

# Recent progress on the gauge theory sector of F-theory

Denis Klevers



*arXiv:1507.05954: M. Cvetič, D.K., H. Piragua, W. Taylor*

*arXiv:1502.06953: M. Cvetič, R. Donagi, D.K., H. Piragua, M. Poretschkin*

*arXiv:1408.4808: D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter*

*arXiv:1303.6970, arXiv:1307.6425: M. Cvetič, D.K., H. Piragua*

*(arXiv:1503.02068: M. Cvetič, D.K., D. Mayorga Peña, P. Oehlmann, J. Reuter*

*arXiv:1310.0463: M. Cvetič, D.K., H. Piragua, P. Song*

*arXiv:1306.3987: M. Cvetič, A. Grassi, D.K., H. Piragua)*

# Motivation

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# Why F-theory?

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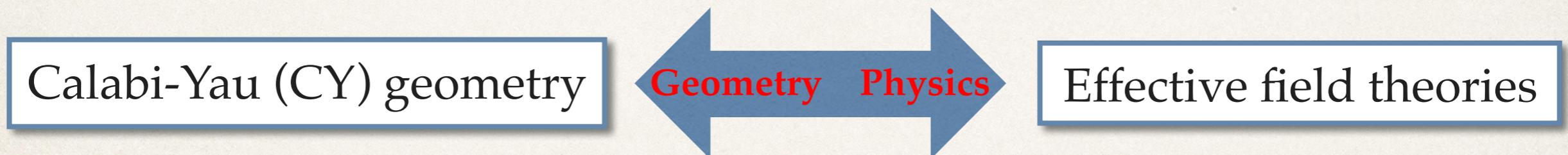
## F-theory

1. describes broad class of non-perturbative vacua of string theory,
2. can produce **GUT models** with **promising particle physics & cosmology**:  
→ **features not** accessible in **perturbative** II strings ( $E_6$  to  $E_8$ ,  $10 \times 10 \times 5, \dots$ ).

**Local:** [Donagi, Wijnholt; Beasley, Heckman, Vafa; ... many works]

**Global:** [Blumenhagen, Grimm, Jurke, Weigand; Marsano, Saulina, Schäfer-Nameki; ... many works]

3. engineers effective field theories coupled to quantum gravity:



- Geometry provides **tools to control** over **non-perturbative physics**.

# Goal of this talk

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Calabi-Yau (CY) geometry

Geometry Physics

Effective field theories

Goal: Use F-theory to **study gauge theory sectors** in  $N=1$  SUGRA theories.

Problem: geometry / physics dictionary **incomplete**

- ❖ **Well-understood for non-Abelian** groups & **simple matter** representations.
- ❖ **less known** about  $U(1)$ 's, **discrete gauge groups** & more **complicated matter** representations.

Today: **develop** some **missing pieces**

- ❖ **Arithmetic of CY-elliptic fibrations** ↔ **global gauge group**
- ❖ **Enlarge matter sector:** **new** Abelian & non-Abelian **representations**

1) What is an F-theory vacuum?

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# Defining data: torus-fibered Calabi-Yau manifold $X$

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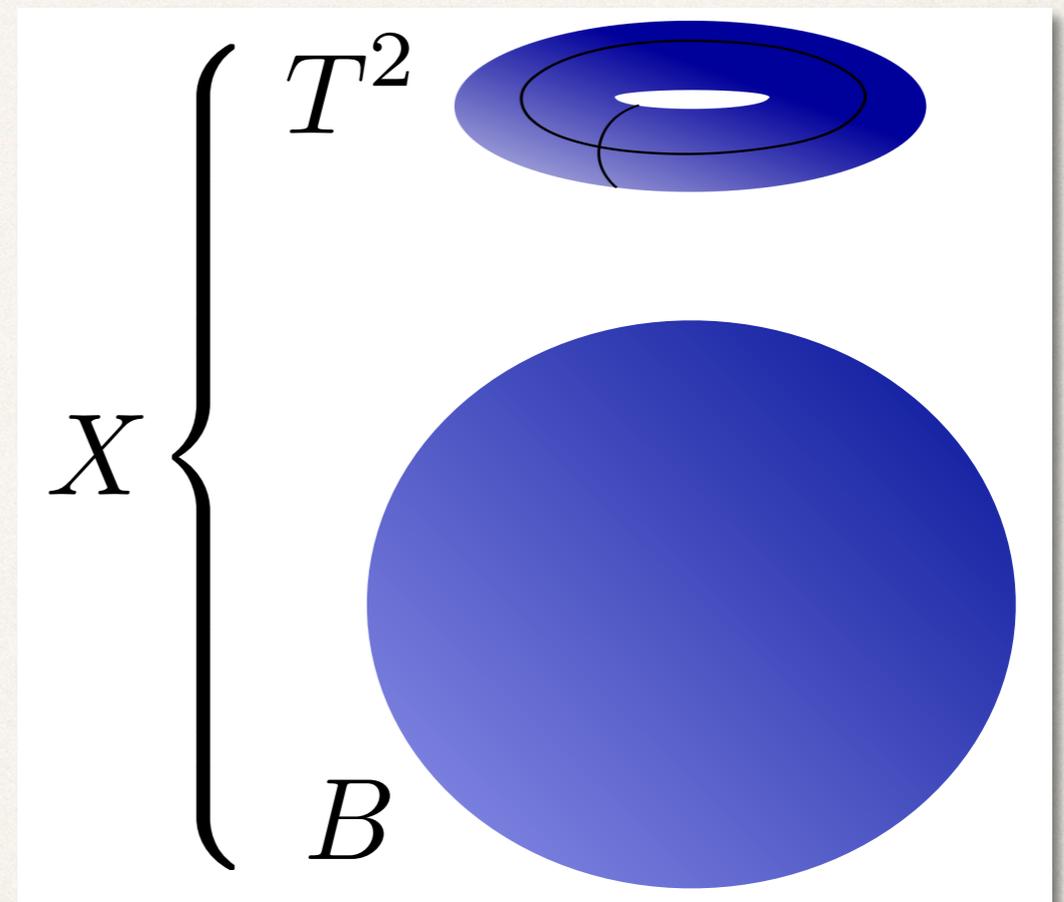
1. **Base  $B$**  of  $X$

