Volume (in-)dependence in $SU(N)$ gauge theories with twisted boundary conditions

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Large-N volume independence

- Eguchi, Kawai, 82: Consider two pure gauge lattice theories:
  - $S_{YM} = \beta \sum_{x,\mu<\nu}(1 - \frac{1}{N}\mathrm{ReTr}\, U^{\square}_{x,\mu\nu})$
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- In fact, the symmetry *is* broken in EK model (Bhanot, Heller, Neuberger, 82).
- One possible fix (Gonzalez-Arroyo, Okawa, 83, 10): add twisted boundary conditions to the model:
  - $S_{TEK} = \beta \sum_{\mu<\nu}(1 - \frac{1}{N} z_{\mu\nu}\mathrm{ReTr}\, U_\mu U_\nu U^\dagger_\mu U^\dagger_\nu)$

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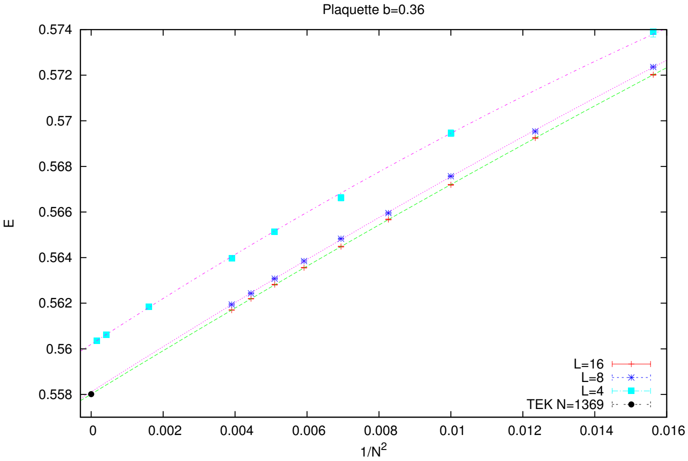
- Can be used to calculate Wilson loop expectation values, as well as meson correlators in momentum space.

Large-N volume independence

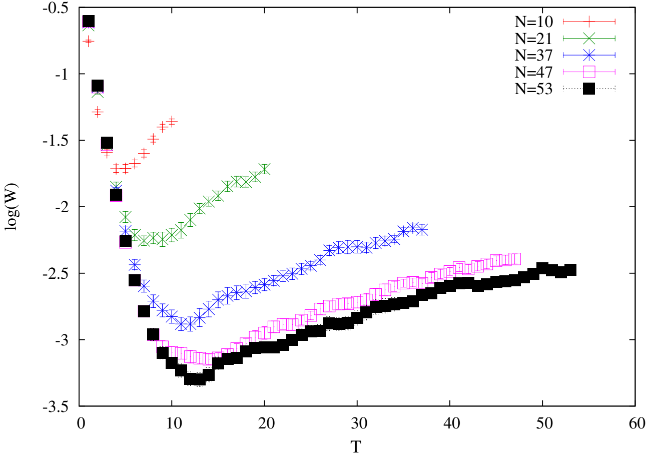


Comparison of plaquette in TEK and p.b.c. lattice calculations
Gonzalez-Arroyo, Okawa, 14.

Finite-N volume (in-)dependence

- Large-N volume independence strictly true only in the limit $N \to \infty$.
- What is the situation when working with $N < \infty$? Can we quantitatively define some effective system size $L_{\text{eff}}(N)$?

Finite-N volume (in-)dependence



Adjoint Eguchi-Kawai model (Bringoltz, MK, Sharpe, 2011):
Log of $1 \times T$ Wilson loops vs T for various values of N

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- We have three interesting possibilities:
  - 2+1 dim, spatial dimensions compact, $\propto NL$
  - 3+1 dim, all dimensions compact, $\propto \sqrt{N}L$
  - 3+1 dim, two spatial dimensions compact, $\propto NL$

x-scaling conjecture

- Theory: pure $SU(N)$ gauge theory on a spatial two-torus of size L with twisted boundary conditions.
- Dimensionless scaling variable $x = \frac{NL}{4\pi b}$, where $b = \frac{1}{g^2 N}$ is the inverse 't Hooft coupling

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- x-scaling conjecture: physical quantities in the theory depend only on x and the angle $\tilde{\theta}$ given by the parameters of the twist: $\tilde{\theta} = \frac{2\pi \bar{k}}{N}$, with integer $\bar{k}$ defined as: $k\bar{k} = 1 \pmod N$, where k is the magnetic flux.
- Can be thought of as a strong form of TEK-like volume independence also valid at finite N.

x-scaling conjecture cntd.

