Production of $c\bar{c}c\bar{c}$ in single and double parton scattering in collinear and k_t-factorization approaches in hadronic collisions

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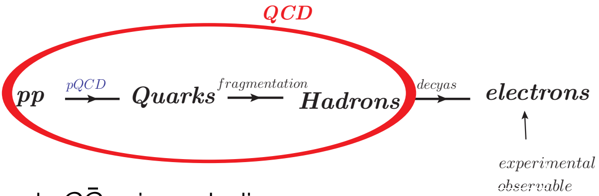
Outline

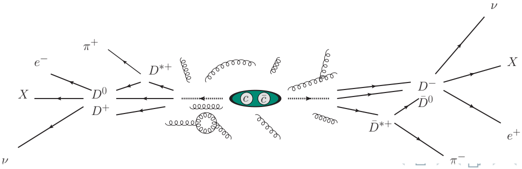
1. Production of $c\bar{c}$
2. D meson production
3. Production of $c\bar{c}c\bar{c}$ in DPS
4. Collinear SPS production of $c\bar{c}c\bar{c}$
5. SPS $c\bar{c}c\bar{c}$ production in k_t-factorization
6. Perturbative parton splitting
7. Conclusions

3-step process



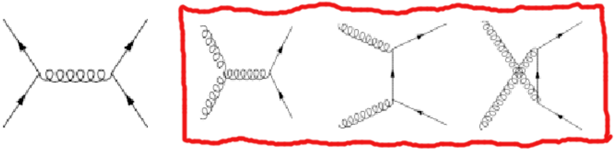
1. Heavy quarks $Q\bar{Q}$ pairs production
   - $m_c = 1.5$ GeV, $m_b = 4.75$ GeV $\longrightarrow$ perturbative QCD
2. Heavy quarks hadronization (fragmentation)
3. Semileptonic decays of D and B mesons

$$\frac{d\sigma^e}{dyd^2p} = \frac{d\sigma^Q}{dyd^2p} \otimes D_{Q\to H} \otimes f_{H\to e}$$

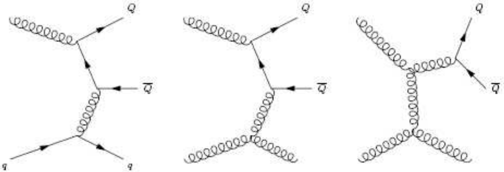


Dominant mechanisms of $Q\bar{Q}$ production

- Leading order processes contributing to $Q\bar{Q}$ production:

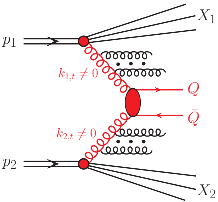


- **gluon-gluon fusion** dominant at high energies
- $q\bar{q}$ anihilation important only near the threshold
- some of next-to-leading order diagrams:



NLO contributions → K-factor

k_t-factorization (semihard) approach



- charm and bottom quarks production at high energies $\longrightarrow$ gluon-gluon fusion
- QCD collinear approach $\rightarrow$ only inclusive one particle distributions, total cross sections

LO k_t-factorization approach $\longrightarrow \kappa_{1,t}, \kappa_{2,t} \neq 0$
$\Rightarrow Q\bar{Q}$ correlations

- multi-differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \sum_{i,j} \int \frac{d^2\kappa_{1,t}}{\pi} \frac{d^2\kappa_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}_{ij\to Q\bar{Q}}|^2}$$
$$\times \; \delta^2\left(\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}\right) \mathcal{F}_i(x_1, \kappa_{1,t}^2)\, \mathcal{F}_j(x_2, \kappa_{2,t}^2)$$

- off-shell $\overline{|\mathcal{M}_{gg\to Q\bar{Q}}|^2} \longrightarrow$ Catani, Ciafaloni, Hautmann (rather long formula)
- major part of NLO corrections automatically included
- $\mathcal{F}_i(x_1, \kappa_{1,t}^2)$, $\mathcal{F}_j(x_2, \kappa_{2,t}^2)$ - unintegrated parton distributions
- $x_1 = \frac{m_{1,t}}{\sqrt{s}} \exp(y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(y_2)$,

  $x_2 = \frac{m_{1,t}}{\sqrt{s}} \exp(-y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(-y_2)$, where $m_{i,t} = \sqrt{p_{i,t}^2 + m_Q^2}$.

