Recent progress on the gauge theory sector of F-theory

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Motivation
Why F-theory?

F-theory

1. describes **broad class** of **non-perturbative vacua** of string theory,

2. can produce **GUT models** with **promising particle physics & cosmology**:
   - **features not accessible in perturbative II strings** ($E_6$ to $E_8$, $10\times10\times5$, ...).
   - **Local**: [Donagi, Wijnholt; Beasley, Heckman, Vafa; ... many works]
   - **Global**: [Blumenhagen, Grimm, Jurke, Weigand; Marsano, Saulina, Schäfer-Nameki; ... many works]

3. engineers **effective field theories** coupled to quantum gravity:

   ![Diagram](Calabi-Yau (CY) geometry - Geometry - Physics - Effective field theories)

   - **Geometry provides tools to control over non-perturbative physics.**
Goal of this talk

Goal: Use F-theory to study gauge theory sectors in $N=1$ SUGRA theories.

Problem: geometry / physics dictionary incomplete

- Well-understood for non-Abelian groups & simple matter representations.
- Less known about $U(1)$'s, discrete gauge groups & more complicated matter representations.

Today: develop some missing pieces

- Arithmetic of CY-elliptic fibrations
- Enlarge matter sector: new Abelian & non-Abelian representations
1) What is an F-theory vacuum?
1. **Base** $B$ of $X$  
   $\Rightarrow$ part of **physical space-time** of string theory

2. **Torus fiber** $T^2$ of $X$  
   $\Rightarrow$ **book-keeping** device for Type IIB complexified string coupling $\tau \equiv C_0 + ig_s^{-1}$
Singularities of CY manifolds & physics

Singularities of $T^2$-fibration of Calabi-Yau $X$ over base $B$ ↔ globally well-defined setup of intersecting $(p,q)7$-branes

Gauge theory in 8D: co-dim. one singularity (7-branes)

Matter in 6D: co-dim. two sing. (intersec. 7-branes)

4D Yukawa: co-dim three
$pt = S \cap S' \cap S''$

[Katz, Vafa]
Singularities of CY manifolds & physics

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Gauge theory in 8D: co-dim. one singularity (7-branes)

obtain only non-Abelian groups, no U(1)'s

[Johnson, Tate, Vafa, Morrison, Vafa; Bershadsky, Intriligator, Kachru, Morrison, Sadov, Vafa]

Matter in 6D: co-dim. two sing. (intersec. 7-branes)

4D Yukawa: co-dim three

$pt = S \cap S' \cap S''$
2) Global F-theory compactifications with U(1) symmetries
U(1)’s in F-theory & the Mordell-Weil group

- U(1)’s arise by KK-reduction of M-theory three-form $C_3 \supset A^m \omega_m$.
- Not from codimension one singularities: otherwise again non-Abelian groups.

$(1,1)$-form $\omega_m \leftrightarrow$ rational section of $X$ [Morrison, Vafa II]

Rational section = map $\hat{s}_Q : B \to X$ induced by rational point $Q$ on $T^2 = \text{elliptic curve } E$.

- Rational points form Abelian group: Mordell-Weil (MW) group of rational sections of $X$
- $\hat{s}_Q$ gives rise to a second copy of $B$ in $X$: new divisor $B_Q$ in $X$

$(1,1)$-form $\omega_m$ constructed from divisor $B_Q$. 
U(1)’s in F-theory & the Mordell-Weil group

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Rational section = map $\hat{s}_Q : B \rightarrow X$ induced by rational point $Q$ on $T^2 = $ elliptic curve $E$.

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Systematic construction of F-theory vacua with U(1)’s

- $n$ rational sections of CY-manifold $X \rightarrow$ F-theory with $U(1)^n$ gauge group
- Deligne: Systematic construction of CY $X$ with $n$ rational sections

![elliptic curve $E$ embedded into $W\mathbb{P}^m$]

Examples:

one $U(1)$: elliptic curve $E$ is generic CY in $\text{Bl}_1\mathbb{P}^2(1,1,2)$  [Morrison,Park]

$\rightarrow$ Construction yields only matter with $U(1)$-charge $q=2$,

$\rightarrow$ Extension to models with $q=3$ matter: $E$ is cubic CY in $dP_1$. [DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]
Systematic construction of F-theory vacua with U(1)’s

\( U(1)^2: \) \( E \) is non-generic cubic in \( \mathbb{P}^2 \)

