Theory of Lepton Flavour Violation

Paride Paradisi

University of Padua

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Open questions

- **The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:**
  - **Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?**
  - **Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?**
- **Related important questions are:**
  - **Which is the role of flavor physics in the LHC era?**
  - **Do we expect to understand the (SM and NP) flavor puzzles through the synergy and interplay of flavor physics and the LHC?**

NP search strategies

- **High-energy frontier: A unique effort to determine the NP scale**

- **High-intensity frontier (flavor physics): A collective effort to determine the flavor structure of NP**

Where to look for New Physics at the low energy?

- **Processes very suppressed or even forbidden in the SM**
  - FCNC processes ($\mu \to e\gamma$, $\mu \to eee$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $B^0_{s,d} \to \mu^+\mu^-$...)
  - CPV effects in the electron/neutron EDMs, $d_{e,n}$...
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes

- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_{\mu,e}$: $a_\mu^{exp} - a_\mu^{SM} \approx (3 \pm 1) \times 10^{-9}$, a discrepancy at 3σ!
  - LU in $R_M^{e/\mu} = \Gamma(M \to e\nu)/\Gamma(M \to \mu\nu)$ with $M = \pi, K$

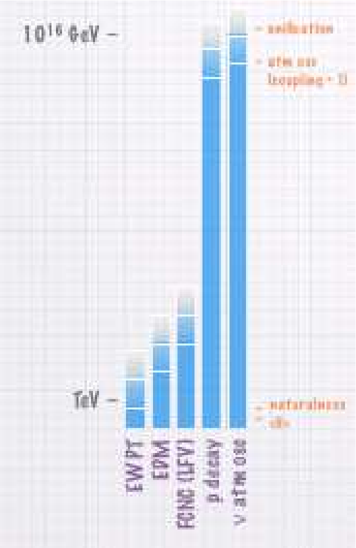
Experimental status

LFV process	Experiment	Future limits	Year (expected)
BR($\mu \to e\gamma$)	MEG	$\mathcal{O}(10^{-14})$	~ 2017
	Project X	$\mathcal{O}(10^{-15})$	> 2021
BR($\mu \to eee$)	Mu3e	$\mathcal{O}(10^{-15})$	~ 2017
	Mu3e	$\mathcal{O}(10^{-16})$	> 2017
	MUSIC	$\mathcal{O}(10^{-16})$	~ 2017
	Project X	$\mathcal{O}(10^{-17})$	> 2021
CR($\mu \to e$)	COMET	$\mathcal{O}(10^{-17})$	~ 2017
	Mu2e	$\mathcal{O}(10^{-17})$	~ 2020
	PRISM/PRIME	$\mathcal{O}(10^{-18})$	~ 2020
	Project X	$\mathcal{O}(10^{-19})$	> 2021
BR($\tau \to \mu\gamma$)	Belle II	$\mathcal{O}(10^{-8})$	> 2020
BR($\tau \to \mu\mu\mu$)	Belle II	$\mathcal{O}(10^{-10})$	> 2020
BR($\tau \to e\gamma$)	Belle II	$\mathcal{O}(10^{-9})$	> 2020
BR($\tau \to \mu\mu\mu$)	Belle II	$\mathcal{O}(10^{-10})$	> 2020

Table: Future sensitivities of next-generation experiments.

The NP "scale"

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}\ \mathrm{GeV}$
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \mathrm{GeV}$
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim \mathrm{TeV}$
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \mathrm{GeV}$
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim \mathrm{TeV}$
- **Dark Matter** $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim \mathrm{TeV}$



SM = effective theory at the EW scale

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}}\, O_{ij}^{(d)}$$

- $\mathcal{L}_{\rm eff}^{d=5} = \dfrac{y_\nu^{ij}}{\Lambda_{\rm see-saw}} L_i L_j \phi\phi,$
- $\mathcal{L}_{\rm eff}^{d=6}$ generates FCNC operators

$$\mathbf{BR}(\ell_i \to \ell_j\gamma) \sim \frac{v^4}{\Lambda_{NP}^4}$$

Why LFV is interesting?