→ part of **physical space-time** of string theory

2. **Torus fiber  $T^2$**  of  $X$

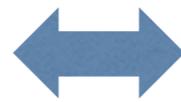
→ **book-keeping** device for Type IIB

**complexified string coupling**  $\tau \equiv C_0 + ig_s^{-1}$



# Singularities of CY manifolds & physics

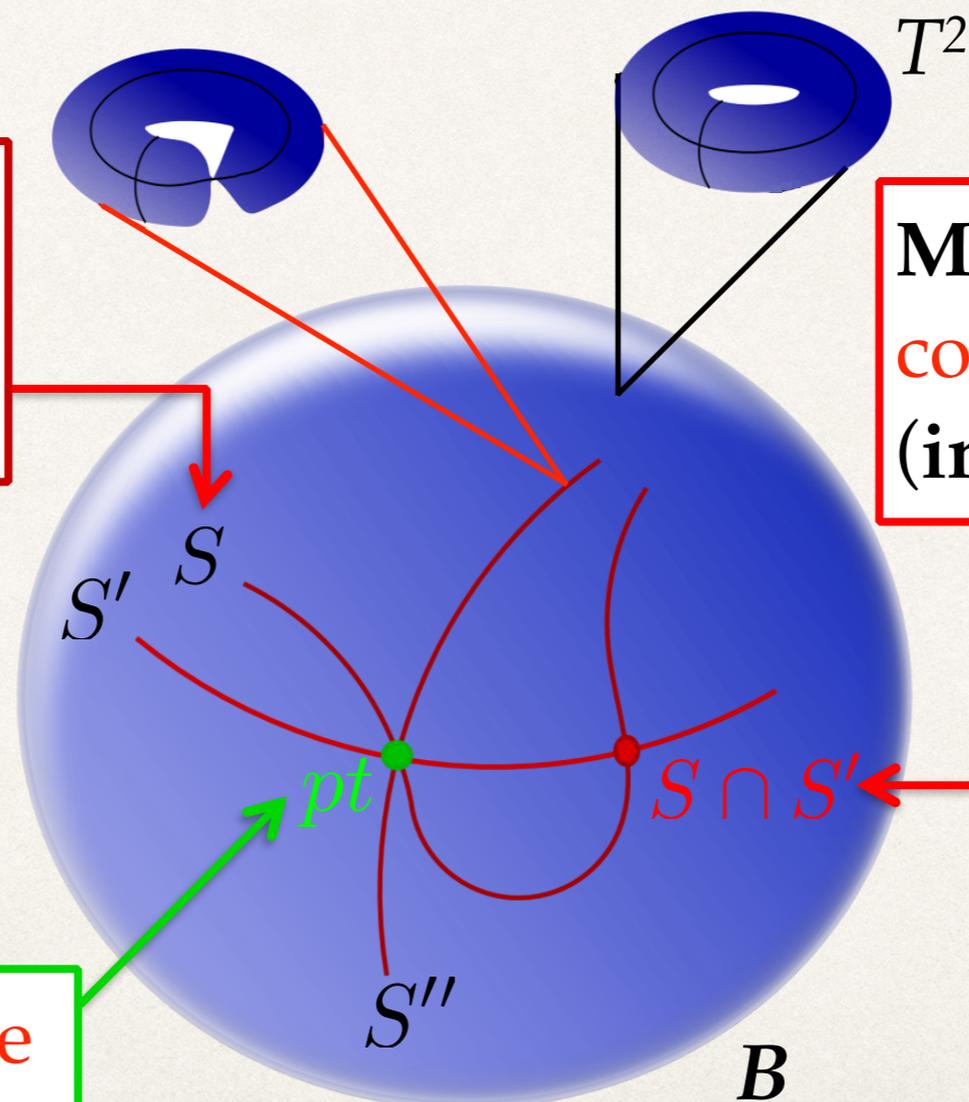
Singularities of  $T^2$ -fibration  
of Calabi-Yau  $X$  over base  $B$



globally well-defined setup  
of intersecting  $(p,q)$  7-branes

Gauge theory in 8D:  
co-dim. one singularity  
(7-branes)