Unintegrated parton distribution functions

- k_t-factorization $\rightarrow$ replacement: $p_k(x, \mu_F^2) \longrightarrow \mathcal{F}_k(x, \kappa_t^2, \mu_F^2)$
- PDFs $\longrightarrow$ UPDFs

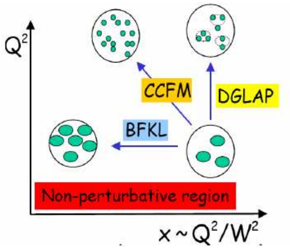
$$xp_k(x, \mu_F^2) = \int_0^\infty d\kappa_t^2 \mathcal{F}(x, \kappa_t^2, \mu_F^2)$$

- UPDFs - needed in less inclusive measurements which are sensitive to the transverse momentum of the parton

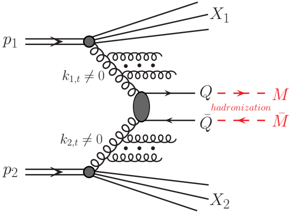
gg-fusion dominance $\Rightarrow$ great test of existing unintegrated gluon densities!
especially at LHC (small-x)

several models:

- Jung, Kwiecinski (CCFM, wide x-range)
- Kimber-Martin-Ryskin (higher x-values)
- Kutak-Stasto (small-x, saturation effects)
- Ivanov-Nikolaev, GBW, Karzeev-Levin, etc.



Fragmentation functions technique



- fragmentation functions extracted from e^+e^- data
- often used: Braaten et al., Kartvelishvili et al., Peterson et al.
- rescalling transverse momentum
  at a constant rapidity (angle)

- from heavy quarks to heavy mesons:

$$\frac{d\sigma(y, p_t^M)}{dy d^2 p_t^M} \approx \int \frac{D_{Q \to M}(z)}{z^2} \cdot \frac{d\sigma(y, p_t^Q)}{dy d^2 p_t^Q} dz$$

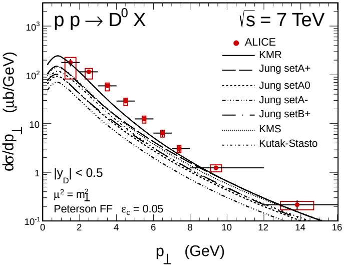
where: $p_t^Q = \frac{p_t^M}{z}$ and $z \in (0, 1)$

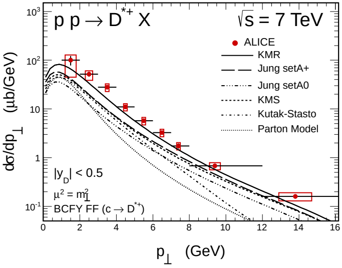
- **approximation:**
  rapidity unchanged in the fragmentation process $\rightarrow y_Q \approx y_M$

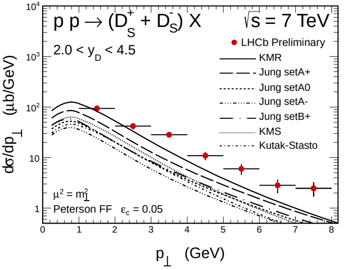
Production of D mesons in this framework:

Maciula, Szczurek, Phys. Rev. **D87** (2013) 094022.

D mesons, different UGDFs

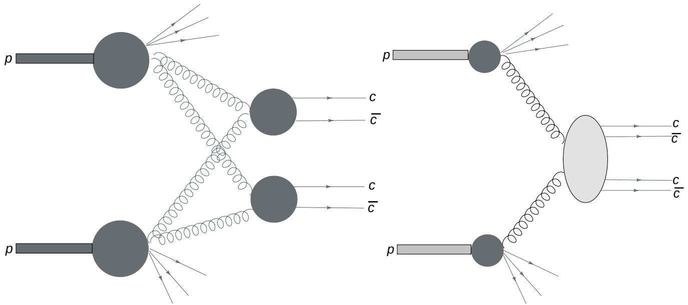






- various UGDFs models → crucial test of their applicability at high energies and small x-values
- only KMR model gives good description of the ALICE and LHCb data
- significant difference between LO parton model and LO k_t-factorization

Production of $c\bar{c}c\bar{c}$



Łuszczak, Maciuła, Szczurek, Phys. Rev. **D85** (2012) 014905.

Formalism

Consider reaction: $pp \to c\bar{c}c\bar{c}X$

Modeling double-parton scattering

Factorized form:

$$\sigma^{DPS}(pp \to c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{eff}}\sigma^{SPS}(pp \to c\bar{c}X_1) \cdot \sigma^{SPS}(pp \to c\bar{c}X_2).$$

The simple formula can be generalized to include differential distributions

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t}} = \frac{1}{2\sigma_{eff}} \cdot \frac{d\sigma}{dy_1 dy_2 d^2p_{1t}} \cdot \frac{d\sigma}{dy_3 dy_4 d^2p_{2t}} .$$

σ_{eff} is a model parameter (15 mb).

Found e.g. from experimental analysis of four jets (see also Siódmok et al.)

