Theory of Lepton Flavour Violation

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The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?
- Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?

Related important questions are:

- Which is the role of flavor physics in the LHC era?
- Do we expect to understand the (SM and NP) flavor puzzles through the synergy and interplay of flavor physics and the LHC?
NP search strategies

- **High-energy frontier**: A unique effort to determine the NP scale

- **High-intensity frontier** (flavor physics): A collective effort to determine the flavor structure of NP

Where to look for **New Physics** at the low energy?

- **Processes very suppressed or even forbidden** in the SM
  - FCNC processes ($\mu \to e\gamma$, $\mu \to eee$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $B_{s,d}^0 \to \mu^+\mu^-$...)
  - CPV effects in the electron/neutron EDMs, $d_{e,n}$...
  - FCNC & CPV in $B_{s,d}$ & $D$ decay/mixing amplitudes

- **Processes predicted with high precision** in the SM
  - EWPO as $(g - 2)_\mu,e$: $a_{\mu}^{exp} - a_{\mu}^{SM} \approx (3 \pm 1) \times 10^{-9}$, a discrepancy at $3\sigma$!
  - LU in $R_{M}^{e/\mu} = \Gamma(M \to e\nu)/\Gamma(M \to \mu\nu)$ with $M = \pi, K$
<table>
<thead>
<tr>
<th>LFV process</th>
<th>Experiment</th>
<th>Future limits</th>
<th>Year (expected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR(µ → eγ)</td>
<td>MEG</td>
<td>$\mathcal{O}(10^{-14})$</td>
<td>~ 2017</td>
</tr>
<tr>
<td></td>
<td>Project X</td>
<td>$\mathcal{O}(10^{-15})$</td>
<td>&gt; 2021</td>
</tr>
<tr>
<td>BR(µ → eee)</td>
<td>Mu3e</td>
<td>$\mathcal{O}(10^{-15})$</td>
<td>~ 2017</td>
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<tr>
<td></td>
<td>Mu3e</td>
<td>$\mathcal{O}(10^{-16})$</td>
<td>&gt; 2017</td>
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<td></td>
<td>MUSIC</td>
<td>$\mathcal{O}(10^{-16})$</td>
<td>~ 2017</td>
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<tr>
<td></td>
<td>Project X</td>
<td>$\mathcal{O}(10^{-17})$</td>
<td>&gt; 2021</td>
</tr>
<tr>
<td>CR(µ → e)</td>
<td>COMET</td>
<td>$\mathcal{O}(10^{-17})$</td>
<td>~ 2017</td>
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<tr>
<td></td>
<td>Mu2e</td>
<td>$\mathcal{O}(10^{-17})$</td>
<td>~ 2020</td>
</tr>
<tr>
<td></td>
<td>PRISM/PRIME</td>
<td>$\mathcal{O}(10^{-18})$</td>
<td>~ 2020</td>
</tr>
<tr>
<td></td>
<td>Project X</td>
<td>$\mathcal{O}(10^{-19})$</td>
<td>&gt; 2021</td>
</tr>
<tr>
<td>BR(τ → µγ)</td>
<td>Belle II</td>
<td>$\mathcal{O}(10^{-8})$</td>
<td>&gt; 2020</td>
</tr>
<tr>
<td>BR(τ → µµµ)</td>
<td>Belle II</td>
<td>$\mathcal{O}(10^{-10})$</td>
<td>&gt; 2020</td>
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<tr>
<td>BR(τ → eγ)</td>
<td>Belle II</td>
<td>$\mathcal{O}(10^{-9})$</td>
<td>&gt; 2020</td>
</tr>
<tr>
<td>BR(τ → µµµ)</td>
<td>Belle II</td>
<td>$\mathcal{O}(10^{-10})$</td>
<td>&gt; 2020</td>
</tr>
</tbody>
</table>

**Table:** Future sensitivities of next-generation experiments.
The NP “scale”

- **Gravity** $\Rightarrow \Lambda_{\text{Planck}} \sim 10^{18-19} \text{ GeV}
- **Neutrino masses** $\Rightarrow \Lambda_{\text{see-saw}} \lesssim 10^{15} \text{ GeV}
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}
  - Leptogenesis $\Rightarrow \Lambda_{\text{see-saw}} \lesssim 10^{15} \text{ GeV}
- **Hierarchy problem**: $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}
- **Dark Matter** $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}

