

Emphasize recent  
highlights since  
EPS-HEP 2013!

# Lattice QCD for the precision era

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Fermilab

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# Where is the new physics?

◆ HEP experimental community searching for **tiny effects of new heavy particles** via broad program of precision measurements, targeting process that are

(1) **Extremely rare**

(or even forbidden)  
in the Standard Model

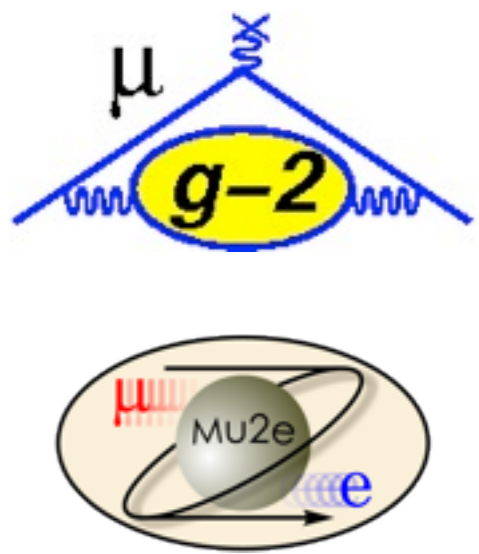
(2) **Predicted to high**

**theoretical precision**  
in the Standard Model

**neutrinos**



**charged leptons**



**quark flavor**



**Higgs**



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neutrinos

We do not know where the new physics lies →  
*cast a wide net!*

quark flavor



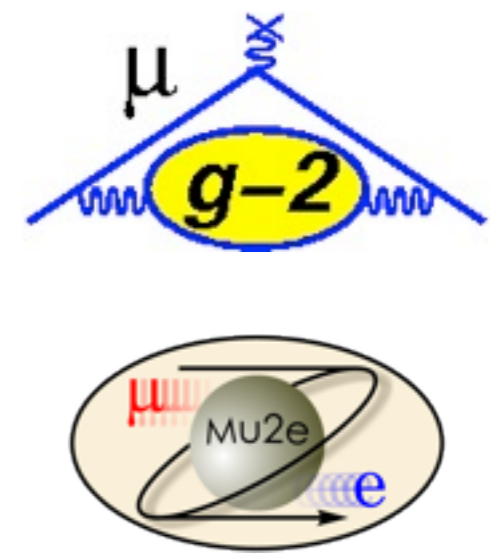
LHCb, Belle II, BES III, KOTO, NA62

Higgs



ILC, FCC TLEP, Muon Accelerator Program

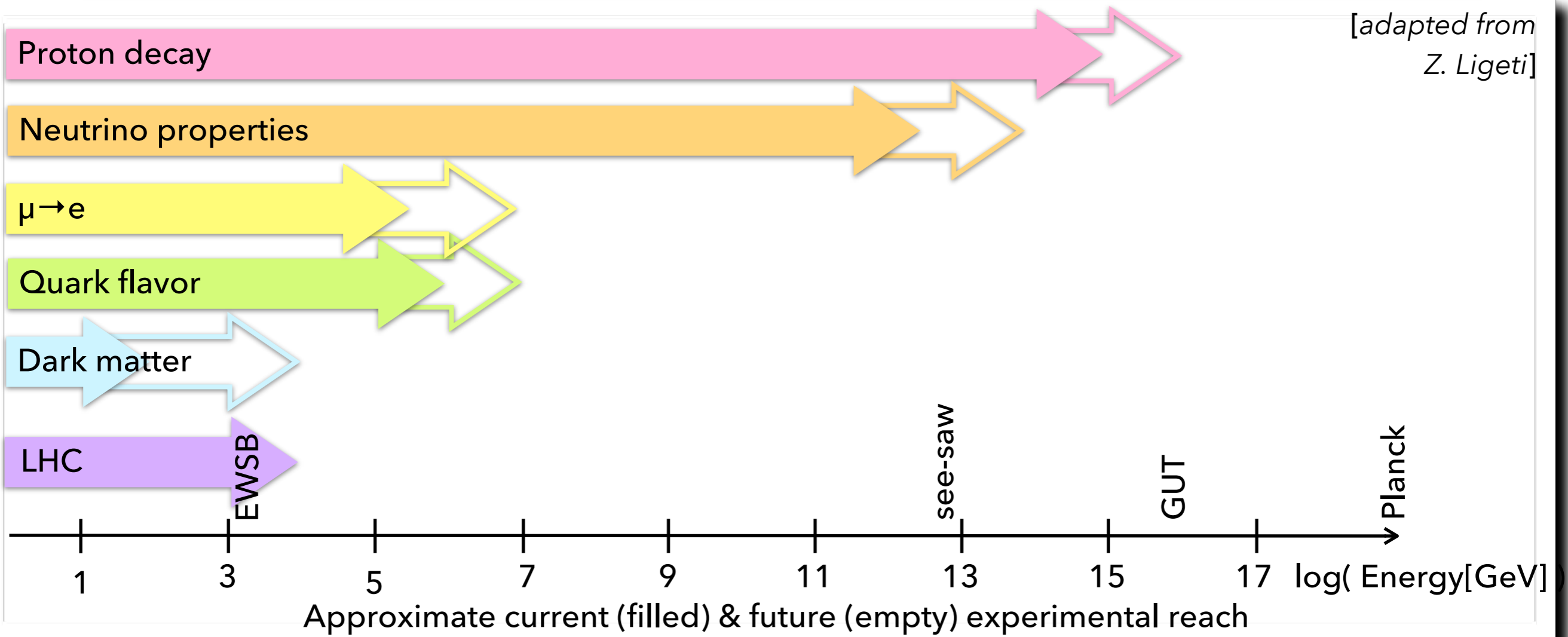
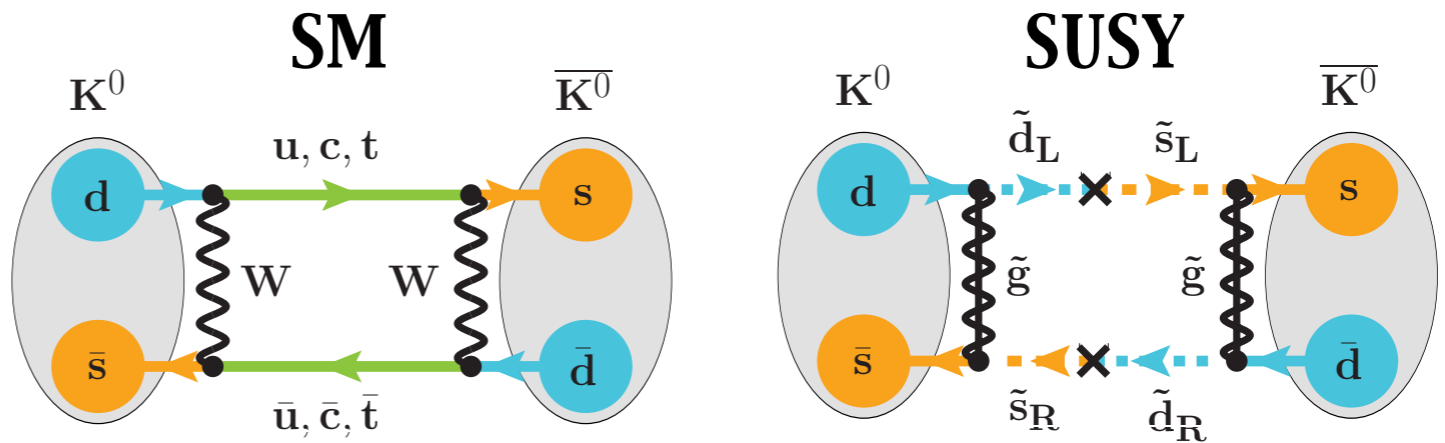
charged leptons



$\mu$ ,  $g-2$ , Mu2e

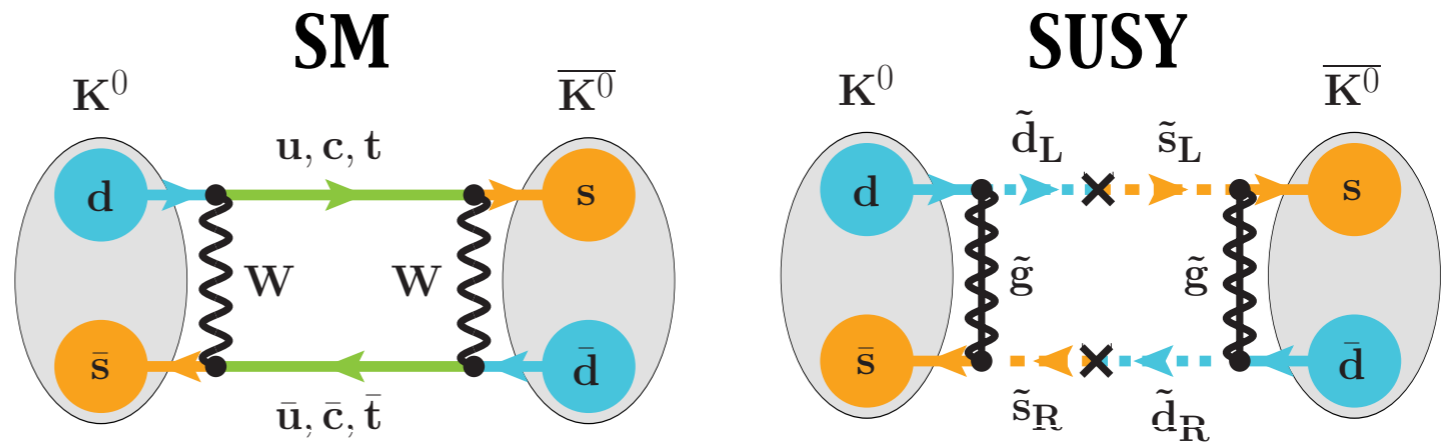
# New-physics reach

Quantum-mechanical loops sensitive to new particles above the TeV scale, e.g. in neutral kaon mixing



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Proton decay

Neutrino prop

$\mu \rightarrow e$

Quark flavor

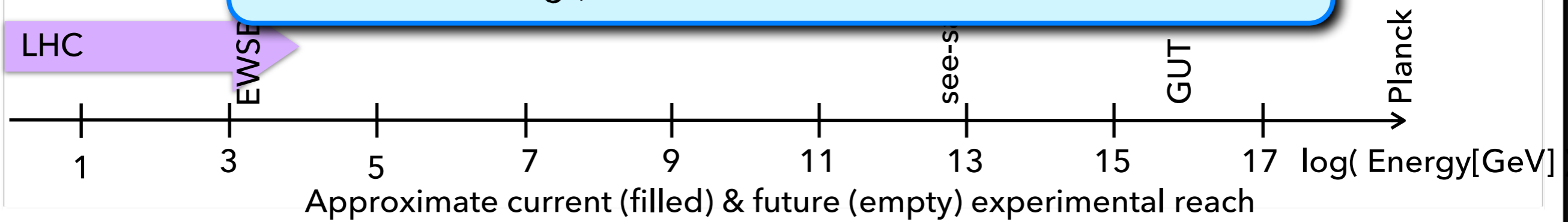
Dark matter

LHC

➔ Precision measurements essential ingredient of experimental program:

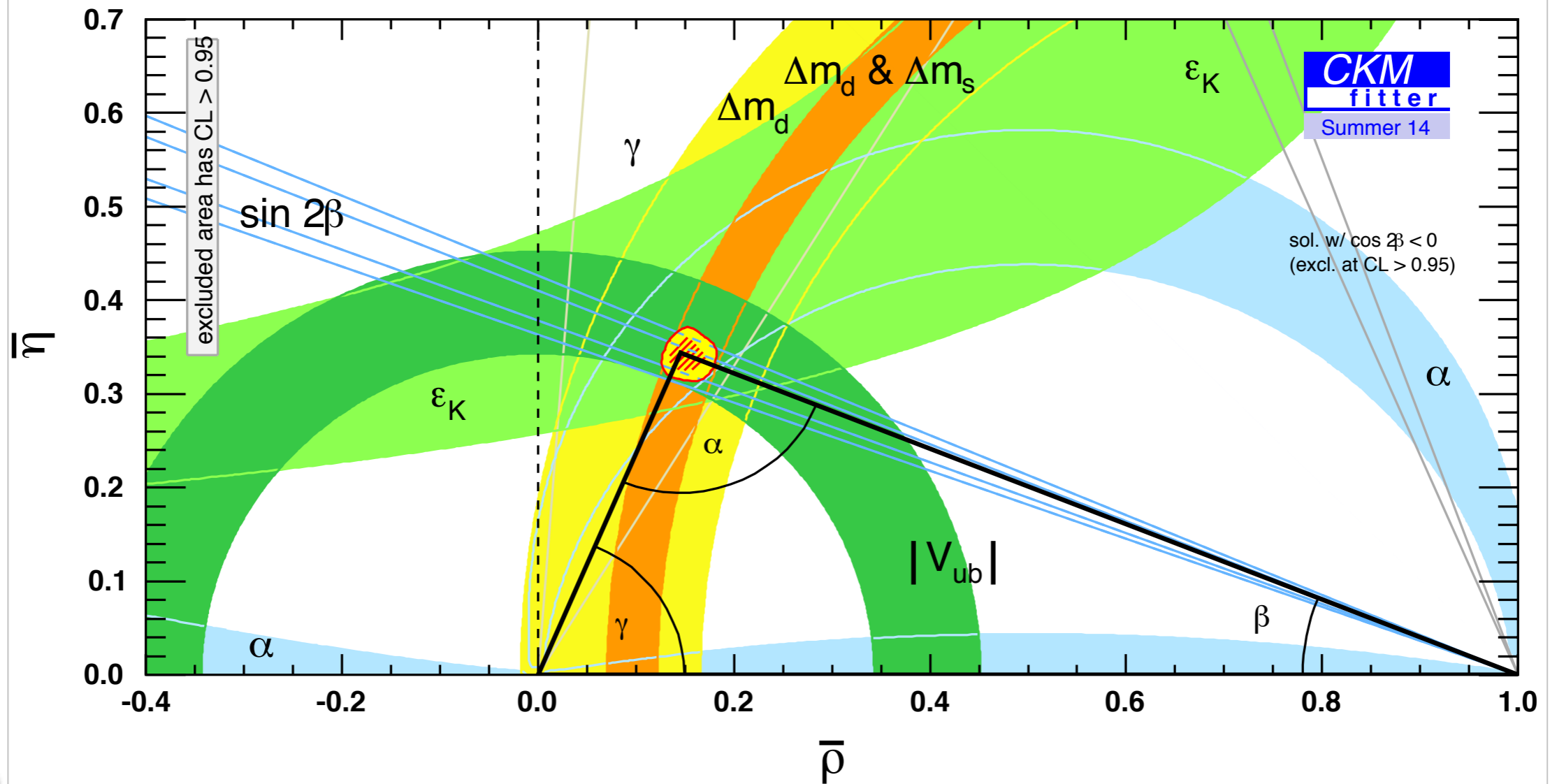
- ❖ If LHC discovers new particles, flavor & CP-violating couplings needed to determine underlying theory
- ❖ If new physics lies above the TeV scale, indirect searches will be only probe!

[adapted from Z. Ligeti]



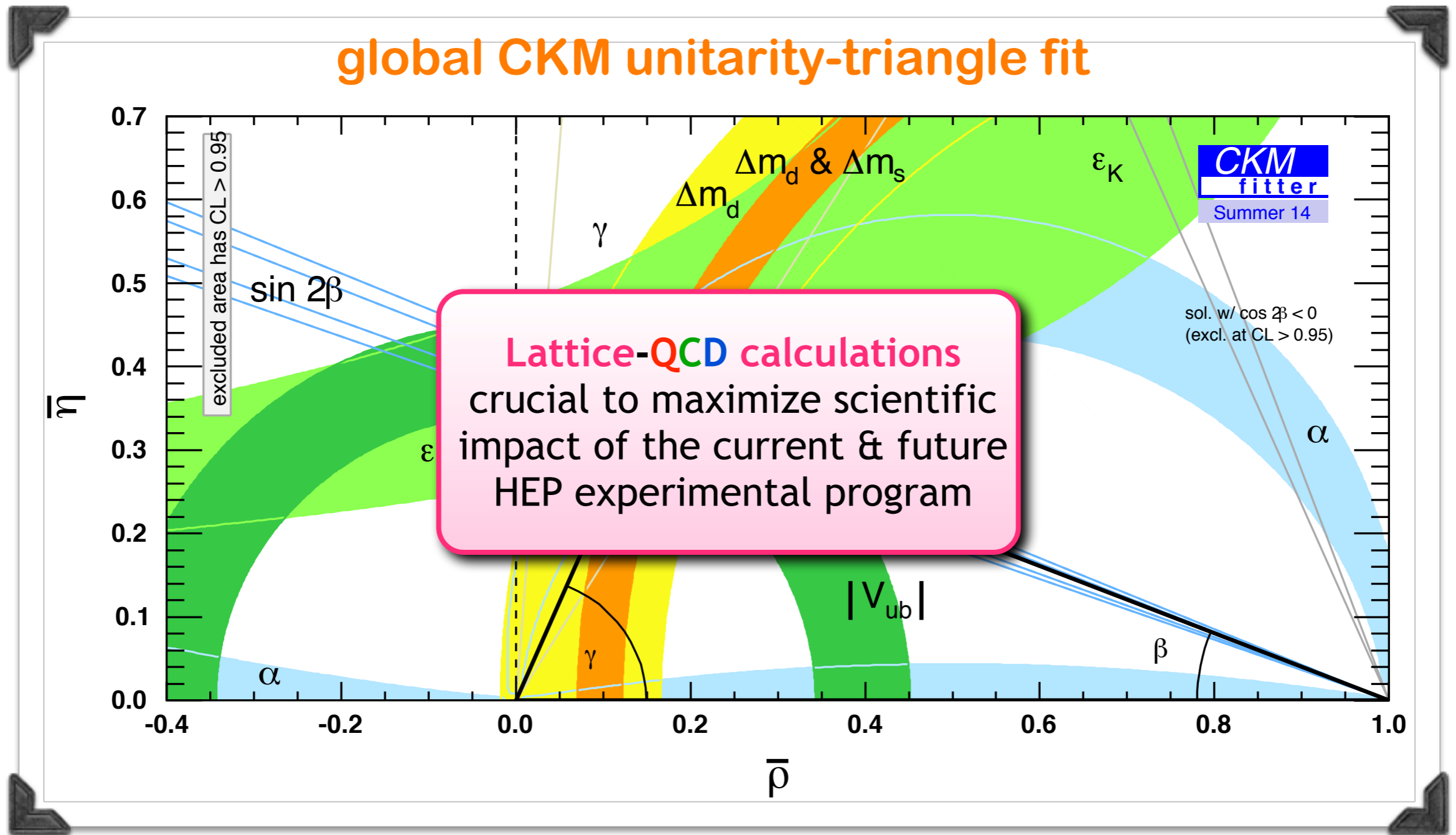
# Why lattice QCD?

## global CKM unitarity-triangle fit



- ◆ Comparison between measurements and Standard-Model predictions limited in most cases by theory, often from hadronic matrix elements

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Brief introduction to  
lattice-QCD simulations



# Quantum ChromoDynamics

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2g^2} \text{tr} [F_{\mu\nu} F^{\mu\nu}] - \sum_{f=1}^{n_f} \bar{\psi}_f (\not{D} + m_f) \psi_f + \underbrace{\frac{i\bar{\theta}}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu} F_{\rho\sigma}]}_{\text{violates } CP}$$

- ◆ QCD Lagrangian contains  $1 + n_f + 1$  parameters that can be fixed from equal number of experimental inputs

## FUNDAMENTAL PARAMETER

- ❖ Gauge coupling  $g^2$
- ❖  $n_f$  quark masses  $m_f$
- ❖  $\theta = 0$

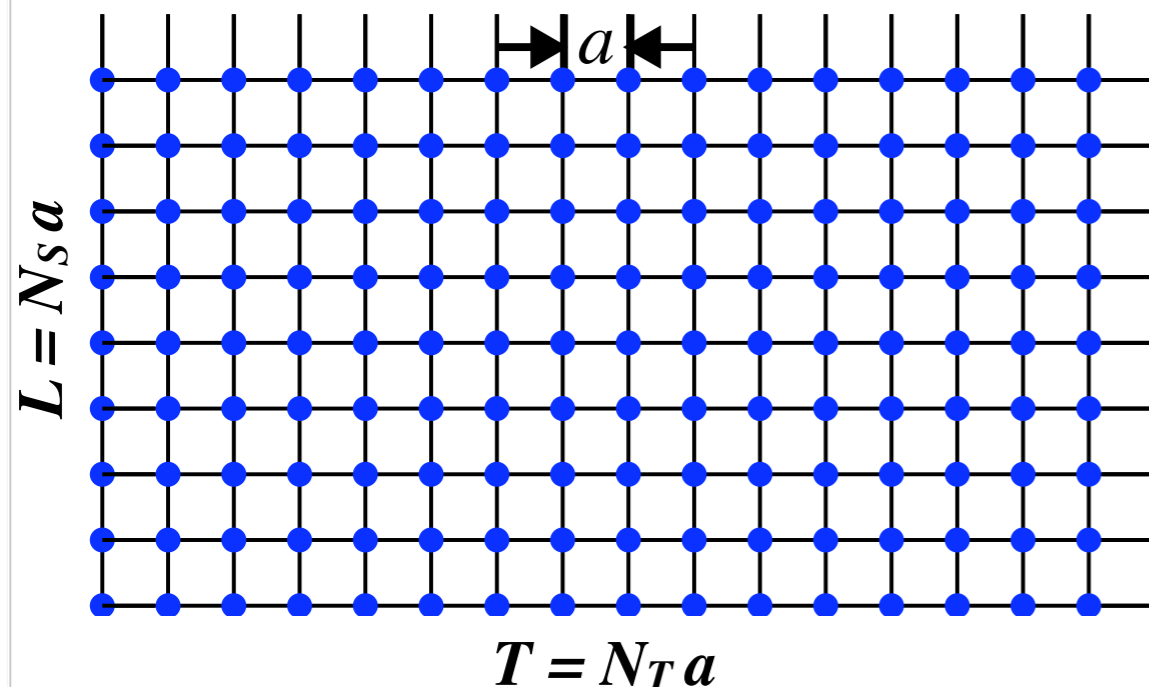
## EXPERIMENTAL INPUT

- $r_1, m_\Omega, Y(2S-1S),$  or  $f_\pi$
- $m_\pi, m_K, m_{J/\psi}, m_Y, \dots$
- neutron EDM ( $|\theta| < 10^{-11}$ )