- **Neutrino Oscillation** $\Rightarrow m_{\nu_i} \neq m_{\nu_j} \Rightarrow$ **LFV**

- **see-saw**: $m_\nu \sim \frac{v^2}{M_R} \sim eV \Rightarrow M_R \sim 10^{14-16}$

- LFV transitions like $\mu \to e\gamma$ @ 1 loop with exchange of

  - W and ν in the SM with $\Lambda_{NP} \equiv M_R \equiv \Lambda_{see-saw}$

  $$Br(\mu \to e\gamma) \sim \frac{v^4}{M_R^4} \leq 10^{-50} \qquad \textbf{GIM}$$

  - If $\Lambda_{NP} \ll \Lambda_{see-saw}$ ($\Lambda_{NP} \equiv m_{susy}$ in the MSSM)

  $$Br(\mu \to e\gamma) \sim \frac{v^4}{\Lambda_{NP}^4}$$

  $$\Downarrow$$

- LFV generally detectable in (multi) TeV scale NP scenarios like the MSSM, ...

The NP “scale” vs. LFV

$$\boxed{\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d \geq 5} \frac{C_{ij}^{(d)}}{\Lambda_{NP}^{d-4}} O_{ij}^{(d)}}$$

$\boxed{\text{BR}(\mu \to e\gamma) < 5 \times 10^{-14}}$

Process	Relevant operators	Present Bound on Λ (TeV) $C = 1/16\pi^2$	Present Bound on Λ (TeV) $C = 1$	Future Bound on Λ (TeV) $C = 1/16\pi^2$	Future Bound on Λ (TeV) $C = 1$
$\mu \to e\gamma$	$\frac{C}{\Lambda^2} \frac{m_\mu}{16\pi^2} \overline{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu}$	50	—	90	—
$\mu \to eee$	$\frac{C}{\Lambda^2} (\overline{\mu}_L \gamma^\mu e_L)(\overline{e}_L \gamma^\mu e_L)$	17	210	170	2100
	$\frac{C}{\Lambda^2} (\overline{\mu}_L e_R)(\overline{e}_R e_L)$	10	120	100	1200
$\mu \to e$ in Ti	$\frac{C}{\Lambda^2} (\overline{\mu}_L \gamma^\mu e_L)(\overline{d}_L \gamma^\mu d_L)$	30	420	580	7300
	$\frac{C}{\Lambda^2} (\overline{\mu}_L e_R)(\overline{d}_R d_L)$	60	750	1000	13000

updated from LC Lalak Pokorski Ziegler '12

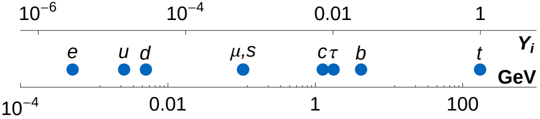
$\boxed{\text{BR}(\mu \to eee) < 10^{-16}}$

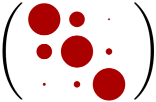
$\boxed{\text{CR}(\mu \to e \text{ in Ti}) < 5 \times 10^{-17}}$

Calibbi @ IFAE2014

SM vs. NP flavor problems

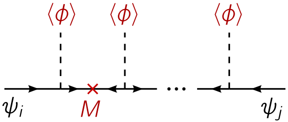
- Can the SM and NP flavour problems have a common explanation?



$V_{\text{CKM}} \sim$ 

- **Froggat-Nielsen '79: Hierarchies from SSB of a Flavour Symmetry**

$$\epsilon = \frac{\langle \phi \rangle}{M} \ll 1 \Rightarrow Y_{ij} \propto \epsilon^{(a_i+b_j)}$$



- **Flavor protection from flavor models:** [Lalak, Pokorski & Ross '10]

Operator	$U(1)$	$U(1)^2$	$SU(3)$	MFV
$(\overline{Q}_L X^Q_{LL} Q_L)_{12}$	λ	λ^5	λ^3	λ^5
$(\overline{D}_R X^D_{RR} D_R)_{12}$	λ	λ^{11}	λ^3	$(y_d y_s) \times \lambda^5$
$(\overline{Q}_L X^D_{LR} D_R)_{12}$	λ^4	λ^9	λ^3	$y_s \times \lambda^5$

- Is this flavor protection enough?
- Can we disentangle flavour models through flavour physics?

Not only $\mu \to e\gamma$...

