

Recent progress on the gauge theory sector of F-theory

Denis Klevers



arXiv:1507.05954: M. Cvetič, D.K., H. Piragua, W. Taylor

arXiv:1502.06953: M. Cvetič, R. Donagi, D.K., H. Piragua, M. Poretschkin

arXiv:1408.4808: D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter

arXiv:1303.6970, arXiv:1307.6425: M. Cvetič, D.K., H. Piragua

(arXiv:1503.02068: M. Cvetič, D.K., D. Mayorga Peña, P. Oehlmann, J. Reuter

arXiv:1310.0463: M. Cvetič, D.K., H. Piragua, P. Song

arXiv:1306.3987: M. Cvetič, A. Grassi, D.K., H. Piragua)

Motivation

Why F-theory?

F-theory

1. describes **broad class** of **non-perturbative vacua** of string theory,
2. can produce **GUT models** with **promising particle physics & cosmology**:
→ **features not** accessible in **perturbative** II strings (E_6 to E_8 , $10 \times 10 \times 5, \dots$).

Local: [Donagi, Wijnholt; Beasley, Heckman, Vafa; ... many works]

Global: [Blumenhagen, Grimm, Jurke, Weigand; Marsano, Saulina, Schäfer-Nameki; ... many works]

3. engineers **effective field theories** coupled to quantum gravity:

Calabi-Yau (CY) geometry

Geometry **Physics**

Effective field theories

- Geometry provides **tools to control** over **non-perturbative physics**.

Goal of this talk

Calabi-Yau (CY) geometry

Geometry Physics

Effective field theories

Goal: Use F-theory to **study gauge theory sectors** in $N=1$ SUGRA theories.

Problem: geometry / physics dictionary **incomplete**

- ❖ **Well-understood for non-Abelian** groups & **simple matter** representations.
- ❖ **less known** about $U(1)$'s, **discrete gauge groups** & more **complicated matter** representations.

Today: **develop** some **missing pieces**

- ❖ **Arithmetic of CY-elliptic fibrations** ↔ **global gauge group**
- ❖ **Enlarge matter sector**: **new** Abelian & non-Abelian **representations**

1) What is an F-theory vacuum?