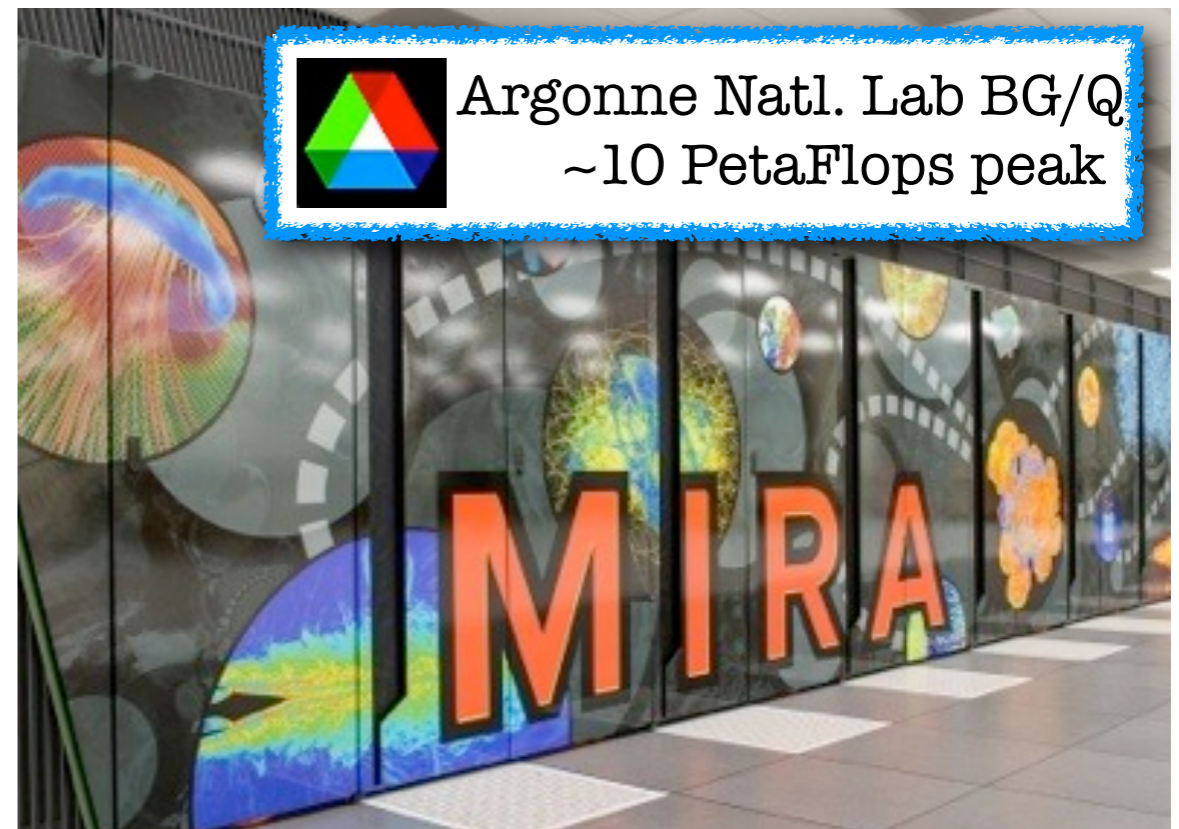
- ◆ Once the parameters are fixed, everything else is a prediction of the theory
- ◆ Calculations of hadronic parameters challenging in practice because low-energy QCD is nonperturbative

# Numerical lattice QCD

- ◆ Systematic method for calculating hadronic parameters from QCD first principles
- ◆ Define QCD on (Euclidean) spacetime lattice and solve path integral numerically
  - ❖ Recover QCD when lattice spacing  $a \rightarrow 0$  and box size  $L \rightarrow \infty$



- ◆ Simulate using Monte-Carlo methods and importance sampling
  - ❖ Sample from all possible field configurations using a distribution given by  $\exp(-S_{\text{QCD}})$
- ◆ Run codes upon supercomputers and dedicated clusters

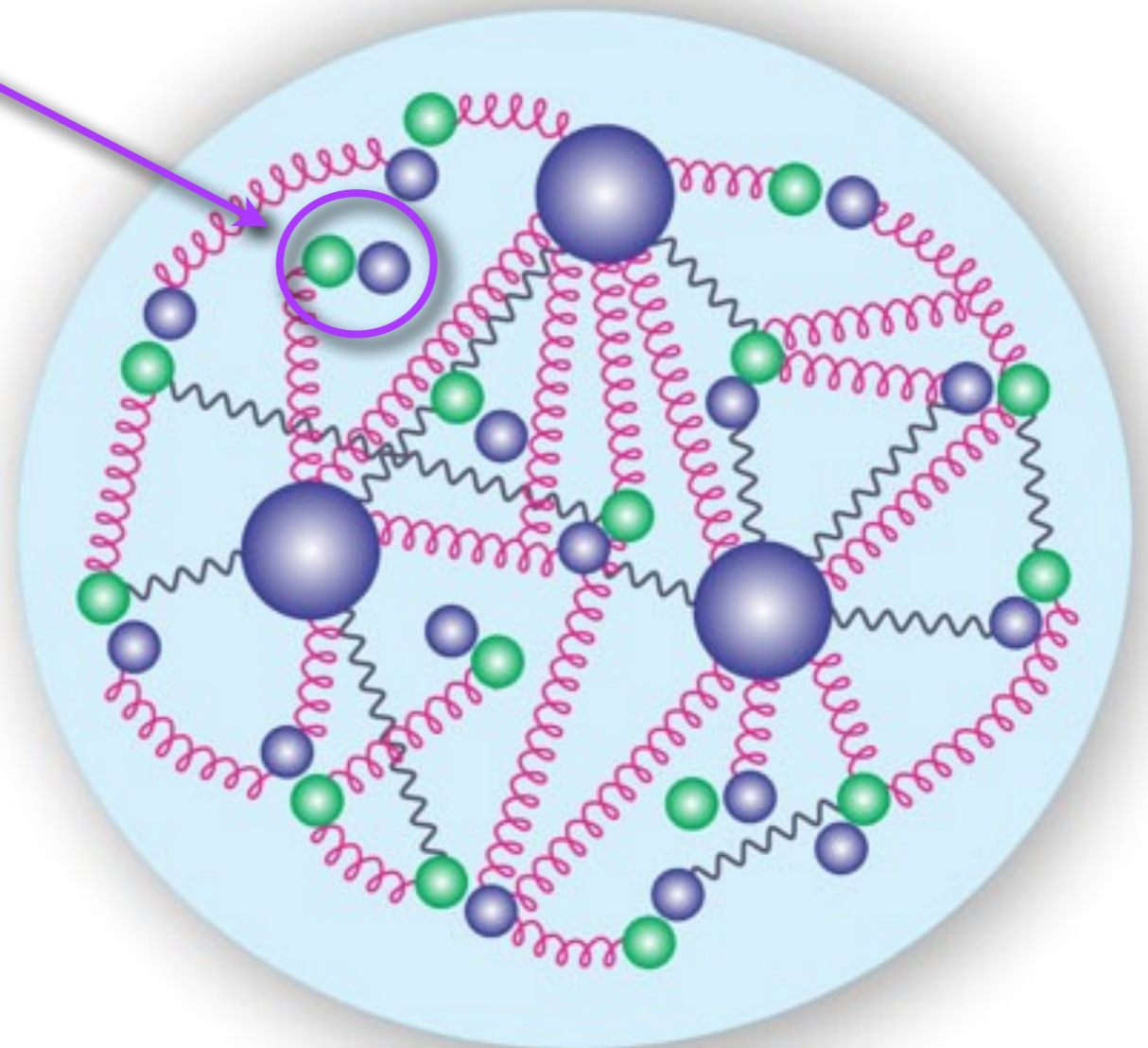


# Modern lattice-QCD simulations

- ◆ Realistic simulations include dynamical  $u$ ,  $d$ ,  $s$  (&  $c$ ) quarks in the vacuum
  - ❖ (Typically sea  $m_u=m_d$ )
- ◆ **Control systematic errors using gauge-field ensembles with different parameters, e.g.:**
  - ❖ **Multiple lattice spacings** to extrapolate to continuum limit ( $a \rightarrow 0$ )
  - ❖ **Multiple up/down-quark masses** to extrapolate to physical  $M_\pi = 135$  MeV

## Test and validate methods by

- (1) Comparison with experiment
- (2) Independent calculations sensitive to different systematics

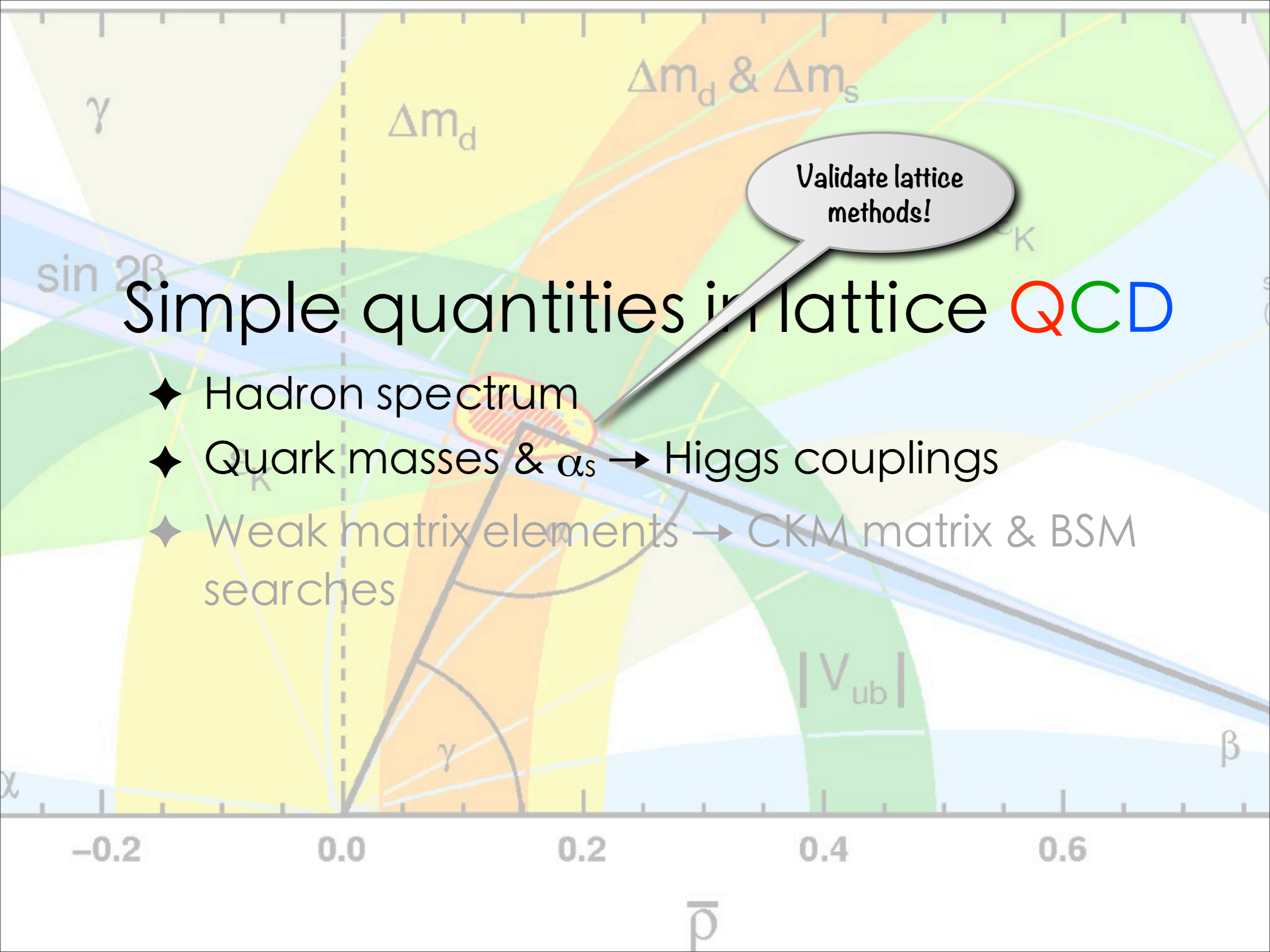


# Simple quantities in lattice QCD

Easiest quantities to compute with controlled systematic errors & high precision have **single hadron in initial state & at most one hadron in final state**, where hadrons are stable under QCD

# Simple quantities in lattice **Q****C****D**

- ◆ Hadron spectrum
- ◆ Quark masses &  $\alpha_s \rightarrow$  Higgs couplings
- ◆ Weak matrix elements  $\rightarrow$  CKM matrix & BSM searches



# Simple quantities in lattice QCD

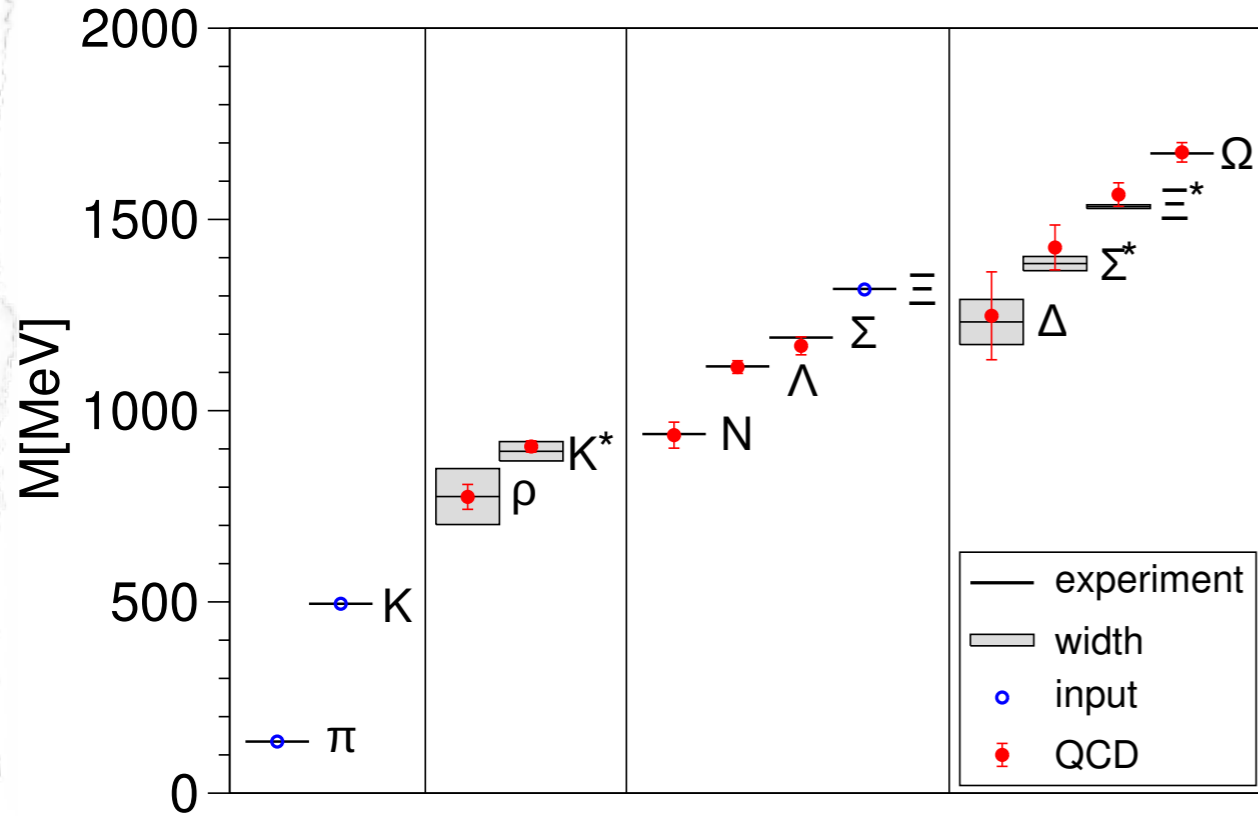
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Validate lattice methods!

# Hadron spectrum

## Light hadrons

[BMW, Science 322 (2008) 1224-1227]

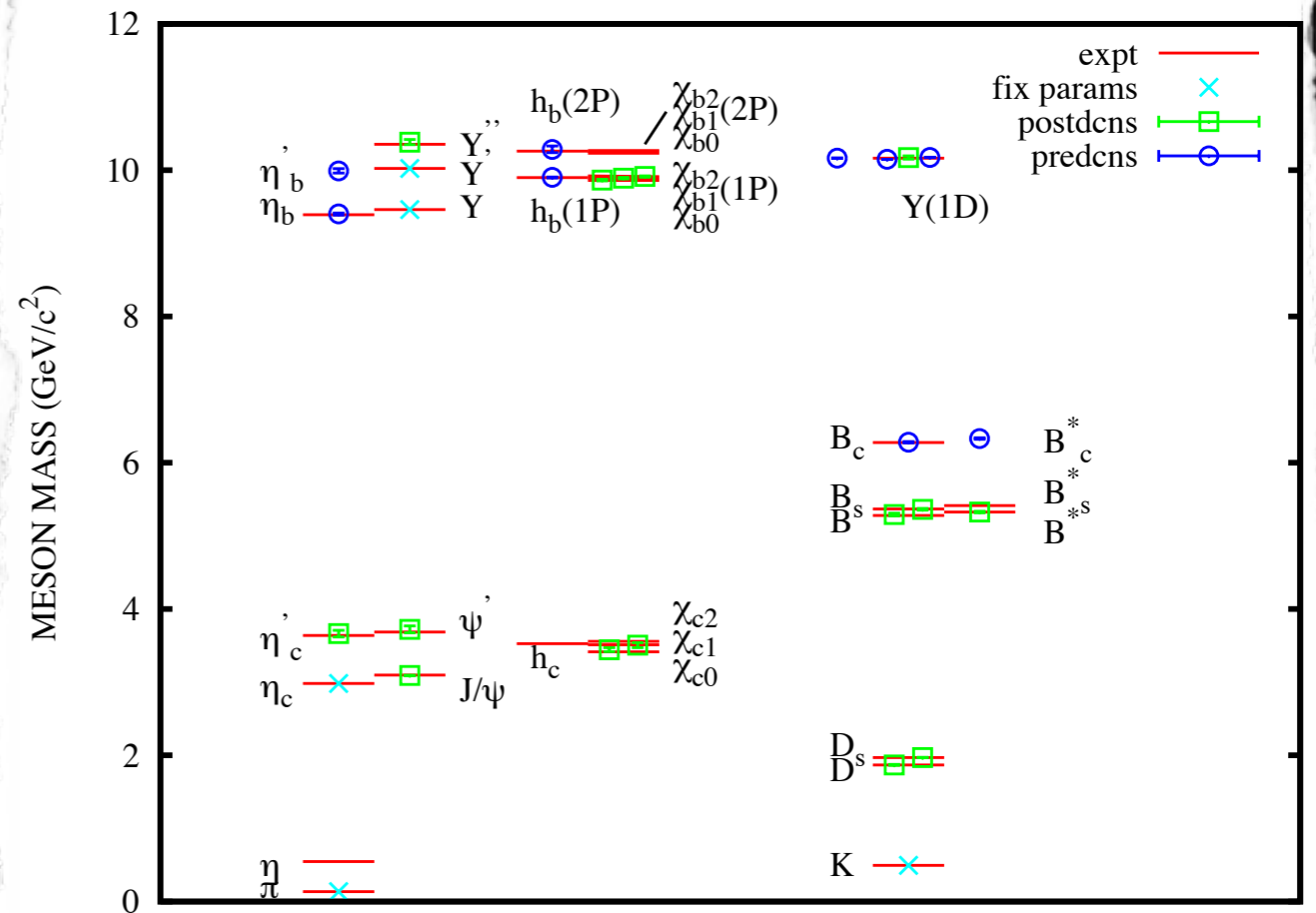


- ◆ Light hadron masses much larger than constituent quark masses, so primarily due to energy stored in gluon field and to quarks' kinetic energy

➔ **Tests nonperturbative QCD dynamics**

## Heavy hadrons

[HPQCD, 1203.3862 (C.Davies Lattice 2011)]

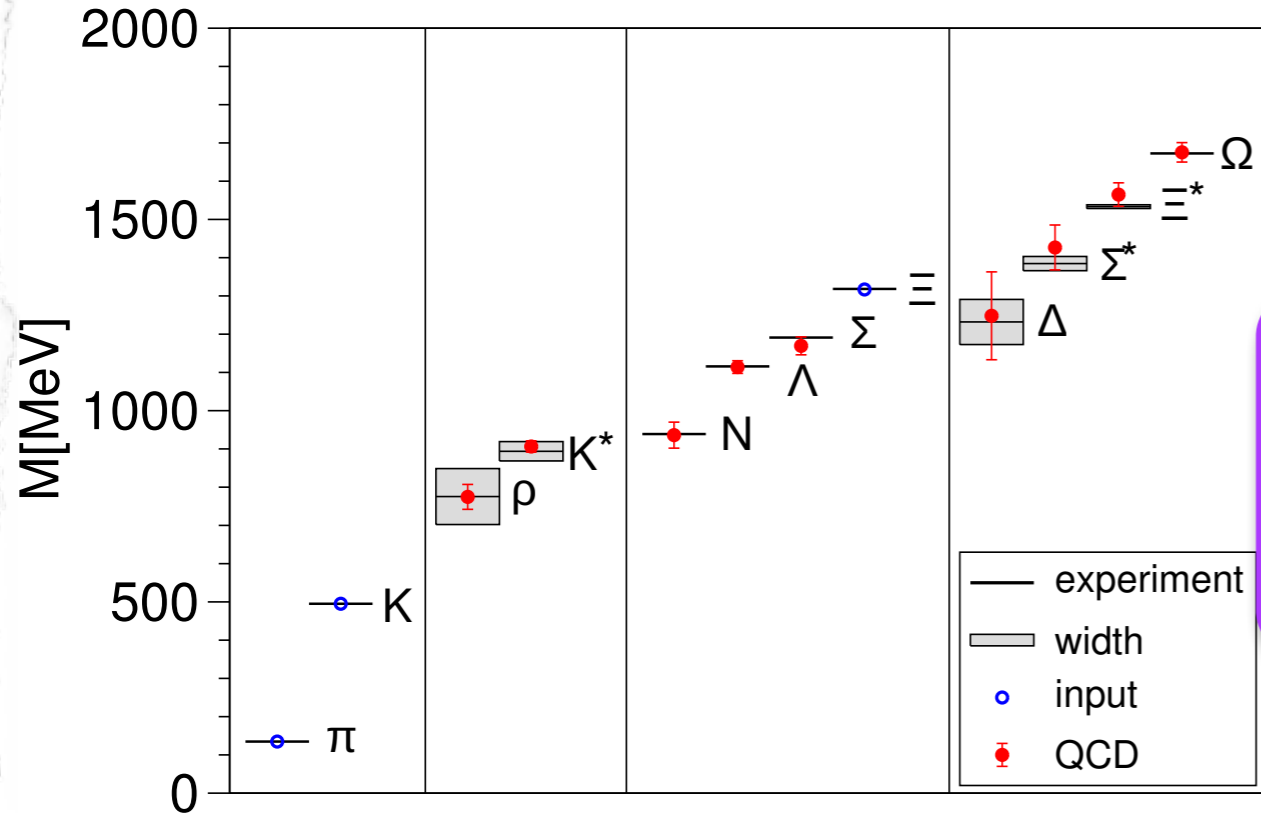


- ◆ Tests lattice methods for charm & bottom quarks, which often rely on effective field theories

