Statistical issues in future neutrino oscillation experiments

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Outline

- Assessing a "discovery"
- A "discovery" in a future experiment

- The case of Mass Hierarchy (MH)
  - the test statistic and its distributions
  - showing sensitivities
  - parameters and systematics

- The case of CP violation (δ_{CP})
  - the test statistic and its distributions
  - impact on future and on current results

- The Bayesian approach for MH

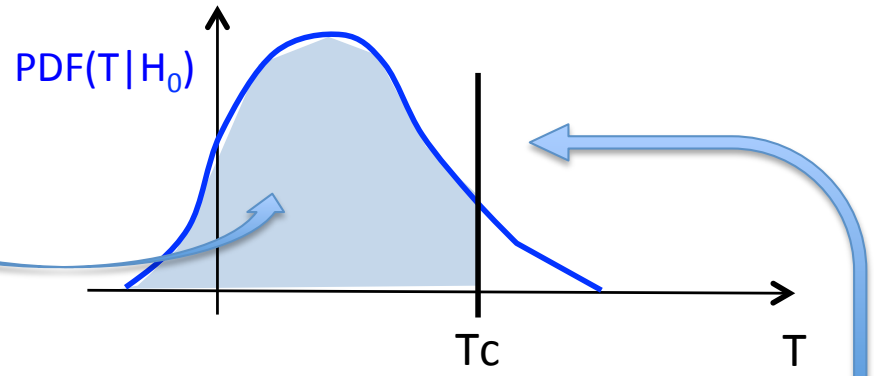
Disclaimer: not showing physics results, focusing on statistical issues !

What is a discovery ?

- Assessing a discovery = excluding a "null hypothesis" H_0
  (e.g.: "MH is inverted", "there is no CPV") with a given CL

- First, construct a test statistic **T**, function of your data
- Given the PDF for the possible values of T, $PDF(T|H_0)$, set the threshold value T_C such that

$$CL = \int_{-\infty}^{T_C} PDF(T \mid H_0)\, dT$$



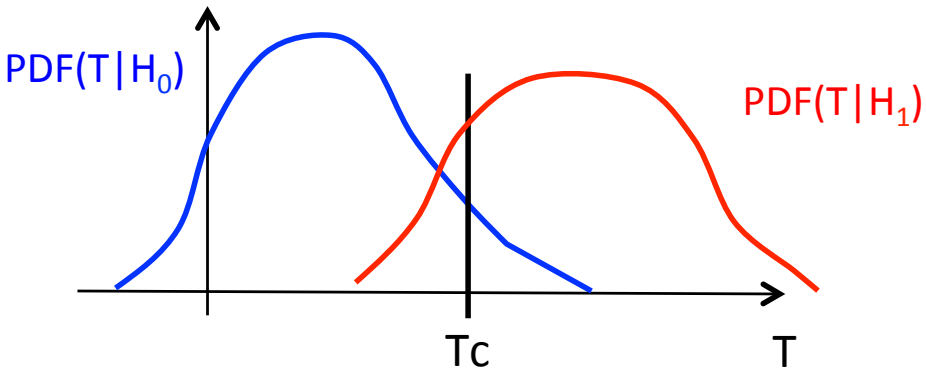
- Exclude H_0 at the given CL if your experiment has provided $T>T_C$

Message 1:

you need to know $PDF(T|H_0)$ to asses the correct T_C for a given CL

What is a discovery in a future experiment ?

- In a future experiment, you still don't know what value of T you will get… need to consider statistical fluctuations
- The possible experimental outcomes if the "alternative hypothesis" H_1 is true will be distributed according to $PDF(T|H_1)$
- You need to make some assumptions on what your actual result will be



Message 2

For future experiments, you also need to know $PDF(T|H_1)$

AND to specify your assumption on the experimental outcome

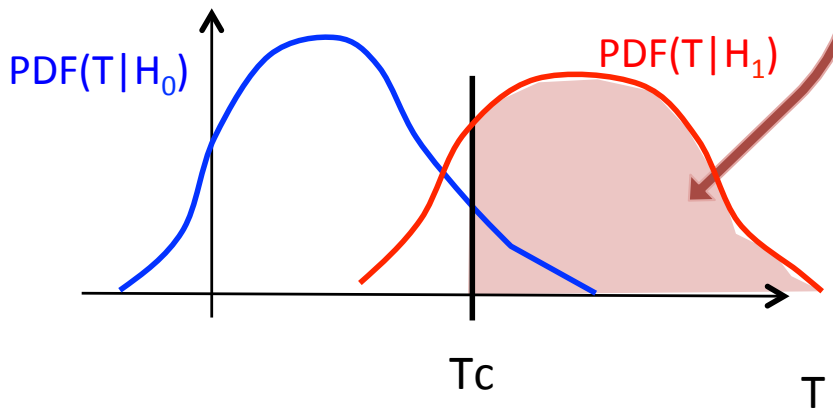
What is a discovery in a future experiment ?