**SM = effective theory at the EW scale**

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{d \geq 5} \frac{C_{ij}^{(d)}}{\Lambda_{\text{NP}}^{d-4}} O_{ij}^{(d)}$$

- $L_{\text{eff}}^{d=5} = \frac{y_{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi \phi$,
- $L_{\text{eff}}^{d=6}$ generates FCNC operators

$$\text{BR}(\ell_i \rightarrow \ell_j \gamma) \sim \frac{v^4}{\Lambda_{\text{NP}}^4}$$
Why LFV is interesting?

- **Neutrino Oscillation** ⇒ $m_{\nu_i} \neq m_{\nu_j} \Rightarrow$ LFV

- **see-saw**: $m_\nu \sim \frac{\nu^2}{M_R} \sim eV \Rightarrow M_R \sim 10^{14-16}$

- LFV transitions like $\mu \rightarrow e\gamma$ @ 1 loop with exchange of
  - $W$ and $\nu$ in the SM with $\Lambda_{NP} \equiv M_R \equiv \Lambda_{\text{see-saw}}$

\[ Br(\mu \rightarrow e\gamma) \sim \frac{\nu^4}{M_R^4} \leq 10^{-50} \quad \text{GIM} \]

  - If $\Lambda_{NP} \ll \Lambda_{\text{see-saw}}$ ($\Lambda_{NP} \equiv m_{\text{susy}}$ in the MSSM)

\[ Br(\mu \rightarrow e\gamma) \sim \frac{\nu^4}{\Lambda_{NP}^4} \]

- LFV generally detectable in (multi) TeV scale NP scenarios like the MSSM, ....
The NP “scale” vs. LFV

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d \geq 5} \frac{C_{ij}}{\Lambda_{NP}^{d-4}} O_{ij}^{(d)} \]

<table>
<thead>
<tr>
<th>Process</th>
<th>Relevant operators</th>
<th>Present Bound on ( \Lambda ) (TeV)</th>
<th>Future Bound on ( \Lambda ) (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu \to e\gamma )</td>
<td>( \frac{C}{\Lambda^2} \frac{m_\mu^2}{16\pi^2} \bar{\mu}_L \gamma^\mu \bar{e}_R e_L )</td>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu \to e e e )</td>
<td>( \frac{C}{\Lambda^2} \bar{\mu}_L \gamma^\mu e_L (\bar{e}_L \gamma^\mu e_L) )</td>
<td>17</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>( \frac{C}{\Lambda^2} \bar{\mu}_L e_R (\bar{e}_R e_L) )</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>( \mu \to e ) in Ti</td>
<td>( \frac{C}{\Lambda^2} \bar{\mu}_L \gamma^\mu e_L (\bar{d}_L \gamma^\mu d_L) )</td>
<td>30</td>
<td>580</td>
</tr>
<tr>
<td></td>
<td>( \frac{C}{\Lambda^2} \bar{\mu}_L e_R (\bar{d}_R d_L) )</td>
<td>60</td>
<td>1000</td>
</tr>
</tbody>
</table>

\( \text{BR}(\mu \to e\gamma) < 5 \times 10^{-14} \)

\( \text{BR}(\mu \to e e e) < 10^{-16} \)

\( \text{CR}(\mu \to e \) in Ti\) \( < 5 \times 10^{-17} \)

Calibbi @ IFAE2014

updated from LC Lalak Pokorski Ziegler ‘12
SM vs. NP flavor problems

- Can the SM and NP flavour problems have a common explanation?