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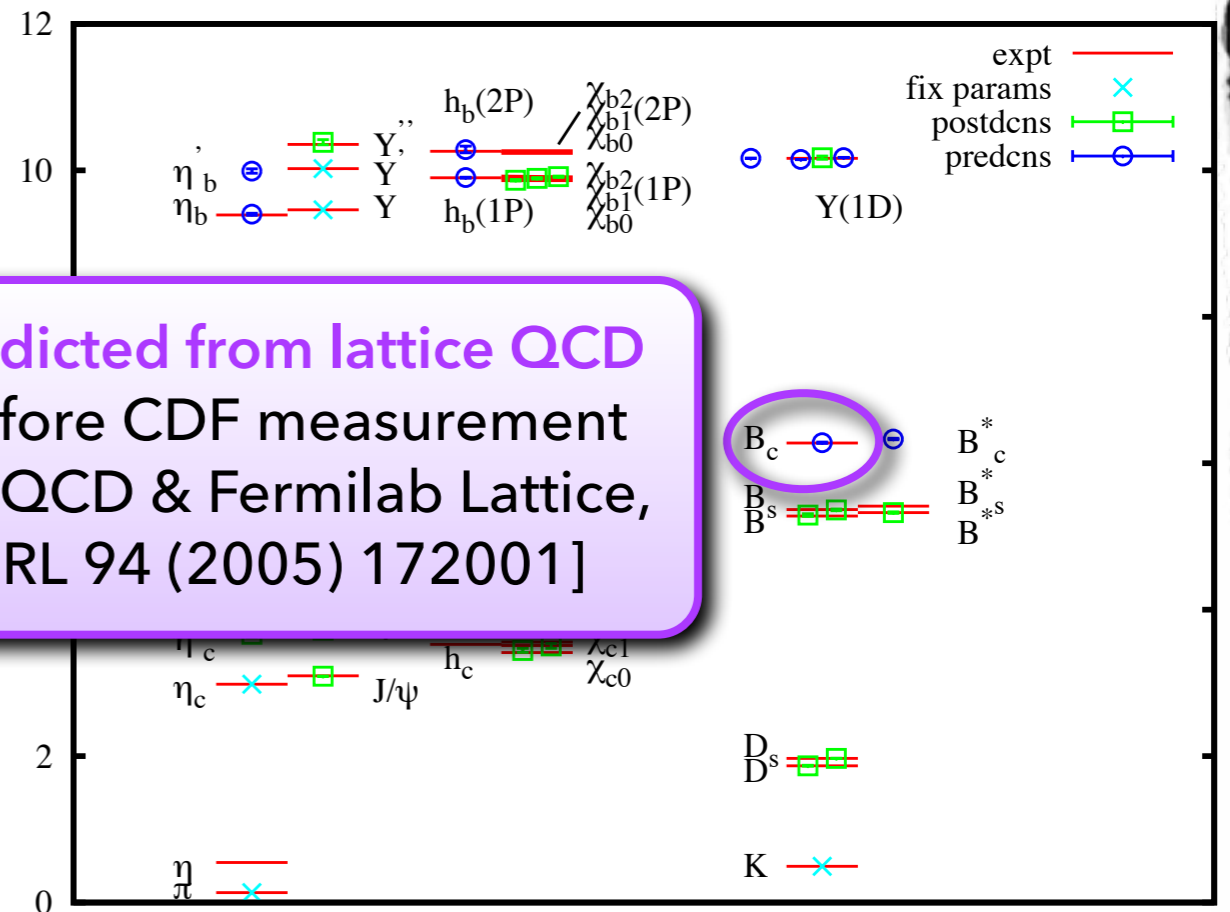
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## Heavy hadrons

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Predicted from lattice QCD  
before CDF measurement  
[HPQCD & Fermilab Lattice,  
PRL 94 (2005) 172001]

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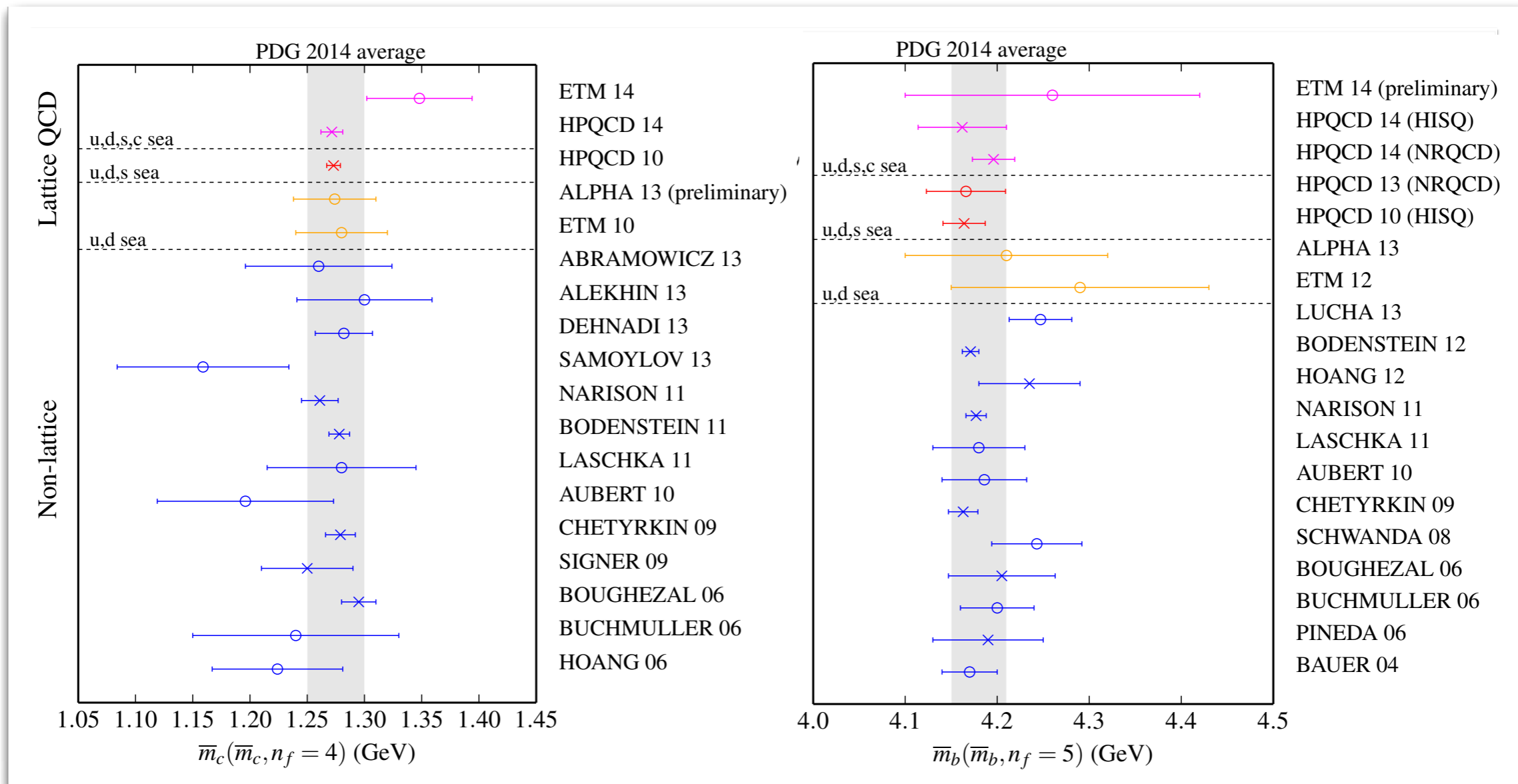
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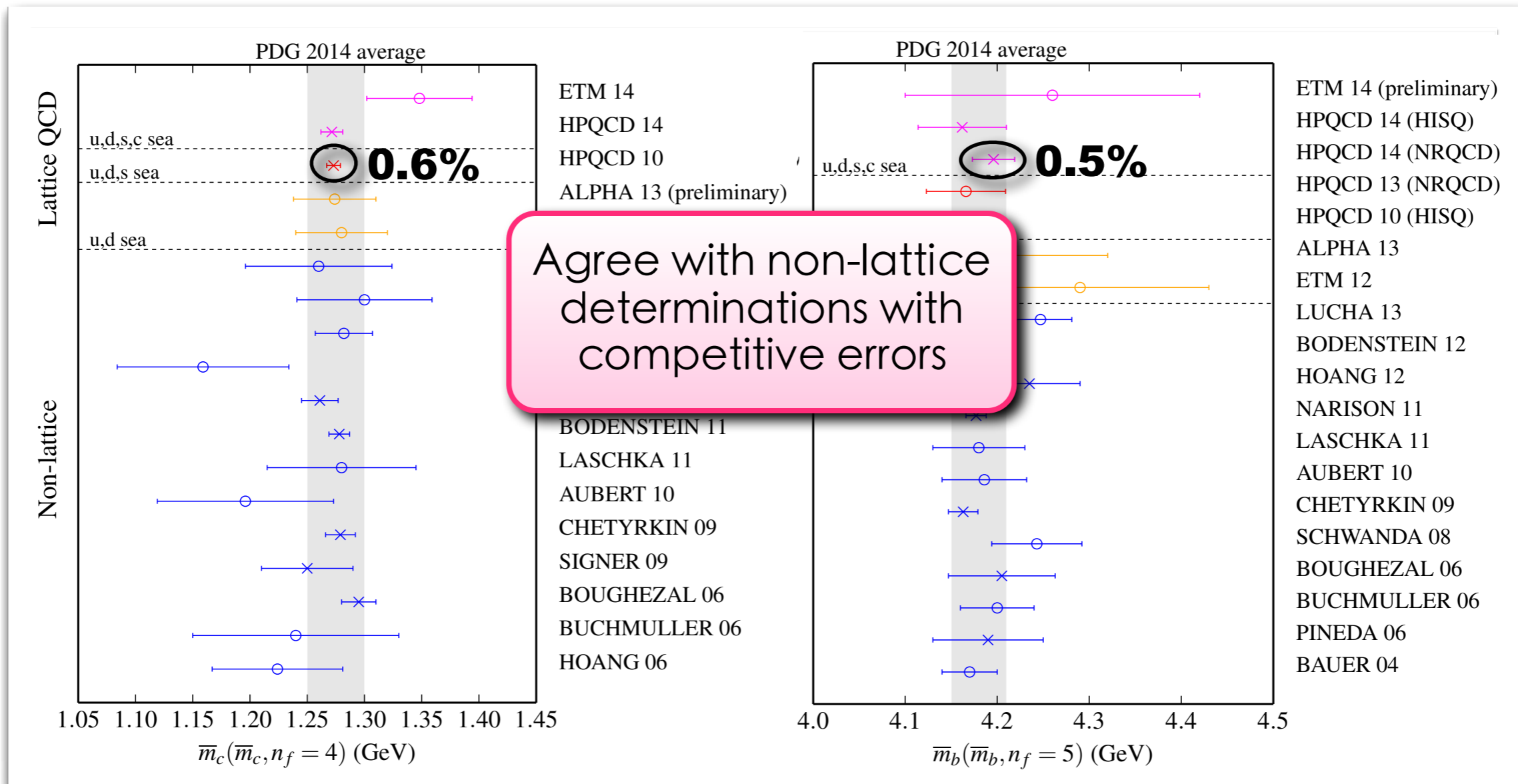
# Heavy-quark masses

- ◆ Most precise  $m_c$  and  $m_b$  obtained by fitting moments of correlation functions of the quarks' electromagnetic current to  $O(\alpha_s^3)$  perturbative expressions
- ◆ Moments can be obtained from experimental  $e^+e^-$  annihilation data, and also **computed numerically with lattice-QCD simulations with negligible statistical uncertainties**



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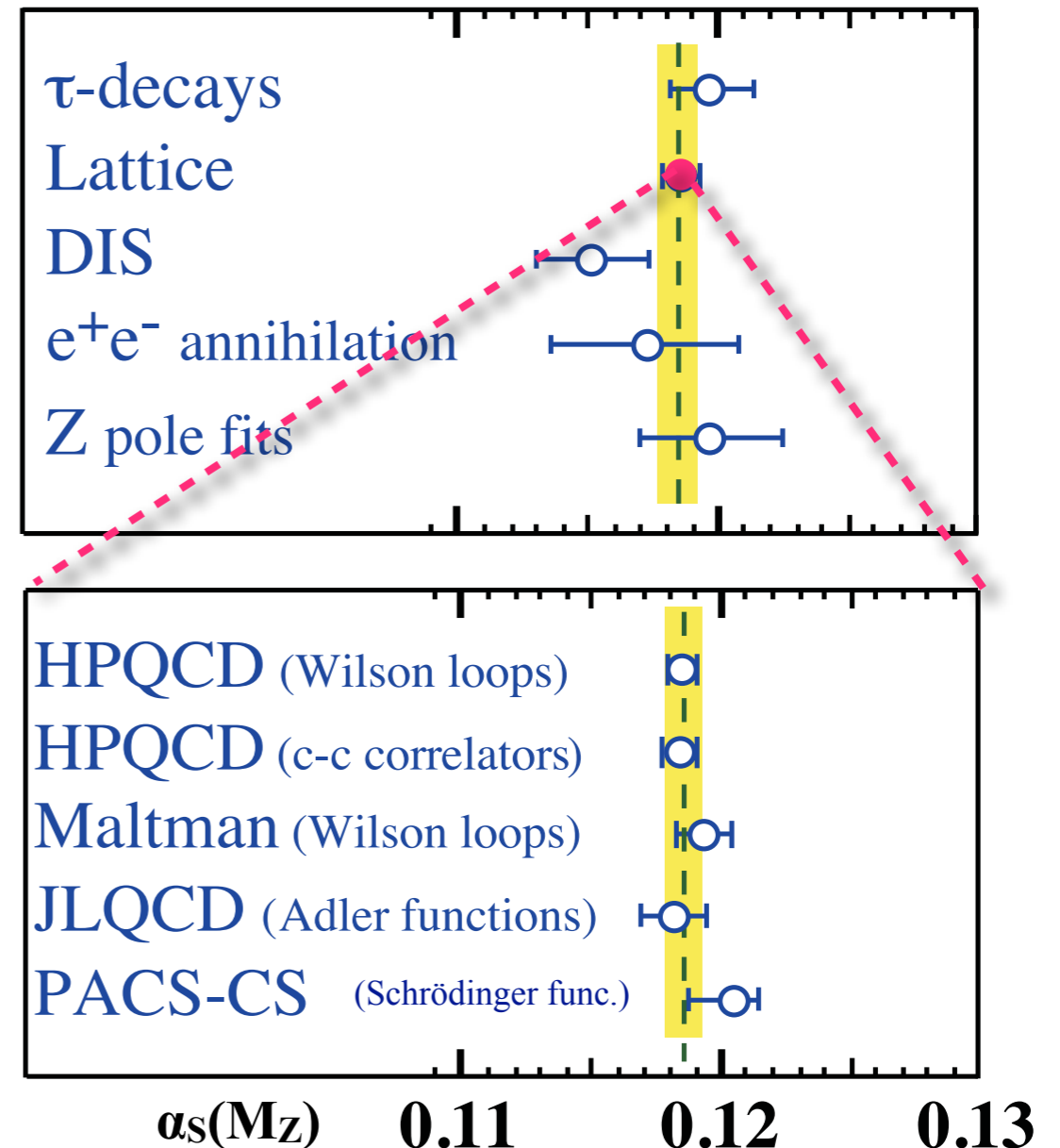
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# Strong coupling constant

- ◆ **Several independent lattice methods available**
  - ❖ Most precise result from fitting NNNLO QCD  $\beta$ -function to short-distance lattice quantities built from Wilson loops (fits of heavy-quark correlators yield similar precision)
  - ❖ Approaches consistent, and **each is more precise than experiment**

[Particle Data Group 2014]

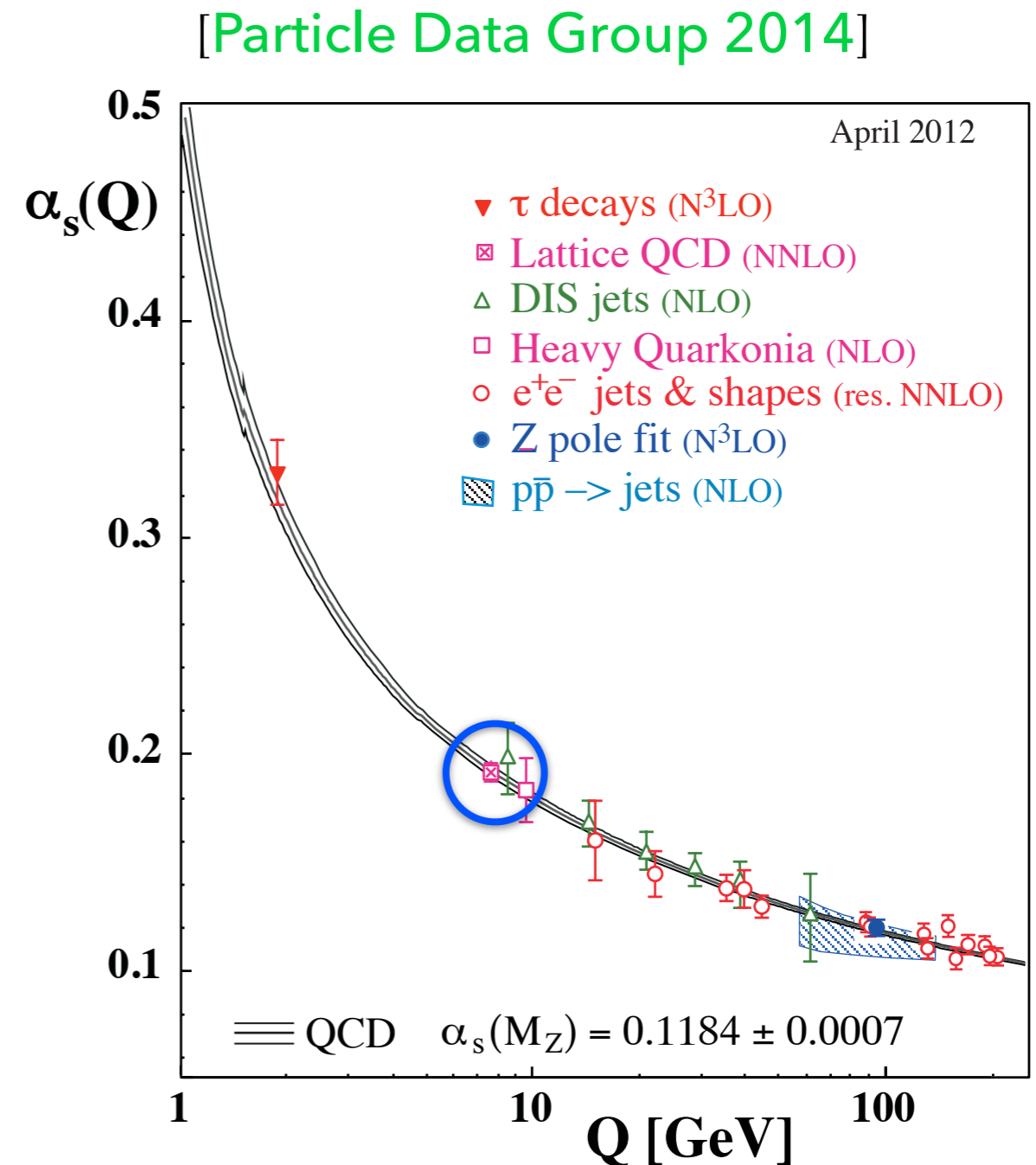


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Agreement with experiment  
nontrivial test that