Common approaches for a future experiment

1) for a given CL, consider the possible fluctuations and quote Power

$$P = \int_{Tc}^{+\infty} PDF(T \mid H_1)\, dT$$

~ probability to do "at least as good"

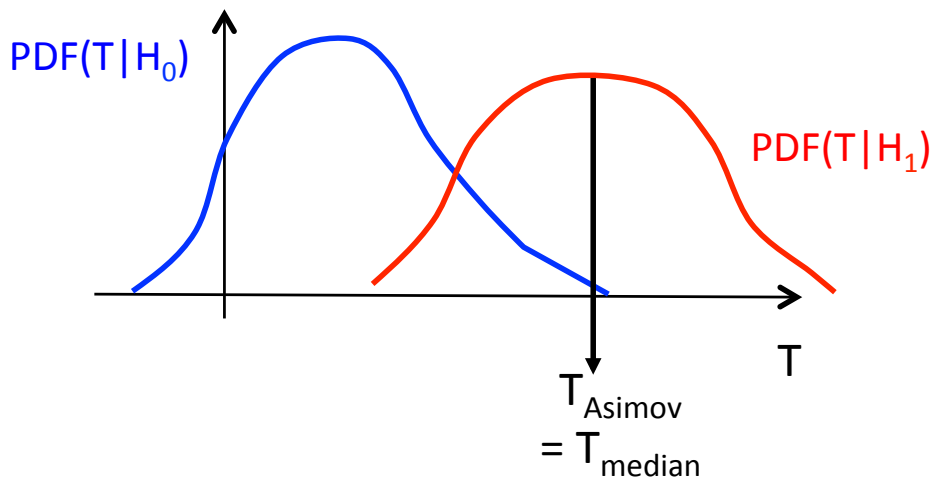


2) Quote the CL with a given outcome

2a) "typical"or "Asimov" experiment: $P(T|H_1)$=max

2b) "median" experiment: P=0.5

coinciding if PDF symmetric



Test statistics for MH

Test statistic for MH : $T = \chi^2_{IH} - \chi^2_{NH}$ ("$\Delta\chi^2$")

where the χ^2 is built on some experimental observable (e.g. energy, Pt, etc.)

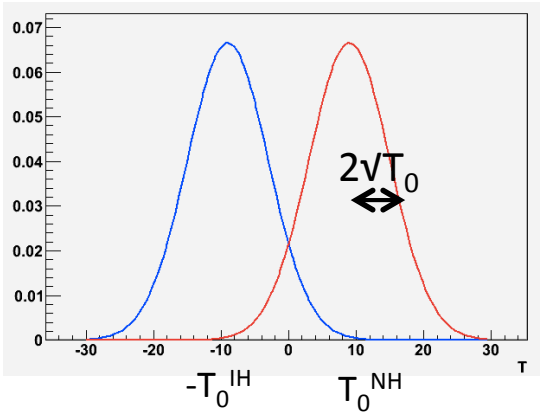
The PDFs of $T=\Delta\chi^2$ are shown to be, in most cases, gaussians with $\sigma=2\sqrt{\mu}$