- Froggat-Nielsen ’79: Hierarchies from SSB of a Flavour Symmetry

\[ \epsilon = \frac{\langle \phi \rangle}{M} \ll 1 \Rightarrow Y_{ij} \propto \epsilon^{(a_i+b_j)} \]

- Flavor protection from flavor models: [Lalak, Pokorski & Ross ’10]

<table>
<thead>
<tr>
<th>Operator</th>
<th>$U(1)$</th>
<th>$U(1)^2$</th>
<th>$SU(3)$</th>
<th>MFV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(Q_L X^Q_{LL} Q_L)_{12}$</td>
<td>$\lambda$</td>
<td>$\lambda^5$</td>
<td>$\lambda^3$</td>
<td>$\lambda^5$</td>
</tr>
<tr>
<td>$(D_R X^D_{RR} D_R)_{12}$</td>
<td>$\lambda$</td>
<td>$\lambda^{11}$</td>
<td>$\lambda^3$</td>
<td>$(y_d y_s) \times \lambda^5$</td>
</tr>
<tr>
<td>$(Q_L X^D_{LR} D_R)_{12}$</td>
<td>$\lambda^4$</td>
<td>$\lambda^9$</td>
<td>$\lambda^3$</td>
<td>$y_s \times \lambda^5$</td>
</tr>
</tbody>
</table>

- Is this flavor protection enough?
- Can we disentangle flavour models through flavour physics?
Not only $\mu \to e\gamma$...

- **LFV operators @ dim-6**

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda_{\text{LFV}}^2} \mathcal{O}^{\text{dim-6}} + \ldots .$$

$$\mathcal{O}^{\text{dim-6}} \ni \bar{\mu}_R \sigma^{\mu\nu} H e_L F_{\mu\nu}, \ (\bar{\mu}_L \gamma^\mu e_L) (\bar{f}_L \gamma^\mu f_L), \ (\bar{\mu}_R e_L) (\bar{f}_R f_L), \ f = e, u, d$$

- the dipole-operator leads to $\ell \to \ell' \gamma$ while 4-fermion operators generate processes like $\ell_i \to \ell_j \ell_k \bar{\ell}_k$ and $\mu \to e$ conversion in Nuclei.

- When the dipole-operator is dominant:

$$\frac{\text{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k)}{\text{BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)} \simeq \frac{\alpha_{\text{el}}}{3\pi} \left( \log \frac{m_{\ell_i}^2}{m_{\ell_k}^2} - 3 \right) \frac{\text{BR}(\ell_i \to \ell_j \gamma)}{\text{BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)},$$

$$\text{CR}(\mu \to e \text{ in N}) \simeq \alpha_{\text{em}} \times \text{BR}(\mu \to e\gamma).$$

- $\text{BR}(\mu \to e\gamma) \sim 5 \times 10^{-13}$ implies

$$\frac{\text{BR}(\mu \to 3e)}{3 \times 10^{-15}} \approx \frac{\text{BR}(\mu \to e\gamma)}{5 \times 10^{-13}} \approx \frac{\text{CR}(\mu \to e \text{ in N})}{3 \times 10^{-15}}$$

- $\mu + N \to e + N$ on different N discriminates the operator at work [Okada et al. 2004].

- An angular analysis for $\mu \to eee$ can test operator which is at work.
Pattern of LFV in NP models

- Ratios like $Br(\mu \rightarrow e\gamma)/Br(\tau \rightarrow \mu\gamma)$ probe the NP flavor structure

- Ratios like $Br(\mu \rightarrow e\gamma)/Br(\mu \rightarrow eee)$ probe the NP operator at work