Matter in 6D :  
co-dim. two sing.  
(intersec. 7-branes)



[Katz, Vafa]

4D Yukawa: co-dim three  
 $pt = S \cap S' \cap S''$

# Singularities of CY manifolds & physics

Singularities of  $T^2$ -fibration of Calabi-Yau  $X$  over base  $B$   $\leftrightarrow$  globally well-defined setup of intersecting  $(p,q)$  7-branes

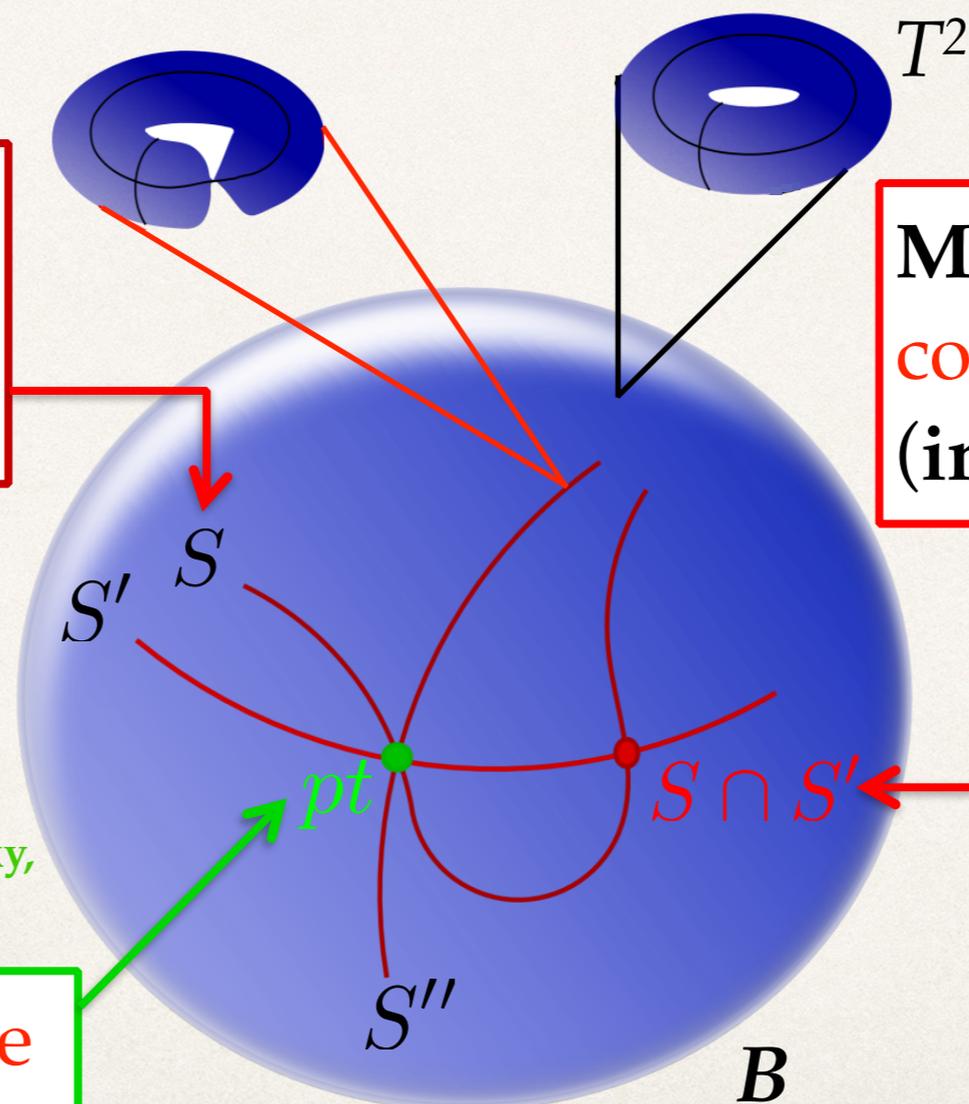
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Matter in 6D:  
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➔ obtain only  
non-Abelian groups,  
no  $U(1)$ 's

[Kodaira; Tate; Vafa, Morrison, Vafa; Bershadsky, Intriligator, Kachru, Morrison, Sadvov, Vafa]

4D Yukawa: co-dim three  
 $pt = S \cap S' \cap S''$



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## 2) Global F-theory compactifications with $U(1)$ symmetries

