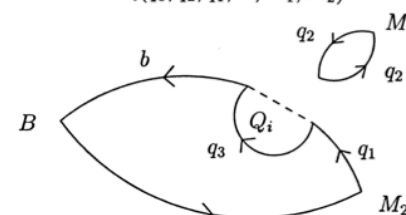
$DA_i(q_3, q_2, q_1; B, M_1, M_2)$



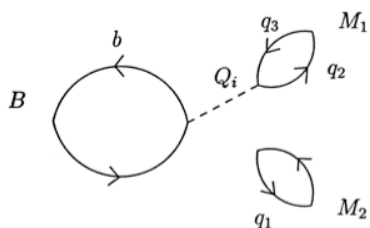
$CA_i(q_3, q_2, q_1; B, M_1, M_2)$



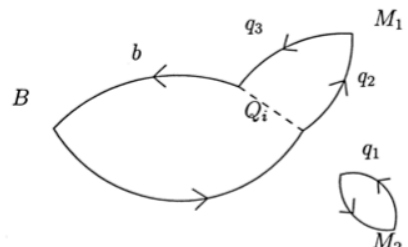
$DPE_i(q_3, q_2, q_1; B, M_1, M_2)$



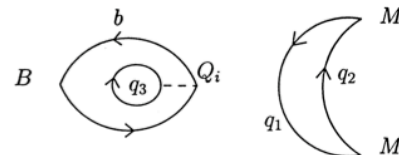
$CPE_i(q_3, q_2, q_1; B, M_1, M_2)$



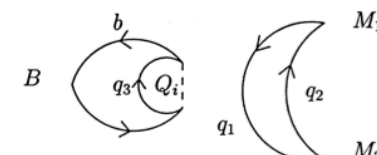
$DEA_i(q_3, q_2, q_1; B, M_1, M_2)$



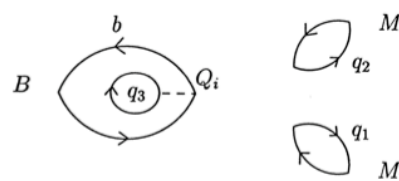
$CEA_i(q_3, q_2, q_1; B, M_1, M_2)$



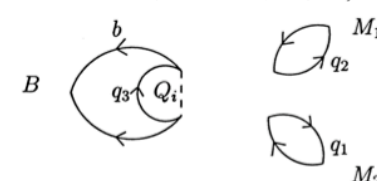
$DPA_i(q_3, q_2, q_1; B, M_1, M_2)$



$CPA_i(q_3, q_2, q_1; B, M_1, M_2)$



$\overline{DPA}_i(q_3, q_2, q_1; B, M_1, M_2)$



$\overline{CPA}_i(q_3, q_2, q_1; B, M_1, M_2)$

an insight into isospin universality

$$R_8^6 = \frac{\sqrt{5}}{2} (-A_1 + A_2 + E_1 - E_2)$$

$$R_8^{15} = \frac{1}{2\sqrt{2}} (5A_1 + 5A_2 + E_1 + E_2)$$

$$R_{27}^{15} = \sqrt{2} (E_1 + E_2)$$

$$R_{8,s}^6 = -\frac{1}{2} (A'_{1,s} + A_{1,s} - A'_{2,s} - A_{2,s} + E_{1,s} - E_{2,s} - 3EA_{1,s} + 3EA_{2,s})$$

$$R_{8,s}^{15} = \frac{1}{2} \sqrt{\frac{5}{2}} (A'_{1,s} + A_{1,s} + A'_{2,s} + A_{2,s} + E_{1,s} + E_{2,s} + 3EA_{1,s} + 3EA_{2,s})$$

- ✓ In the exact SU(3) limit the reduced matrix elements are isospin universal. Hence the Grinstein-Lebed relations hold good.
- ✓ These reduced matrix elements can be extracted from CF and DCS branching fractions.
- ✓ Note that penguin contributions are absent.

an insight into isospin universality

$${}^0R_8^6 = \frac{1}{2\sqrt{5}}(-A_1^{uss} - 4A_1^{uus} + A_2^{uss} + 4A_2^{uus} + 5E_1^{usu} - 5E_2^{usu} - EA_1^{uss} + EA_1^{usu} + EA_2^{uss} - EA_2^{usu})$$

$$\begin{aligned} \frac{1}{2}R_8^6 = & \frac{1}{4\sqrt{5}}(-A_1^{uss} - 3A_1^{usu} - 4A_1^{uus} - 2A_1^{uuu} + 2A_2^{sss} + 3A_2^{sus} + 5A_2^{uuu} + 5E_1^{uss} + 5E_1^{uuu} \\ & - 5E_2^{ssu} - 5E_2^{uuu} - EA_1^{uss} + EA_1^{usu} + 2EA_1^{uus} - 2EA_1^{uuu} + 2EA_2^{sss} - 2EA_2^{ssu} + \underline{5P_1^{us}} \\ & - 5P_1^{uu} - 5P_2^{us} + 5P_2^{uu} + P_3^{ss} + 3P_3^{us} - 4P_3^{uu} + P_4^{ss} - 2P_4^{us} + P_4^{uu}) \end{aligned}$$