Defining data: torus-fibered Calabi-Yau manifold X

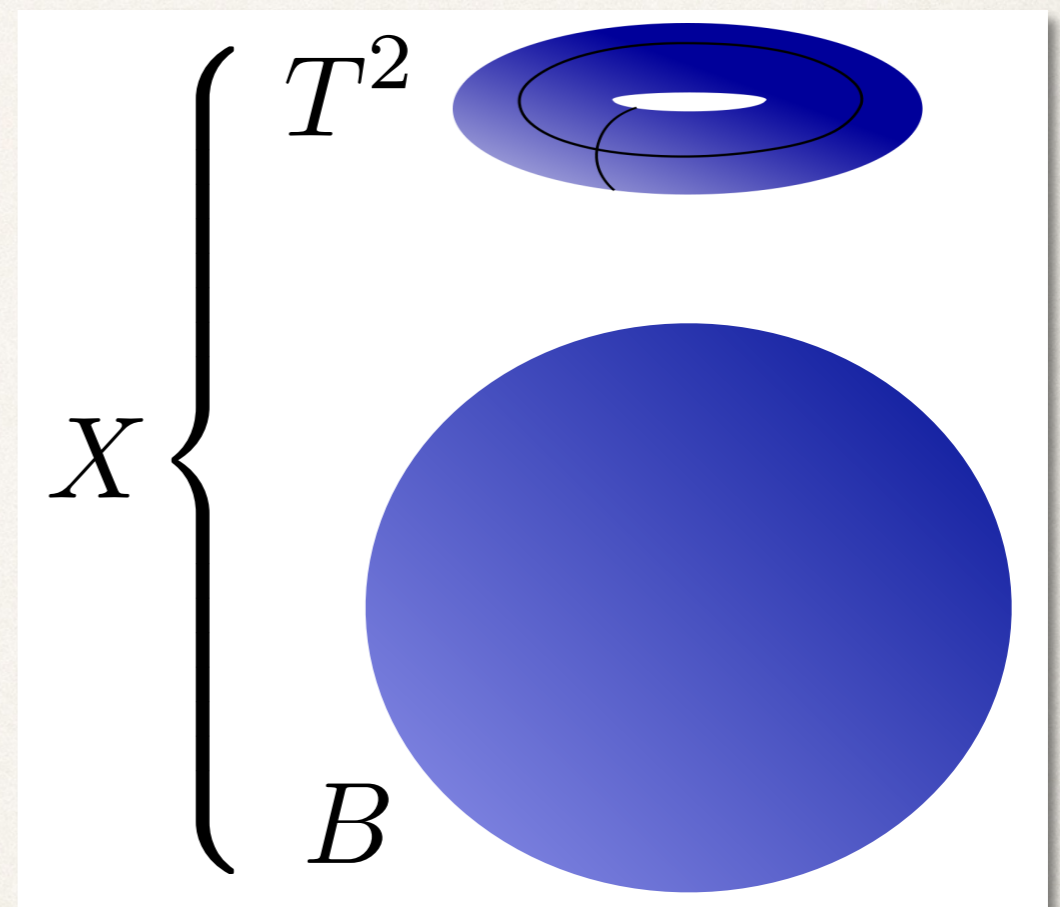
1. **Base B** of X

→ part of **physical space-time** of string theory

2. **Torus fiber T^2** of X

→ **book-keeping** device for Type IIB

complexified string coupling $\tau \equiv C_0 + ig_s^{-1}$

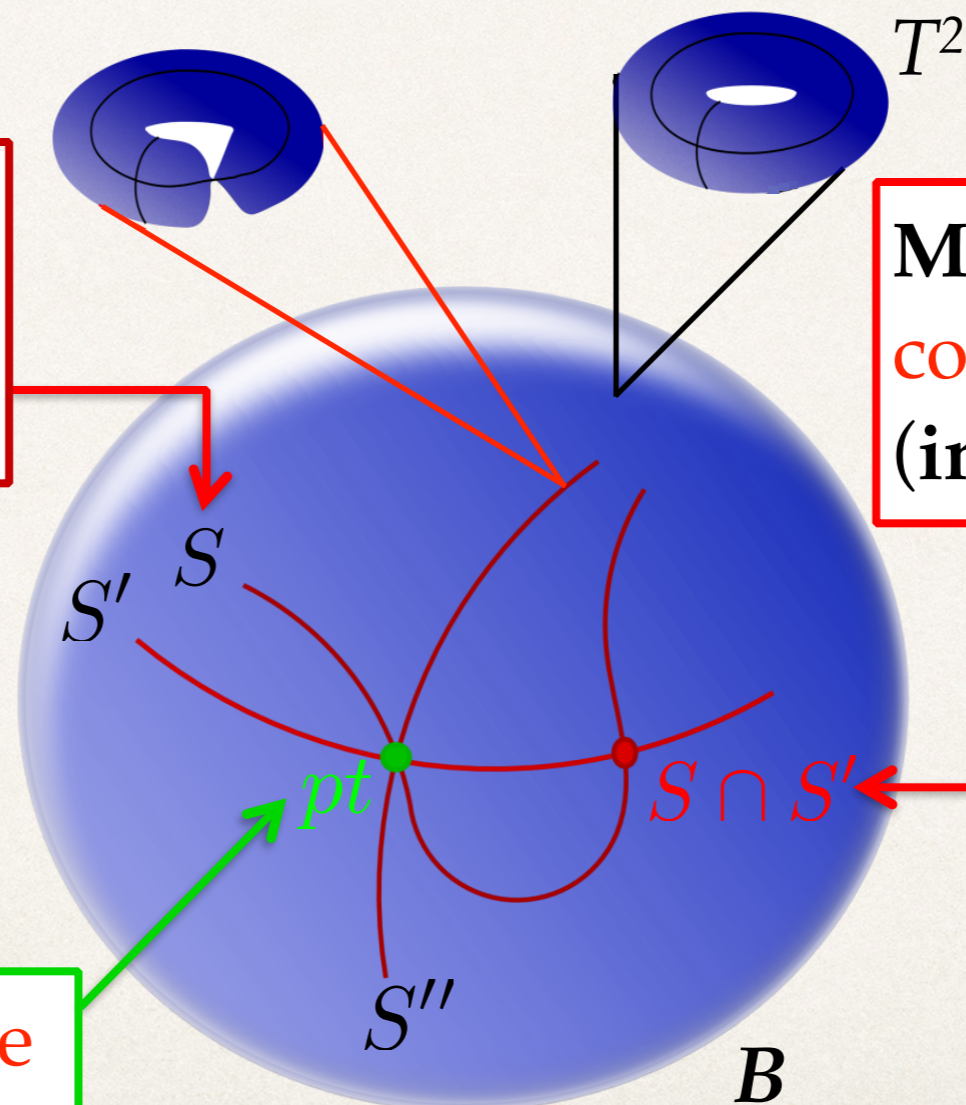


Singularities of CY manifolds & physics

Singularities of T^2 -fibration of Calabi-Yau X over base B \leftrightarrow globally well-defined setup of intersecting (p,q) 7-branes

Gauge theory in 8D:
co-dim. one singularity
(7-branes)

Matter in 6D:
co-dim. two sing.
(intersec. 7-branes)



4D Yukawa: co-dim three
 $pt = S \cap S' \cap S''$

[Katz, Vafa]