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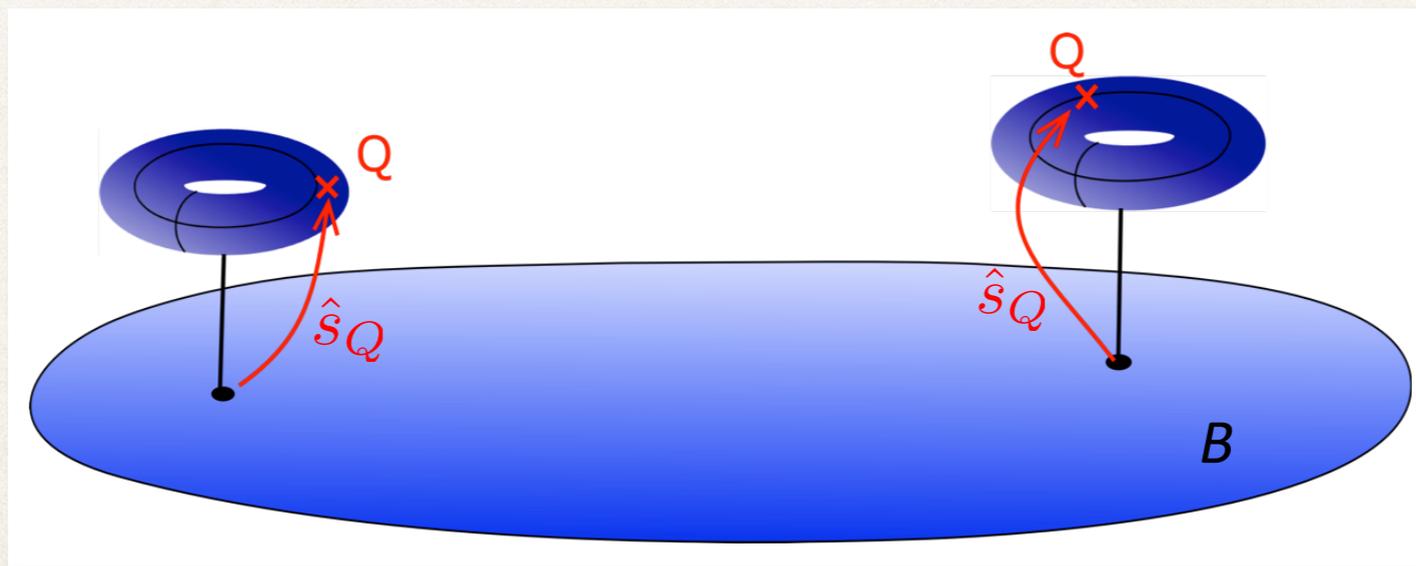
# U(1)'s in F-theory & the Mordell-Weil group

- ❖ U(1)'s arise by **KK-reduction** of M-theory three-form  $C_3 \supset A^m \omega_m$ .
- ❖ **Not from codimension one singularities**: otherwise again non-Abelian groups.

(1,1)-form  $\omega_m$   $\longleftrightarrow$  rational section of  $X$  [Morrison, Vafa II]

**Rational section** = map  $\hat{s}_Q : B \rightarrow X$  induced by **rational point  $Q$**  on  $T^2 =$  elliptic curve  $E$ .

- ❖ Rational points form Abelian group: **Mordell-Weil (MW) group** of rational sections of  $X$
- ❖  $\hat{s}_Q$  gives rise to a **second copy of  $B$**  in  $X$ : **new divisor  $B_Q$**  in  $X$



➔ (1,1)-form  $\omega_m$  constructed from **divisor  $B_Q$** .

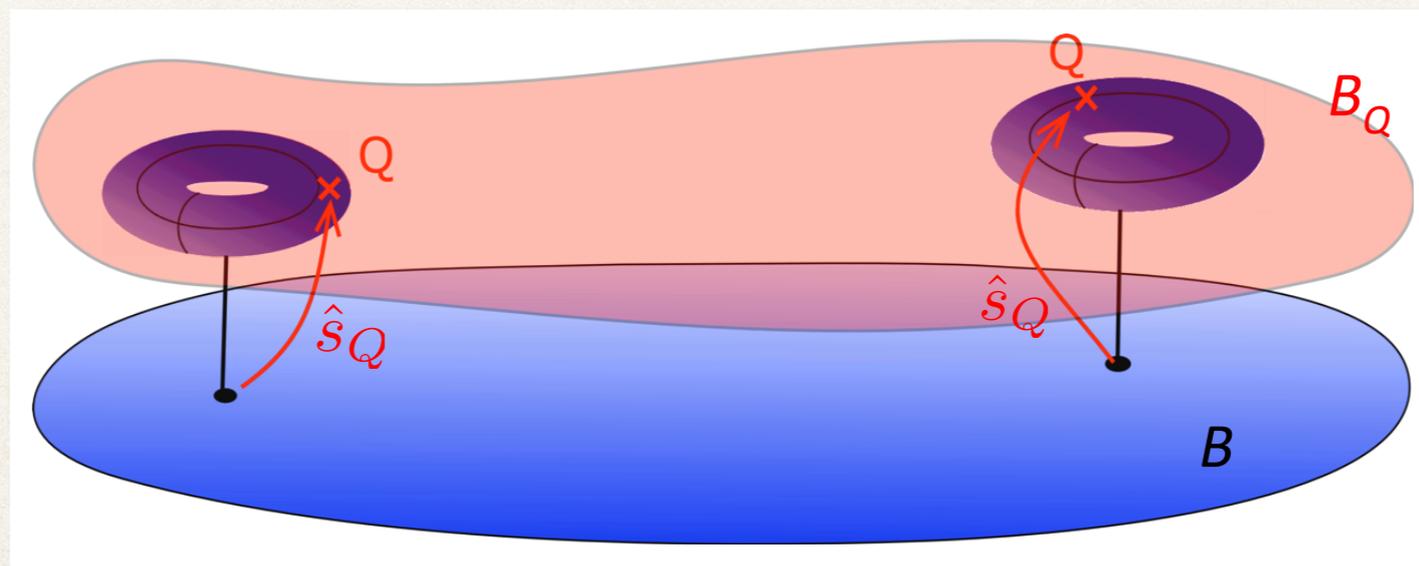
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# Systematic construction of F-theory vacua with $U(1)$ 's