In principle does not need to be universal

Formalism

$$d\sigma^{DPS} = \frac{1}{2\sigma_{eff}} F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2) F_{gg}(x_1' x_2', \mu_1^2, \mu_2^2)$$
$$d\sigma_{gg\to c\bar{c}}(x_1, x_1', \mu_1^2) d\sigma_{gg\to c\bar{c}}(x_2, x_2', \mu_2^2)\; dx_1 dx_2 dx_1' dx_2' \,.$$

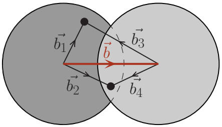
$F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2)$, $F_{gg}(x_1' x_2', \mu_1^2, \mu_2^2)$
are called double parton distributions

dPDF are subjected to special evolution equations

single scale evolution: Snigirev

double scale evolution: Ceccopieri, Gaunt-Stirling

Factorized Ansatz and double-parton distributions (DPDFs)



DPDF - emission of parton i with assumption that second parton j is also emitted:

$$\Gamma_{i,j}(b, x_1, x_2; \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2)\, F_j(x_2, \mu_2^2)\, F(b; x_1, x_2, \mu_1^2, \mu_2^2)$$

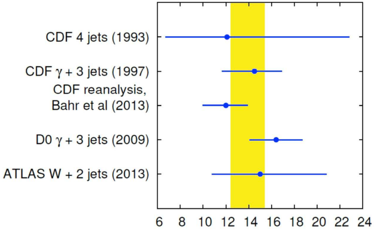
- correlations between two partons
  C. Flensburg et al., JHEP 06, 066 (2011)

in general:

$$\sigma_{eff}(x_1, x_2, x_1', x_2', \mu_1^2, \mu_2^2) = \left(\int d^2b\, F(b; x_1, x_2, \mu_1^2, \mu_2^2)\, F(b; x_1', x_2', \mu_1^2, \mu_2^2) \right)^{-1}$$

factorized Ansatz:

- additional limitations: $x_1 + x_2 < 1$ oraz $x_1' + x_2' < 1$
- DPDF in multiplicative form: $F_{ij}(b; x_1, x_2, \mu_1^2, \mu_2^2) = F_i(x_1, \mu_1^2) F_j(x_2, \mu_2^2) F(b)$
- $\sigma_{eff} = \left[\int d^2b\, (F(b))^2 \right]^{-1}$, $F(b)$ - energy and process independent

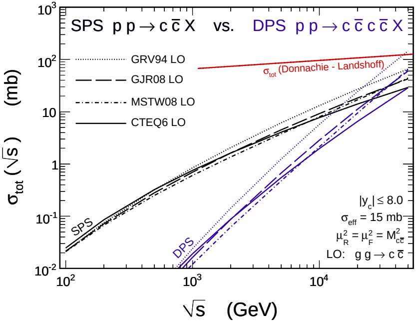


phenomenology: $\sigma_{eff} \Rightarrow$ nonperturbative quantity with a dimension of cross section, connected with transverse size of overlaping protons

$\sigma_{eff} \approx$ **15 mb** ($p_\perp$-independent)

a detailed analysis of σ_{eff}:
Seymour, Siódmok, JHEP 10, 113 (2013)

Energy dependence of $c\bar{c}c\bar{c}$ production



Luszczak,Maciula,Szczurek, Phys. Rev. **C86** (2012) 014905

spectacular result:

Already at the LHC production of two pairs as probable as production of one pair.

Exact LO calculation of SPS $c\bar{c}c\bar{c}$ contribution

- Till recently only approximate (high-energy approx.) SPS contribution to $gg \to c\bar{c}c\bar{c}$
- Recently full calculation of leading-order $2 \to 4$ diagrams for SPS $gg \to c\bar{c}c\bar{c}$ and $q\bar{q} \to c\bar{c}c\bar{c}$ (small)
- Automatic calculation with exact LO matrix elements and full phase space integration (van Hameren code, similar to HELAC)
- Generation of unweighted events and building quark/antiquark distributions
- Hadronization with fragmentation functions
- No K-factor (relatively small for $b\bar{b}b\bar{b}$)

van Hameren, Maciuła, Szczurek, Phys. Rev. **D89** (2014) 054907.

SPS cross section for $c\bar{c}c\bar{c}$

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \overline{|\mathcal{M}_{gg \to c\bar{c}c\bar{c}}|^2} \, d^4PS. \qquad (1)$$

where

$$d^4PS = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} \frac{d^3p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^4\left(p_1 + p_2 + p_3 + p_4\right) \, . \qquad (2)$$

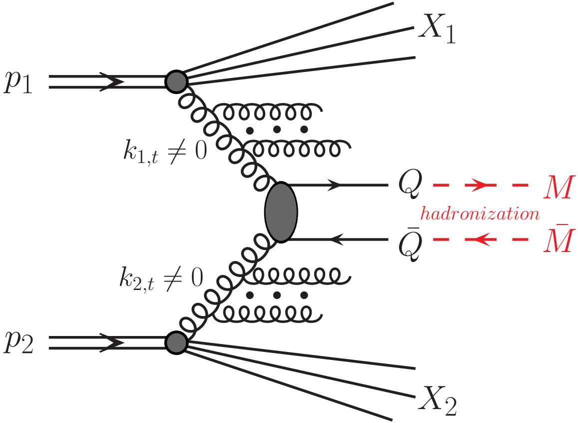
Above p_1, p_2, p_3, p_4 are four-momenta of final c, $\bar{c}$, c, $\bar{c}$ quarks and antiquarks, respectively.
Neglecting small electroweak corrections, the hadronic cross section is