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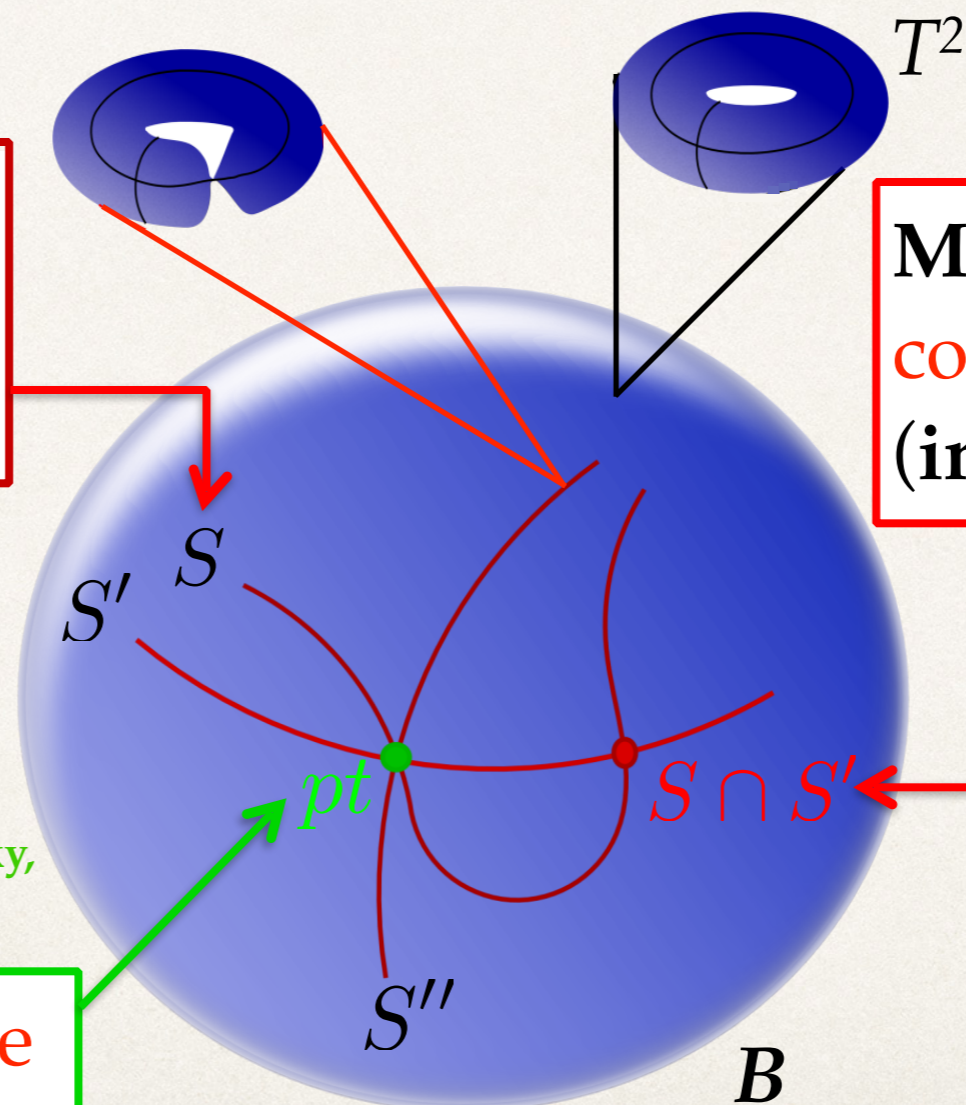
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➔ obtain only
non-Abelian groups,
no $U(1)$'s

[Kodaira; Tate; Vafa, Morrison, Vafa; Bershadsky, Intriligator, Kachru, Morrison, Sadov, Vafa]

4D Yukawa: co-dim three
 $pt = S \cap S' \cap S''$



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2) Global F-theory compactifications with $U(1)$ symmetries