<table>
<thead>
<tr>
<th>ratio</th>
<th>LHT</th>
<th>MSSM</th>
<th>SM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{Br(\mu \rightarrow eee)}{Br(\mu \rightarrow e\gamma)}$</td>
<td>$0.02 \ldots 1$</td>
<td>$\sim 2 \cdot 10^{-3}$</td>
<td>$0.06 \ldots 2.2$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow eee)}{Br(\tau \rightarrow e\gamma)}$</td>
<td>$0.04 \ldots 0.4$</td>
<td>$\sim 1 \cdot 10^{-2}$</td>
<td>$0.07 \ldots 2.2$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow \mu\mu\mu)}{Br(\tau \rightarrow e\gamma)}$</td>
<td>$0.04 \ldots 0.4$</td>
<td>$\sim 2 \cdot 10^{-3}$</td>
<td>$0.06 \ldots 2.2$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow e\mu\mu)}{Br(\tau \rightarrow e\gamma)}$</td>
<td>$0.04 \ldots 0.3$</td>
<td>$\sim 2 \cdot 10^{-3}$</td>
<td>$0.03 \ldots 1.3$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow \mu ee)}{Br(\tau \rightarrow \mu\gamma)}$</td>
<td>$0.04 \ldots 0.3$</td>
<td>$\sim 1 \cdot 10^{-2}$</td>
<td>$0.04 \ldots 1.4$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow eee)}{Br(\tau \rightarrow e\mu\mu)}$</td>
<td>$0.8 \ldots 2$</td>
<td>$\sim 5$</td>
<td>$1.5 \ldots 2.3$</td>
</tr>
<tr>
<td>$\frac{Br(\tau \rightarrow \mu\mu\mu)}{Br(\tau \rightarrow \mu ee)}$</td>
<td>$0.7 \ldots 1.6$</td>
<td>$\sim 0.2$</td>
<td>$1.4 \ldots 1.7$</td>
</tr>
<tr>
<td>$\frac{R(\mu Ti \rightarrow eTi)}{Br(\mu \rightarrow e\gamma)}$</td>
<td>$10^{-3} \ldots 10^{2}$</td>
<td>$\sim 5 \cdot 10^{-3}$</td>
<td>$10^{-12} \ldots 26$</td>
</tr>
</tbody>
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[Buras et al., '07, '10]
On leptonic dipoles: $\ell \rightarrow \ell' \gamma$

- NP effects are encoded in the effective Lagrangian

$$\mathcal{L} = e \frac{m_\ell}{2} \left( \bar{\ell}_R \sigma_{\mu \nu} A_{\ell \ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu \nu} A^{*}_{\ell \ell'} \ell_R \right) F^{\mu \nu} \quad \ell, \ell' = e, \mu, \tau ,$$

$$A_{\ell \ell'} = \frac{1}{(4\pi \Lambda_{NP})^2} \left[ \left( g_{\ell k}^L g^{L*}_{\ell' k} + g_{\ell k}^R g^{R*}_{\ell' k} \right) f_1(x_k) + \frac{v}{m_\ell} \left( g_{\ell k}^L g^{R*}_{\ell' k} \right) f_2(x_k) \right] ,$$

- $\Delta a_\ell$ and leptonic EDMs are given by

$$\Delta a_\ell = 2m_\ell^2 \Re(A_{\ell \ell}), \quad \frac{d_\ell}{e} = m_\ell \Im(A_{\ell \ell}) .$$

- The branching ratios of $\ell \rightarrow \ell' \gamma$ are given by

$$\frac{\text{BR}(\ell \rightarrow \ell' \gamma)}{\text{BR}(\ell \rightarrow \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3 \alpha}{G_F^2} \left( |A_{\ell \ell'}|^2 + |A_{\ell' \ell'}|^2 \right) .$$

- "Naive scaling":

$$\Delta a_{\ell_i}/\Delta a_{\ell_j} = m_{\ell_i}^2/m_{\ell_j}^2, \quad \frac{d_{\ell_i}}{e} = \frac{m_{\ell_i}}{m_{\ell_j}} .$$

(for instance, if the new particles have an underlying SU(3) flavor symmetry in their mass spectrum and in their couplings to leptons, which is the case for gauge interactions).

[Giudice, P.P., & Passera, '12]
Model-independent predictions

- BR($\ell_i \rightarrow \ell_j \gamma$) vs. $(g - 2)_\mu$

  \[
  \text{BR}(\mu \rightarrow e \gamma) \approx 3 \times 10^{-13} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{e\mu}}{10^{-5}} \right)^2 ,
  \]

  \[
  \text{BR}(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-8} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{\ell\tau}}{10^{-2}} \right)^2 .
  \]

- EDMs assuming “Naive scaling” $d_\ell_i / d_\ell_j = m_\ell_i / m_\ell_j$

  \[
  d_e \approx \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-24} \tan \phi_e \ e \ cm ,
  \]

  \[
  d_\mu \approx \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \tan \phi_\mu \ e \ cm ,
  \]

  \[
  d_\tau \approx \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 4 \times 10^{-21} \tan \phi_\tau \ e \ cm ,
  \]

- $(g - 2)_\ell$ assuming “Naive scaling” $\Delta a_\ell_i / \Delta a_\ell_j = m_\ell_i / m_\ell_j$

  \[
  \Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13} , \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6} .
  \]