$PDF(T|IH)=N(-T_0^{IH},2\sqrt{T_0^{IH}})$

$PDF(T|NH)=N(T_0^{NH},2\sqrt{T_0^{NH}})$

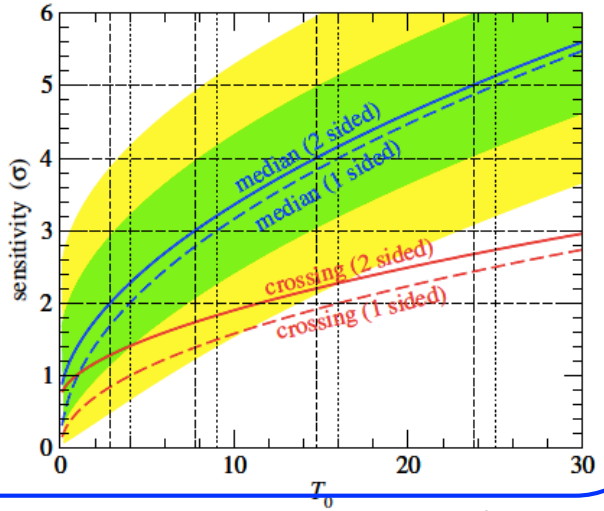
$T_0^{IH/NH}$ increase with exposure

Qian et al., Phys.Rev. D86 (2012) 113011
Ciuffoli et al., JHEP 1401 (2014) 095

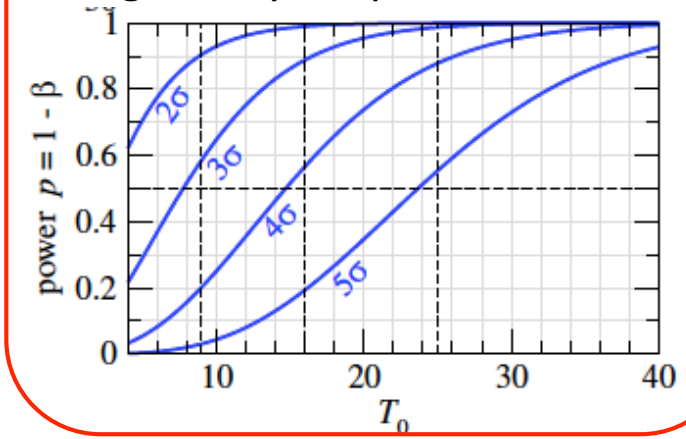


➔ The "median" or "typical" sensitivity is $\#\sigma = \sqrt{T_0}$

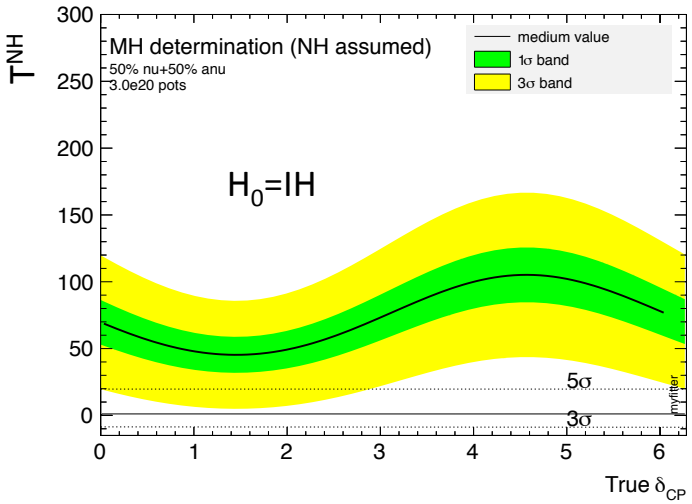
Blennow et al., JHEP 1403 (2014) 028



➔The Power for fixed CL is given by simple formulae



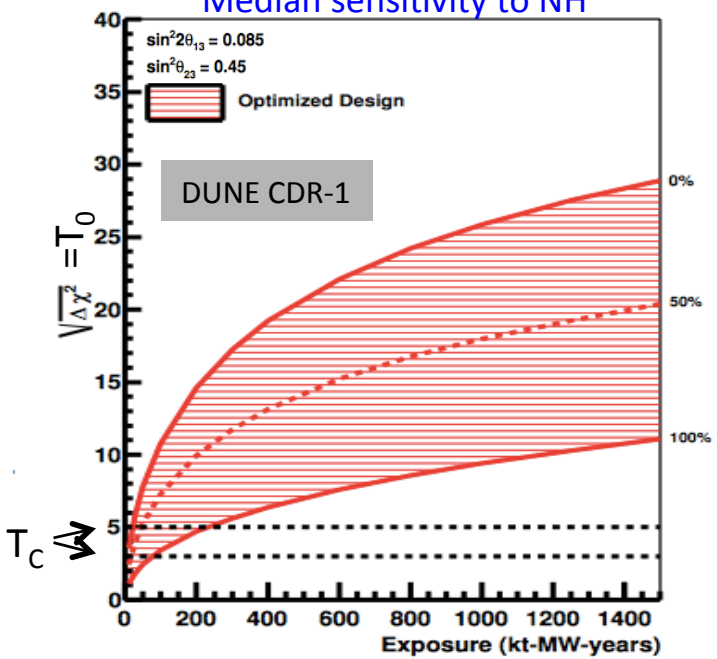
Sensitivity to MH : parameters

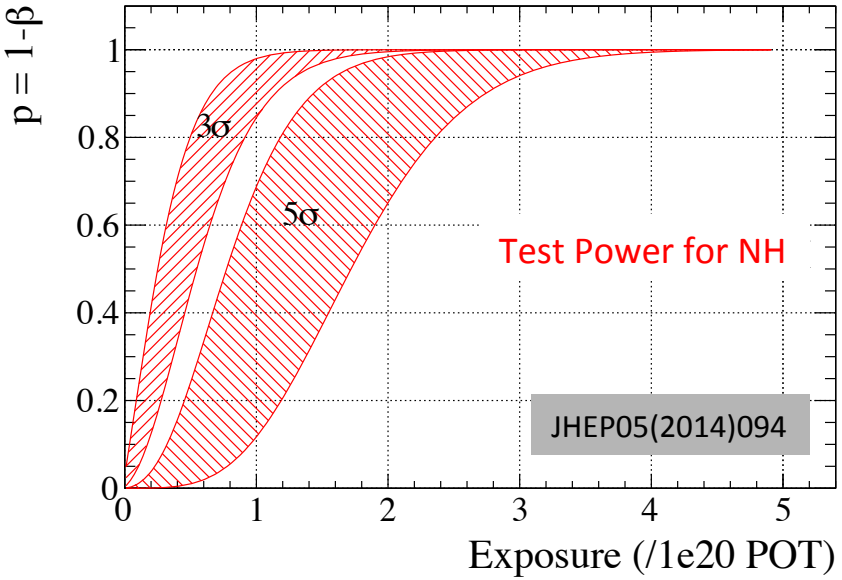


The values of T depend on **parameters** (mixing angles, δ_{CP}, etc.)