- **LFV** operators **@ dim-6**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\rm LFV}}\, \mathcal{O}^{\rm dim-6} + \ldots .$$

$$\mathcal{O}^{\rm dim-6} \ni \;\; \bar{\mu}_R\, \sigma^{\mu\nu}\, H\, e_L\, F_{\mu\nu}\,, \;\; (\bar{\mu}_L \gamma^\mu e_L)\left(\bar{f}_L \gamma^\mu f_L\right)\,, \;\; (\bar{\mu}_R e_L)\left(\bar{f}_R f_L\right)\,, \;\; f = e, u, d$$

- the dipole-operator leads to $\ell \to \ell'\gamma$ while 4-fermion operators generate processes like $\ell_i \to \ell_j \bar{\ell}_k \ell_k$ and $\mu \to e$ conversion in Nuclei.
- When the dipole-operator is dominant:

$$\frac{{\rm BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k)}{{\rm BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)} \simeq \frac{\alpha_{el}}{3\pi}\left(\log\frac{m^2_{\ell_i}}{m^2_{\ell_k}} - 3\right)\frac{{\rm BR}(\ell_i \to \ell_j \gamma)}{{\rm BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)}\,,$$

$${\rm CR}(\mu \to e \text{ in N}) \simeq \alpha_{\rm em} \times {\rm BR}(\mu \to e\gamma)\,.$$

- ${\rm BR}(\mu \to \mathbf{e}\gamma) \sim \mathbf{5 \times 10^{-13}}$ implies

$$\frac{{\rm BR}(\mu \to \mathbf{3e})}{\mathbf{3 \times 10^{-15}}} \approx \frac{{\rm BR}(\mu \to \mathbf{e}\gamma)}{\mathbf{5 \times 10^{-13}}} \approx \frac{{\rm CR}(\mu \to \mathbf{e} \text{ in N})}{\mathbf{3 \times 10^{-15}}}$$

- $\mu + N \to e + N$ on different N discriminates the operator at work [Okada et al. 2004].
- An angular analysis for $\mu \to eee$ can test operator which is at work.

Pattern of LFV in NP models

- **Ratios like $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ probe the NP flavor structure**
- **Ratios like $Br(\mu \to e\gamma)/Br(\mu \to eee)$ probe the NP operator at work**

ratio	LHT	MSSM	SM4
$\frac{Br(\mu\to eee)}{Br(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{Br(\tau\to eee)}{Br(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{Br(\tau\to \mu\mu\mu)}{Br(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{Br(\tau\to e\mu\mu)}{Br(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.03\ldots 1.3$
$\frac{Br(\tau\to \mu ee)}{Br(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{Br(\tau\to eee)}{Br(\tau\to e\mu\mu)}$	$0.8\ldots 2$	~ 5	$1.5\ldots 2.3$
$\frac{Br(\tau\to \mu\mu\mu)}{Br(\tau\to \mu ee)}$	$0.7\ldots 1.6$	~ 0.2	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{Br(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$10^{-12}\ldots 26$

[Buras et al., '07, '10]

On leptonic dipoles: $\ell \to \ell' \gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e \frac{m_\ell}{2} \left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R \right) F^{\mu\nu} \qquad \ell, \ell' = e, \mu, \tau \,,$$

$$A_{\ell\ell'} = \frac{1}{(4\pi\,\Lambda_{\rm NP})^2} \left[\left(g^L_{\ell k}\, g^{L*}_{\ell' k} + g^R_{\ell k}\, g^{R*}_{\ell' k} \right) f_1(x_k) + \frac{v}{m_\ell} \left(g^L_{\ell k}\, g^{R*}_{\ell' k} \right) f_2(x_k) \right] ,$$

  - **Δa_ℓ and leptonic EDMs are given by**

$$\Delta a_\ell = 2 m_\ell^2 \,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell \,\mathrm{Im}(A_{\ell\ell}) \,.$$

  - **The branching ratios of $\ell \to \ell' \gamma$ are given by**

$$\frac{\mathrm{BR}(\ell \to \ell' \gamma)}{\mathrm{BR}(\ell \to \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3 \alpha}{G_F^2} \left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2 \right) \,.$$

- **“Naive scaling”:**

$$\Delta a_{\ell_i} / \Delta a_{\ell_j} = m^2_{\ell_i} / m^2_{\ell_j}, \qquad d_{\ell_i} / d_{\ell_j} = m_{\ell_i} / m_{\ell_j} \,.$$

(for instance, if the new particles have an underlying SU(3) flavor symmetry in their mass spectrum and in their couplings to leptons, which is the case for gauge interactions).