[Giudice, P.P., & Passera, '12]
A concrete SUSY scenario: “Disoriented A-terms”

- **Challenge**: Large effects for $g-2$ keeping under control $\mu \rightarrow e\gamma$ and $d_e$


$$ (\delta_{LR}^{ij})_f \sim \frac{A_f \theta_{ij}^f m_{f_j}}{m_f^f} \quad f = u, d, \ell, $$

  - Flavor and CP violation is restricted to the trilinear scalar terms.
  - Flavor bounds of the down-sector are naturally satisfied thanks to the smallness of down-type quark/lepton masses.
  - This ansatz arises in scenarios with partial compositeness (where a natural prediction is $\theta_{ij}^\ell \sim \sqrt{m_i/m_j}$ [Rattazzi et al.,’12]) or, as shown in [Calibbi, P.P. and Ziegler,’13], in Flavored Gauge Mediation models [Shadmi and collaborators].

- $\mu \rightarrow e\gamma$ and $d_e$ are generated only by $U(1)$ interactions

$$ \text{BR}(\mu \rightarrow e\gamma) \sim \left(\frac{\alpha}{\cos^2 \theta_W}\right)^2 |\delta_{LR}^{\mu e}|^2, \quad \frac{d_e}{e} \sim \frac{\alpha}{\cos^2 \theta_W} \text{Im} \delta_{LR}^{ee}. $$

- $(g-2)_\mu$ is generated by $SU(2)$ interactions and is $\tan \beta$ enhanced

$$ \Delta a_\ell \sim \frac{\alpha}{\sin^2 \theta_W} \tan \beta $$

- $(g-2)_\mu$ is enhanced by $\approx 100 \times (\tan \beta/30)$ w.r.t. $\mu \rightarrow e\gamma$ and $d_e$ amplitudes
A concrete SUSY scenario: “Flavored Gauge Mediation”

- LFV processes with an underlying $\tau - \mu$ and $\tau - e$ are unobservable

[Calibbi, P.P., & Ziegler, '14]
• The light-blue (yellow) area is excluded by ATLAS (LEP) and the dashed line refers to the limits by LHC14 with $\mathcal{L} = 100 \text{ fb}^{-1}$. The green band explains the $(g - 2)_\mu$ anomaly at 2σ. The red-shaded area is excluded by a stau LSP.

[Calibbi, Galon, Masiero, P.P., & Shadmi, ’15]
The light-blue (yellow) area is excluded by ATLAS (LEP) and the dashed line refers to the limits by LHC14 with $\mathcal{L} = 100 \, fb^{-1}$. The green band explains the $(g - 2)_\mu$ anomaly at $2\sigma$. In the grey area the LSP is not neutral.

[Calibbi, Galon, Masiero, P.P., & Shadmi, '15]
Conclusions and future prospects

• **Important questions in view of ongoing/future experiments are:**

  ▶ What are the expected deviations from the SM predictions induced by TeV NP?
  ▶ Which observables are not limited by theoretical uncertainties?
  ▶ In which case we can expect a substantial improvement on the experimental side?
  ▶ What will the measurements teach us if deviations from the SM are [not] seen?

• **(Personal) answers:**

  ▶ The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes.
  ▶ On general grounds, we can expect any size of deviation below the current bounds.
  ▶ cLFV processes, leptonic EDMs and LFU observables $R_{K,\pi}^{e/\mu}$ do not suffer from theoretical limitations (clean th. observables).
  ▶ On the experimental side there are still excellent prospects of improvements in several clean channels especially in the leptonic sector: $\mu \rightarrow e\gamma$, $\mu N \rightarrow eN$, $\mu \rightarrow eee$, $\tau$-LFV, EDMs and leptonic $(g-2)$ and also $R_{K,\pi}^{e/\mu}$.
  ▶ The origin of the $(g-2)_\mu$ discrepancy can be understood testing new-physics effects in the electron $(g-2)_e$. This would require improved measurements of $(g-2)_e$ and more refined determinations of $\alpha$ in atomic-physics experiments.
Conclusions

The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?

- Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?

Irrespective of whether the LHC will discover or not new particles, flavor physics in the leptonic sector (especially cLFV, leptonic $g - 2$ and EDMs) will teach us a lot...