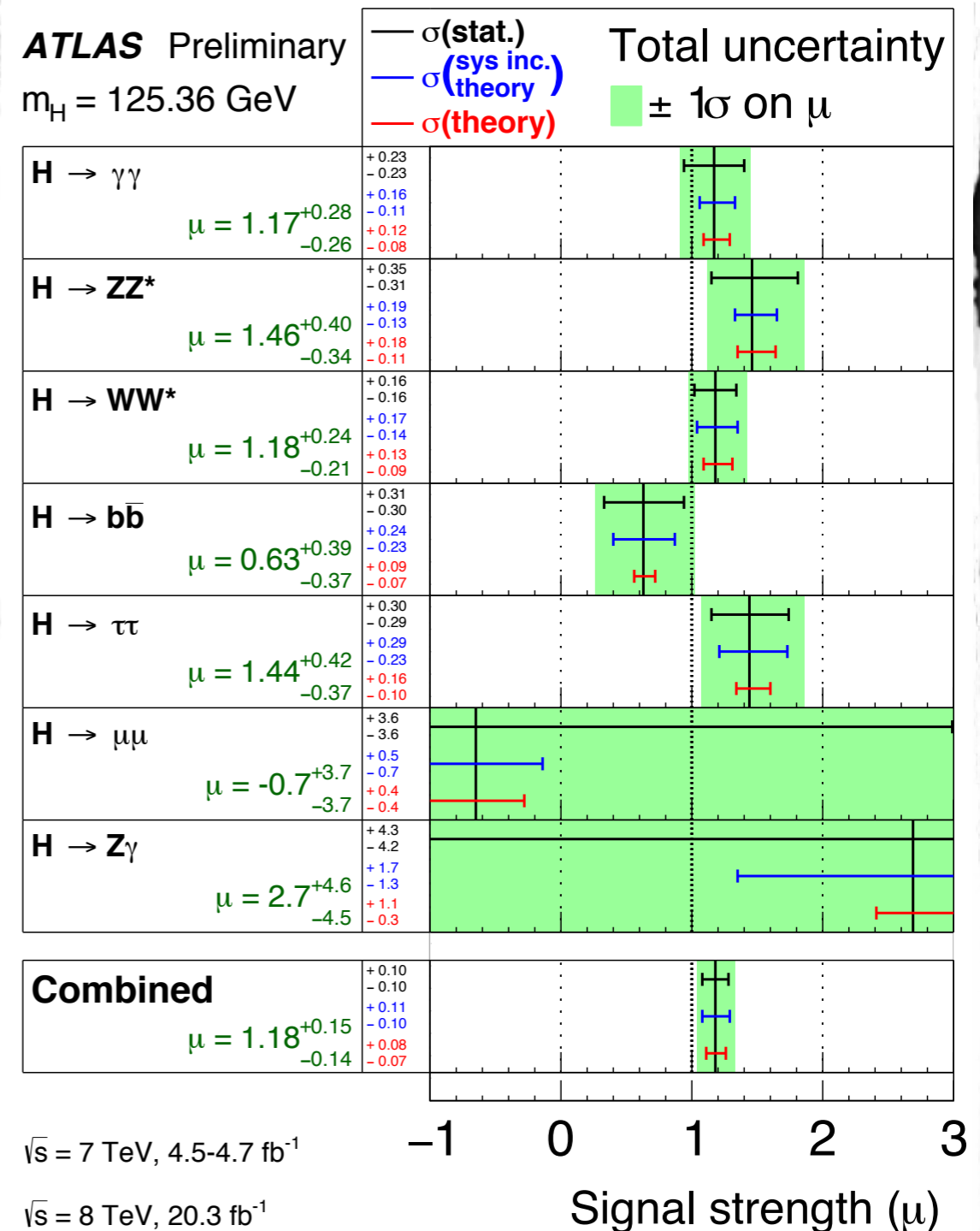
**QCD of partons = QCD of hadrons**



# Implications for Higgs couplings

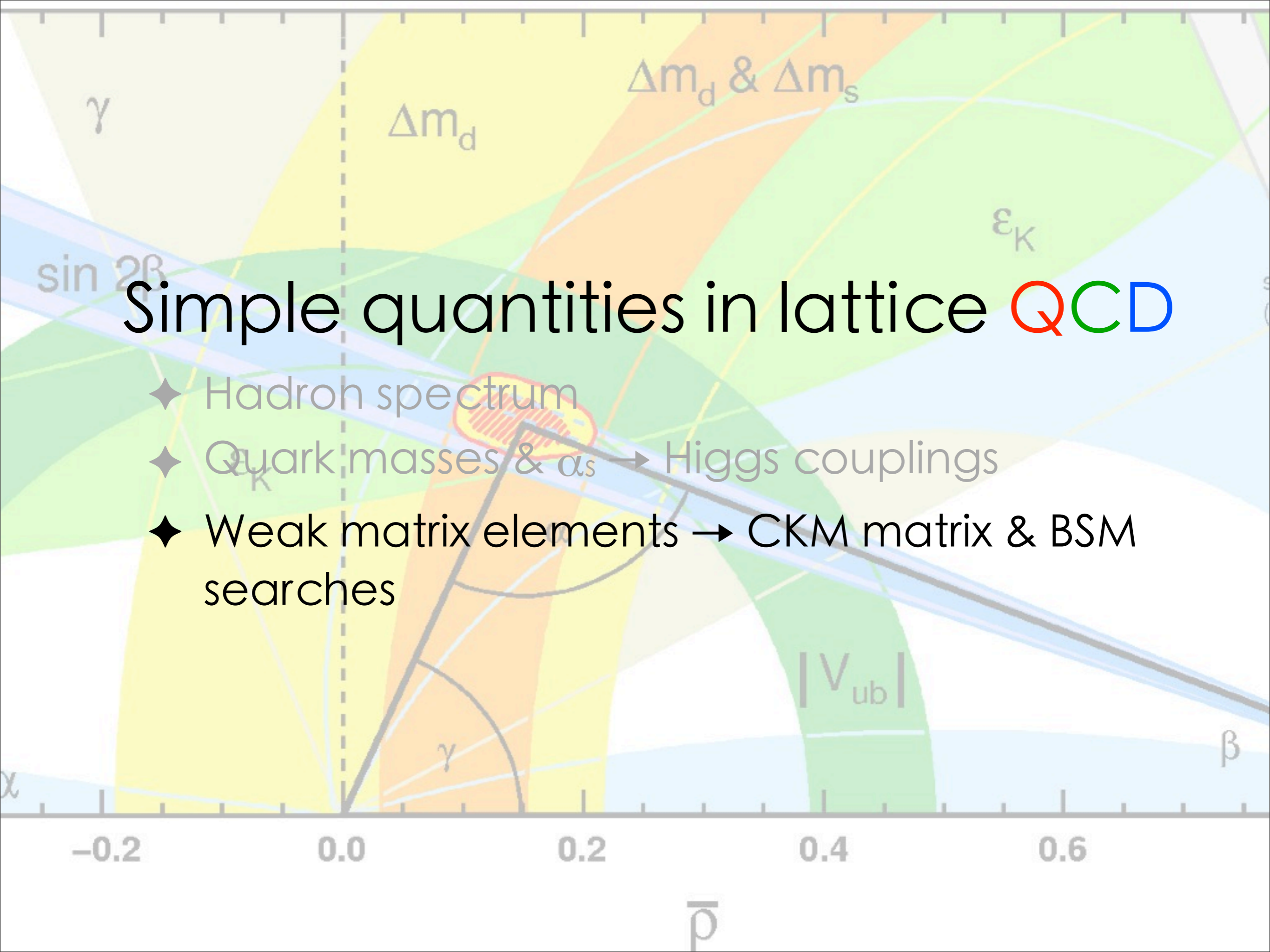


- ◆ Observation of new physics effects at next-generation colliders requires theory errors at the sub-percent level
- ◆ Standard-Model Higgs predictions currently limited by parametric errors from  $b, c$ -quark masses and  $\alpha_s$  [LHC Higgs X-Section WG, EPJ C71 (2011) 1753]
- ◆ Lepage, Mackenzie, & Peskin [1404.0319] showed with toy Monte Carlo that existing lattice methods + anticipated computing resources in next decade will enable straightforward reduction in errors on  $m_c, m_b, & \alpha_s$  to needed precision



# Simple quantities in lattice QCD

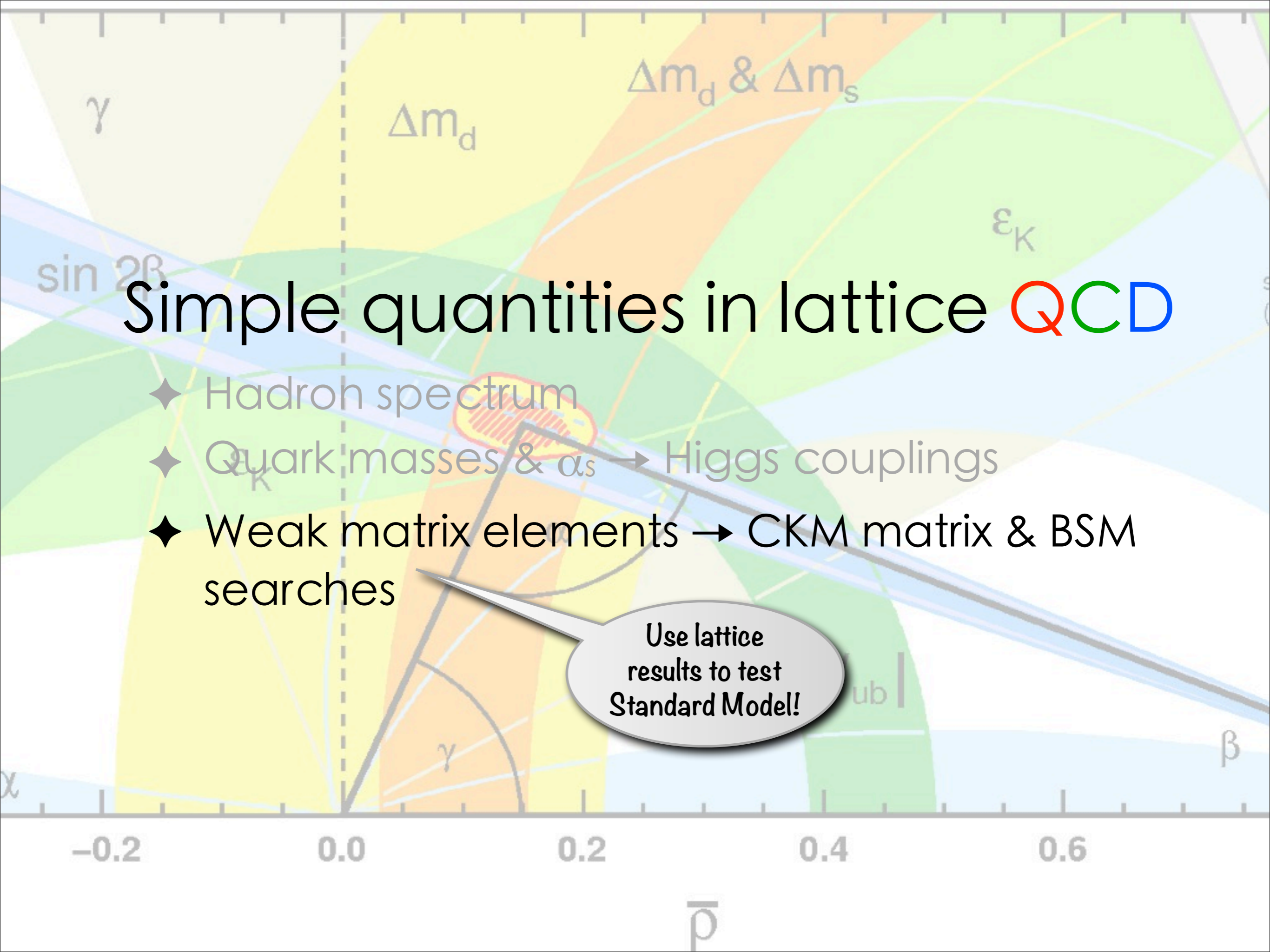
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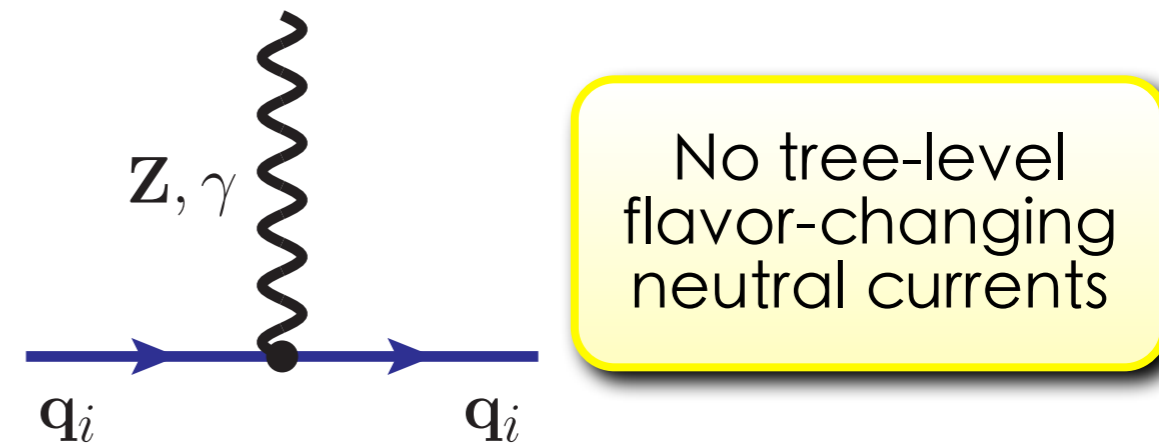
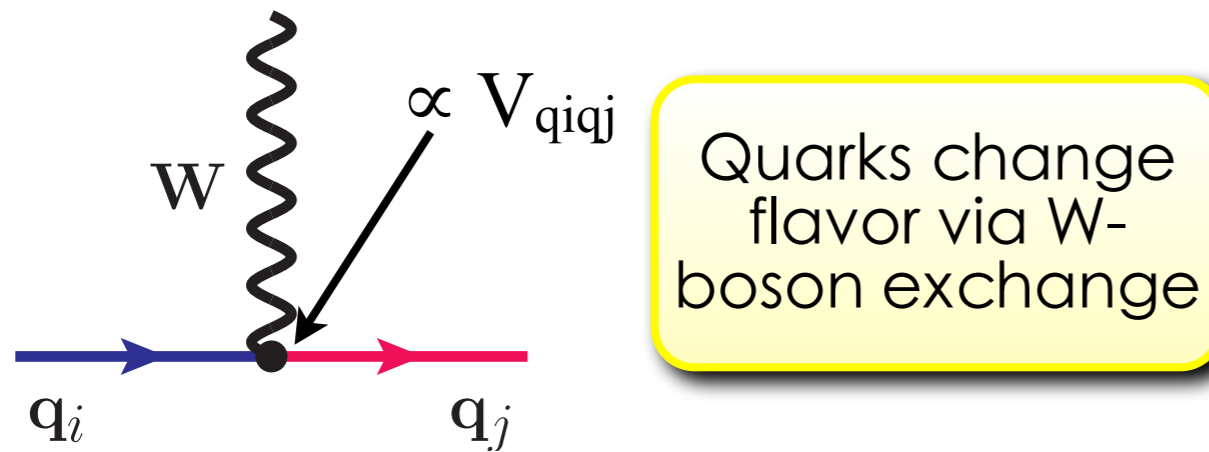
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Use lattice results to test Standard Model!



# Standard-Model flavor structure



- ◆ Mixing between quark flavors under charged weak interactions parameterized by **Cabibbo-Kobayashi-Maskawa (CKM) matrix:**

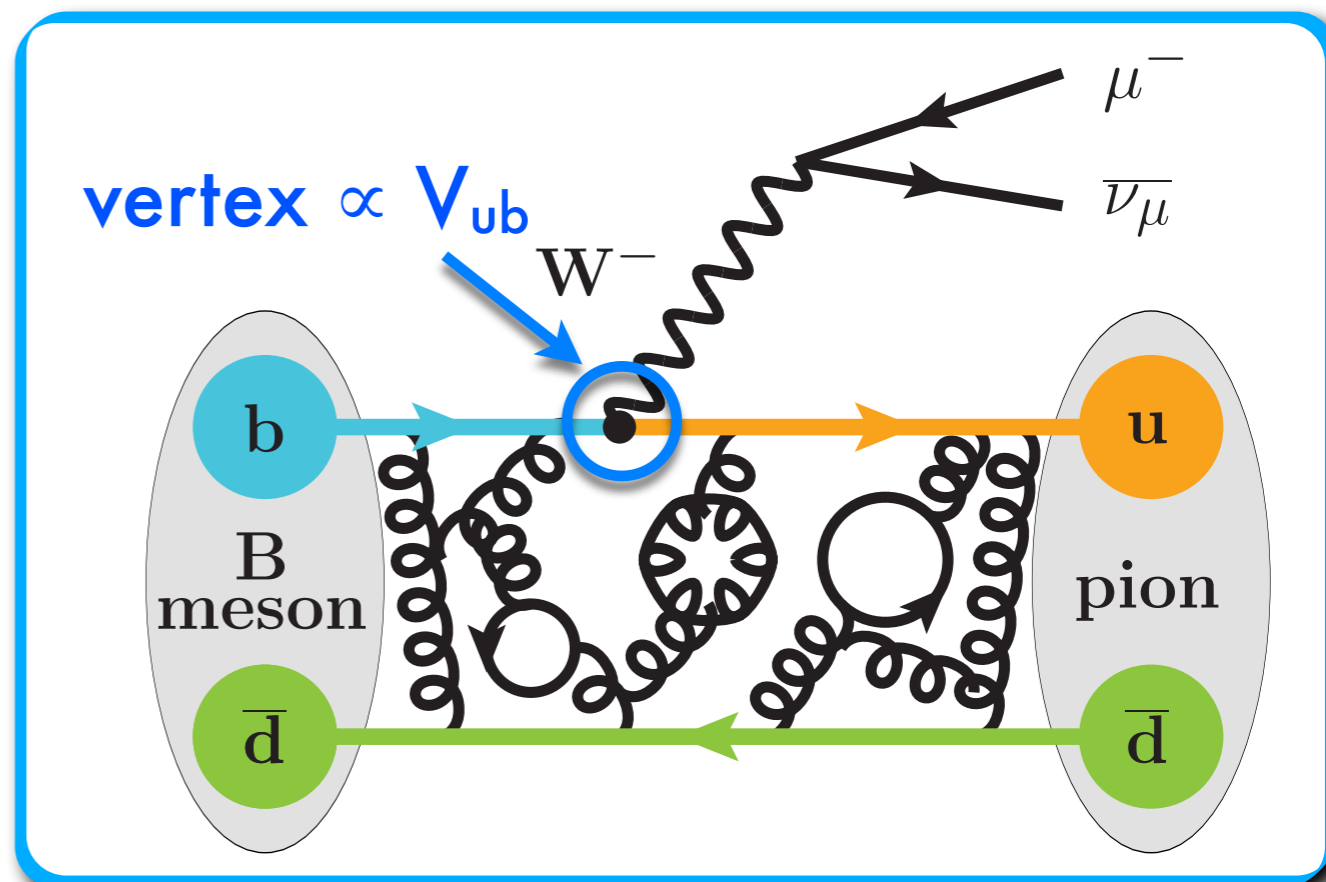
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} \text{d} & \text{s} & \text{b} \\ 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix} \begin{pmatrix} \text{u} \\ \text{c} \\ \text{t} \end{pmatrix}$$

- ◆ CKM elements & phase are **parametric inputs to Standard Model predictions for many flavor-changing processes** such as neutral kaon mixing and rare kaon decays



# “Measuring” the CKM matrix

- ◆ Measure flavor-changing processes involving hadrons
  - ❖ Absorb nonperturbative QCD dynamics into hadronic parameters
- ◆ Infer CKM elements within Standard Model by comparing experimental measurements of flavor-changing interactions with theory predictions



$$\Delta m_{(d,s)}, \frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow D^{(*)} \ell \nu)}{dw}, \dots$$

Compute nonperturbative QCD parameters (decay constants, form factors, B-parameters,...) numerically with **LATTICE QCD**

$$(\text{Experiment}) = (\text{known}) \times (\text{CKM factors}) \times (\text{Hadronic Matrix Element})$$

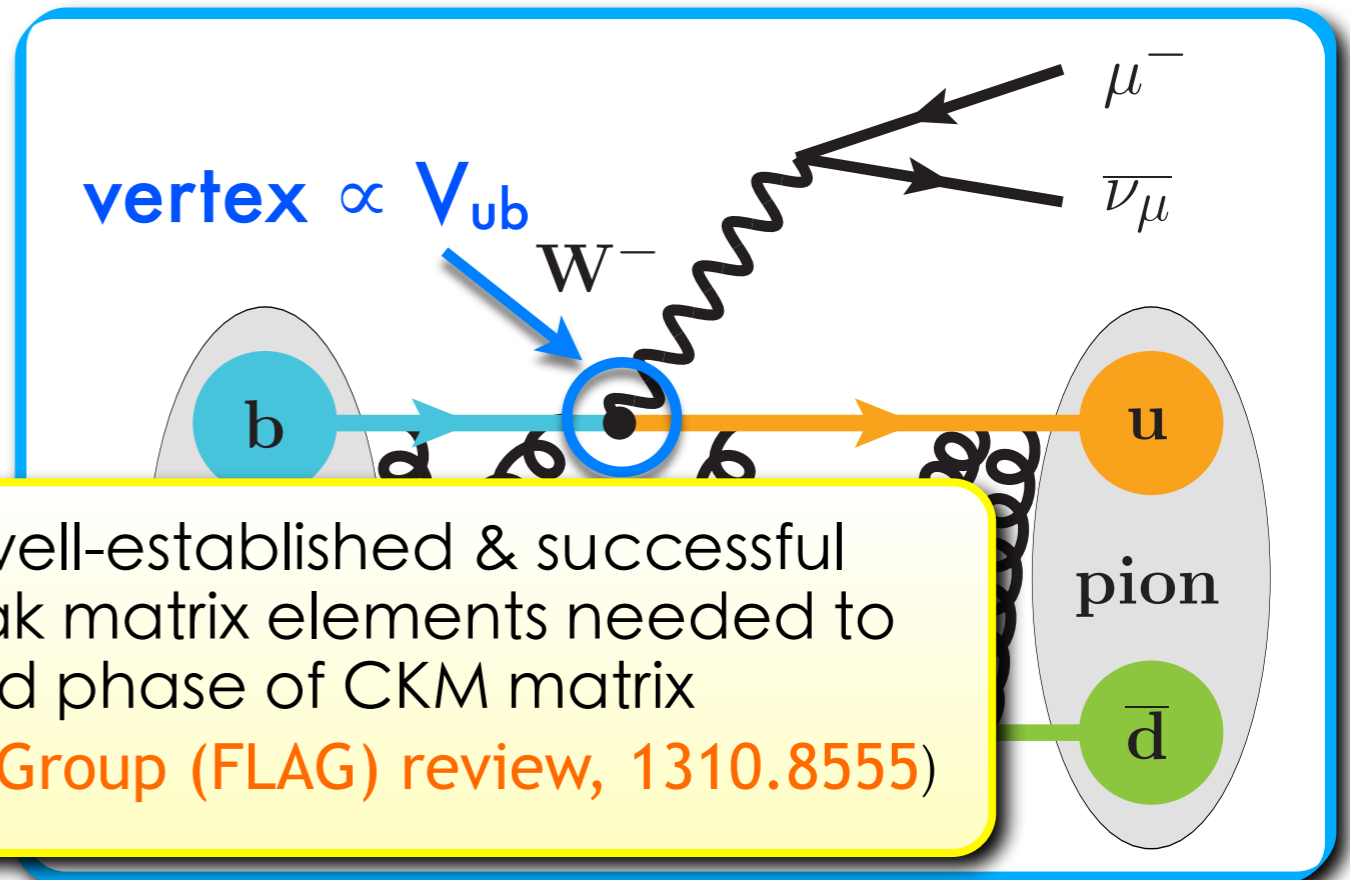
# “Measuring” the CKM matrix

- ◆ Measure flavor-changing processes involving hadrons

- ❖ Absorb nonperturbative QCD dynamics into hadronic parameters

- ◆ Infer CKM parameters from measurements of hadronic interactions

Lattice community has well-established & successful program to calculate weak matrix elements needed to obtain elements and phase of CKM matrix  
(see Flavor Lattice Averaging Group (FLAG) review, 1310.8555)



$$\Delta m_{(d,s)}, \frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow D^{(*)} \ell \nu)}{dw}, \dots$$