[Giudice, P.P., & Passera, ’12]

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j\gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2,$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\ell\tau}}{10^{-2}}\right)^2.$$

- **EDMs assuming "Naive scaling"** $d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}$

$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-24} \tan\phi_e \; e\,\mathrm{cm},$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 2 \times 10^{-22} \tan\phi_\mu \; e\,\mathrm{cm},$$

$$d_\tau \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 4 \times 10^{-21} \tan\phi_\tau \; e\,\mathrm{cm},$$

- $(g-2)_\ell$ **assuming "Naive scaling"** $\Delta a_{\ell_i}/\Delta a_{\ell_j} = m^2_{\ell_i}/m^2_{\ell_j}$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}, \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}.$$

[Giudice, P.P., & Passera, '12]

A concrete SUSY scenario: “Disoriented A-terms”

- **Challenge**: Large effects for $g-2$ keeping under control $\mu \to e\gamma$ and d_e
- **“Disoriented A-terms”** [Giudice, Isidori & P.P., ’12]:

$$(\delta^{ij}_{LR})_f \sim \frac{A_f \theta^f_{ij} m_{f_j}}{m_{\tilde{f}}} \qquad f = u, d, \ell \,,$$

  - Flavor and CP violation is restricted to the trilinear scalar terms.
  - Flavor bounds of the down-sector are naturally satisfied thanks to the smallness of down-type quark/lepton masses.
  - This ansatz arises in scenarios with partial compositeness (where a natural prediction is $\theta^\ell_{ij} \sim \sqrt{m_i/m_j}$ [Rattazzi et al.,’12]) or, as shown in [Calibbi, P.P. and Ziegler,’13], in Flavored Gauge Mediation models [Shadmi and collaborators].

- $\mu \to e\gamma$ and d_e are generated only by $U(1)$ interactions

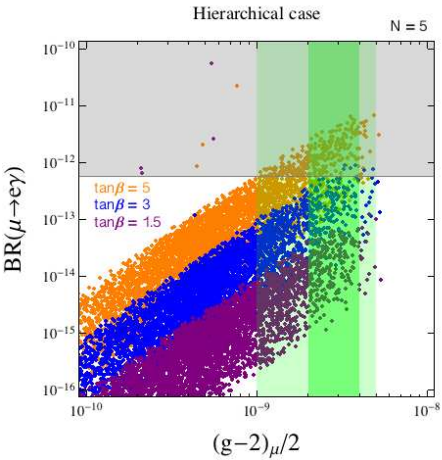
$$\mathrm{BR}(\mu \to e\gamma) \sim \left(\frac{\alpha}{\cos^2\theta_W}\right)^2 \, |\delta^{\mu e}_{LR}|^2 \,, \qquad \frac{d_e}{e} \sim \frac{\alpha}{\cos^2\theta_W} \, \mathrm{Im}\delta^{ee}_{LR} \,.$$

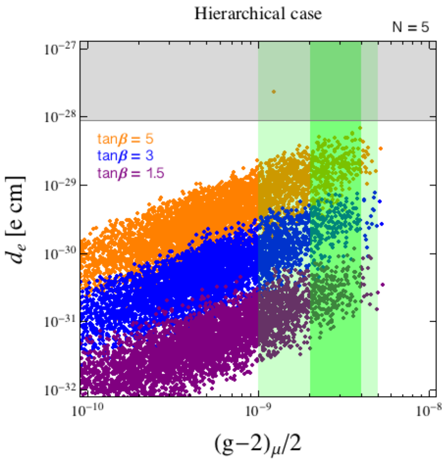
- $(g-2)_\mu$ is generated by $SU(2)$ interactions and is $\tan\beta$ enhanced

$$\Delta a_\ell \sim \frac{\alpha}{\sin^2\theta_W} \, \tan\beta$$

- $(g-2)_\mu$ is enhanced by $\approx 100 \times (\tan\beta/30)$ w.r.t. $\mu \to e\gamma$ and d_e amplitudes