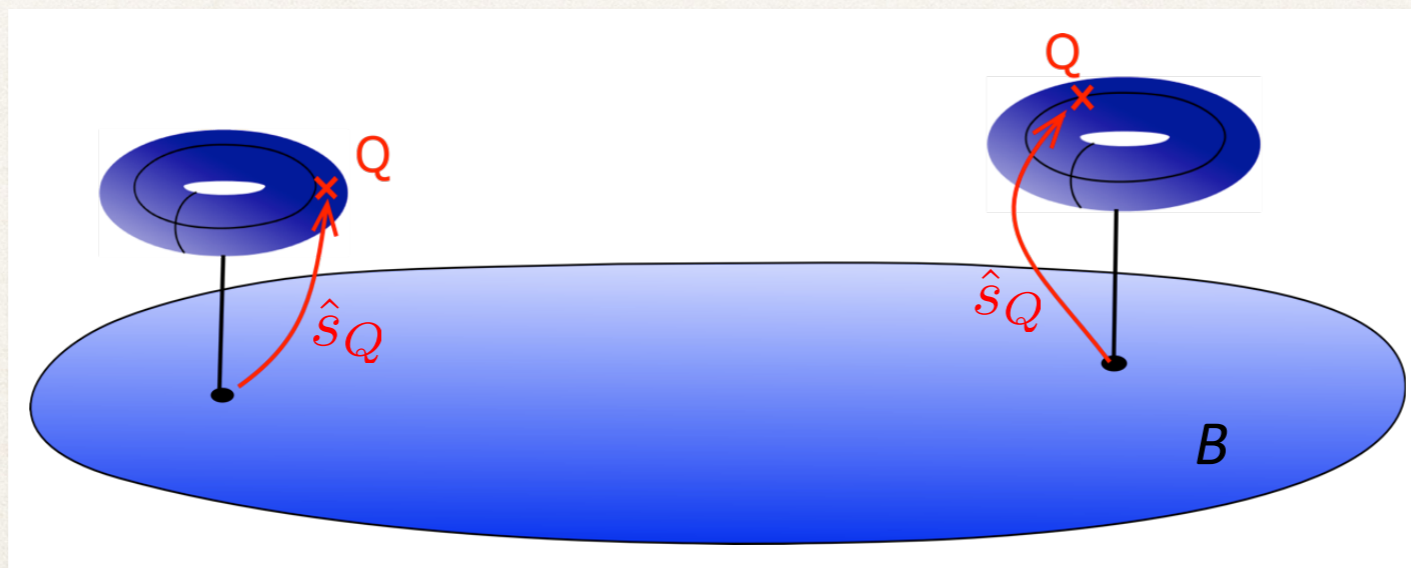
U(1)'s in F-theory & the Mordell-Weil group

- ❖ U(1)'s arise by **KK-reduction** of M-theory three-form $C_3 \supset A^m \omega_m$.
- ❖ **Not from codimension one singularities**: otherwise again non-Abelian groups.

(1,1)-form ω_m \longleftrightarrow rational section of X [Morrison, Vafa II]

Rational section = map $\hat{s}_Q : B \rightarrow X$ induced by **rational point Q** on $T^2 =$ elliptic curve E .

- ❖ Rational points form Abelian group: **Mordell-Weil (MW) group** of rational sections of X
- ❖ \hat{s}_Q gives rise to a **second copy of B** in X : **new divisor B_Q** in X



➔ (1,1)-form ω_m constructed from **divisor B_Q** .

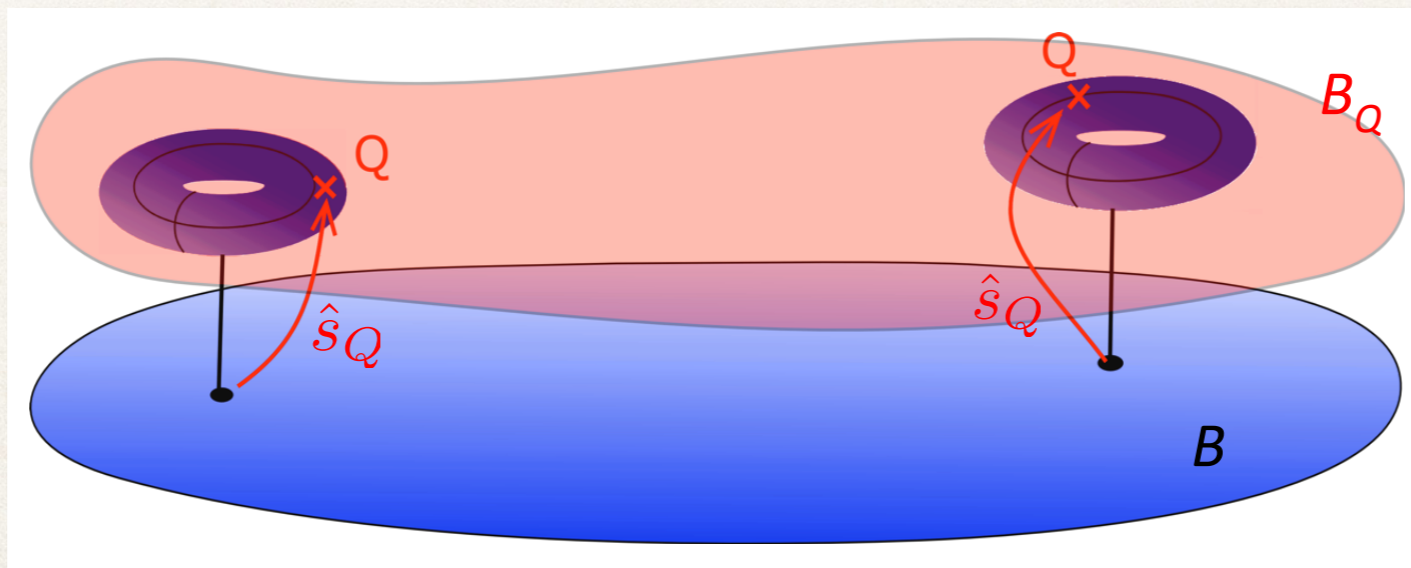
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Systematic construction of F-theory vacua with $U(1)$'s