- Garcia, Gonzalez-Arroyo, Okawa 2013, 14: conjecture satisfied in PT for the non-zero electric flux sector ($\propto$ k-string tensions), also strong lattice confirmation.
- Can avoid tachyonic instabilities by suitably scaling $k, \bar{k} \propto N$, analogous to the Twisted Eguchi-Kawai model.
- What about the zero electric flux sector ($\propto$ glueballs/torelons)?
- Here $1/N^2$ corrections can arise in higher orders of PT.
- Also known to be approximate at large x (large volume), but $1/N^2$ coefficients "remarkably small" (Teper et al. 2015)

Numerical calculation

- Goal: numerically verify the conjecture, particularly in the glueball sector.
- For numerical investigation: lattice model with Wilson action:

$$S = Nb \sum_{n \in \mathbb{Z}^3_{(L,L,T)}} \sum_{\mu \neq \nu} \left(N - z^*_{\mu\nu}(n) U^{\Box}_{\mu\nu}(n) \right),$$

where $z_{\mu\nu}(n) = \exp\left(i\epsilon_{\mu\nu}\frac{2\pi k}{N}\right)$ at corner plaquettes in each (1,2)-plane, and 1 everywhere else.

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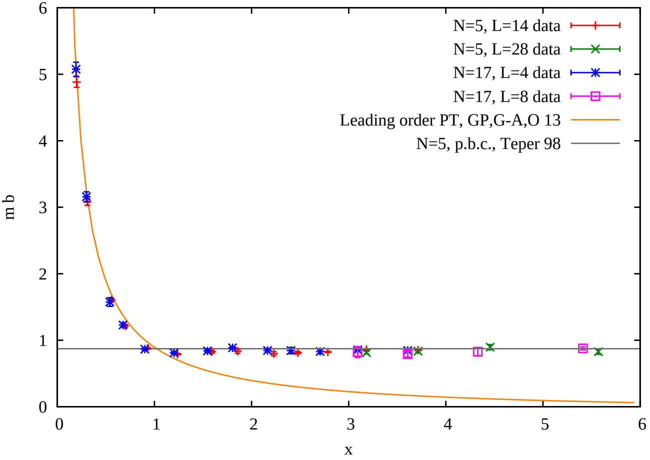
- Numerical agenda:
  1. Take theories with $N = 5, 7, 11, 17$ and approx. matching NL.
  2. Take all values of k and wide range of couplings, ranging from small-volume perturbative regime (small x), to large-volume non-perturbative one (large x).
  3. Calculate lightest scalar [and tensor] glueball masses, as well as electric flux energies.

Numerical calculation cntd.

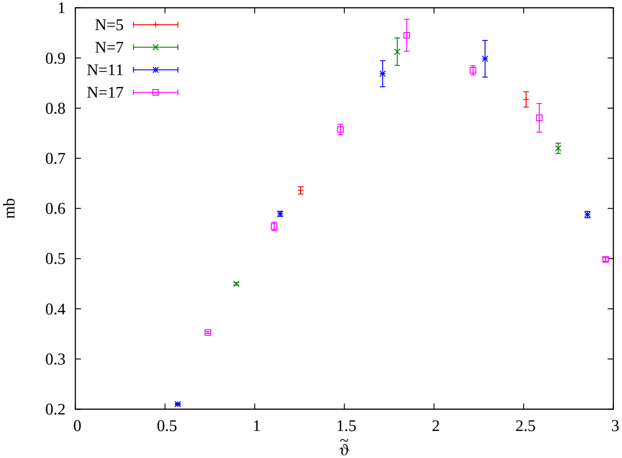
- Electric flux energies: find energies from plateaux of the Polyakov loop correlators with winding number $\bar{k}$.
- Glueballs: variational analysis, use basis of large rectangular Wilson loops and moduli of multi-winding Polyakov loops $|\mathrm{Tr}\, P^n|^2$, with different levels of smearing.
- Construct $C_{ij}(t) = \sum_{t'} \langle O_i(t'+t) O_j(t') \rangle - \langle O_i(t'+t) \rangle \langle O_j(t') \rangle$ and do GEVP to find improved plateaux.
- Look out for finite-temperature effects!