$$\begin{aligned} d\sigma &= \int dx_1 dx_2 (g(x_1, \mu_F^2) g(x_2, \mu_F^2) \, d\sigma_{gg \to c\bar{c}c\bar{c}} \\ &+ \Sigma_f \, q_f(x_1, \mu_F^2) \bar{q}_f(x_2, \mu_F^2) \, d\sigma_{q\bar{q} \to c\bar{c}c\bar{c}} \\ &+ \Sigma_f \, \bar{q}_f(x_1, \mu_F^2) q_f(x_2, \mu_F^2) \, d\sigma_{\bar{q}q \to c\bar{c}c\bar{c}}) \, . \end{aligned} \qquad (3)$$

In the calculation below we include $u\bar{u}$, $\bar{u}u$, $d\bar{d}$, $\bar{d}d$, $s\bar{s}$, $\bar{s}s$ annihilation terms.

DPS in k_t-factorization

each step:



DPS in k_t-factorization

Generalize the LO collinear approach to
k_t-factorization approach.
More complicated (more kinematical variables) as momenta of outgoing partons are less correlated
We need information about each quark and antiquark

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t} dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{1}{2\sigma_{eff}} \cdot \frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} \cdot \frac{d\sigma}{dy_3 dy_3 d^2p_{3,t} d^2p_{4,t}} \tag{4}$$

DPS in k_t-factorization

Each individual scattering in the k_t-factorization approach

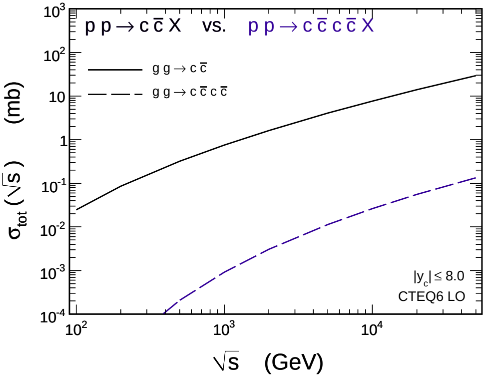
$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \overline{|\mathcal{M}_{off}|^2} \delta\left(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t}\right) \mathcal{F}(x_1, k_{1t}^2, \mu^2) \mathcal{F}(x_2, k_{2t}^2, \mu^2) \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi}$$

$$\frac{d\sigma}{dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \overline{|\mathcal{M}_{off}|^2} \delta\left(\vec{k}_{3t} + \vec{k}_{4t} - \vec{p}_{3t} - \vec{p}_{4t}\right) \mathcal{F}(x_3, k_{3t}^2, \mu^2) \mathcal{F}(x_4, k_{4t}^2, \mu^2) \frac{d^2k_{3t}}{\pi} \frac{d^2k_{4t}}{\pi}$$

Effectively 16 dimensions, Monte Carlo method

Maciula-Szczurek, hep-ph-1301.4469, Phys. Rev.**D87** (2013) 074039.

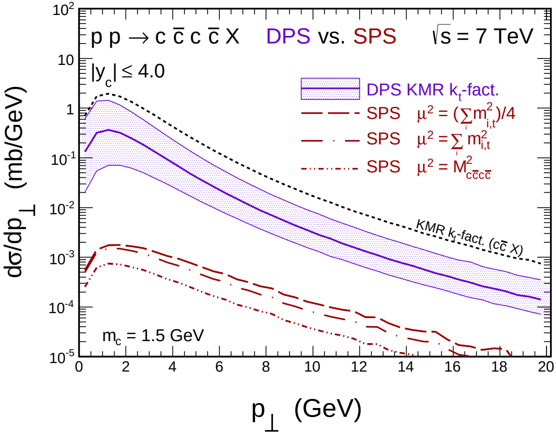
Single parton scattering $2 \to 4$ process?



Only about 1 % at high energies

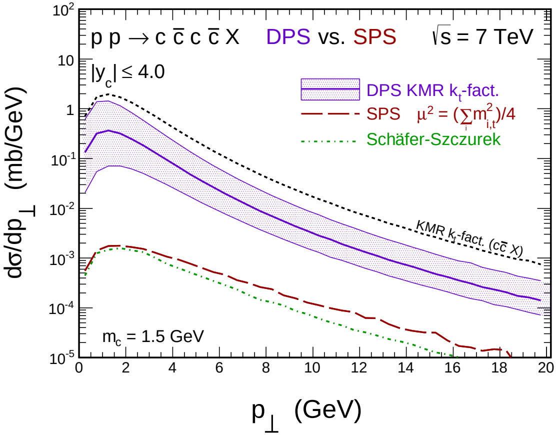
Much smaller than DPS production of $c\bar{c}c\bar{c}$