- ❖ n rational sections of CY-manifold X \rightarrow F-theory with $U(1)^n$ gauge group
- ❖ Deligne: Systematic construction of CY X with n rational sections

\rightarrow elliptic curve E embedded into WP^m

Examples:

one $U(1)$: elliptic curve E is generic CY in $Bl_1\mathbb{P}^2(1, 1, 2)$ [Morrison, Park]

\rightarrow Construction yields only matter with $U(1)$ -charge $q=2$,

\rightarrow extension to models with $q=3$ matter: E is cubic CY in dP_1 . [DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]

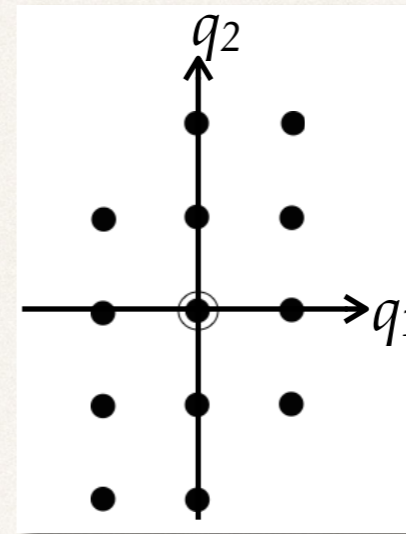
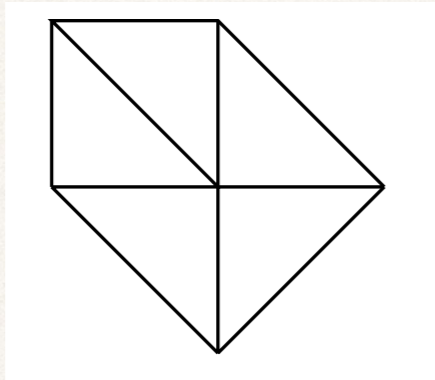
Systematic construction of F-theory vacua with $U(1)$'s

$U(1)^2$: E is **non-generic cubic** in \mathbb{P}^2

- generic CY in dP_2** : has **restricted matter** spectrum with $U(1)^2$ charges (q_1, q_2)

[Borchmann, Mayrhofer, Palti, Weigand; Cvetič, DK, Piragual]

toric dP_2 :



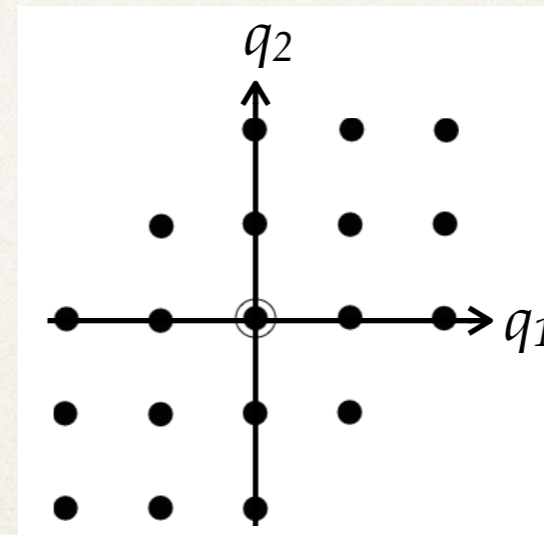
- generalization**: CY with $U(1)^2$ has fully **symmetric matter spectrum**

[Cvetic, DK, Piragua, Taylor]

non-toric model:

$$u f_2(u, v, w) + \prod_{i=1}^3 (a_i v + b_i w) = 0$$

$$f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2$$



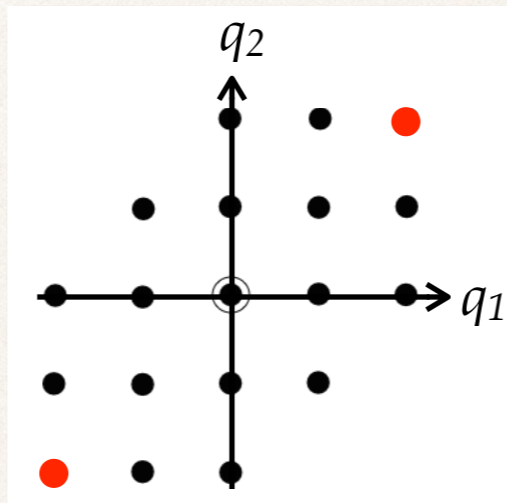
➔ **All spectra automatically anomaly-free.**