1. **generic CY in \( dP_2 \):** has restricted matter spectrum with \( U(1)^2 \) charges \((q_1,q_2)\)
   
   \[ \text{toric } dP_2: \]
   
   \[
   u f_2(u, v, w) + \prod_{i=1}^{3} (a_i v + b_i w) = 0
   \]
   
   \[
   f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2
   \]

2. **generalization:** CY with \( U(1)^2 \) has fully symmetric matter spectrum
   
   \[ \text{non-toric model:} \]
   
   \[
   u f_2(u, v, w) + \prod_{i=1}^{3} (a_i v + b_i w) = 0
   \]
   
   All spectra automatically anomaly-free.
New realizations of matter singularities

“UnHiggs” \( U(1)^2 \rightarrow SU(3) \)

- \((q_1,q_2) = (2,2)\) matter becomes symmetric representation 6 of \( SU(3) \): first concrete construction in global F-theory.

- related to new algebraic description of \( I_3^s \) singularities over divisor \( T \)

locus of 6 matter:
ordinary double point
New realizations of matter singularities
[Cvetic,DK,Piragua,Taylor]

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locus of $6$ matter: ordinary double point
3) Global F-theory compactifications with discrete gauge groups
F-theory with discrete gauge groups

Torus fibration $X$ has no sections, only $n$-section: genus-one fibration

$\Rightarrow$ $X$ has $n$ rational sections “locally” but they are interchanged globally.

$\Rightarrow$ only sum well-defined globally

$$Q^{(n)} = Q_1 + \ldots + Q_n$$

Obstruction to gluing points together globally: Tate-Shafarevich (TS) group visible in physics as discrete gauge group of F-theory.

[DeBoer,Dijkgraaf,Hori,Keurentjes,Morgan,Morrison,Sethi]
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All known examples

F-theory vacua with $\mathbb{Z}_n$ discrete gauge groups identified

- $\mathbb{Z}_2$ gauge group: $T^2 = \text{quartic in } \mathbb{P}^2(1,1,2)$
  \[ \mathbb{Z}_2 \text{ and } (\text{SU}(2) \times \mathbb{Z}_4)/\mathbb{Z}_2 \]
  [Braun, Morrison; Morrison, Taylor Anderson, García-Etxebarria, Grimm, Keitel; DK, Mayorga-Pena, Oehlmann, Piragua, Reuter; García-Etxebarria, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand]

- $\mathbb{U}(1) \times \mathbb{Z}_2$ gauge group: $T^2 = \text{bi-quadric in } \mathbb{P}^1 \times \mathbb{P}^1$
  [DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]

- $\mathbb{Z}_3$ discrete gauge group: $T^2 = \text{cubic in } \mathbb{P}^2$
  [DK, Mayorga-Pena, Oehlmann, Piragua, Reuter; Cvetič, Donagi, DK, Piragua, Poretschkin]

In all cases we have found

- matter carrying non-trivial $\mathbb{Z}_n$ discrete charge,
- all gauge invariant Yukawas exist, including $\mathbb{Z}_n$ selection rules.
4) Conclusions
Summary

- Systematic construction of F-theory vacua with $U(1)^n$ ($n \leq 3$):
  
  | Mordell-Weil group | U(1)’s |
  |

- Construction of F-theory vacua with $\mathbb{Z}_n$ ($n \leq 4$) discrete gauge groups:
  
  | Tate-Shafarevich group | discrete gauge group |
  |

- New matter representations:
  
  $\Rightarrow$ charge $q=3$ for one $U(1)$, new $U(1)^2$ models with up to charges $(2,2)$
  
  $\Rightarrow$ matter with charges under discrete gauge group,
  
  $\Rightarrow$ first concrete construction of symmetric representation of $SU(3)$. 
Things I didn’t have time to talk about

Construction of $U(1)^3$: $E$ is pencil of non-generic quadrics in $\mathbb{Bl}_4\mathbb{P}^4$. [Cvetic,DK,Piragua,Song]

Pheno applications: Use $U(1)$’s for construction of SM in F-theory

Construction in [Cvetič,DK,Mayorga-Pena,Oehlmann,Reuter]

- $U_Y(1)$ rational section
- SM non-Abelian gauge group $SU(3) \times SU(2)$ is automatically present
- add $G_4$-flux to generate 4D chirality following [Cvetic,Grassi,DK,Piragua]
- solve D3-brane tadpole

⇒ get 4D three-family Standard models in F-theory.

⇒ natural embedding into Pati-Salam & Trinification
Thank you