Exact collinear SPS, quark level



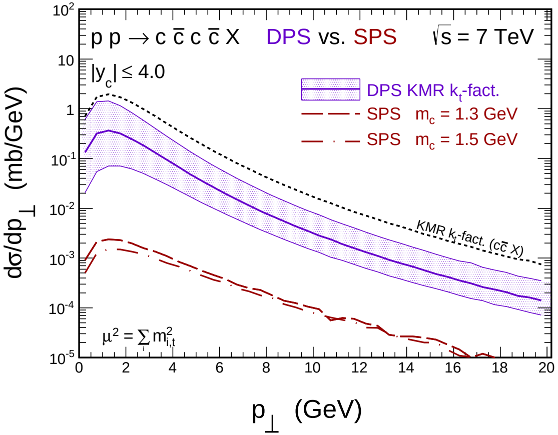
dependence onf renormalization and factorization scale of SPS

Exact collinear SPS, quark level



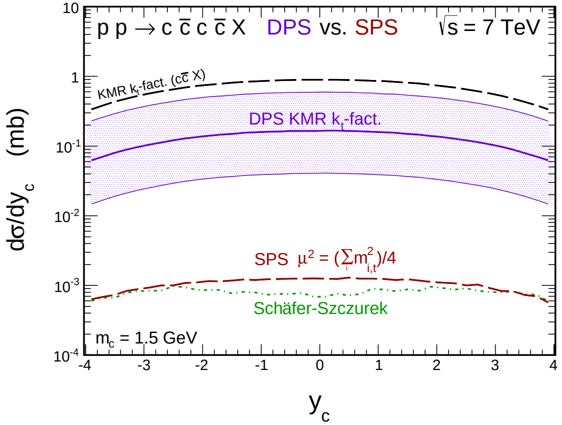
similar shape of SPS and DPS

Exact collinear SPS, quark level



weak dependence on the quark mass for SPS

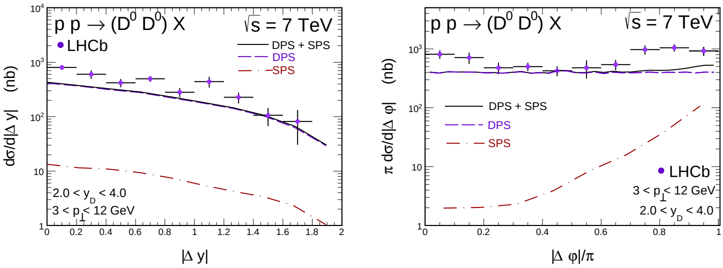
Exact collinear SPS, quark level



Agreement of high-energy approx. and exact at large quark rapidities

Comparison to LHCb data

After hadronization

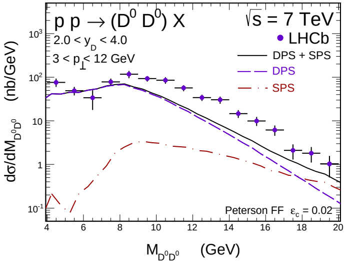


single-parton $c\bar{c}c\bar{c}$ production:

van Hameren, Maciuła, Szczurek, Phys. Rev. **D89** (2014) 094019;

Schafer, Szczurek, Phys. Rev. **D85** (2012) 094029.

Comparison to LHCb data

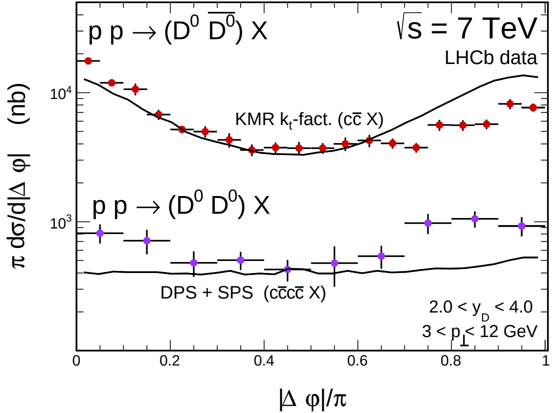


single-parton $c\bar{c}c\bar{c}$ production:

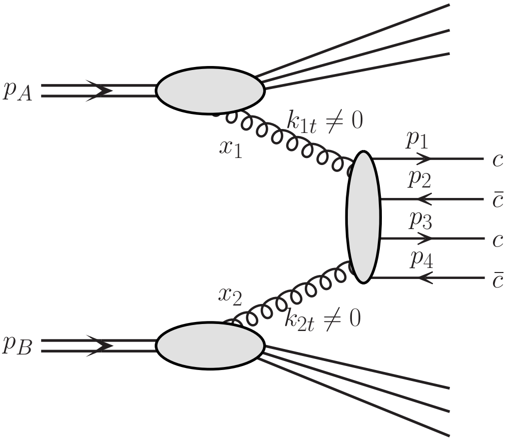
van Hameren, Maciuła, Szczurek, Phys. Rev. **D89** (2014) 094019.