1) marginalize on the parameters to compute T_C
2) show the range of (median) sensitivity or of power given the range of the parameters

Median sensitivity to NH



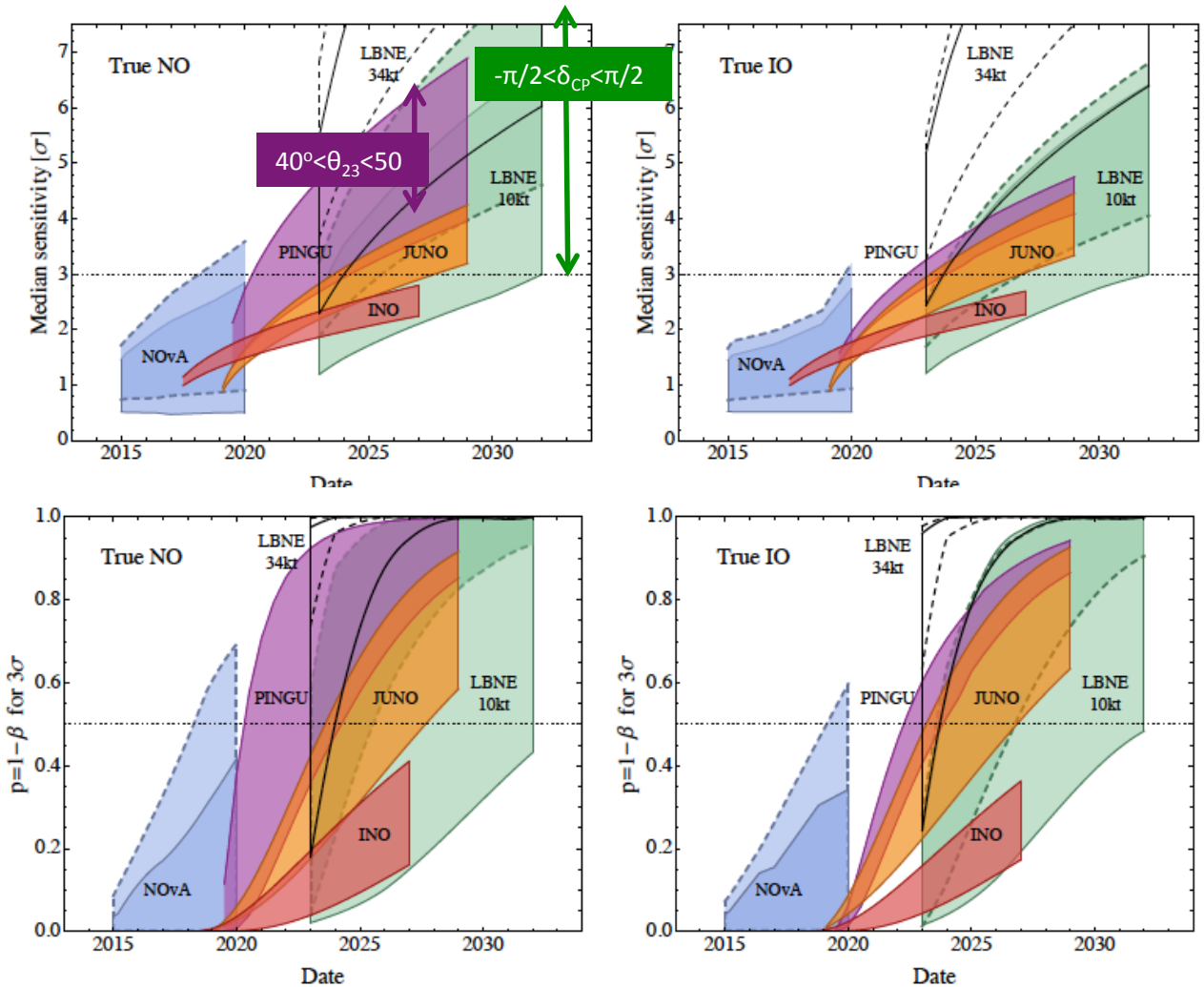


Sensitivity to MH : NH ≠ IH

In general $T_0^{IH} \neq -T_0^{NH}$, so **two sets of information** must be provided:
one for NH discovery (IH exclusion), one for IH discovery (NH exclusion)