New realizations of matter singularities

[Cvetic,DK,Piragua,Taylor]

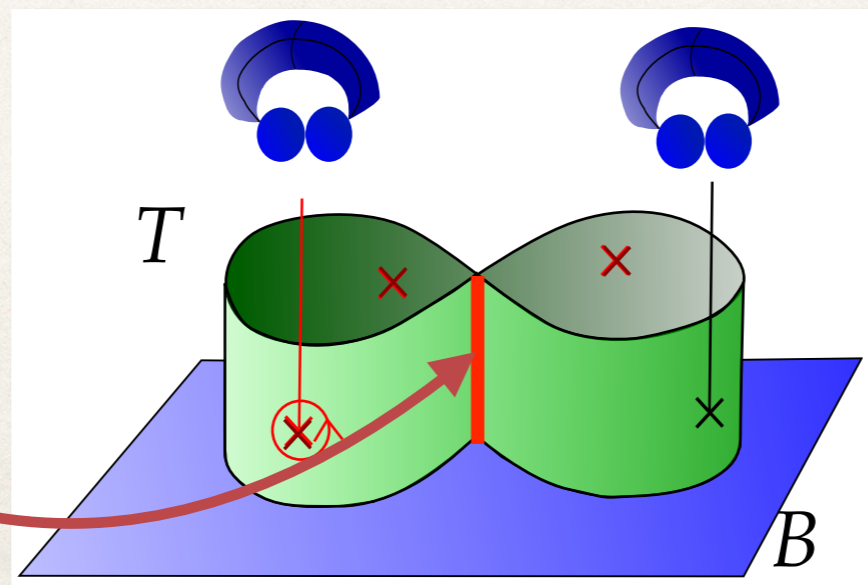
“UnHiggs” $U(1)^2 \rightarrow SU(3)$

- ❖ $(q_1, q_2) = (2, 2)$ matter becomes **symmetric representation 6 of $SU(3)$** :
first concrete construction in global F-theory.



- ❖ related to **new algebraic description** of I_3^s singularities over divisor T

locus of **6** matter:
ordinary double point

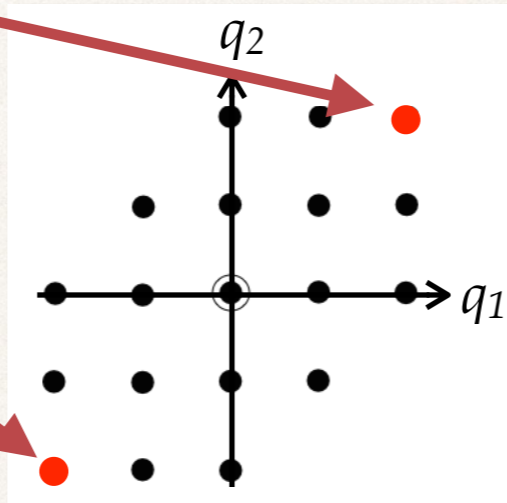


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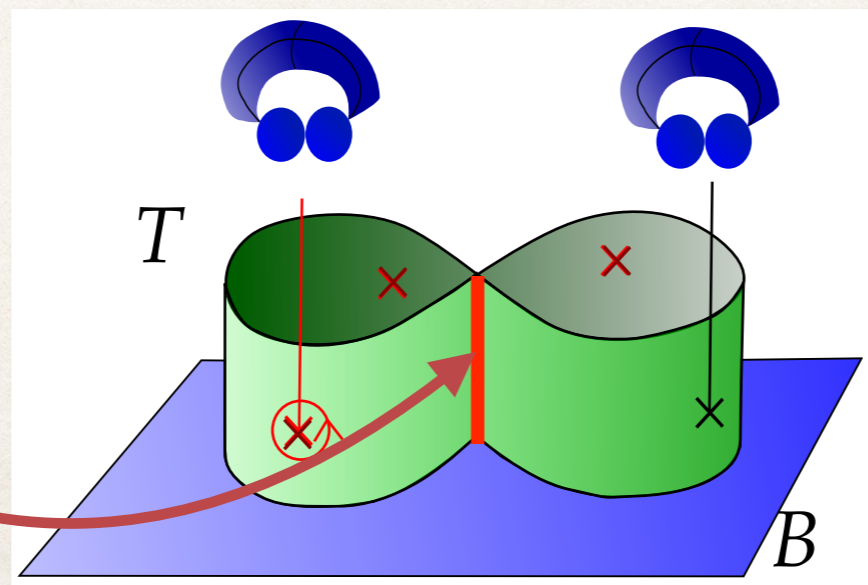
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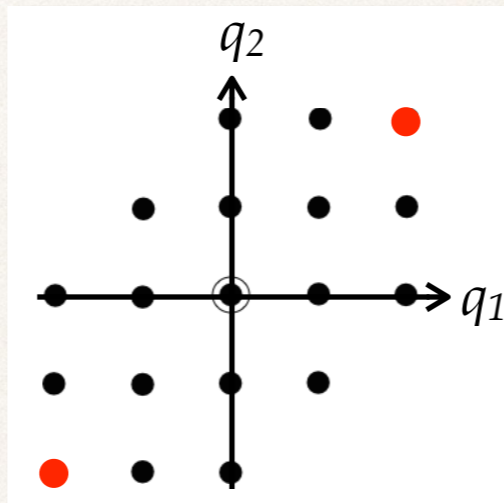