Exact collinear SPS, D meson level

$D^0 D^0$ versus $D^0 \bar{D}^0$ correlations



SPS in k_t-factorization approach



include gluon transverse momenta

A. van Hameren, R. Maciula and A. Szczurek,

arXiv:1504.06490, Phys. Lett. **B748** (2015) 737.

Basic formulae

Within the k_t-factorization approach the SPS cross section for $pp \to c\bar{c}c\bar{c}X$ reaction can be written as

$$d\sigma_{pp\to c\bar{c}c\bar{c}} = \int dx_1 \frac{d^2k_{1t}}{\pi} dx_2 \frac{d^2k_{2t}}{\pi} \mathcal{F}(x_1, k_{1t}^2, \mu^2) \mathcal{F}(x_2, k_{2t}^2, \mu^2) d\hat{\sigma}_{gg\to c\bar{c}c\bar{c}} \, . \tag{5}$$

The elementary cross section in Eq. (5) can be written somewhat formally as:

$$d\hat{\sigma} = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} \frac{d^3p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^4(p_1+p_2+p_3+p_4-k_1-k_2) \times \frac{1}{\text{flux}} \overline{|\mathcal{M}_{g^*g^*\to c\bar{c}c\bar{c}}(k_1,k_2)|^2} \, , \tag{6}$$

where only dependence of the matrix element on four-vectors of gluons k_1 and k_2 is made explicit.

Matrix element squared

Denoting by $\mathcal{M}^a$ the amplitude with the color of one off-shell gluon highlited explicitly we have

$$\sum_a |\mathcal{M}^a|^2 = \sum_a \left| \sqrt{2} \sum_{i,j} \mathcal{M}_{ij} T^a_{ij} \right|^2 = \sum_{i,j,k,l} \mathcal{M}_{ij} \mathcal{M}^*_{kl} \left(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right) = \sum_{i,j} |\mathcal{M}_{ij}|^2 \quad (7)$$

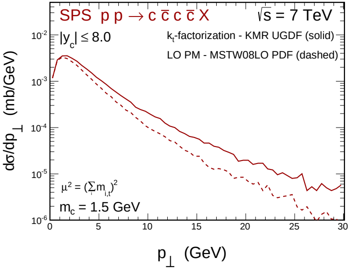
Matrix element squared with off-shell initial gluons calculated with an automated code of A. van Hameren

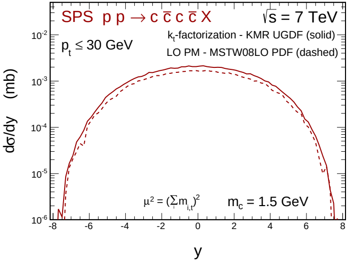
(Dyson-Schwinger recursion method, JHEP. 01 (2013) 078.)

Spinor helicity representation

Monte Carlo generation of events and constructing distributions from the kinematically complete weighted events

First results for k_t-factorization approach





$$\mu_f^2 = \left(\sum_i^4 m_{i,t}\right)^2$$

k_t-factorization and collinear results similar

First results for k_t-factorization approach

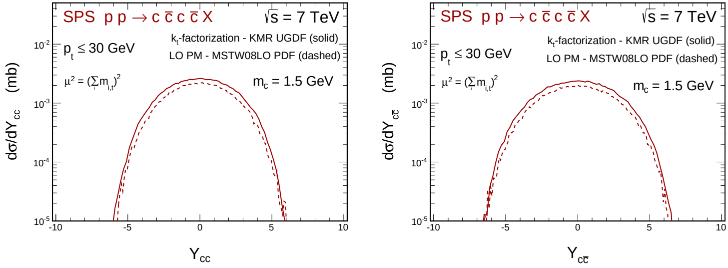


Figure: $Y_{cc} = (y_c + y_c)/2$ (left panel) and $Y_{c\bar{c}} = (y_c + y_{\bar{c}})/2$ (right panel).

$$\mu_f^2 = \left(\sum_i^4 m_{i,t}\right)^2$$

k_t-factorization and collinear results similar

First results for k_t-factorization approach

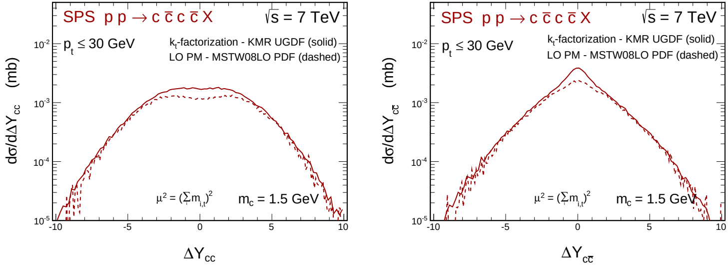


Figure: $\Delta Y_{cc} = y_c - y_c$ (left panel) and $\Delta Y_{c\bar{c}} = y_c - y_{\bar{c}}$ (right panel).