for Median sensitivity

for Power



Blennow et al., JHEP 1403 (2014) 028

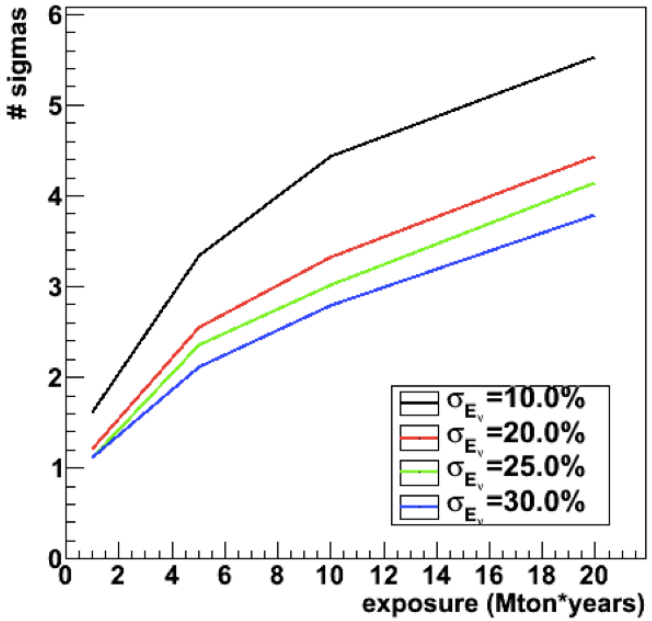
MANY ISSUES CLARIFIED !
values to be updated...

Sensitivity to MH : systematics

<u>Systematics</u> play an important role

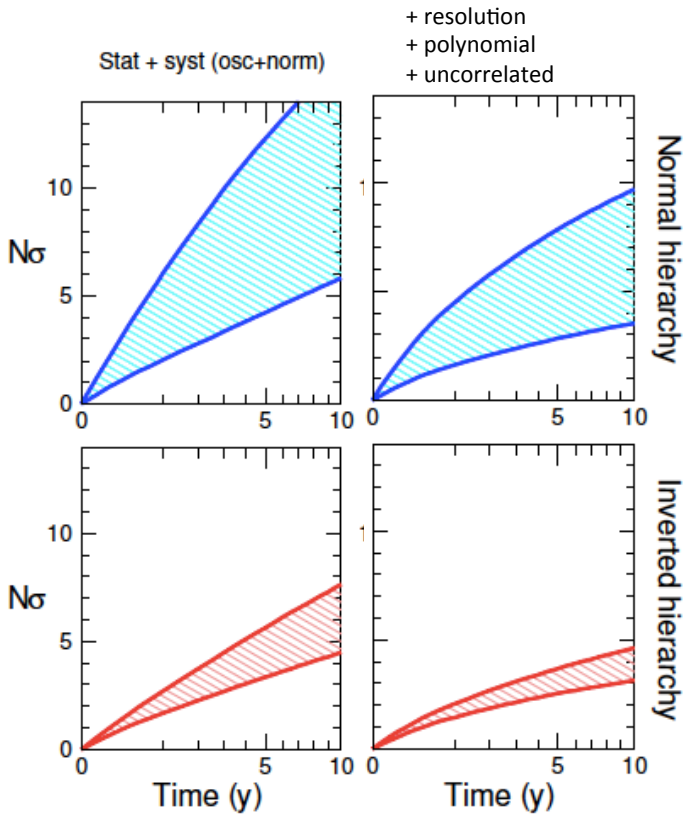
=> they must be accounted for correctly in the PDFs

ORCA



Katz., ArXiv:1402.1022

PINGU



Capozzi et al., ArXiv:1503.01999

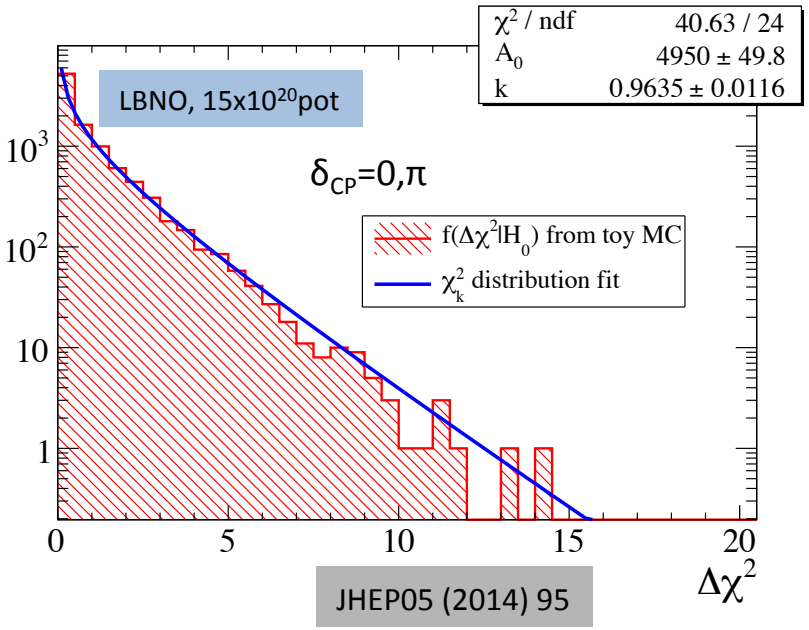
Test statistic for δ_{CP}

Test statistic $\Delta\chi^2 = \min(\Delta\chi^2_{\delta CP=0}, \Delta\chi^2_{\delta CP=\pi})$