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- ❖  $n$  rational sections of CY-manifold  $X$   $\rightarrow$  F-theory with  $U(1)^n$  gauge group
- ❖ Deligne: Systematic construction of CY  $X$  with  $n$  rational sections

$\rightarrow$  elliptic curve  $E$  embedded into  $WP^m$

## Examples:

one  $U(1)$ : elliptic curve  $E$  is generic CY in  $Bl_1\mathbb{P}^2(1, 1, 2)$  [Morrison, Park]

$\rightarrow$  Construction yields only matter with  $U(1)$ -charge  $q=2$ ,

$\rightarrow$  extension to models with  $q=3$  matter:  $E$  is cubic CY in  $dP_1$ . [DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]

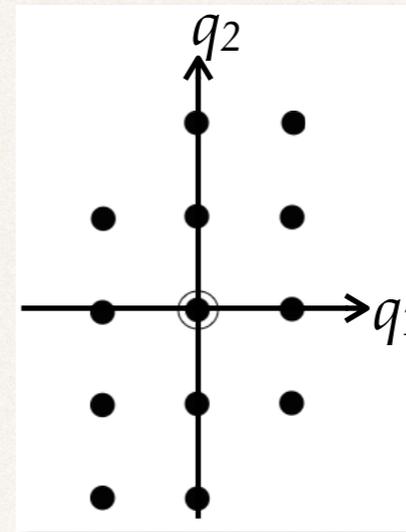
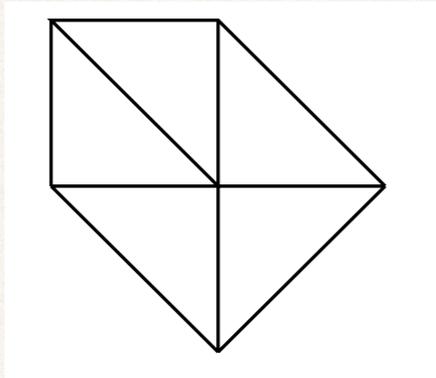
# Systematic construction of F-theory vacua with $U(1)$ 's

$U(1)^2$ :  $E$  is **non-generic cubic** in  $\mathbb{P}^2$

- generic CY in  $dP_2$** : has **restricted matter** spectrum with  $U(1)^2$  charges  $(q_1, q_2)$

[Borchmann, Mayrhofer, Palti, Weigand; Cvetič, DK, Piragual]

toric  $dP_2$ :



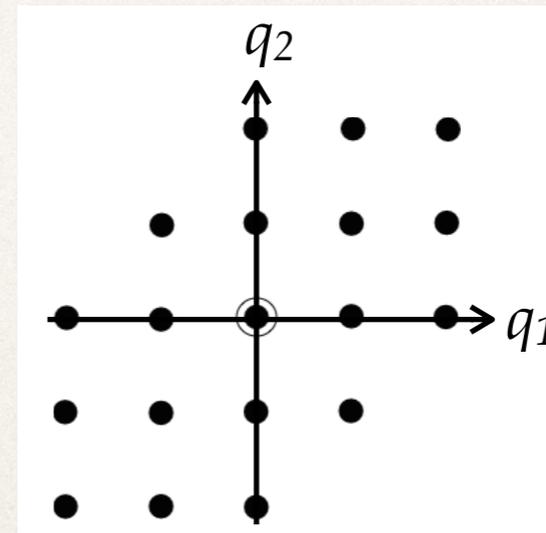
- generalization**: CY with  $U(1)^2$  has fully **symmetric matter spectrum**