$$\mu_f^2 = \left(\sum_i^4 m_{i,t}\right)^2$$

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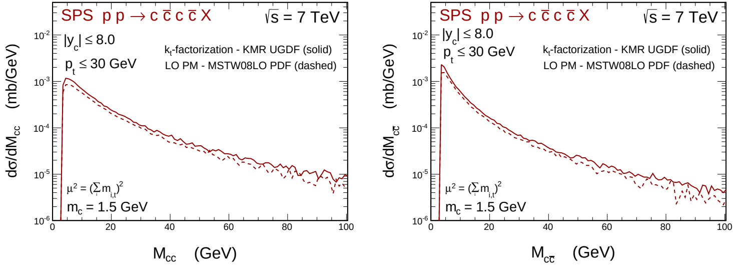


Figure: M_{cc} (left panel) and $M_{c\bar{c}}$ (right panel).

$$\mu_f^2 = \left(\sum_i^4 m_{i,t}\right)^2$$

First results for k_t-factorization approach

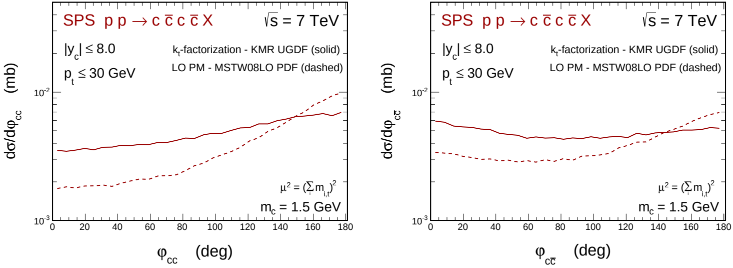
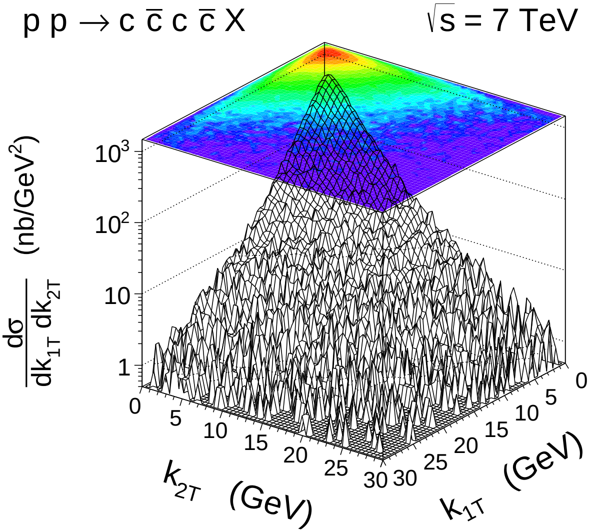


Figure: Azimuthal angle correlations between two c quarks (left panel) and between c and $\bar{c}$ (right panel).

$$\mu_f^2 = \left(\sum_i^4 m_{i,t}\right)^2$$

k_t-factorization gives more decorrelation than collinear-factorization

First results for k_t-factorization approach



First results for k_t-factorization approach

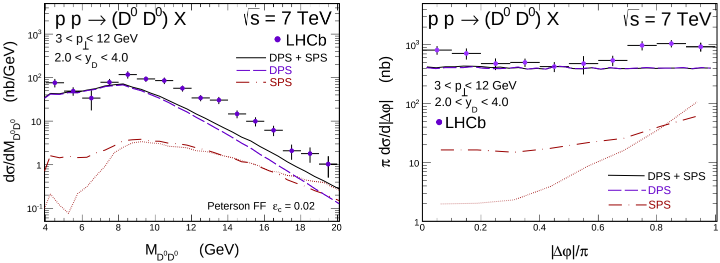
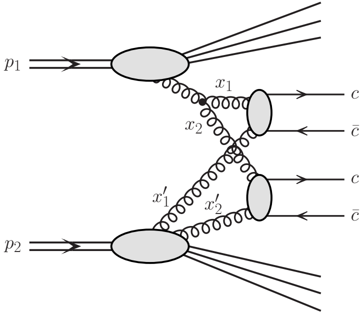


Figure: Distributions in $D^0 D^0$ invariant mass (left) and in azimuthal angle between both D^0's (right) within the LHCb acceptance. The DPS contribution (dashed line) and the SPS contribution within the k_t-factorization approach (dashed-dotted line). The collinear SPS result from our previous studies (dotted line).

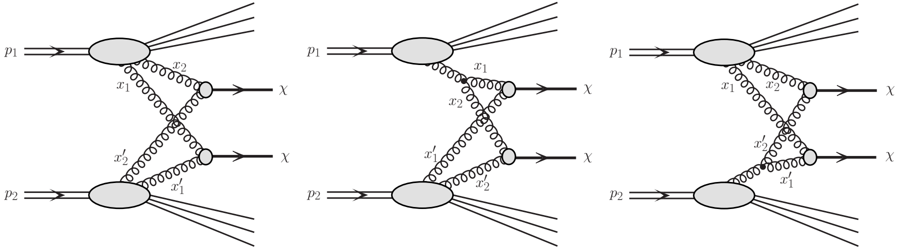
Parton splitting mechanism

There are perturbative mechanisms not included in conventional DPS.