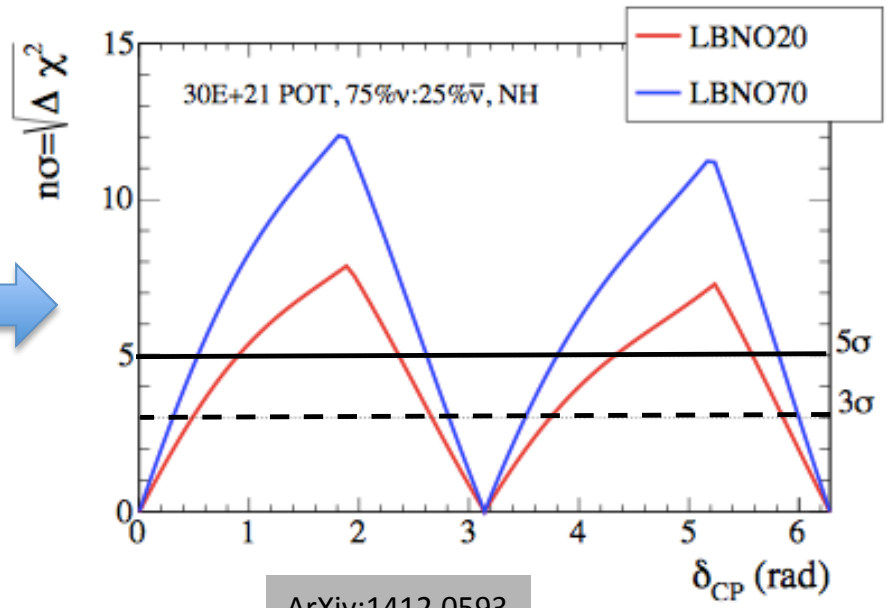
This is a case of "nested hypotheses": $H_0 = \{\delta_{CP}=0 \text{ or } \pi\}$, $H_1 = \{0<\delta_{CP}<2\pi\}$

$\Rightarrow$ Wilk's theorem $PDF(\Delta\chi^2|H_0)=X^2(1dof)$ => $T_C=(\#\text{ of }\sigma)^2$

independent of exposure (unlike for MH)