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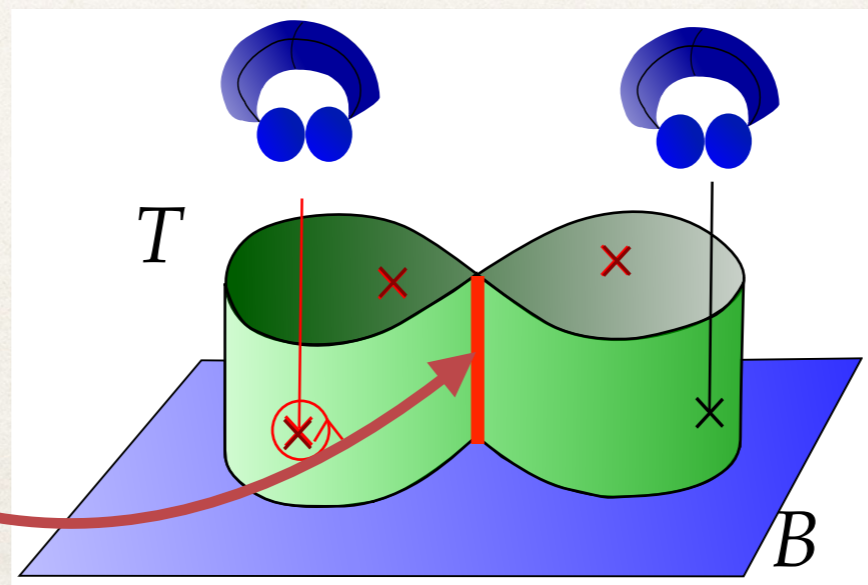
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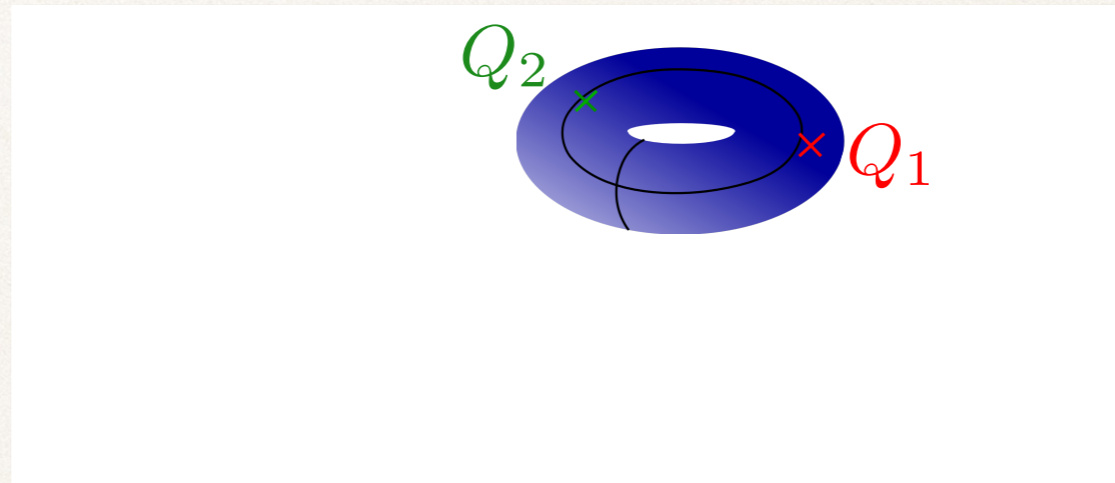


3) Global F-theory compactifications with discrete gauge groups

F-theory with discrete gauge groups

Torus fibration X has **has no sections**, only n -section: genus-one fibration

→ X has n rational **sections** “locally” but they are **interchanged globally**.