Gaunt, Maciuła, Szczurek, Phys. Rev. **D90** (2014) 054017.

Double quarkonium production



Gaunt, Maciuła, Szczurek, Phys. Rev. **D90** (2014) 054017.

A bit of formalism for parton splitting

Conventional DPS:

$$\sigma(2v2) = \frac{1}{2}\frac{1}{\sigma_{eff,2v2}}\int dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t} \frac{1}{16\pi\hat{s}^2}\overline{|\mathcal{M}(gg\to c\bar{c})|^2}\, x_1 x_1' x_2 x_2'$$
$$\times\, D^{gg}(x_1,x_2,\mu_1^2,\mu_2^2)\; D^{gg}(x_1,x_2,\mu_1^2,\mu_2^2)$$

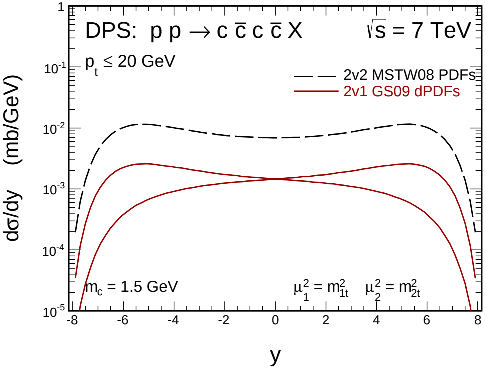
Parton splitting DPS

$$\sigma(2v1) = \frac{1}{2}\frac{1}{\sigma_{eff,2v1}}\int dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t} \frac{1}{16\pi\hat{s}^2}\overline{|\mathcal{M}(gg\to c\bar{c})|^2}\, x_1 x_1' x_2 x_2'$$
$$\times\, \Big(\hat{D}^{gg}(x_1',x_2',\mu_1^2,\mu_2^2) D^{gg}(x_1,x_2,\mu_1^2,\mu_2^2) + D^{gg}(x_1',x_2',\mu_1^2,\mu_2^2)\hat{D}^{gg}(x_1,x_2,\mu_1^2$$

There are two different normalization parameters. They are related in a geometrical picture.

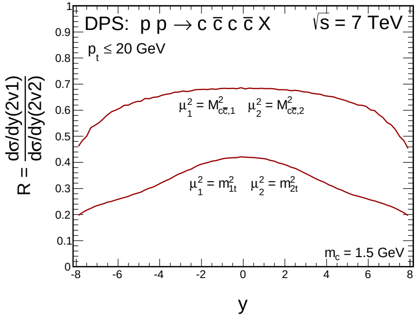
Presence of the two components leads to a dependence of effective parameter on different kinematical variables.

Parton splitting vs conventional DPS



Asymmetric 1v2 and 2v1 contributions

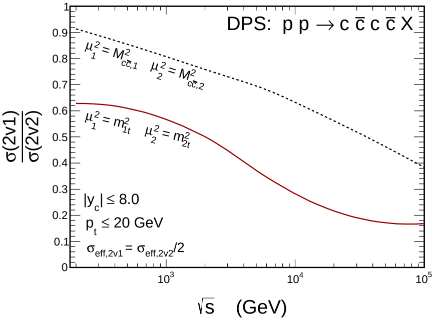
Parton splitting vs conventional DPS



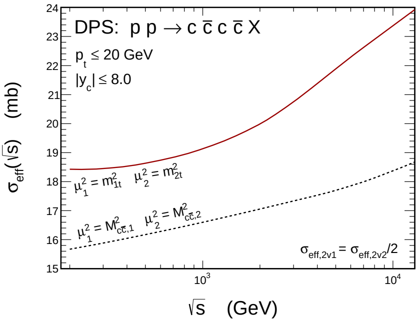
Rapidity and factorization scale dependence

There could be also transverse momentum dependence.

Parton splitting vs conventional DPS



Parton splitting vs conventional DPS



σ_{eff} is no longer a constant

Conclusions

- k_t-factorization provides good description of charm production at RHIC and LHC.
- Surprisingly large cross sections for inclusive $c\bar{c}c\bar{c}$ due to DPS.
- Relatively small cross sections for SPS $c\bar{c}c\bar{c}$.
- Multiple $c\bar{c}$ pairs can be produced in p p collisions at the LHC and FCC.
- Look at correlations between same flavour charmed mesons such as $D^0 D^0$.
- Look at correlations between $e^+\mu^+$ or $e^-\mu^-$ from semileptonic decays (ALICE, CMS).
- Enhancement of the number of $c\bar{c}$ pairs in AA collisions
  $\longrightarrow$ important for recombination/coalescence
  $\longrightarrow$ further enhancement of hidden-charm meson production $(J/\psi, \psi')$ at higher energies.