JHEP05 (2014) 95



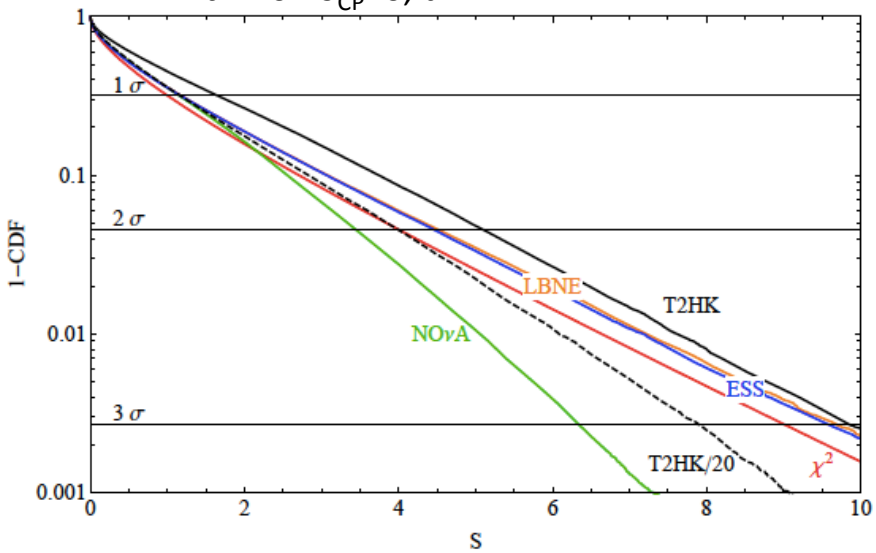
ArXiv:1412.0593

Sensitivity to δ_{CP} : future experiments

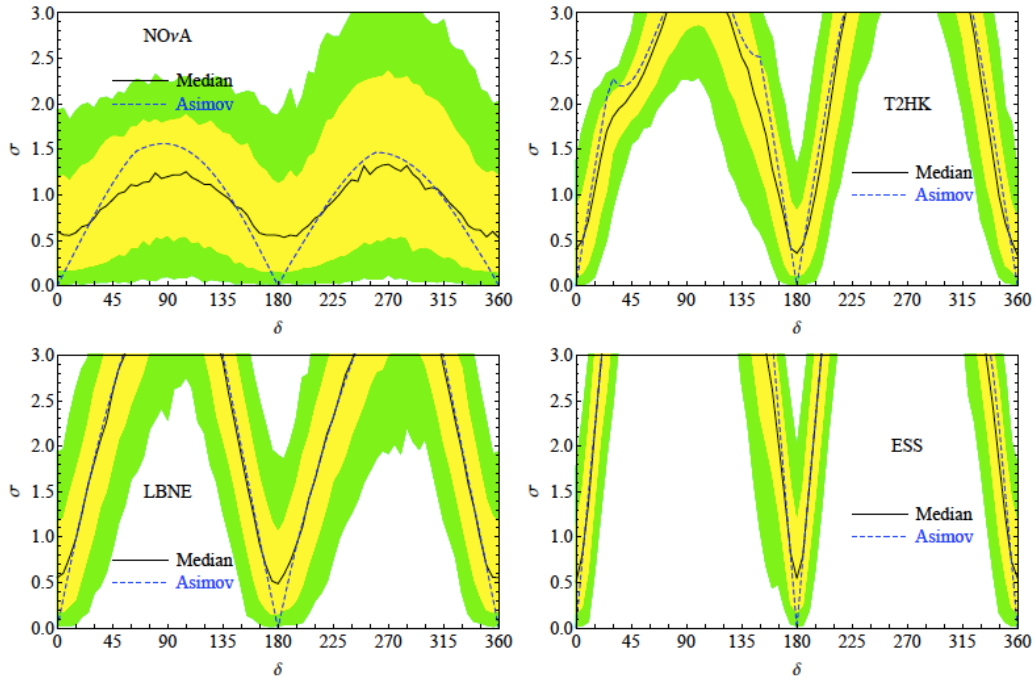
However, Wilk's theorem does not always hold when the experiment has limited sensitivity to δ_{CP} => PDF(T|H_0) is not $\chi^2(1)$

