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Outline

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Motivations

- Recent studies suggest that DM is not collisionless
- The DM temperature, $T_\chi$, is not necessarily to be the same as the core temperature of the Sun, $T_c$
- Thermal energy can exchange between DM-DM and DM-nucleus or even dissipates through annihilation
- For much weaker $\sigma_{\chi p}$, the thermal energy exchange between DM-nucleus is less efficient
- In such case, $T_\chi$ can be distinct from $T_c$ eventually
- As a consequence, it may alter the DM annihilation rate
How does thermal energy transport?
Energy flow via capture

The capture could be due to scattering with nucleus or DM themselves

$u$: DM velocity in the halo
$w(r)$: DM velocity when arrives at layer $r$ in the Sun
DM kinetics after scattering

- When DM reaches layer $r$ in the Sun, it carries velocity

$$w = \sqrt{u^2 + v_{\text{esc}}^2(r)}$$

- Thus, the average DM kinetic energy after scattering with nucleus is given by

$$\bar{E}_K(r) = \frac{m_\chi}{4} \left( \frac{m_\chi - m_A}{m_\chi + m_A} \right)^2 u^2 + \frac{m_\chi}{2} \frac{m_\chi^2 + m_A^2}{(m_\chi + m_A)^2} v_{\text{esc}}^2(r)$$

- However, the scattering among DM themselves gives

$$\bar{E}_K(r) = \frac{1}{4} m_\chi v_{\text{esc}}^2(r)$$

- $m_A$ is the nucleus mass of element $A$ in the Sun
Energy flow via thermal contact

$T_x > T_A$
Heat flow

$T_x < T'_x$
Heat flow

Thermal contact via DM and nucleus A

Thermal contact via DM themselves

Mean collision time

- The mean collision time is used to determine which one reaches thermal equilibrium earlier, DM-DM or DM-nucleus.

- Thereupon, by $\tau \sim 1 / (n \sigma v)$ we have:

$$
\begin{align*}
\tau_{\chi \chi}(t) & \simeq \frac{V_\odot}{N_\chi(t) \sigma_{\chi \chi} v}, \quad \text{for DM-DM} \\
\tau_{\chi \odot} & \simeq \frac{V_\odot}{\sum_i N_i \sigma_{\chi A_i} v}, \quad \text{for DM-nucleus}
\end{align*}
$$

- Suppose the time scale for DM to reach thermal equilibrium is $t = \tau_{\chi}^{\text{eq}}$, we have:

$$
\tau_{\chi}^{\text{eq}} \simeq \tau_{\chi \chi}(\tau_{\chi}^{\text{eq}}) = \frac{V_\odot}{N_\chi(\tau_{\chi}^{\text{eq}}) \sigma_{\chi \chi} v}
$$
Thermal equilibrium time scale

- Assuming when DM reaches its own thermal equilibrium, $N_\chi$ is still its early stage of accumulation.
- In other words, the capture by DM-nucleus scattering is dominant, $N_\chi \approx C_c t$.
- By such assumption, we can define the ratio $r$

$$r \equiv \frac{\tau_\chi^{eq}}{\tau_\chi^\odot} \approx 10^9 \sqrt{\frac{\sigma_{xp}}{\sigma_{xx}}}$$

- For $r < 1$, DMs will reach their own thermal equilibrium quicker than with the solar medium.
- Thus it is sufficient to just consider the thermal contact between DM and nucleus after $t = \tau_\chi^{eq}$.
Energy dissipation via annihilation

Annihilation takes DM away and transports them into the final SM products.
Defining physical quantities

Energy injections due to gravitational capture and self-capture

Energy dissipation due to annihilation

Heat bath

DM reservoir $N_x$

Thermal contact between DM-nucleus

$J_c$ & $J_s$

$J_x$
DM evolution process and thermal exchange
DM evolution in the Sun

- The common DM evolution equation:
  \[
  \frac{dN_\chi}{dt} = C_c + C_s N_\chi - C_a N_\chi^2
  \]

- Additionally, \( N_\chi \) contributes to the total energy evolving as well. Thus, we have the energy evolution equation:
  \[
  \frac{d(N_\chi \bar{E})}{dt} = J_c + (J_\chi + J_s)N_\chi - J_a N_\chi^2
  \]
  \[
  \bar{E} = \frac{s}{2} k_B T_\chi(t)
  \]

- \( T_\chi(t) \) is the average DM temperature and \( s \) denotes the d.o.f. of DM. We take \( s = 3 \) for subsequent discussions.
$T_\chi$-dependent quantity in $dN_\chi/dt$

- The two differential equations are mutual dependent since $C_a$ is $T_\chi$-dependent\(^2\)

\[
C_a = \frac{\langle \sigma v \rangle V_2}{V_1^2} \quad \quad V_j \propto \left( \frac{T_\chi}{T_c} \frac{10 \text{ GeV}}{j m_\chi} \right)^{3/2} \quad j = 1, 2
\]

- How $T_\chi$ evolving will affect $N_\chi$ accumulation and then feedback to $T_\chi$ itself

- The factors $J_{c,s,\chi,a}$ are all $T_\chi$-dependent

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Physical implications
\(N_\chi\) and \(T_\chi\) evolutions: Stronger \(\sigma_{\chi p}\)

- \(r \sim 0.01\) for \((\sigma_{\chi p}, \sigma_{\chi \chi}) = (10^{-45}, 10^{-23})\) cm\(^2\)
- Although \(J_c\) and \(J_s N_\chi\) transport energy into the DM reservoir, the thermal contact \(J_\chi N_\chi\) makes the heat out of the reservoir due to larger \(\sigma_{\chi p}\) as well as the \(J_a N_\chi^2\)
- Eventually \(T_\chi\) will be balanced by the Sun with the core temperature \(T_c\)
$N_\chi$ and $T_\chi$ evolutions: Weaker $\sigma_{\chi p}$

- $r \sim 10^{-4}$ for $(\sigma_{\chi p}, \sigma_{\chi\chi}) = (10^{-47}, 10^{-23})$ cm$^2$
- In such case, the energy injection via $J_c$ and $J_s$ can overcome the dissipation due to $J_\chi$ and $J_a$
- Moreover, energy injects to the DM reservoir constantly and makes them form their own thermal system
- The $T_\chi$ is distinct from $T_c$ in this case
The total annihilation rate $\Gamma_A$

- The total annihilation rate
  \[ \Gamma_A = \frac{1}{2} C_a N^2 \]
- Since $C_a \propto T_\chi^{-1.5}$, larger $T_\chi$ makes smaller $C_a$.
- However, smaller $C_a$ leads to more $N_\chi$ accumulation once $dN_\chi/dt = 0$ is attained\[^{[3]}\]
- When $N_\chi^2$ increment can overcome the $C_a$ suppression, we have larger $\Gamma_A$ compared to those without temperature correction.
- For some cases, $dN_\chi/dt = 0$ does not attain at current epoch. It makes the corresponding $\Gamma_A$ smaller than those without temperature correction.

\[^{[3]}\] C.-S. Chen et al., JCAP 10, 049 (2014)
Summary
To summarize so far...

- The DM temperature does not necessarily equal to the Sun’s core temperature for collisional DM.
- As a consequence of $\tau^{\text{eq}}_{\chi} < \tau^{\odot}_{\chi}$, DM reaches thermal equilibrium before it starts thermal exchange with the solar nucleus efficiently.
- If $\sigma_{\chi p}$ is small enough, eventually DM will have its own temperature inside the Sun regardless of $T_c$.
- For more accurate prediction on the DM signal, the temperature correction factor should be considered.