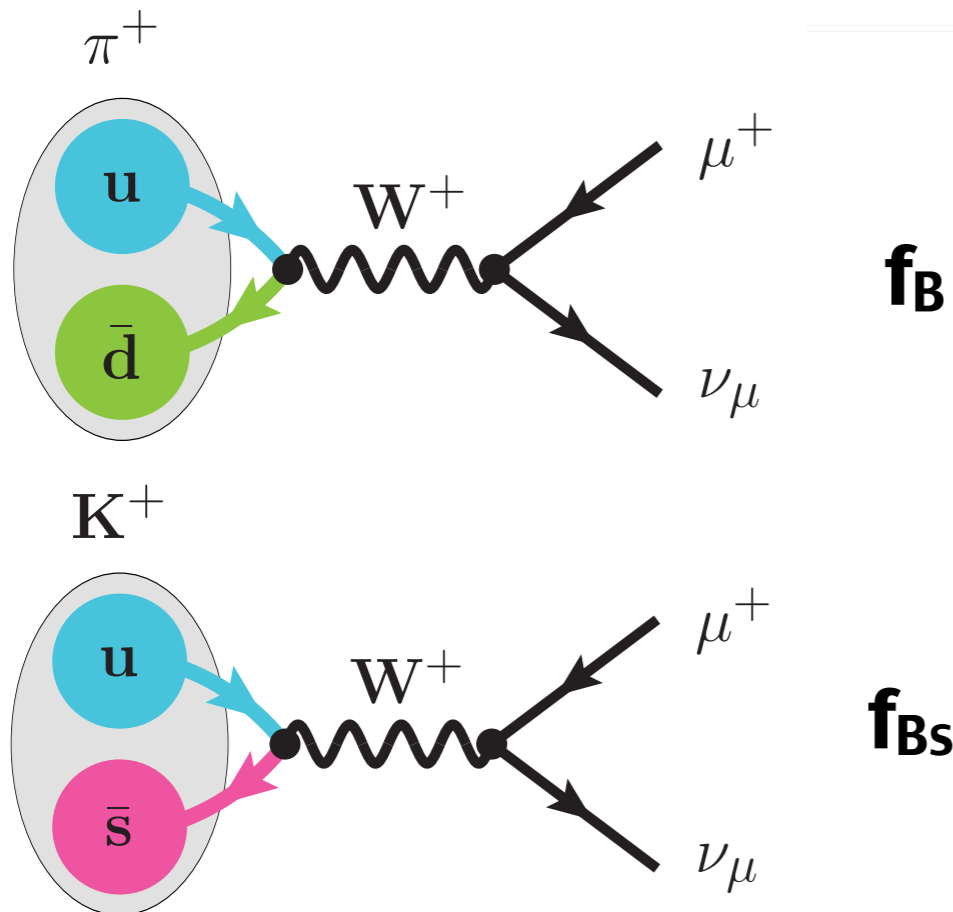
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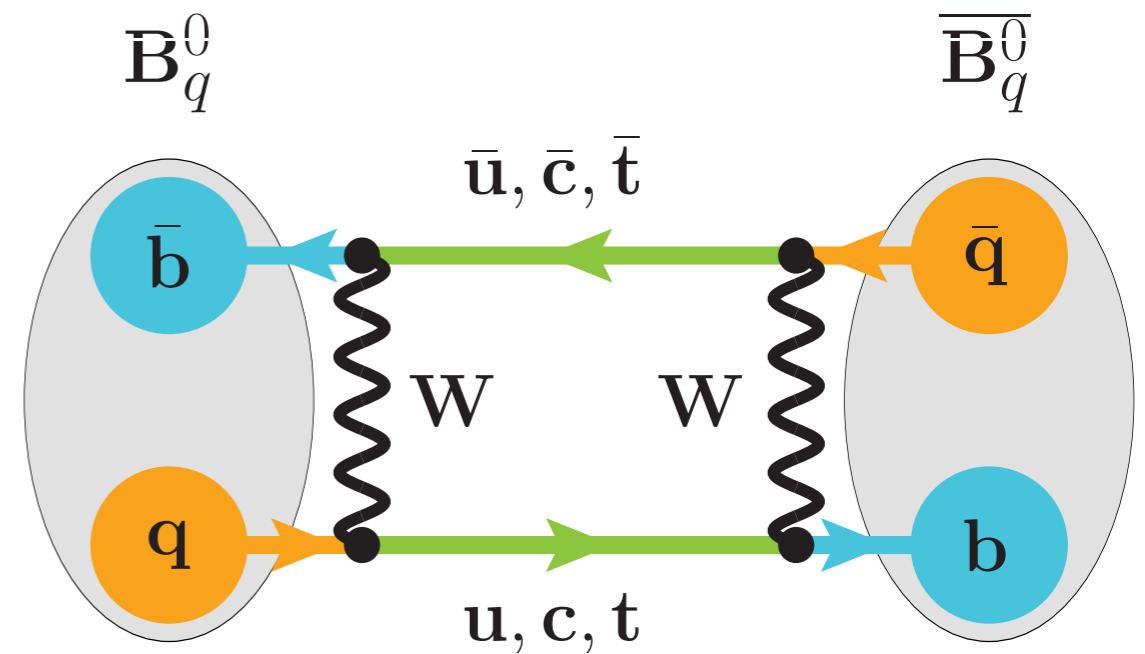
# B-meson overview

See [Pena Lattice '15](#) & [Bouchard Lattice '14](#) heavy-flavor reviews

- ◆ Control discretization errors associated with large b-quark mass using effective theory
- ◆ Few calculations use highly improved b-quark action + very fine lattice spacings
- ◆ Some physical light-quark mass results



$$\xi \equiv f_B (B_{Bd})^{1/2} / f_{Bs} (B_{Bs})^{1/2}$$



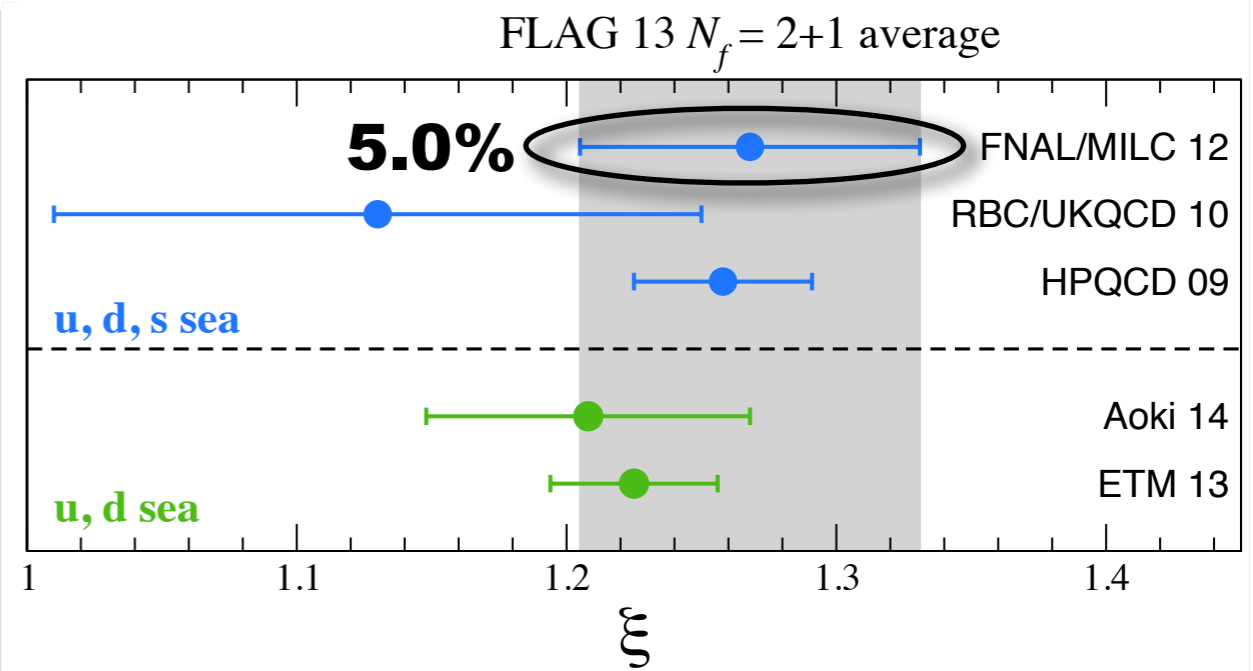
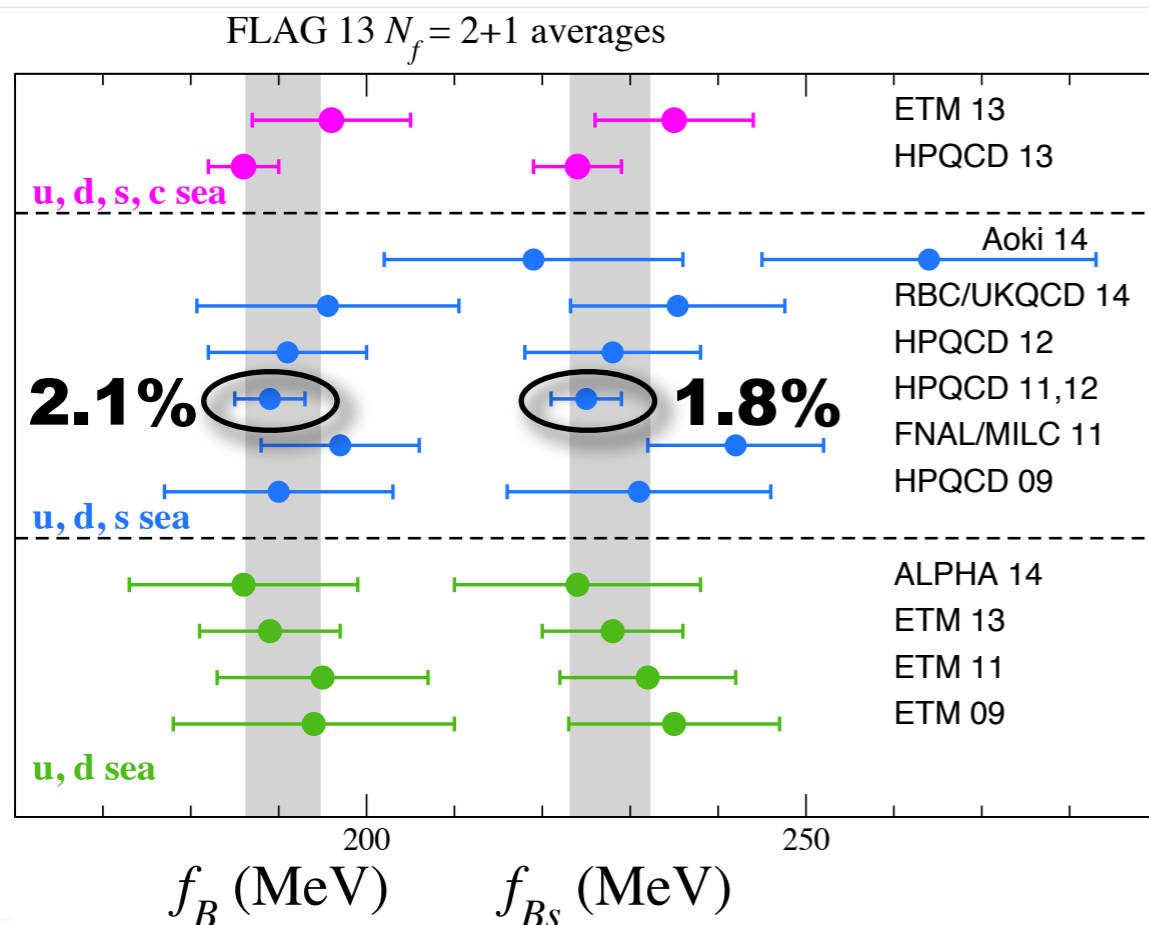
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## New results:

- ◆ First 3-flavor  $f_{B^*}$ ,  $f_{B_s^*}$ , and  $f_{B_c^*}$  [[HPQCD, arXiv:1503.05762](#)]
- ◆ First 3-flavor  $B_s \rightarrow K l \nu$  form factors [[HPQCD, PRD90 \(2014\) 5, 054506](#); [RBC/UKQCD, PRDD91 \(2015\) 7, 074510](#)]
- ◆ Update of [FNAL/MILC](#)  $B \rightarrow D^* l \nu$  zero-recoil form factor [[PRD89 \(2014\) 11, 114504](#)]



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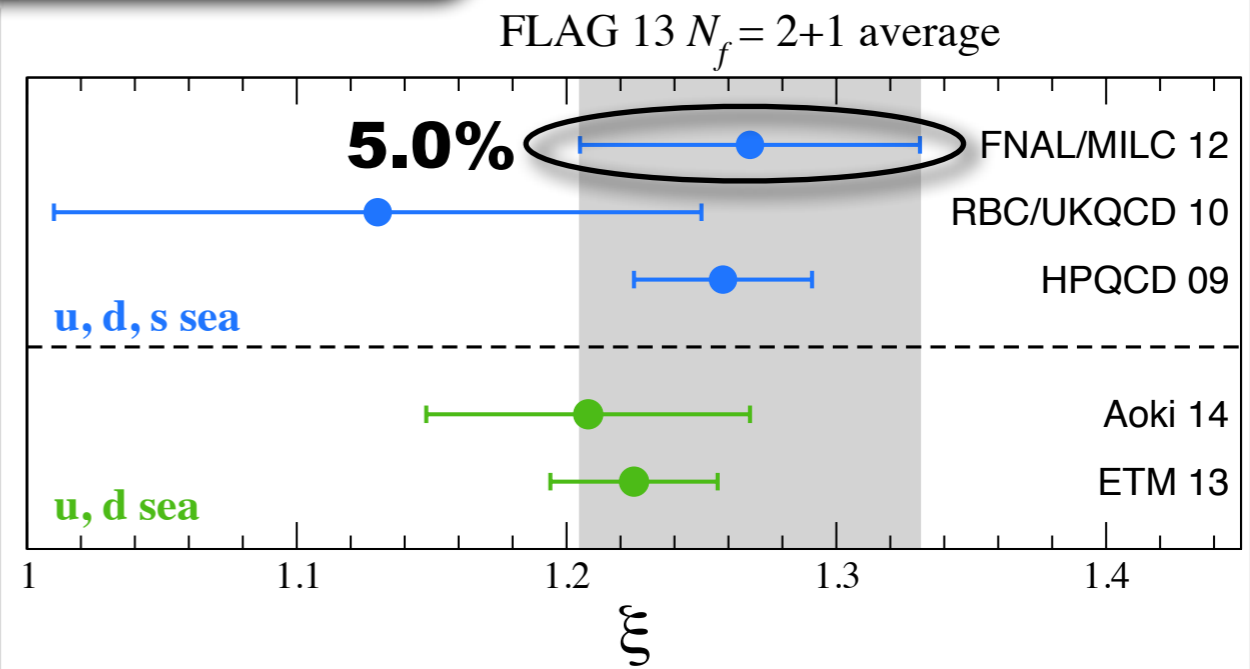
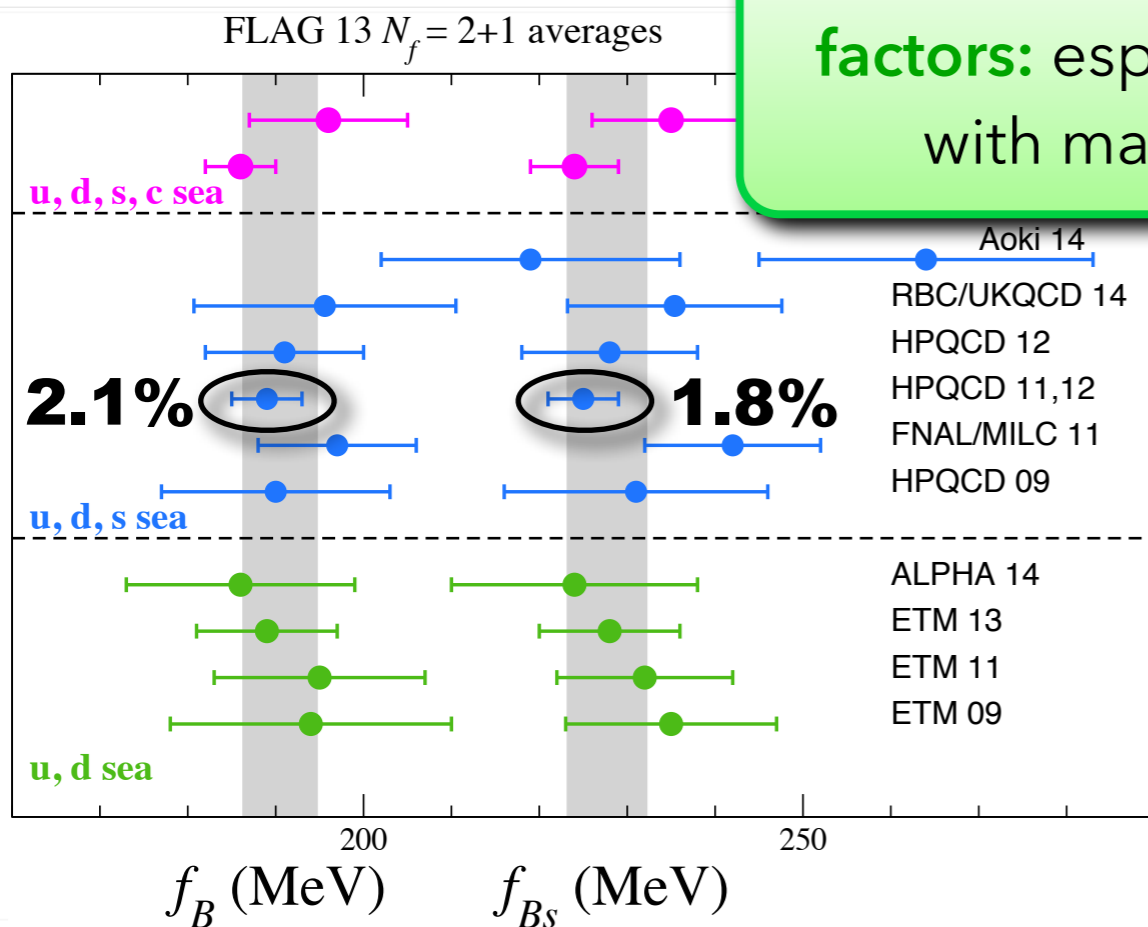
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**Focus on semileptonic form factors:** especially active area with many new results

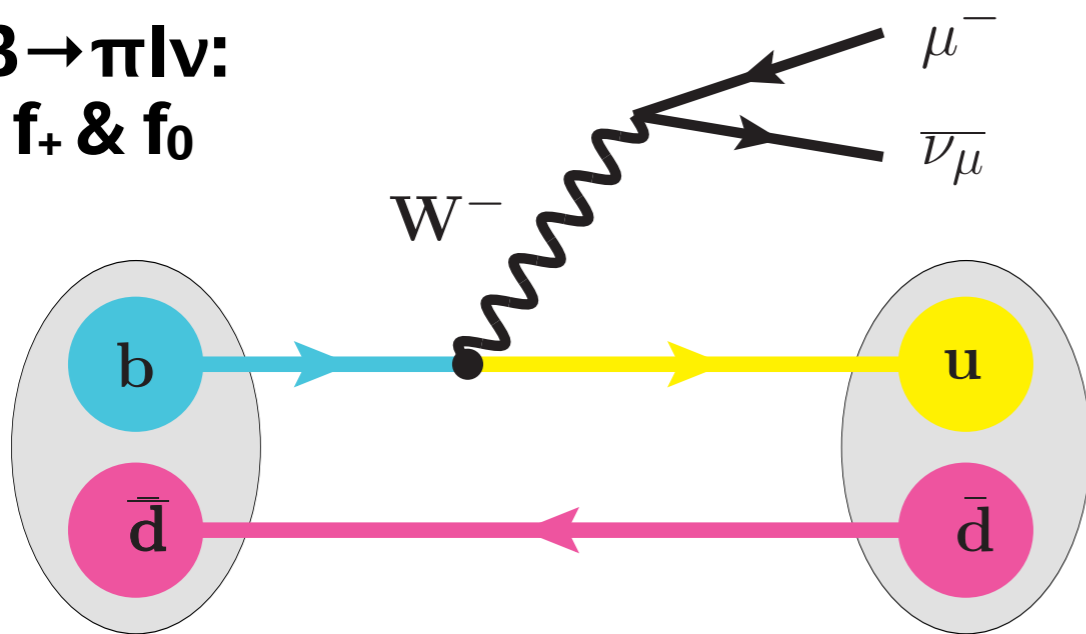


# New! $B \rightarrow \pi$ form factors

## Vector ( $f_+$ ) and scalar ( $f_0$ ) form factors

- ◆ Two new three-flavor calculations using different light & b-quarks actions [RBC/UKQCD, RDD91 (2015) 7, 074510; FNAL/MILC arXiv:1503.07839]
- ◆ Vector form factor gives dominant contribution to  $B \rightarrow \pi l \nu$  decay rate