Results, glueball masses, scan in x



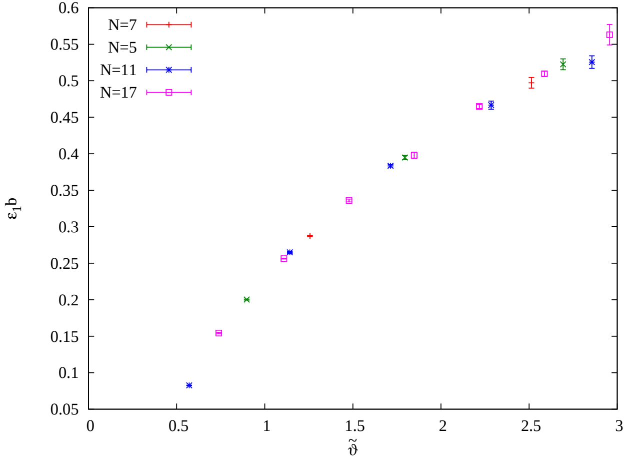
Comparison of lightest scalar glueball mass for $N = 5$, $\tilde{\theta} = 2.513$ and $N = 17$, $\tilde{\theta} = 2.587$, across a wide range of x.

Results, glueball masses, $x = 2.785$ $(b \approx 2)$



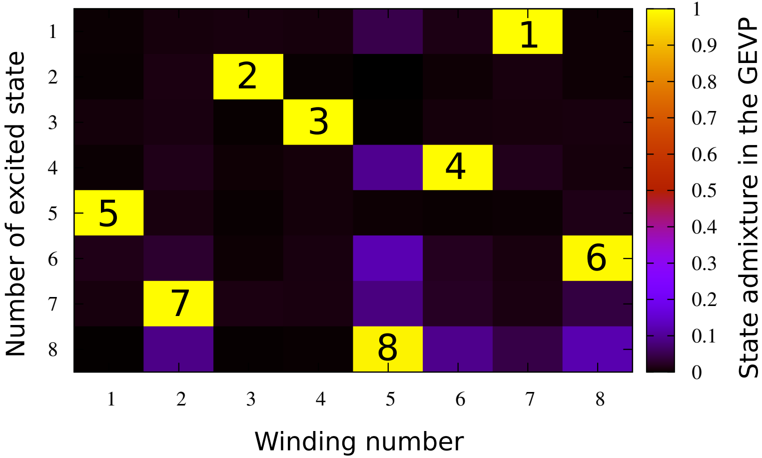
Lightest scalar glueball mass as function of $\tilde{\theta}$ for $N = 5, 7, 11, 17$, all values of k.

Results, electric flux energy, $x = 2.785$ ($b \approx 2$)



Lowest electric flux energy as function of $\tilde{\theta}$ for $N = 5, 7, 11, 17$, all values of k.

Results, overlap of operators, $x = 0.199$ ($b \approx 28$)



State admixture (= absolute value of the eigenvector) in the GEVP for $N = 17$ in the perturbative region. Black numbers on squares denote the PT expectation.

Summary & outlook

- x-scaling conjecture: in a 2+1 dimensional Yang-Mills theory with twisted boundary conditions physics is governed by product $NL\lambda$
- x-scaling with strong confirmation in the non-zero electric flux sector
- Strong hints that the same applies to zero-electric flux (glueball) sector, at least to a good approximation.
- Outlook: continue runs, particularly to reach large-volume regime.
- In principle, straightforward to generalize to $3+1$, modulo technical details.