[Cvetič, DK, Piragua, Taylor]

non-toric model:

$$u f_2(u, v, w) + \prod_{i=1}^3 (a_i v + b_i w) = 0$$

$$f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2$$



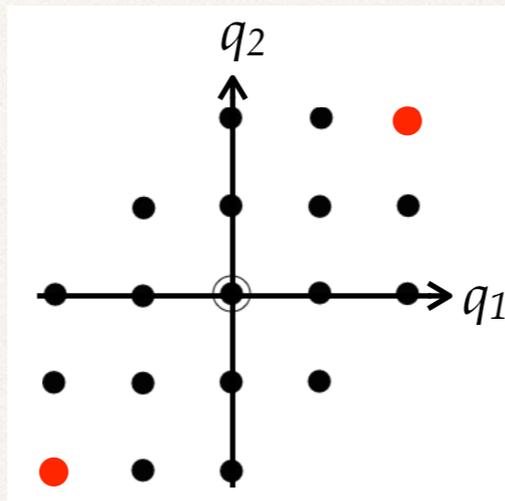
→ **All spectra automatically anomaly-free.**

# New realizations of matter singularities

[Cvetic,DK,Piragua,Taylor]

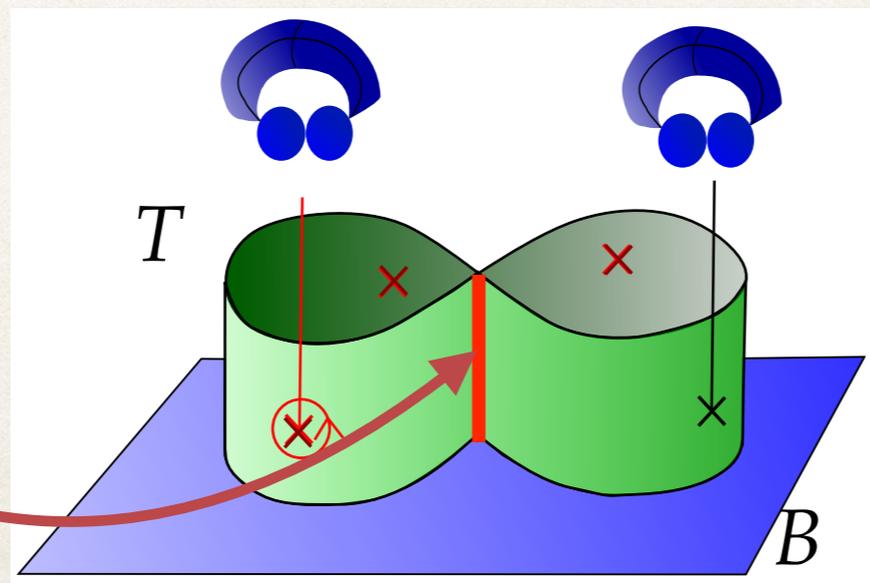
“UnHiggs”  $U(1)^2 \rightarrow SU(3)$

- ❖  $(q_1, q_2) = (2, 2)$  matter becomes **symmetric representation 6 of  $SU(3)$** :  
**first concrete construction** in global F-theory.



- ❖ related to **new algebraic description** of  $I_3^s$  singularities over divisor  $T$

locus of **6** matter:  
**ordinary double point**

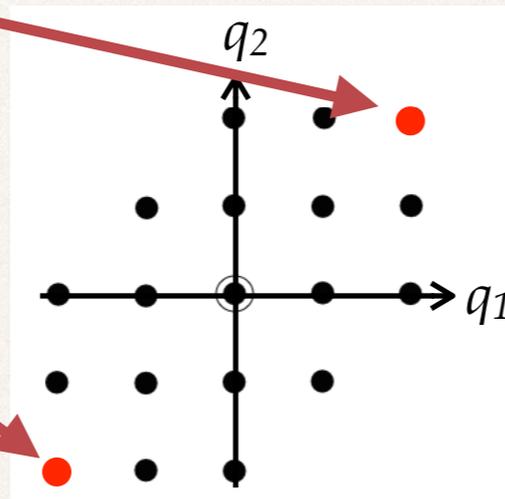


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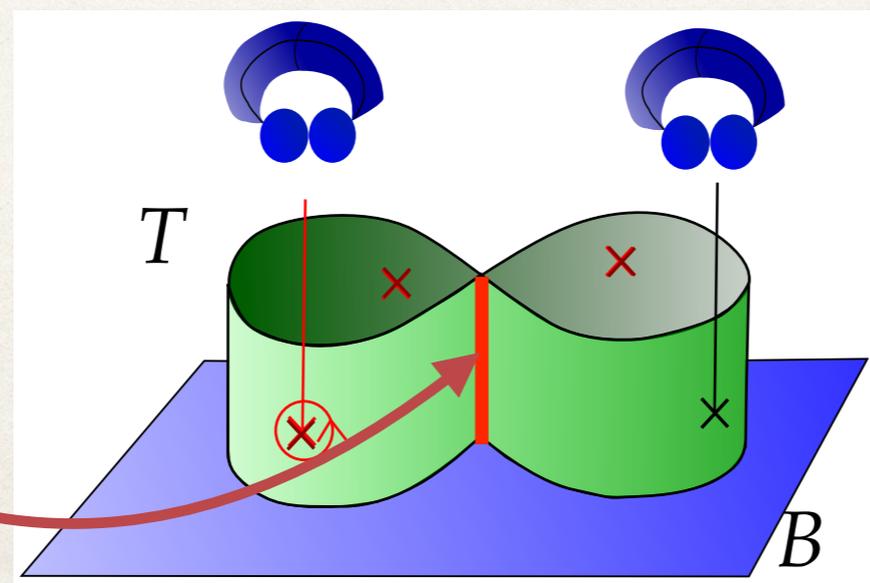
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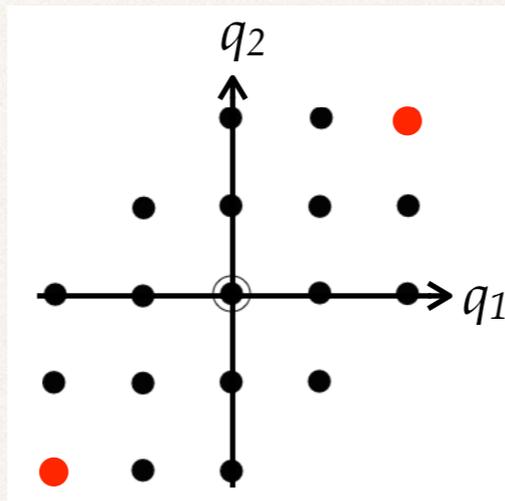


