Statistical issues in future neutrino oscillation experiments

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Outline

• Assessing a “discovery”
• A “discovery” in a future experiment

• The case of Mass Hierarchy (MH)
  – the test statistic and its distributions
  – showing sensitivities
  – parameters and systematics

• The case of CP violation ($\delta_{CP}$)
  – the test statistic and its distributions
  – impact on future and on current results

• The Bayesian approach for MH

Disclaimer: not showing physics results, focusing on statistical issues!
What is a discovery?

- Assessing a discovery = excluding a “null hypothesis” $H_0$
  (e.g.: “MH is inverted”, “there is no CPV”) with a given CL

- First, construct a test statistic $T$, function of your data
- Given the PDF for the possible values of $T$, $PDF(T|H_0)$, set the threshold value $T_C$ such that

$$CL = \int_{-\infty}^{T_C} PDF(T | H_0) dT$$

- Exclude $H_0$ at the given CL if your experiment has provided $T > T_C$

Message 1:

you need to know $PDF(T | H_0)$ to assess the correct $T_C$ for a given CL
What is a discovery in a future experiment?

• In a future experiment, you still don’t know what value of T you will get… need to consider statistical fluctuations
• The possible experimental outcomes if the “alternative hypothesis” $H_1$ is true will be distributed according to $PDF(T|H_1)$
• You need to make some assumptions on what your actual result will be

Message 2
For future experiments, you also need to know $PDF(T|H_1)$ AND to specify your assumption on the experimental outcome
What is a discovery in a future experiment?

Common approaches for a future experiment

1) for a given CL, consider the possible fluctuations and quote Power

\[
P = \int_{T_c}^{+\infty} PDF(T \mid H_1) dT
\]

~ probability to do “at least as good”

2) Quote the CL with a given outcome
2a) “typical” or “Asimov” experiment:
\( P(T \mid H_1) = \text{max} \)
2b) “median” experiment: \( P = 0.5 \)

coinciding if PDF symmetric

\[T_{\text{Asimov}} = T_{\text{median}}\]
The PDFs of $T=\Delta \chi^2$ are shown to be, in most cases, gaussians with $\sigma=2\sqrt{\mu}$

$PDF(T|IH)=N(-T_0^{IH},2\sqrt{T_0^{IH}})$

$PDF(T|NH)=N(T_0^{NH},2\sqrt{T_0^{NH}})$

$T_0^{IH/NH}$ increase with exposure

Ciuffoli et al., JHEP 1401 (2014) 095

The “median” or “typical” sensitivity is 
$#\sigma = \nu T_0$

Blennow et al., JHEP 1403 (2014) 028

The Power for fixed CL is given by simple formulae

Blennow et al., JHEP 1403 (2014) 028

Test statistic for MH: $T = \chi^2_{IH} - \chi^2_{NH}$ ("$\Delta \chi^2$")

where the $\chi^2$ is built on some experimental observable (e.g. energy, Pt, etc.)
The values of $T$ depend on parameters (mixing angles, $\delta_{CP}$, etc.)

1) marginalize on the parameters to compute $T_C$
2) show the range of (median) sensitivity or of power given the range of the parameters
Sensitivity to MH : NH ≠ IH

In general $T_0^{IH} \neq T_0^{NH}$, so **two sets of information** must be provided:
one for NH discovery (IH exclusion), one for IH discovery (NH exclusion)

**MANY ISSUES CLARIFIED !**
values to be updated...

Blennow et al., JHEP 1403 (2014) 028
**Sensitivity to MH : systematics**

**Systematics** play an important role

=> they must be accounted for correctly in the PDFs

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**ORCA**

Katz., ArXiv:1402.1022

**PINGU**

Capozzi et al., ArXiv:1503.01999

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EPS Vienna, July 24th, 2015

A.Tonazzo : Statistical issues in ν oscillation experiments
Test statistic for $\delta_{\text{CP}}$

Test statistic \[ \Delta \chi^2 = \min(\Delta \chi^2_{\delta_{\text{CP}}=0}, \Delta \chi^2_{\delta_{\text{CP}}=\pi}) \]

This is a case of “nested hypotheses”: $H_0 = \{\delta_{\text{CP}}=0 \text{ or } \pi\}$, $H_1 = \{0<\delta_{\text{CP}}<2\pi\}$

$\Rightarrow$ Wilk’s theorem \[ \text{PDF}(\Delta \chi^2|H_0)=X^2(1\text{dof}) \Rightarrow T_C=(\# \text{ of } \sigma)^2 \]

independent of exposure (unlike for MH)
However, Wilk’s theorem does not always hold when the experiment has limited sensitivity to $\delta_{CP} \Rightarrow$ PDF(T|H$_0$) is not $\chi^2(1)$

$\Rightarrow$ need to get PDFs from (toy)-MC

1-cumulative of PDF(T|H$_0$) from MC with H$_0$: $\delta_{CP}=0,\pi$

Median sensitivity

- using $\sqrt{T}$
- using PDF(T|H$_1$) from MC

Blennow et al., ArXiv:1407.3274
Sensitivity to $\delta_{CP}$: existing results

Deviations of $PDF(T|H_0)$ from $\chi^2(1)$ also affect existing results.

Conf. Intervals for $\delta_{CP}$ at $1\sigma$ $2\sigma$ $3\sigma$ using $PDF(T|H_0)$ from MC [NO,IO] and using $\chi^2(1)$ approxim. [Gaussian] for different $\sin^2\theta_{23}$.

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Elevant&Schwetz, ArXiv:1506.07685
The Bayesian approach for MH

In the “frequentist approach” discussed so far, two sets of information must be provided (for NH discovery and for IH discovery).

With a Bayesian approach, a single set of values can contain all the information on the test: the “Odds” or “Ratio of posterior probabilities”

• Need a “prior” P on each hypothesis, providing a relative normalization between the two PDFs
• P(NH)=P(IH)=0.5 is the most conservative and common choice (default)

NOTE: answers a different question than frequentist!
Summary

• To exclude a “null” hypothesis $H_0$ with a given CL, you need to get the correct **PDF of the test statistic under $H_0$**

  ➔ this also affects current results on $\delta_{CP}$!

• For a **future experiment**, you also need to know the PDF of the test statistic under the “alternative” hypothesis

  **AND** to **specify your assumptions** on the outcome

  • Asimov ? Median ? quote Power ?

  ➔ For comparison between experiments at a given CL, any choice is fine as long as it’s the same for all

• In “real life”, treat correctly dependence on parameters, systematics, etc.

• The **Bayesian approach** answers a different question than the frequentist one

  ➔ Many papers in the past couple of years have nicely clarified many issues

  ➔ The bottom line: always state clearly what you are doing / assuming