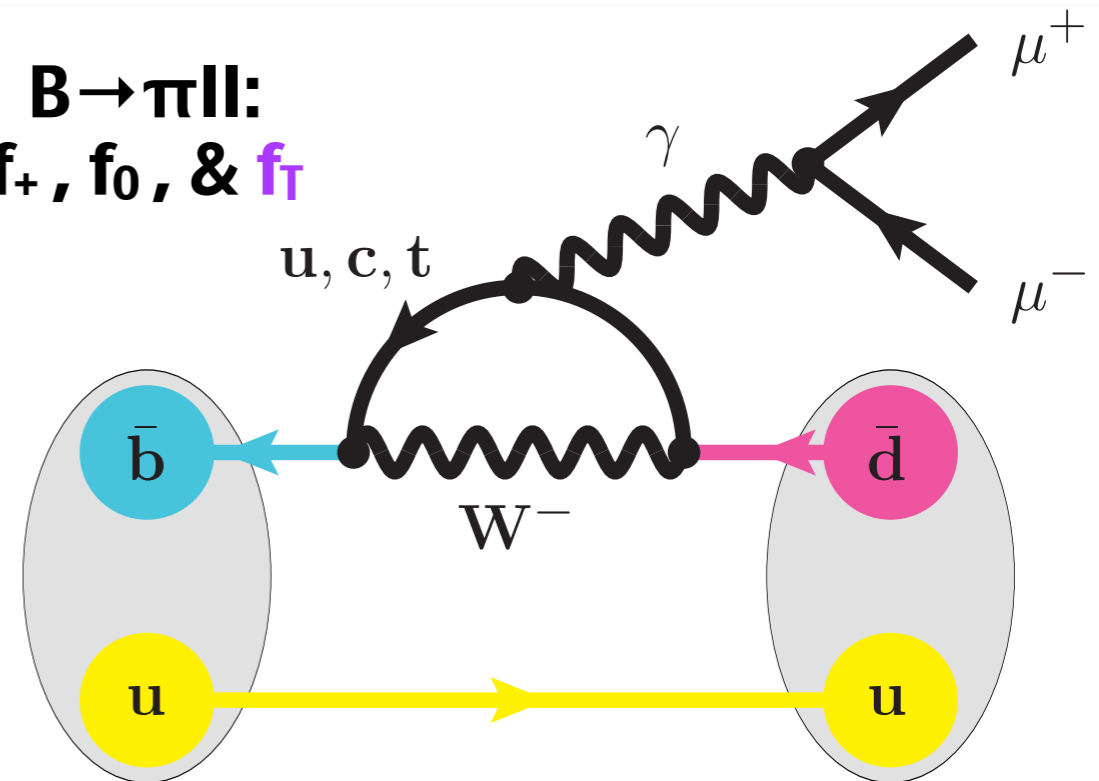
$B \rightarrow \pi l \nu$ :  
 $f_+$  &  $f_0$



## Tensor form factor ( $f_T$ )

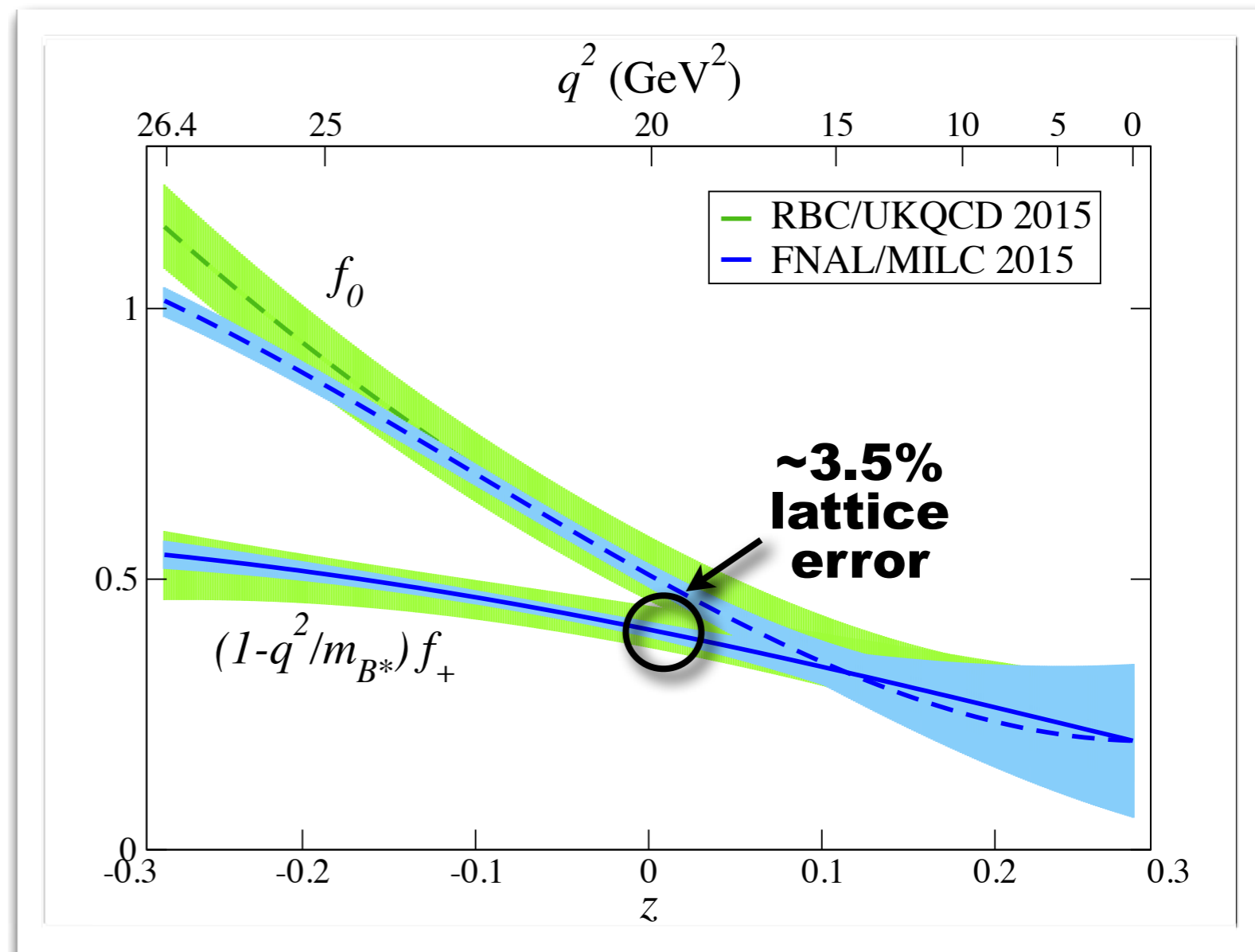
- ◆ First three-flavor lattice calculation [FNAL/MILC, arXiv:1503.07839]
- ◆ Enables predictions of observables for all  $B \rightarrow \pi$  semileptonic decay processes in Standard Model & all BSM theories

$B \rightarrow \pi l l$ :  
 $f_+$ ,  $f_0$ , &  $f_T$



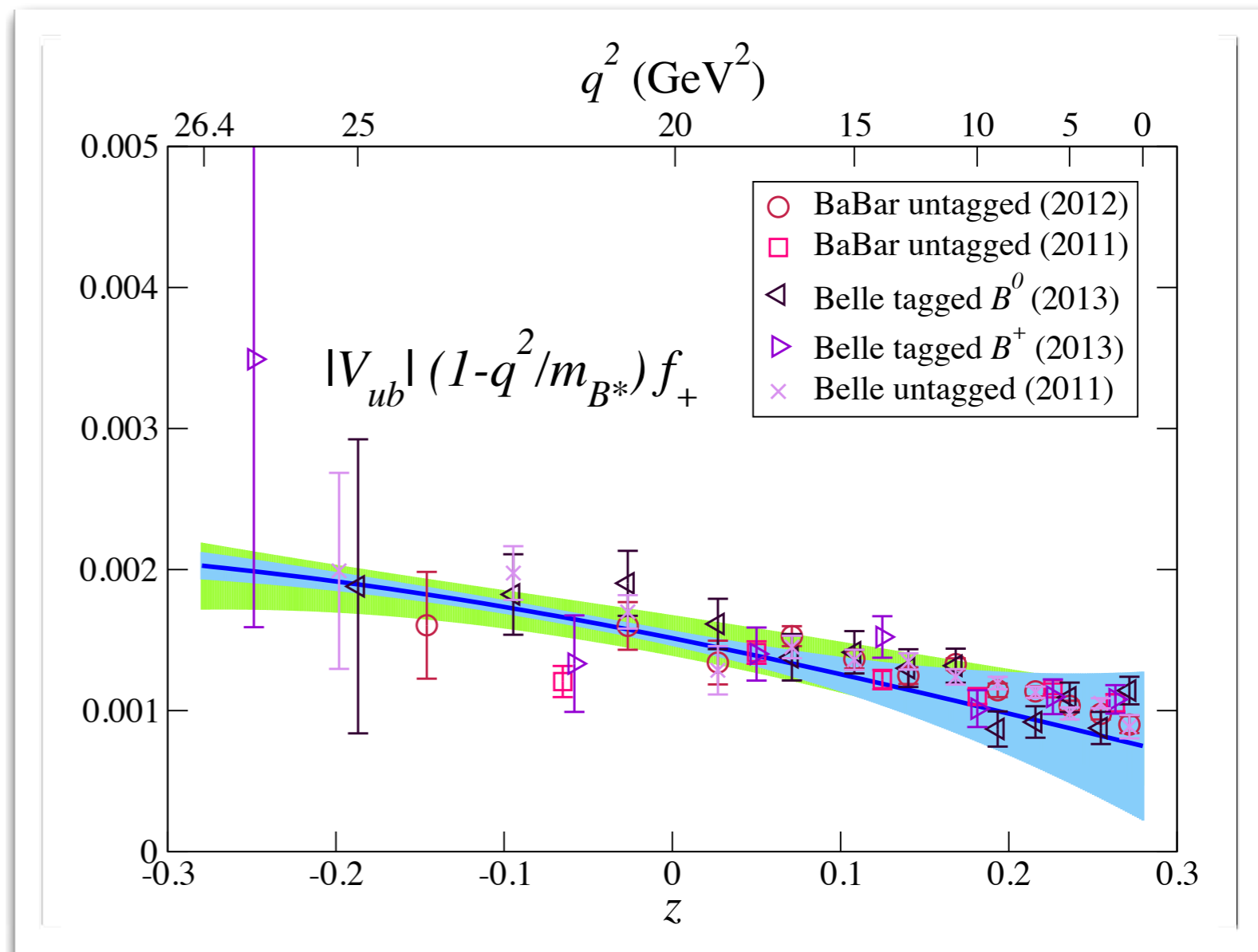
# New! $B \rightarrow \pi \ell \nu$ form factors

- ◆ **RBC/UKQCD**: first results for  $f_+$  and  $f_0$  [[PRDD91 \(2015\) 7, 074510](#)]
- ◆ **FNAL/MILC**: significant update of 2008  $f_+$  with increased statistics and finer lattice spacings; also first  $f_0$  [[arXiv:1503.07839](#)]
- ◆ Independent calculations  $\rightarrow$  cross-check



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- ◆ Independent calculations → *cross-check*
- ◆  $f_+(q^2)$  shape consistent with measured  $B \rightarrow \pi \ell \nu$  differential decay rate ( $dB/dq^2$ )
- ◆ **Obtain  $|V_{ub}|$  from joint lattice + experiment fit to model-independent  $z$ -expansion**

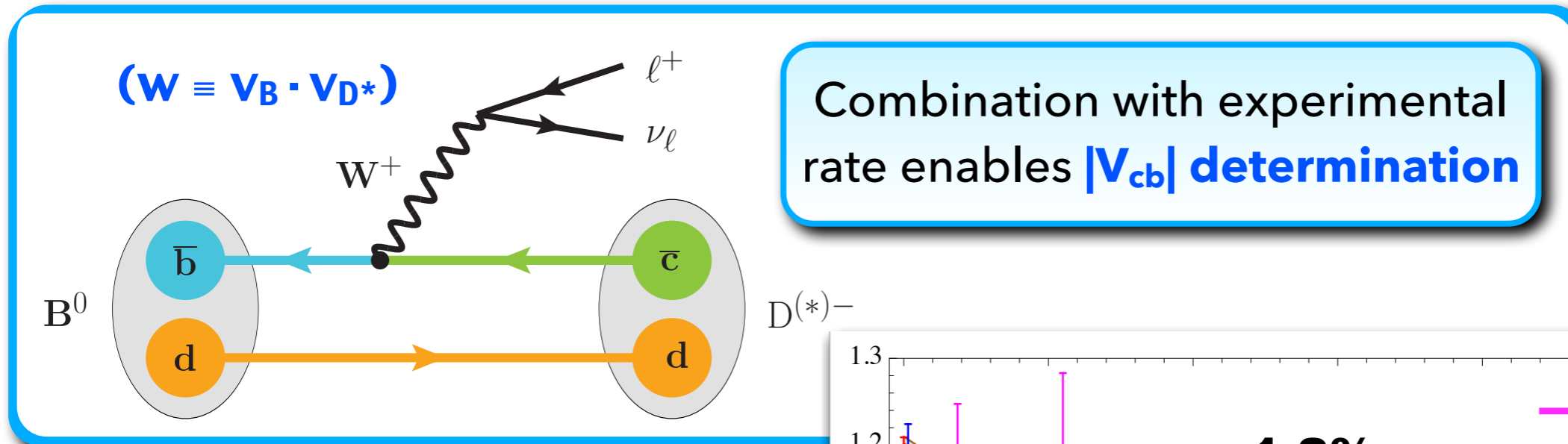


Use of all experimental & theoretical information minimizes error on  $|V_{ub}|$

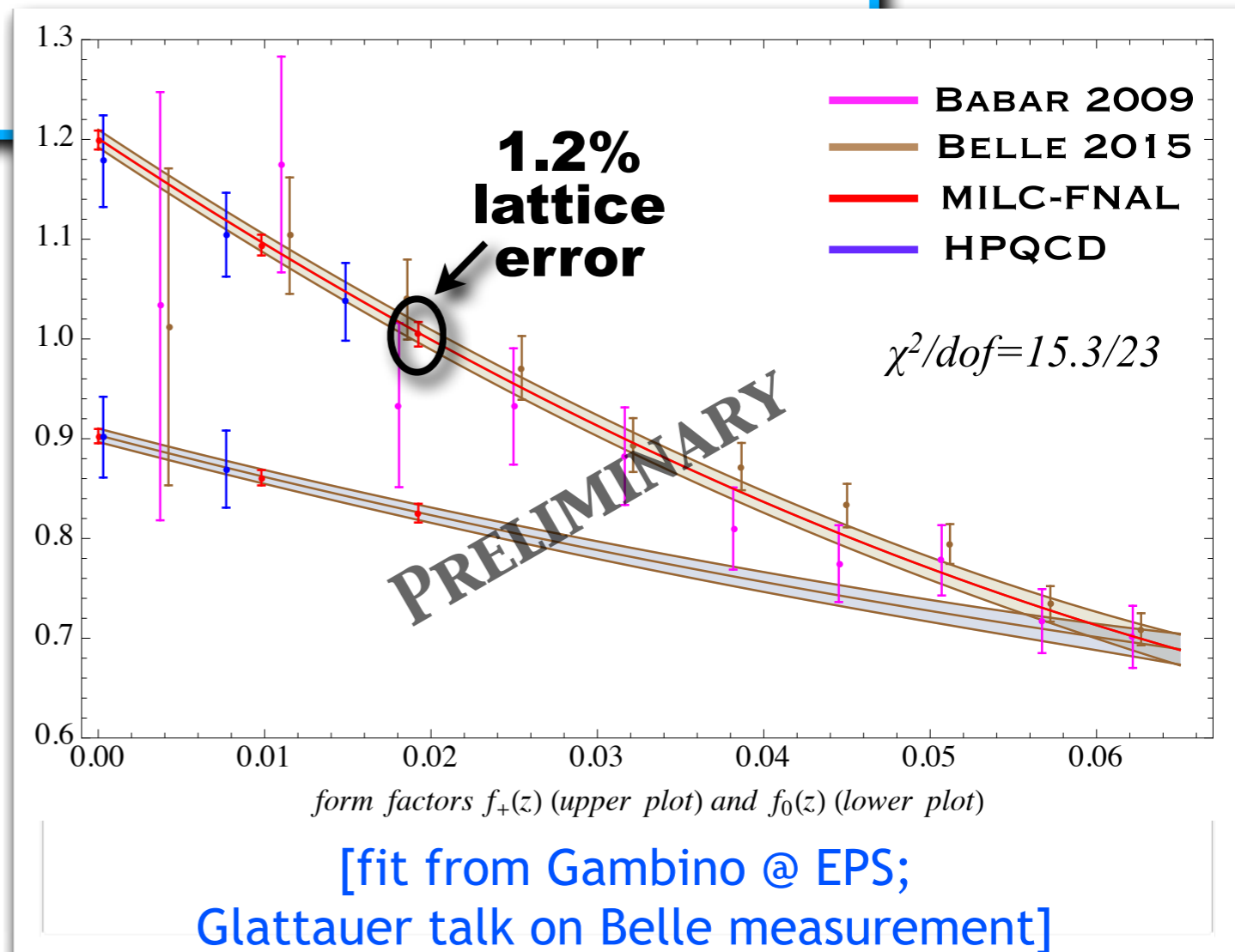


# New! $B \rightarrow D\ell\nu$ form factors @ nonzero recoil

[Bailey *et al.* [FNAL/MILC], arXiv:1503.07237; Na *et al.* [HPQCD], arXiv:1505.03925]



- ◆ Traditionally obtain  $|V_{cb}|$  from comparison of theory & experiment at zero recoil ( $w=1$ )
- ◆ **First three-flavor form factors at  $w>1$** 
  - ❖ Confirmation from two independent calculations
  - ❖ Shape consistent with experiment
- ◆ **Joint lattice + experiment fit using  $w>1$  data reduces error on  $|V_{cb}|$**



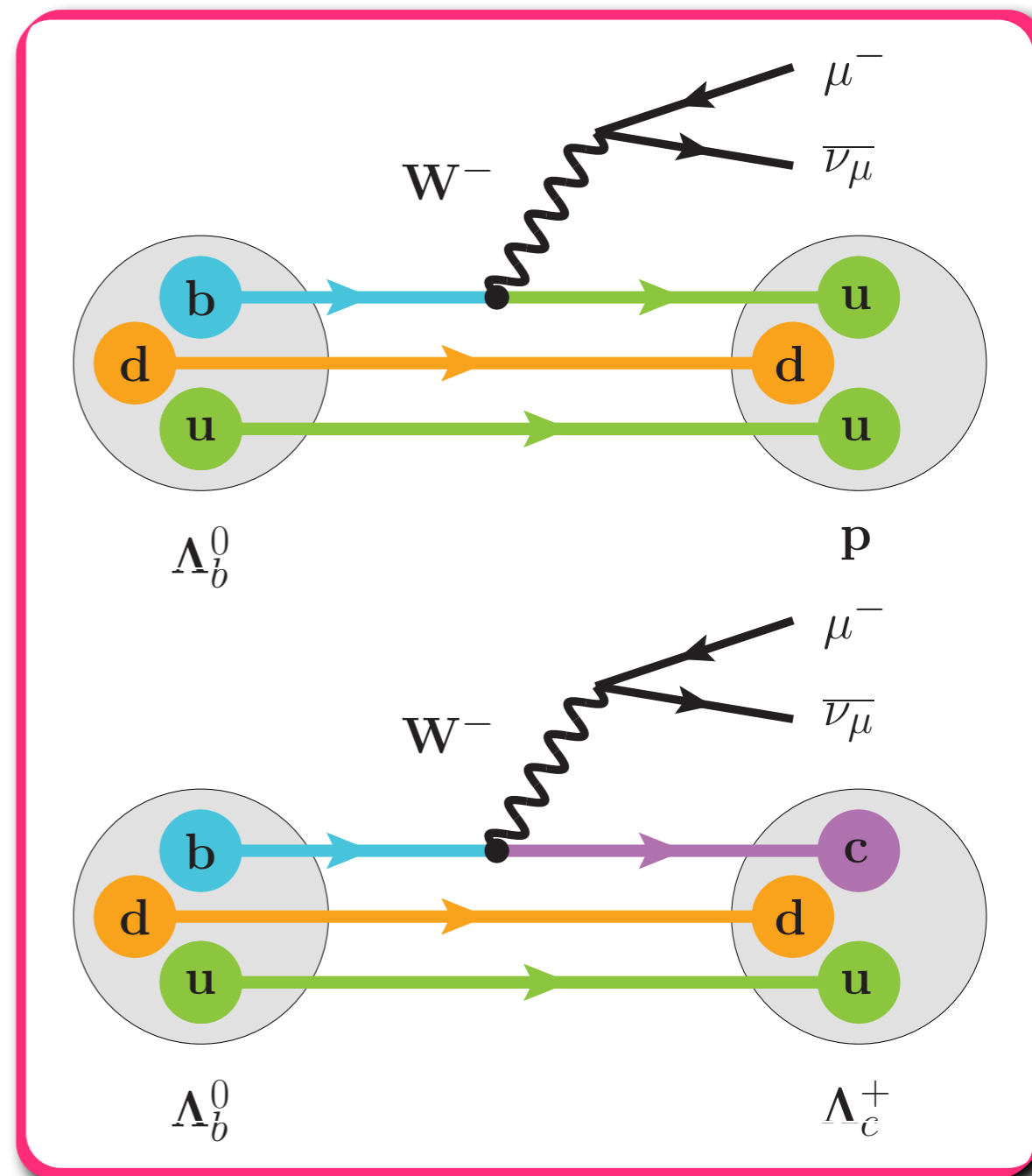
# New! $\Lambda_b \rightarrow p \ell \nu$ and $\Lambda_b \rightarrow \Lambda_c \ell \nu$ form factors

[Detmold, Lehner, & Meinel, arXiv:1503.01421]

Combination with ratio of experimental rates **enables determination of  $|V_{ub}|/|V_{cb}|$**   
 → first from baryon decay!