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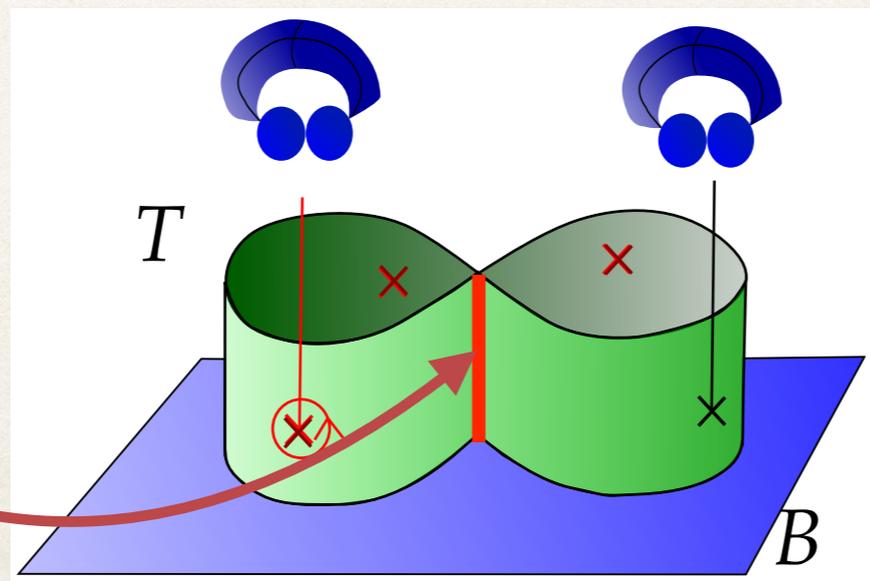
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# 3) Global F-theory compactifications with discrete gauge groups

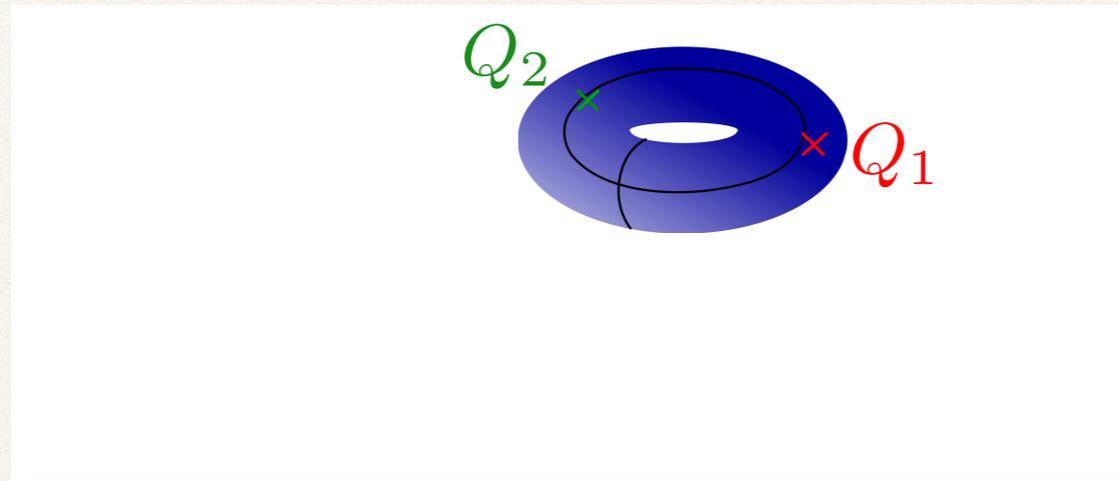
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# F-theory with discrete gauge groups

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Torus fibration  $X$  has **has no sections**, only  $n$ -section: genus-one fibration

→  $X$  has  $n$  rational **sections** “locally” but they are **interchanged globally**.