→ **only sum well-defined globally**

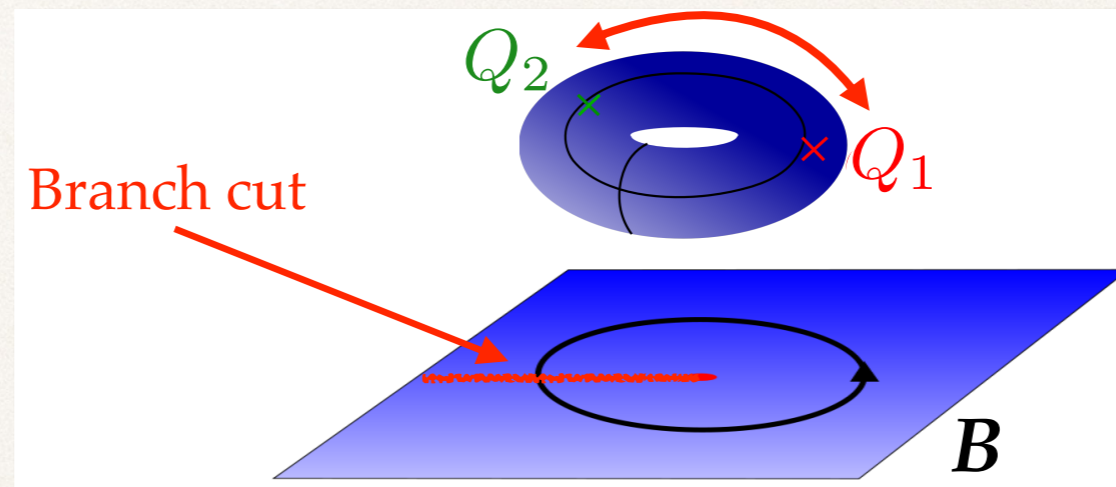
$$Q^{(n)} = Q_1 + \dots + Q_n$$

Obstruction to gluing points together globally: Tate-Shafarevich (TS) group → visible in physics as **discrete gauge group of F-theory**.

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All known examples

F-theory vacua with \mathbb{Z}_n **discrete gauge groups** identified

❖ \mathbb{Z}_2 gauge group: $T^2 =$ **quartic** in $\mathbb{P}^2(1, 1, 2)$

➔ \mathbb{Z}_2 and $(\mathrm{SU}(2) \times \mathbb{Z}_4)/\mathbb{Z}_2$

[Braun, Morrison; Morrison, Taylor
Anderson, García-Etxebarria, Grimm, Keitel; DK, Mayorga-Pena, Oehlmann, Piragua, Reuter; García-Etxebarria, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand]

❖ $\mathrm{U}(1) \times \mathbb{Z}_2$ gauge group: $T^2 =$ **bi-quadric** in $\mathbb{P}^1 \times \mathbb{P}^1$

[DK, Mayorga-Pena, Oehlmann, Piragua, Reuter]

❖ \mathbb{Z}_3 discrete gauge group: $T^2 =$ **cubic** in \mathbb{P}^2

[DK, Mayorga-Pena, Oehlmann, Piragua, Reuter;
Cvetič, Donagi, DK, Piragua, Poretschkin]

In all cases we have found

❖ **matter carrying** non-trivial \mathbb{Z}_n **discrete charge**,

❖ all gauge invariant **Yukawas exist**, including \mathbb{Z}_n **selection rules**.

4) Conclusions

Summary

- ❖ Systematic construction of F-theory vacua with $U(1)^n$ ($n \leq 3$):

Mordell-Weil group \longleftrightarrow U(1)'s

- ❖ Construction of F-theory vacua with \mathbb{Z}_n ($n \leq 4$) discrete gauge groups:

Tate-Shafarevich group \longleftrightarrow discrete gauge group

- ❖ New matter representations:

- ➔ charge $q=3$ for one $U(1)$, new $U(1)^2$ models with up to charges (2,2)
- ➔ matter with charges under discrete gauge group,
- ➔ first concrete construction of symmetric representation of $SU(3)$.

Things I didn't have time to talk about

Construction of $U(1)^3$: E is pencil of non-generic quadrics in $Bl_4\mathbb{P}^4$.
[Cvetic,DK,Piragua,Song]

Pheno applications: Use $U(1)$'s for construction of SM in F-theory

Construction in [Cvetič,DK,Mayorga-Pena,Oehlmann,Reuter]

- ❖ $U_Y(1) \longleftrightarrow$ rational section
- ❖ SM non-Abelian gauge group $SU(3) \times SU(2)$ is automatically present
- ❖ add G_4 -flux to generate 4D chirality following [Cvetic,Grassi,DK,Piragua]
- ❖ solve D3-brane tadpole
- ➔ get 4D three-family Standard models in F-theory.
- ➔ natural embedding into Pati-Salam & Trinification

Thank
You