- ◆ First three-flavor  $\Lambda_b \rightarrow p$  form factors with relativistic b-quark at physical mass; first three-flavor  $\Lambda_b \rightarrow \Lambda_c$  form factors
- ◆ Combine chiral-continuum extrapolation with  $q^2$  fit via modified z-expansion
- ➔ **5.3% LQCD error on  $|V_{ub}|/|V_{cb}|$ :**

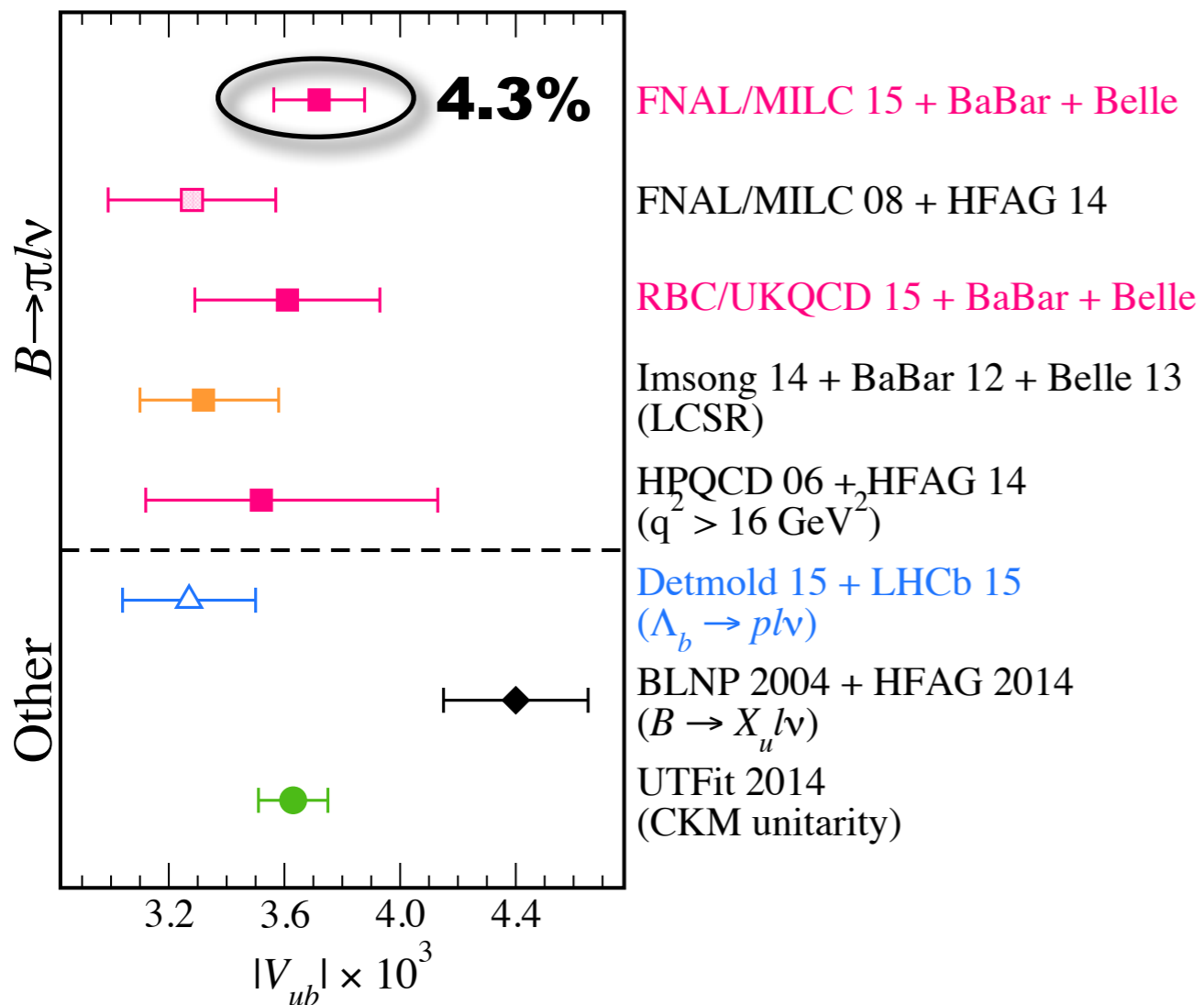
$$\frac{|V_{cb}|^2 \int_{15 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu)}{dq^2} dq^2}{|V_{ub}|^2 \int_7 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)}{dq^2} dq^2 = 1.470 \pm 0.115 \pm 0.104$$



[See Artuso EPS-HEP talk on corresponding LHCb measurement]

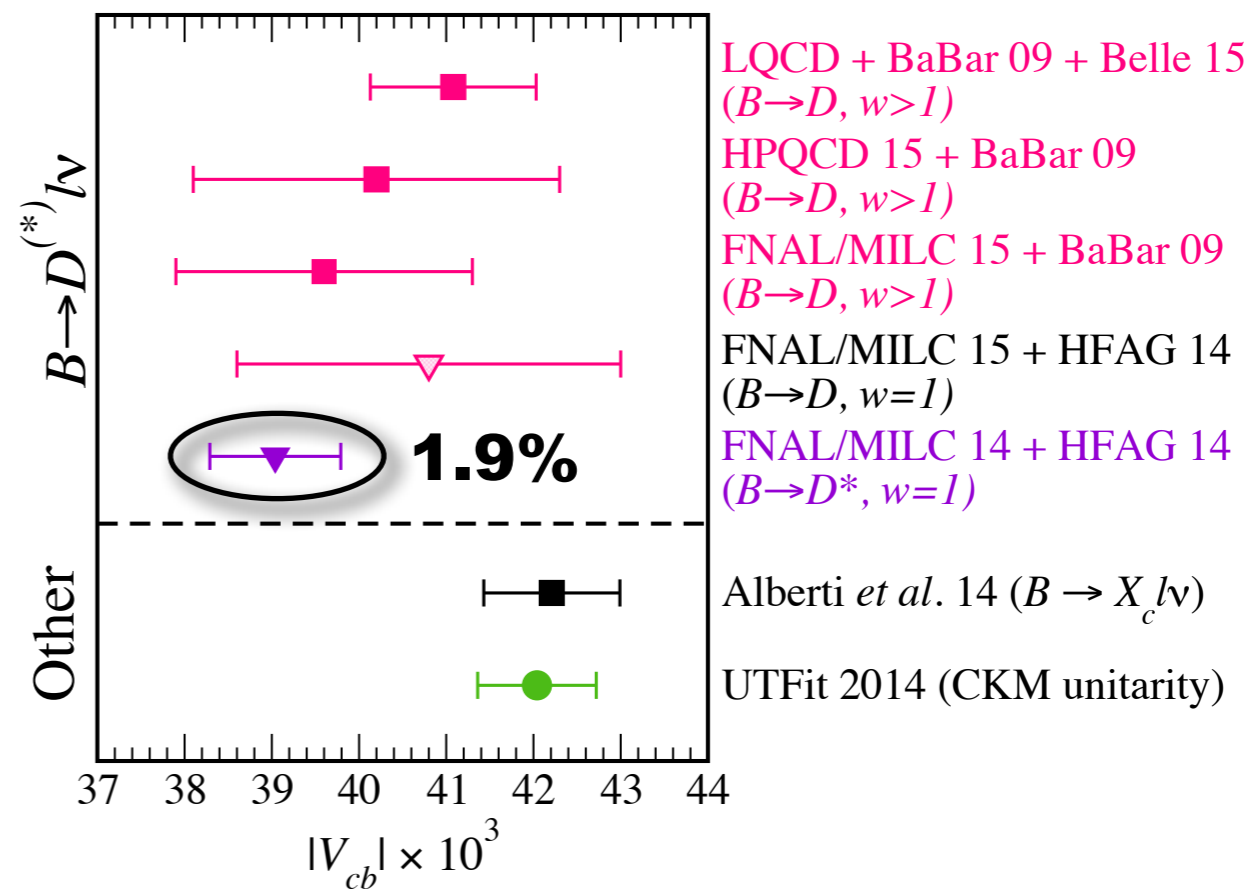
# Implications for $|V_{ub}|$ & $|V_{cb}|$

## $|V_{ub}|$



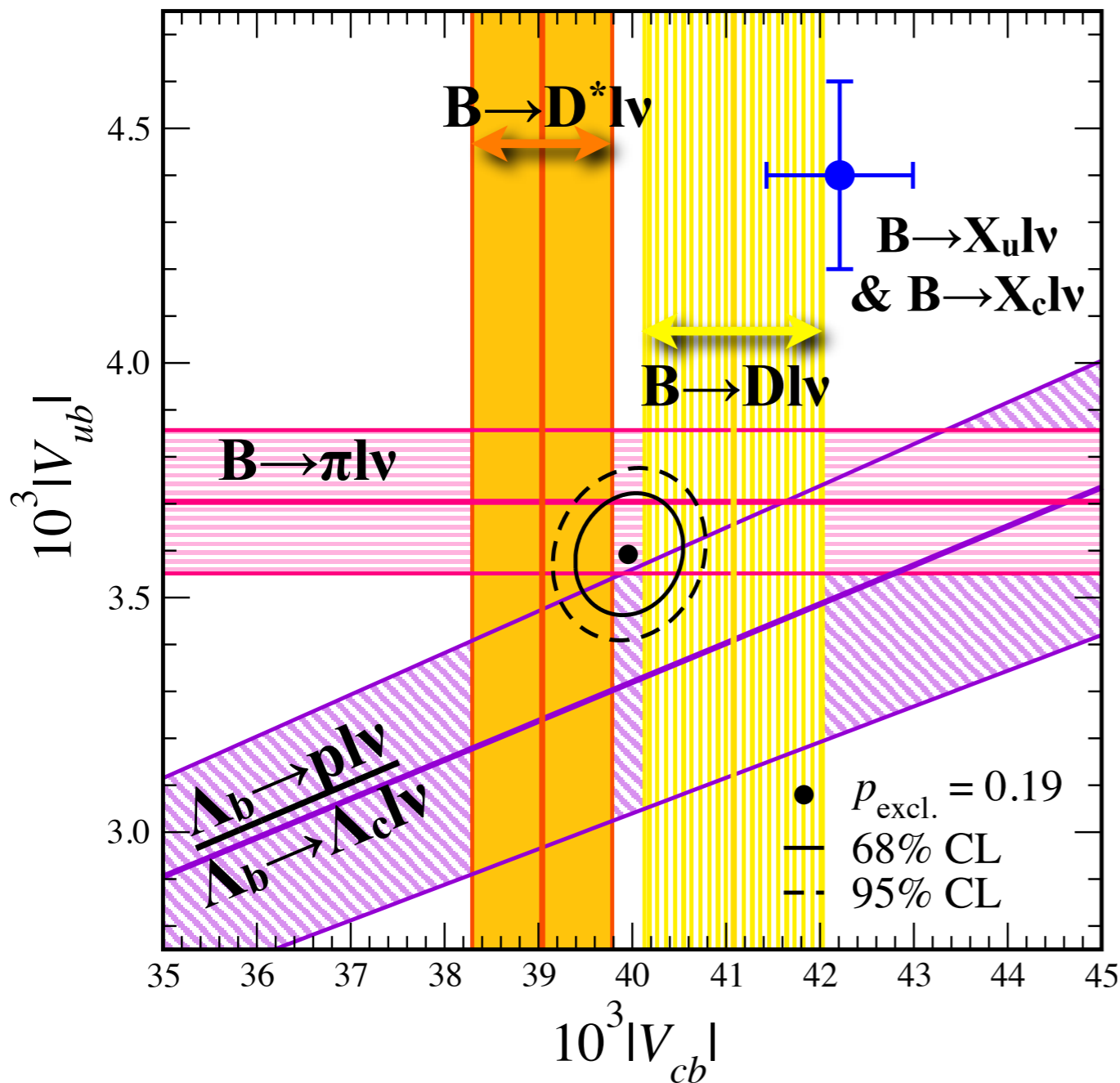
- ◆ Error on  $|V_{ub}|$  from  $B \rightarrow \pi l \nu$  cut by a bit more than half

## $|V_{cb}|$



- ◆ QCD error in  $|V_{cb}|$  from  $B \rightarrow D^* l \nu$  now commensurate with experimental error
- ◆ New Belle nonzero-recoil  $B \rightarrow D$  measurement raises  $|V_{cb}|$

# Implications for the “ $|V_{xb}|$ puzzle”

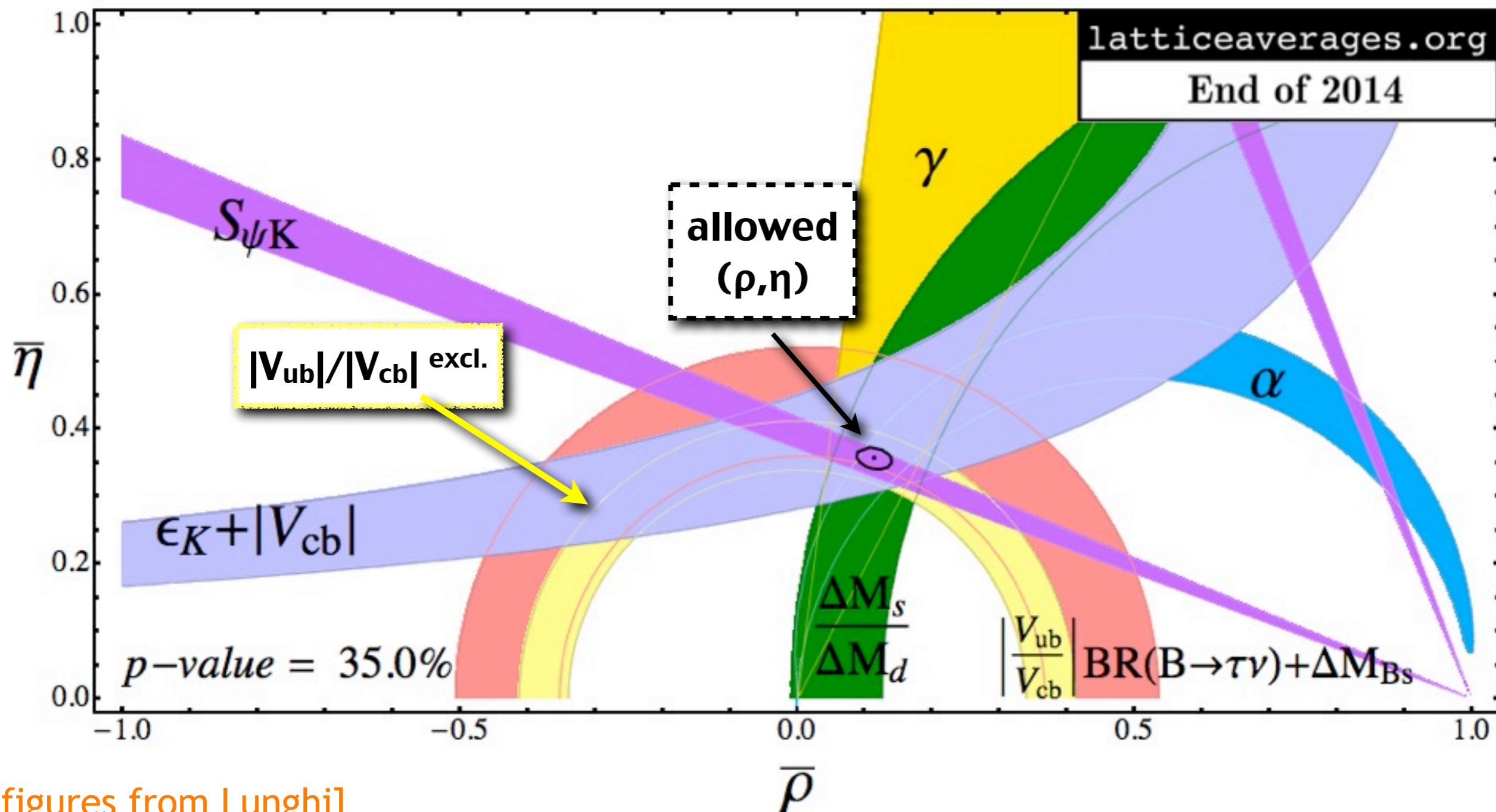


[fit from Kronfeld]

Increased lattice-QCD precision + new measurements sharpening picture of inclusive-exclusive tensions

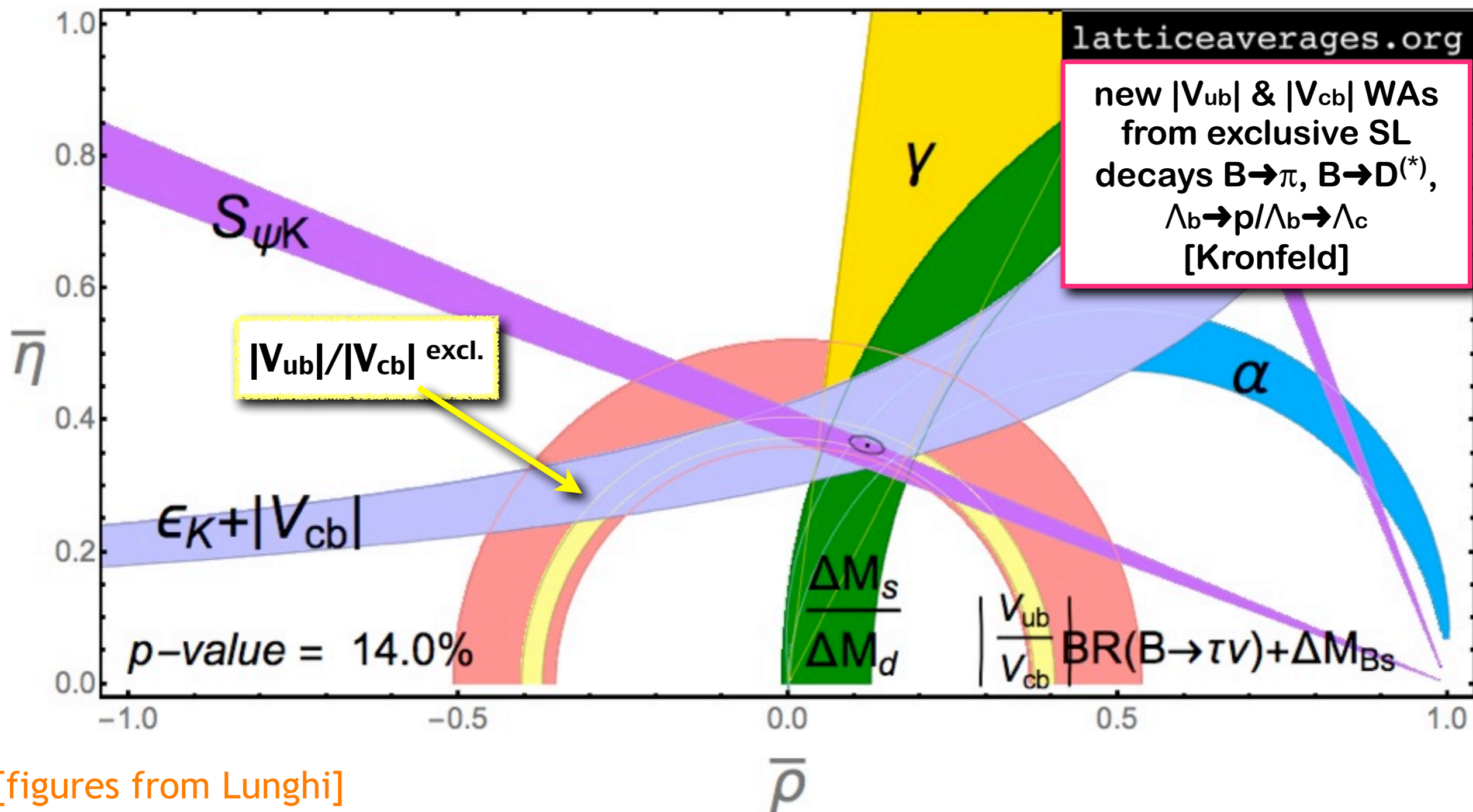
- ◆ LQCD will continue to address “ $V_{xb}$ ” puzzle through:
  - ❖ **New  $b \rightarrow u$  decays** (e.g.  $B_s \rightarrow K l \nu$ )
  - ❖ **Independent  $\Lambda_b \rightarrow p$  &  $\Lambda_b \rightarrow \Lambda_c$  form factors**
  - ❖  **$B \rightarrow D^* l \nu$  form factors at  $w > 1$**
- ◆  $|V_{cb}|$  from  $B \rightarrow D^* l \nu$  extrapolates measurement to zero recoil using CLN parameterization → **time for model-independent analysis!**

# Impact on global CKM UT fit



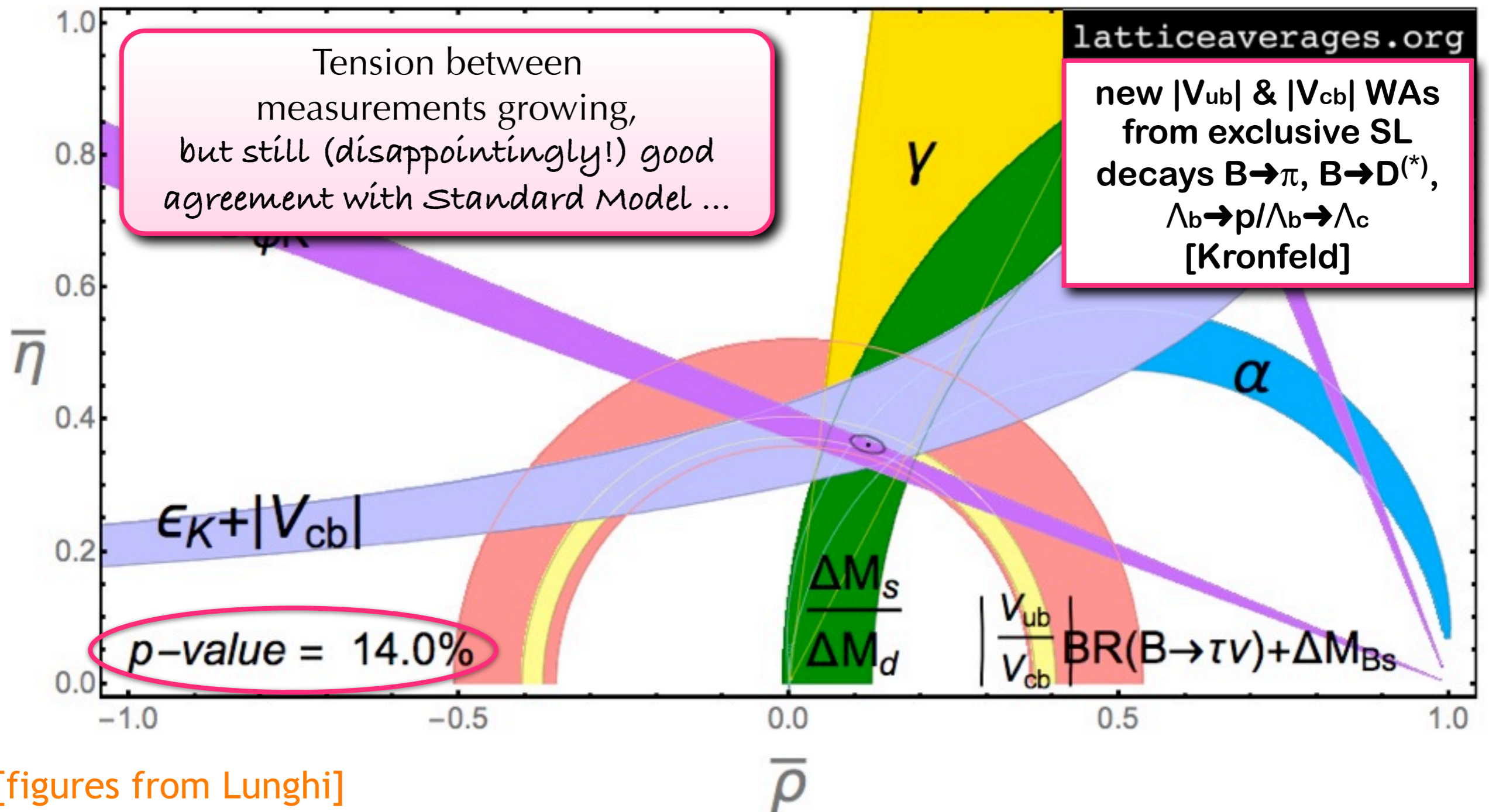
[figures from Lunghi]