→ **only sum well-defined globally**

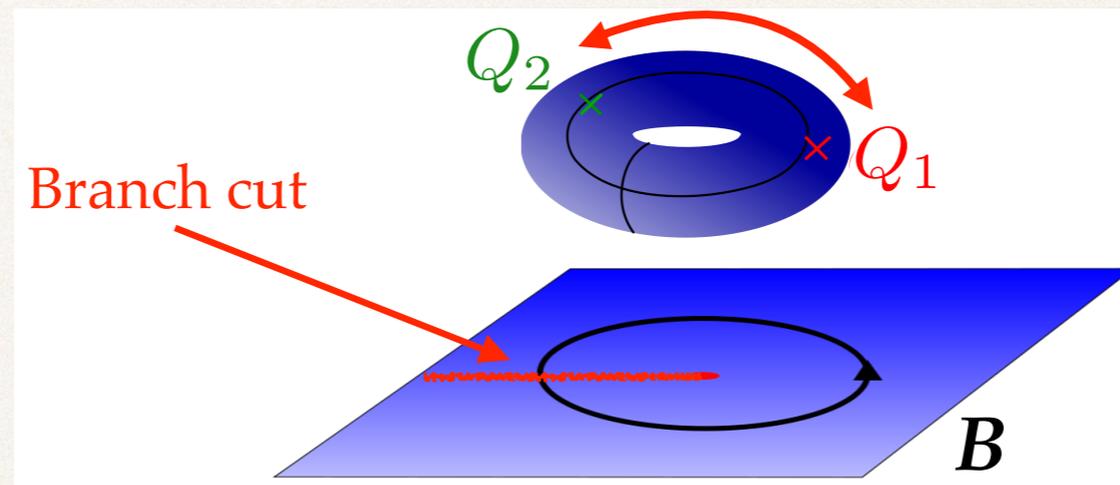
$$Q^{(n)} = Q_1 + \dots + Q_n$$

**Obstruction** to gluing points together globally: Tate-Shafarevich (TS) group → visible in physics as **discrete gauge group of F-theory**.

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# All known examples

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F-theory vacua with  $\mathbb{Z}_n$  **discrete gauge groups** identified

❖  $\mathbb{Z}_2$  gauge group:  $T^2 =$  **quartic** in  $\mathbb{P}^2(1, 1, 2)$

➔  $\mathbb{Z}_2$  and  $(\mathrm{SU}(2) \times \mathbb{Z}_4)/\mathbb{Z}_2$

[Braun, Morrison; Morrison, Taylor  
Anderson, García-Etxebarria, Grimm, Keitel; DK, Mayorga-Pena, Oehlmann, Piragua, Reuter; García-Etxebarria, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand]

❖  $\mathrm{U}(1) \times \mathbb{Z}_2$  gauge group:  $T^2 =$  **bi-quadric** in  $\mathbb{P}^1 \times \mathbb{P}^1$

[DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]

❖  $\mathbb{Z}_3$  discrete gauge group:  $T^2 =$  **cubic** in  $\mathbb{P}^2$

[DK, Mayorga-Pena, Oehlmann, Piragua, Reuter;  
Cvetič, Donagi, DK, Piragua, Poretschkin]

In all cases we have found

❖ **matter carrying** non-trivial  $\mathbb{Z}_n$  **discrete charge**,

❖ all gauge invariant **Yukawas exist**, including  $\mathbb{Z}_n$  **selection rules**.

## 4) Conclusions

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## Summary

- ❖ Systematic construction of F-theory vacua with  $U(1)^n$  ( $n \leq 3$ ):

Mordell-Weil group  $\longleftrightarrow$  U(1)'s

- ❖ Construction of F-theory vacua with  $\mathbb{Z}_n$  ( $n \leq 4$ ) discrete gauge groups:

Tate-Shafarevich group  $\longleftrightarrow$  discrete gauge group

- ❖ New matter representations:

- ➔ charge  $q=3$  for one  $U(1)$ , new  $U(1)^2$  models with up to charges (2,2)
- ➔ matter with charges under discrete gauge group,
- ➔ first concrete construction of symmetric representation of  $SU(3)$ .

# Things I didn't have time to talk about

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Construction of  $U(1)^3$ :  $E$  is pencil of non-generic quadrics in  $Bl_4\mathbb{P}^4$ .  
[Cvetic,DK,Piragua,Song]

Pheno applications: Use  $U(1)$ 's for construction of SM in F-theory

Construction in [Cvetič,DK,Mayorga-Pena,Oehlmann,Reuter]

- ❖  $U_Y(1) \longleftrightarrow$  rational section
- ❖ SM non-Abelian gauge group  $SU(3) \times SU(2)$  is automatically present
- ❖ add  $G_4$ -flux to generate 4D chirality following [Cvetic,Grassi,DK,Piragua]
- ❖ solve D3-brane tadpole
- ➔ get 4D three-family Standard models in F-theory.
- ➔ natural embedding into Pati-Salam & Trinification

Thank  
You