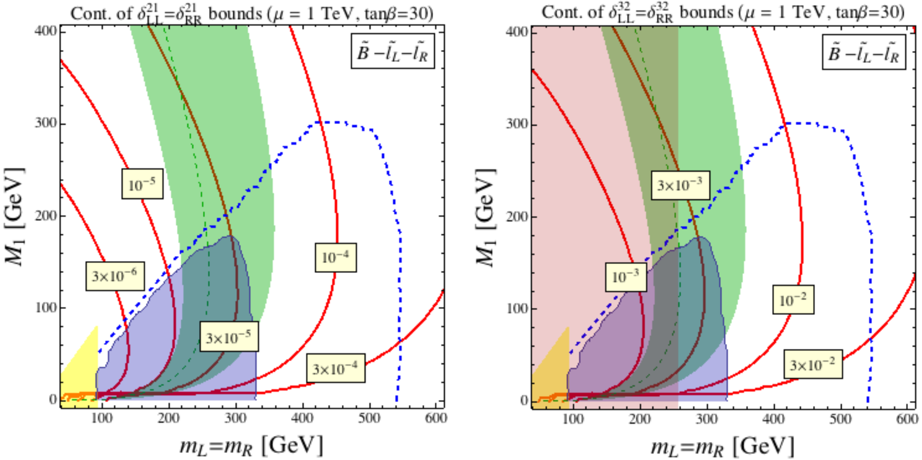
A concrete SUSY scenario: "Flavored Gauge Mediation"





- LFV processes with an undelying $\tau - \mu$ and $\tau - e$ are unobservable

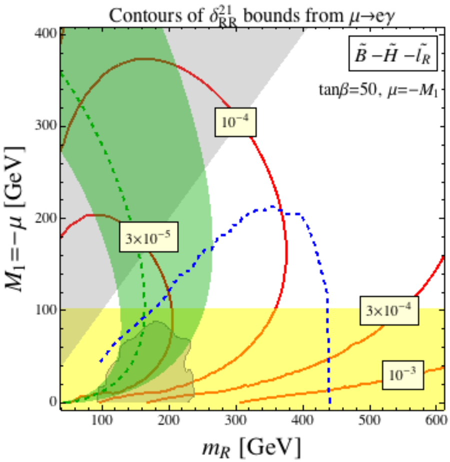
[Calibbi, P.P., & Ziegler, '14]

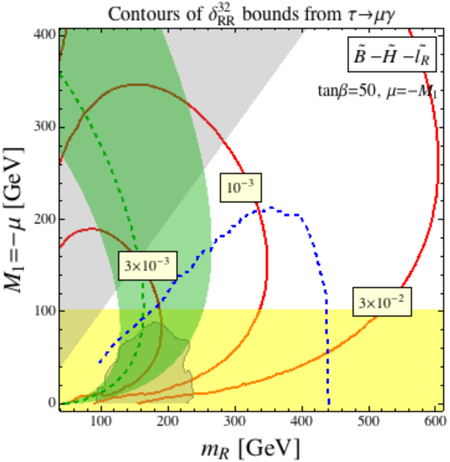


- The light-blue (yellow) area is excluded by ATLAS (LEP) and the dashed line refers to the limits by LHC14 with $\mathcal{L} = 100\ \mathrm{fb}^{-1}$. The green band explains the $(g-2)_\mu$ anomaly at 2σ. The red-shaded area is excluded by a stau LSP.

[Calibbi, Galon, Masiero, P.P., & Shadmi, '15]

LFV vs. LHC





- The light-blue (yellow) area is excluded by ATLAS (LEP) and the dashed line refers to the limits by LHC14 with $\mathcal{L} = 100\ \mathrm{fb}^{-1}$. The green band explains the $(g-2)_\mu$ anomaly at 2σ. In the grey area the LSP is not neutral.

[Calibbi, Galon, Masiero, P.P., & Shadmi, '15]

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes.
  - On general grounds, we can expect any size of deviation below the current bounds.
  - cLFV processes, leptonic EDMs and LFU observables $R^{e/\mu}_{K,\pi}$ do not suffer from theoretical limitations (clean th. observables).
  - On the experimental side there are still excellent prospects of improvements in several clean channels especially in the leptonic sector: $\mu \to e\gamma$, $\mu N \to eN$, $\mu \to eee$, τ-LFV, EDMs and leptonic $(g-2)$ and also $R^{e/\mu}_{K,\pi}$.
  - The the origin of the $(g-2)_\mu$ discrepancy can be understood testing new-physics effects in the electron $(g-2)_e$. This would require improved measurements of $(g-2)_e$ and more refined determinations of α in atomic-physics experiments.

Conclusions

The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- **Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?**

- **Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?**

Irrespectively of whether the LHC will discover or not new particles, flavor physics in the leptonic sector (especially cLFV, leptonic $g-2$ and EDMs) will teach us a lot...