=> need to get PDFs from (toy)-MC

1-cumulative of PDF(T|H_0) from MC with H0 : δ_{CP}=0,π



Blennow et al., ArXiv:1407.3274



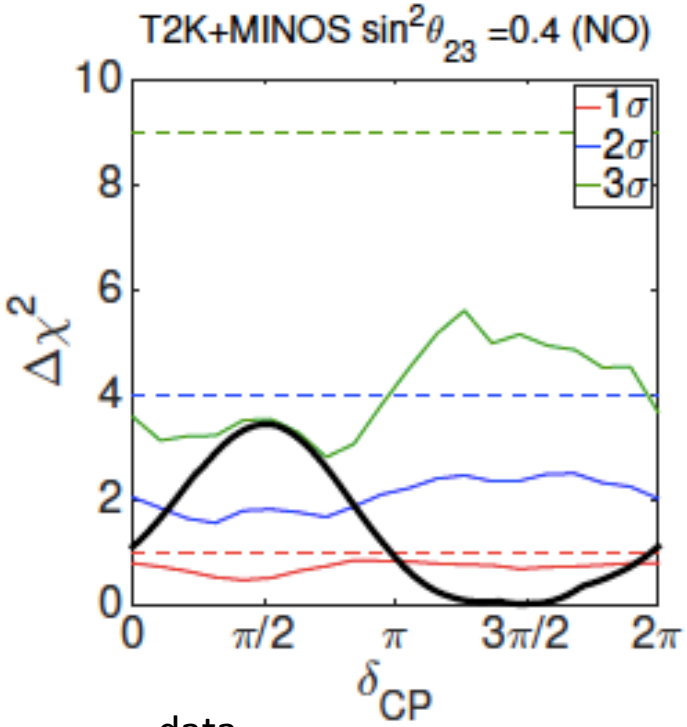
Median sensitivity

- - - - using √T

—— using PDF(T|H_1) from MC

Sensitivity to δ_{CP} : existing results

Deviations of PDF(T|H_0) from $\chi^2(1)$ also affect existing results

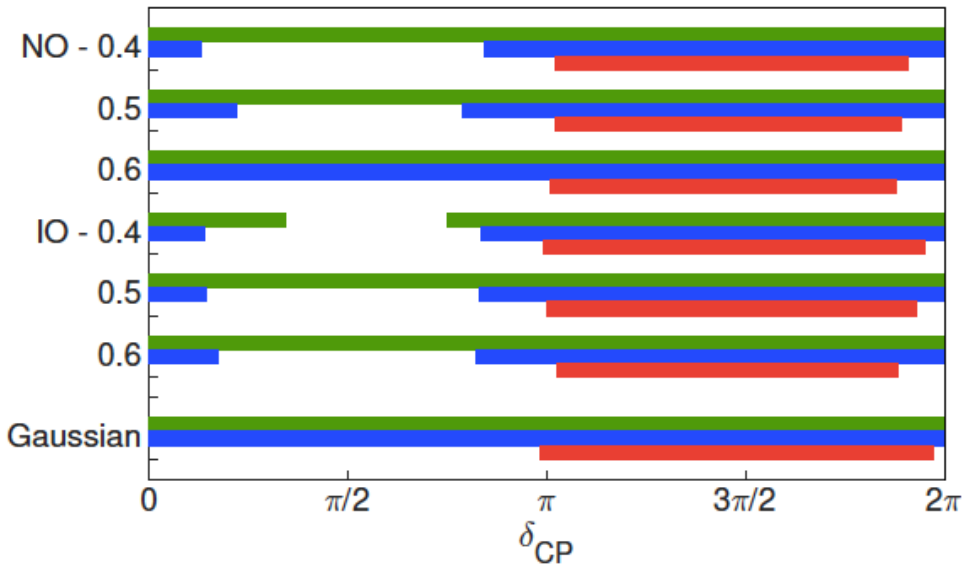


data

T_C with $\chi^2(1)$

T_C with PDF(T|H_0) from MC

Elevant&Schwetz, ArXiv:1506.07685

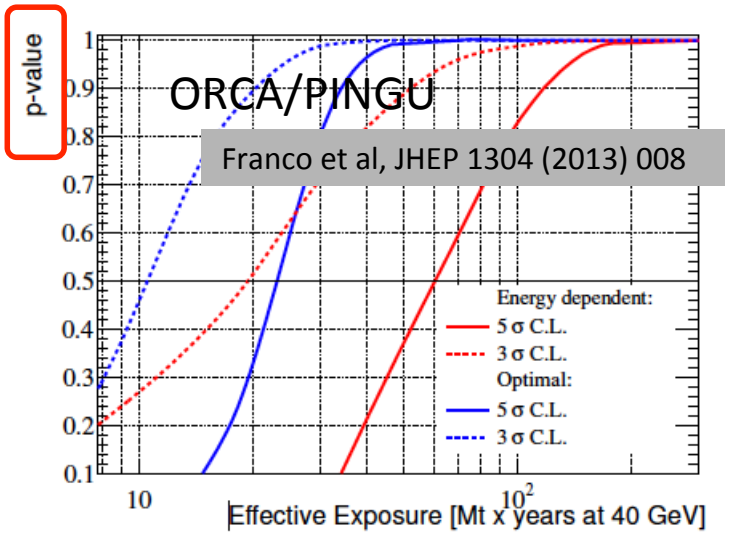


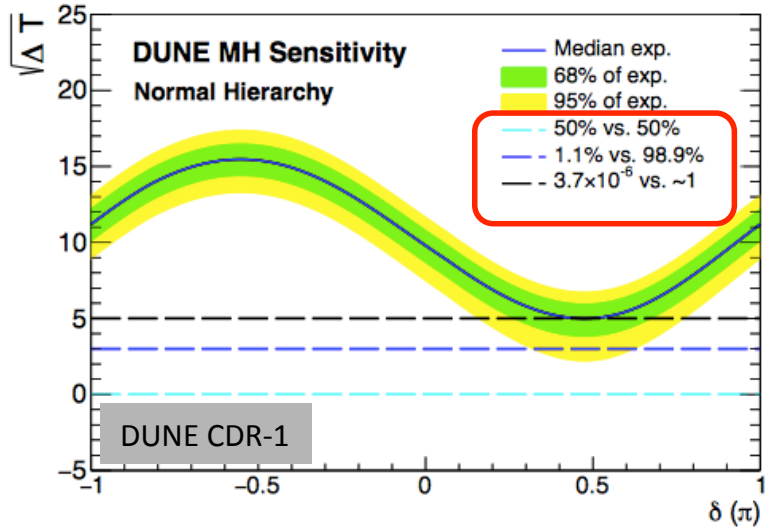
Conf. Intervals for δ_{CP} at 1σ 2σ 3σ using PDF(T|H_0) from MC **[NO,IO]** and using χ^2 (1) approxim. **[Gaussian]** for different $\sin^2\theta_{23}$

The Bayesian approach for MH

In the "frequentist approach" discussed so far, two sets of information must be provided (for NH discovery and for IH discovery).

With a Bayesian approach, a single set of values can contain all the information on the test : the "Odds" or "Ratio of posterior probabilities" Blennow JHEP 1401 (2014) 139





- Need a "prior" P on each hypothesis, providing a relative normalization between the two PDFs
- P(NH)=P(IH)=0.5 is the most conservative and common choice (default)

NOTE: answers a different question than frequentist !

Summary

- To exclude a "null" hypothesis H_0 with a given CL, you need to get the correct **PDF of the test statistic under H_0**
  - ➔ this also affects **current results** on δ_{CP} !
- For a **future experiment**,

  you also need to know the PDF of the test statistic under the "alternative" hypothesis

  AND to **specify your assumptions** on the outcome
  - Asimov ? Median ? quote Power ?
  - ➔ For comparison between experiments at a given CL, any choice is fine as long as it's the same for all
- In "real life", treat correctly dependence on parameters, systematics, etc.

- The **Bayesian approach** answers a different question than the frequentist one

➔ Many papers in the past couple of years have nicely clarified many issues

➔ The bottom line : always state clearly what you are doing / assuming