$${}^1R_8^6 = \frac{1}{2\sqrt{5}}(3A_1^{usu} + 2A_1^{uuu} - A_2^{ssu} - 4A_2^{suu} - 5E_1^{uus} + 5E_2^{suu} - 2EA_1^{uus} + 2EA_1^{uuu} - EA_2^{sus} + EA_2^{suu})$$

$$\begin{aligned} \frac{1}{2}R_1^3 = & \frac{1}{6}(-2A_2^{sss} - 6A_2^{sus} + 3A_2^{usu} + 5A_2^{uuu} - 3E_1^{uss} + 3E_1^{uuu} + E_2^{ssu} - E_2^{uuu} - 2EA_2^{sss} \\ & + 2EA_2^{ssu} - EA_2^{uus} + EA_2^{uuu} - \underline{3P_1^{us} - 5P_1^{uu} + P_2^{us} - P_2^{uu} - P_3^{ss} - 6P_3^{us} - 5P_3^{uu}} \\ & - P_4^{ss} + 2P_4^{us} - P_4^{uu}) \end{aligned}$$

$$\begin{aligned} \frac{1}{2}R_8^3 = & \frac{1}{6\sqrt{10}}(-3A_1^{uss} + 9A_1^{usu} - 12A_1^{uus} + 6A_1^{uuu} + 2A_2^{sss} + 3A_2^{sus} - 6A_2^{usu} + A_2^{uuu} - 15E_1^{uss} \\ & + 15E_1^{uuu} + 5E_2^{ssu} - 5E_2^{uuu} - 3EA_1^{uss} + 3EA_1^{usu} - 6EA_1^{uus} + 6EA_1^{uuu} + 2EA_2^{sss} \\ & - 2EA_2^{ssu} + 4EA_2^{uus} - 4EA_2^{uuu} - \underline{15P_1^{us} - 25P_1^{uu} + 5P_2^{us} - 5P_2^{uu} + P_3^{ss} + 3P_3^{us}} \\ & - 4P_3^{uu} + P_4^{ss} - 2P_4^{us} + P_4^{uu}) \end{aligned}$$

writing these are an isospin universal + isospin non-universal part
increases the number of parameters and decouples the sectors

the validity of the SU(3) formalism

- ✓ The number of reduced matrix elements are much larger hence one has to make assumptions:

- Isospin universality can be assumed. This leads to a new set of Grinstein-Lebed relationships

this assumption seems to break down when one take a careful look at it

- One can assume the reduced matrix elements from the conserving and breaking part of the Hamiltonian are not independent.

the number of reduced matrix elements can not be sufficiently reduced with just the last two assumptions

- One can further assume that the breaking is determined by a single parameter representative of the strange quark mass and independent of the representation of the reduced matrix element.

the question of final states

- ✓ The SU(3) framework requires that the hadronic final states be identified and there be no variation in the reduced matrix elements due to just the final states.
- ✓ Not making this assumption renders the SU(3) framework completely useless.
- ✓ Identifying the charm meson as a triplet of SU(3) and the pseudoscalars as an octet of SU(3) allows for the set up of the SU(3) framework
- ✓ However, the strength of the SU(3) framework like in its dynamical constructions, i.e. , the Hamiltonian.
- ✓ While we know how to deal with the weak part of the Hamiltonian, we depend solely on motivated arguments and data for the QCD part.

what can sum rules say?

- Sum rules come from two sources:
 1. From the zeroes in the reduced matrix elements.
 2. From the isospin association of the reduced matrix elements in the SU(3) limit. (another manifestation of the Grinstein-Lebed relations)
- The first is well documented. (Grossman-Robinson)
- The second gives additional relations.
- These relations are amplitude relations that are broken when SU(3) is broken
- Branching fractions relations are more important as they can be directly measured. (Grossman-Robinson)
- Gronau presented a branching fraction relation that holds till second order in SU(3) breaking – a very precise test
- There is another such relation but it involves $\eta - \eta'$ mixing.

diagrammatic approach + factorization

- The big question is whether factorization can be applied to charmed meson decays...
- If we assume it is possible then the machinery from B physics can be used
- A recent approach made by Müller, Nierste and Schacht. (next talk)
- Another approach made by Biswas, Sinha and Abbas which includes:
 - An estimation of the topologies within the factorization ansatz
 - A parameterization of the non-factorizable part fit to data
 - A data-driven parameterization of the final state interactions through a scattering matrix.
 - All $\eta - \eta'$ channels considered.
- Fits prove to be quite agreeable except in some channels.
- Singlet contribution in $\eta - \eta'$ channels not considered. (left as future work)

Thank you...!!



To my Mother and Father, who showed me what I could do,
and to Ikaros, who showed me what I could not.

“To know what no one else does, what a pleasure it can be!”

– adopted from the words of
Eugene Wigner.