# Impact on global CKM UT fit



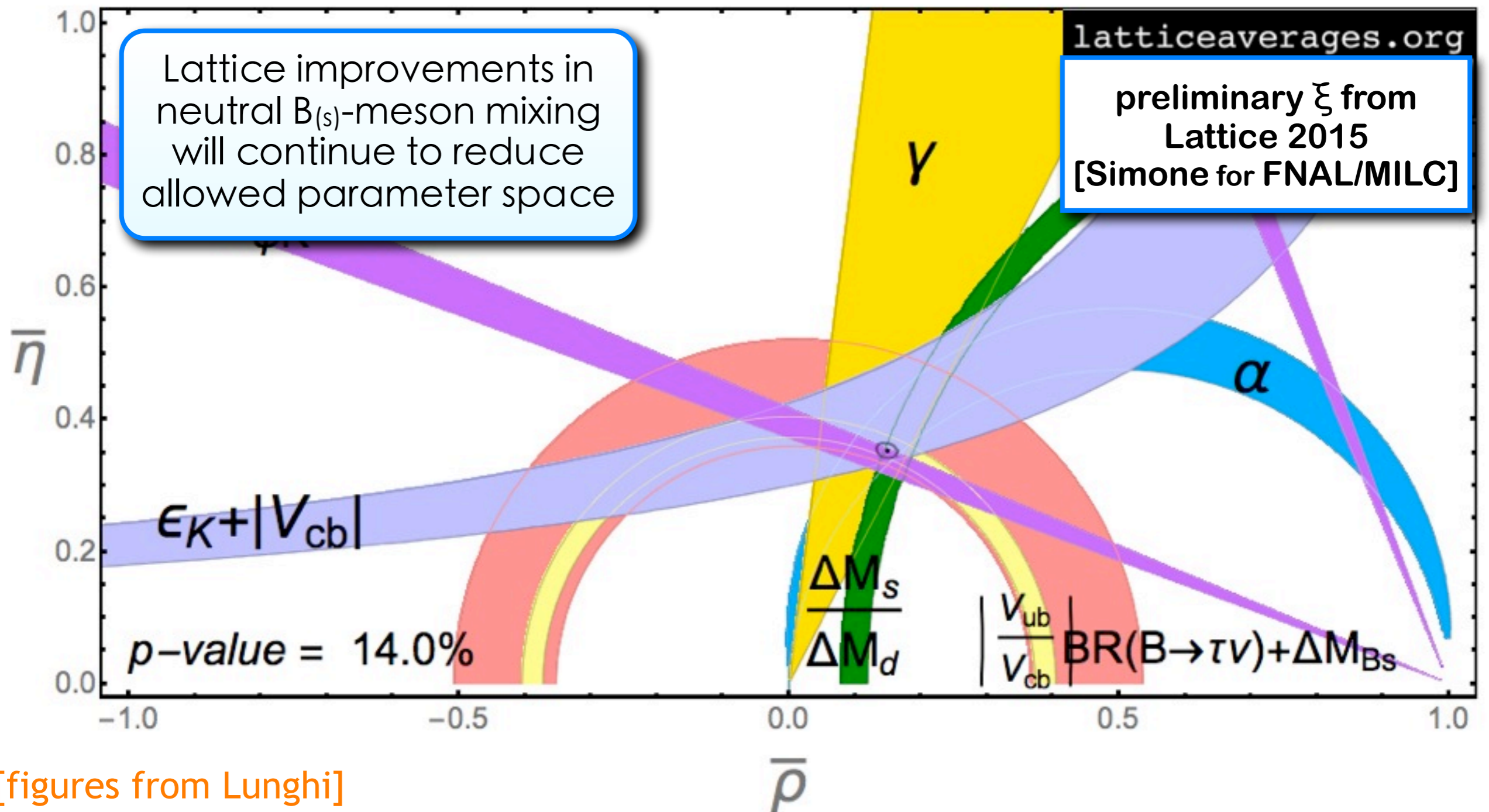
[figures from Lunghi]

# Impact on global CKM UT fit




[figures from Lunghi]

... UT fit at the end of the year?



[figures from Lunghi]





*Unable to cover numerous other exciting advances in lattice-QCD calculations of*

*❖  $K \rightarrow \pi\pi$  decays,*

*❖ long-distance matrix elements,*

*❖ nucleon structure,*

*❖ QCD resonances, ... and more!*

Beyond simple quantities: Muon  $g-2$

Unable to cover numerous other exciting advances in lattice-QCD calculations of

- ❖  $K \rightarrow \pi\pi$  decays,
- ❖ long-distance matrix elements,
- ❖ nucleon structure,
- ❖ QCD resonances,

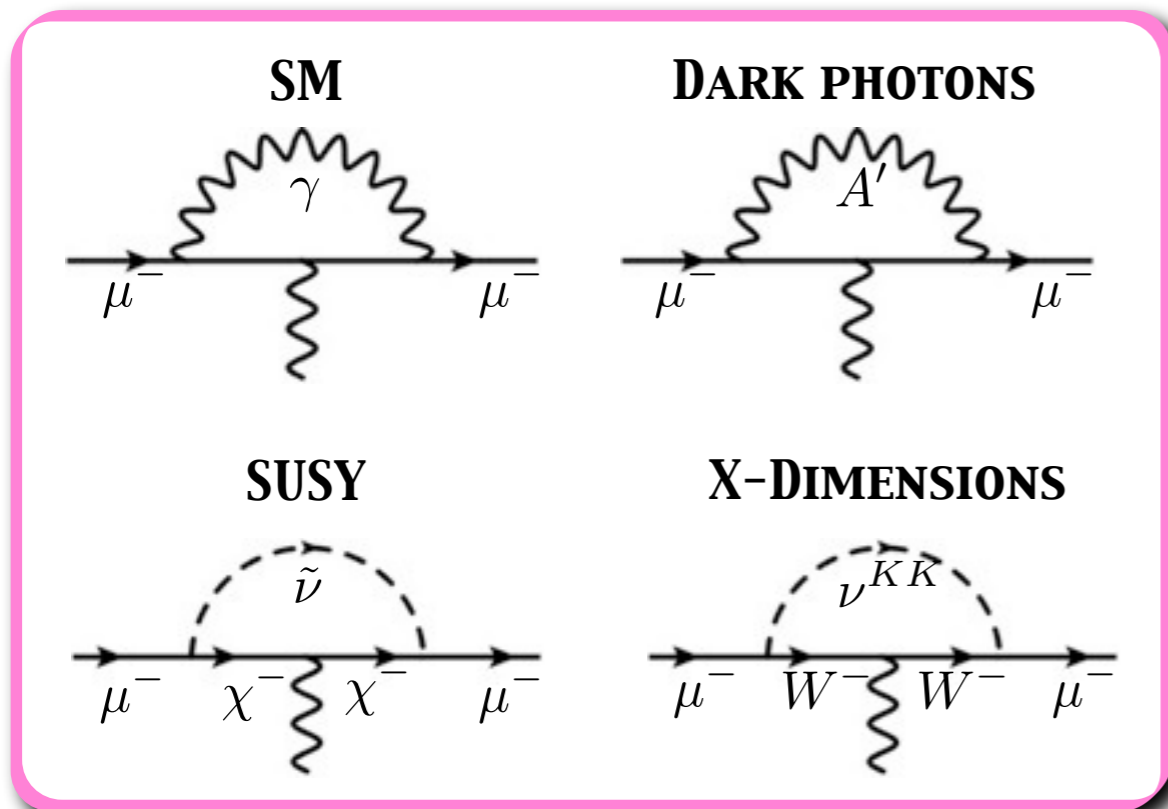
First  $\text{Re}(\epsilon'/\epsilon)$  in Standard Model with controlled errors!  
[RBC/UKQCD, 1505.07863;  
Soni talk @ EPS-HEP]

Beyond simple quantities:

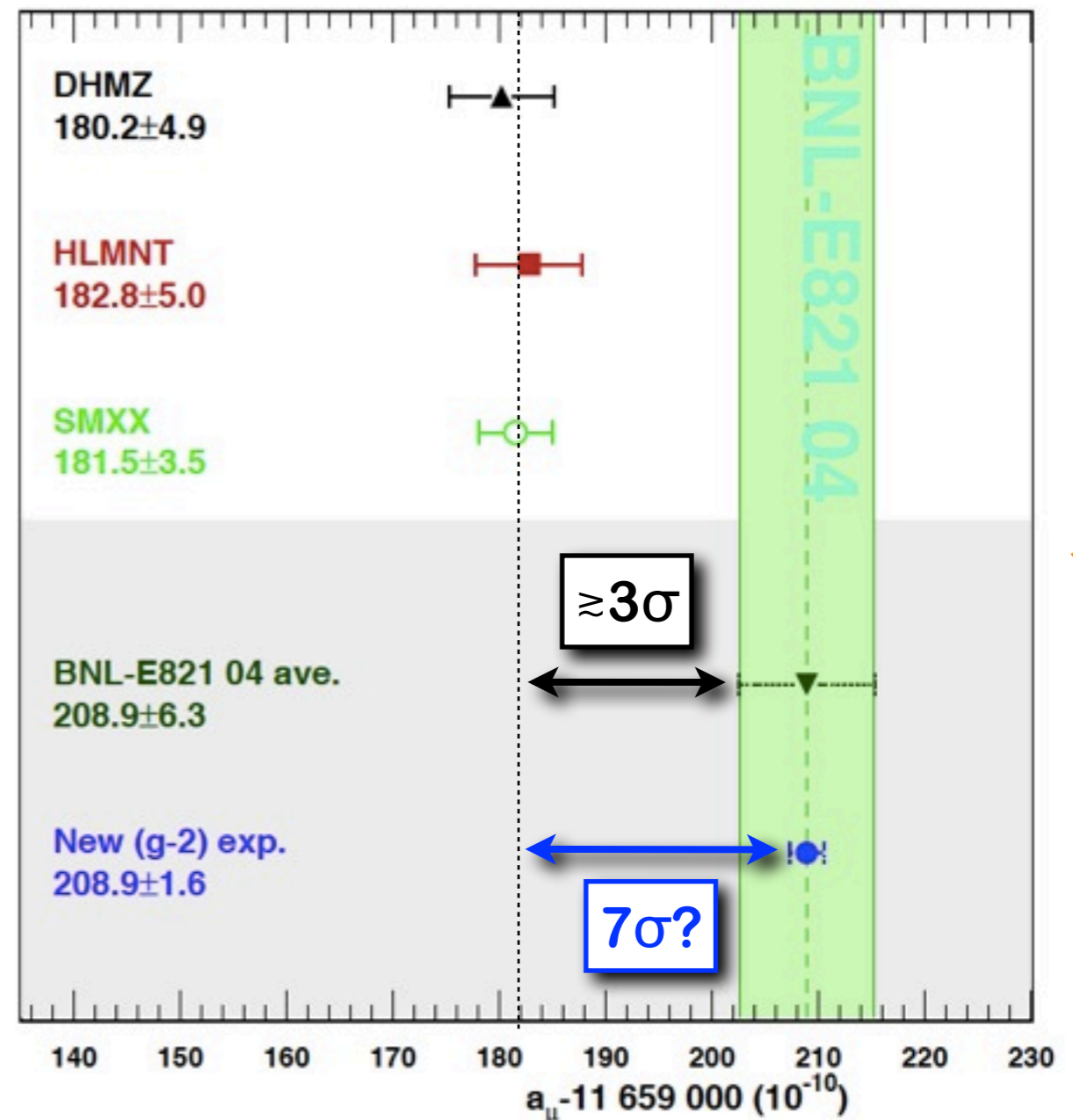


# Experimental status of $g-2$

- ◆ Muon anomalous magnetic moment ( $g-2$ ) among best-measured quantities (0.54 ppm!)  
 → provides some of most precise constraints on extensions of Standard Model



- ◆ **BNL measurement** disagrees with Standard Model by  $\approx 3\sigma$ , while **Fermilab Muon  $g-2$  Experiment** aims to reduce error by factor of four



[Blum *et al.*, arXiv:1311.2198]

# Theoretical status of $g-2$

◆ Uncertainty on SM prediction dominated by **nonperturbative hadronic contributions**

Contribution	Result ( $\times 10^{11}$ )		Error
QED (leptons)	116 584 718	$\pm 0.14 \pm 0.04_\alpha$	0.00 ppm
<b>Hadronic Vacuum Polarization</b>	923	$\pm 42$	<b>0.36 ppm</b>
HVP(ho)	-98	$\pm 0.9_{\text{exp}} \pm 0.3_{\text{rad}}$	0.01 ppm
<b>Hadronic Light-by-Light</b>	105	$\pm 26$	<b>0.22 ppm</b>
EW	154	$\pm 2 \pm 1$	0.02 ppm
Total SM	116 591 802	$\pm 49$	0.42 ppm

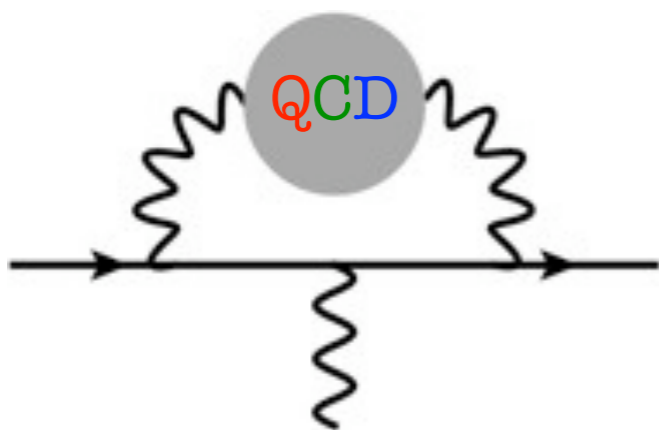
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

# Theoretical status of $g-2$

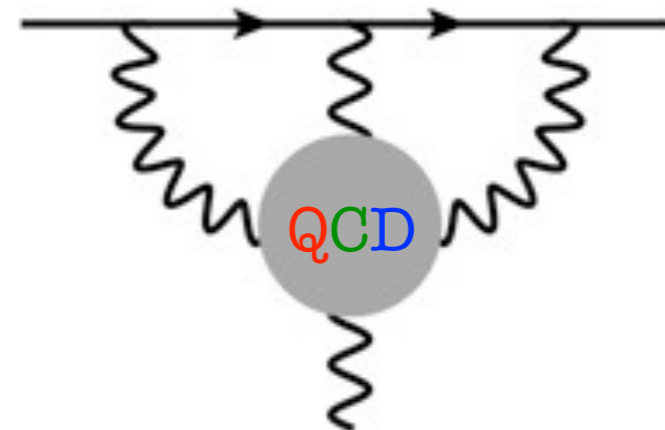
- ◆ Uncertainty on SM prediction dominated by **nonperturbative hadronic contributions**
- ◆ To leverage the anticipated experimental measurement, the **SM theory error must be shored-up and brought to a comparable precision**

## Hadronic vacuum polarization (HVP)



obtained from experimental result for  $e^+e^- \rightarrow \text{hadrons}$  plus dispersion relation

## Hadronic light-by-light (HLbL)

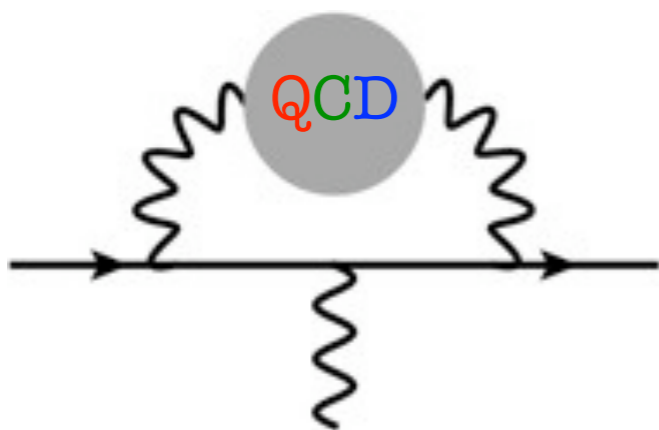


estimated from QCD models such as large  $N_c$ ,  $\chi$ PT, vector meson dominance, etc...  
(subjective and somewhat controversial)

# Theoretical status of $g-2$

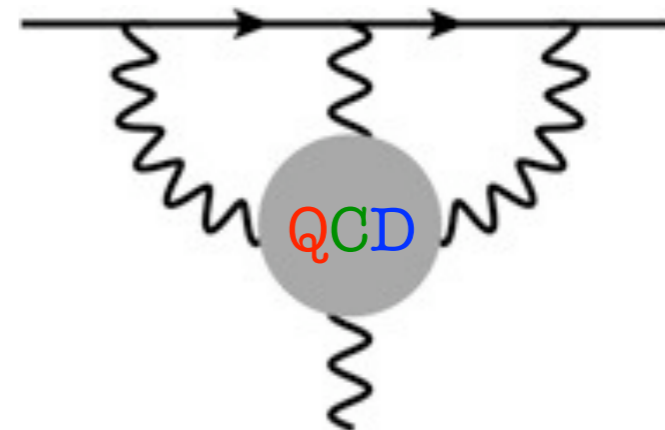
- ◆ Uncertainty on SM prediction dominated by **nonperturbative hadronic contributions**
- ◆ To leverage the anticipated experimental measurement, the **SM theory error must be shored-up and brought to a comparable precision**
- ◆ **Both hadronic contributions are calculable, in principle, with lattice QCD**

## Hadronic vacuum polarization (HVP)



methods are in place, and first results have been reported from different collaborations and with different approaches

## Hadronic light-by-light (HLbL)



very difficult, but methods being developed and tested

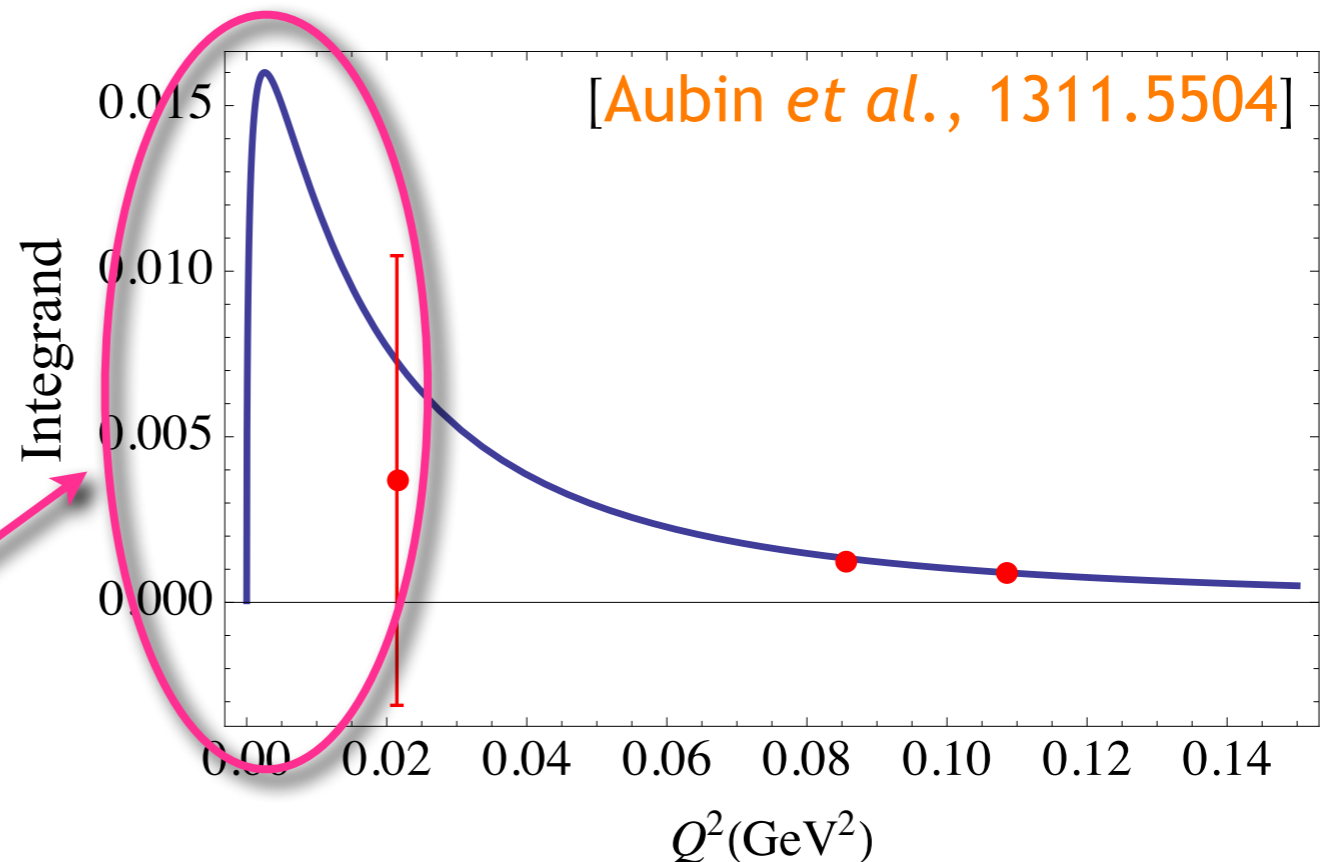
# Standard lattice method for $a_\mu^{\text{HVP}}$

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

- ◆ Calculate  $a_\mu^{\text{HVP}}$  directly from Euclidean space vacuum polarization function  $\Pi(Q^2)$ 
  - ❖  $\Pi(Q^2)$  captures nonperturbative QCD effects → compute numerically in lattice QCD from simple two-point correlation function of electromagnetic quark current
  - ❖ 1-loop QED integral accounts for interaction between muon and external photon field
- ◆ Challenging because integrand  $f(Q^2)[\Pi(Q^2)-\Pi(0)]$  peaks around  $Q^2 \approx (m_\mu/2)^2$ , where lattice data is sparse and noisy

$$a_\mu^{\text{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) [\Pi(Q^2) - \Pi(0)]$$

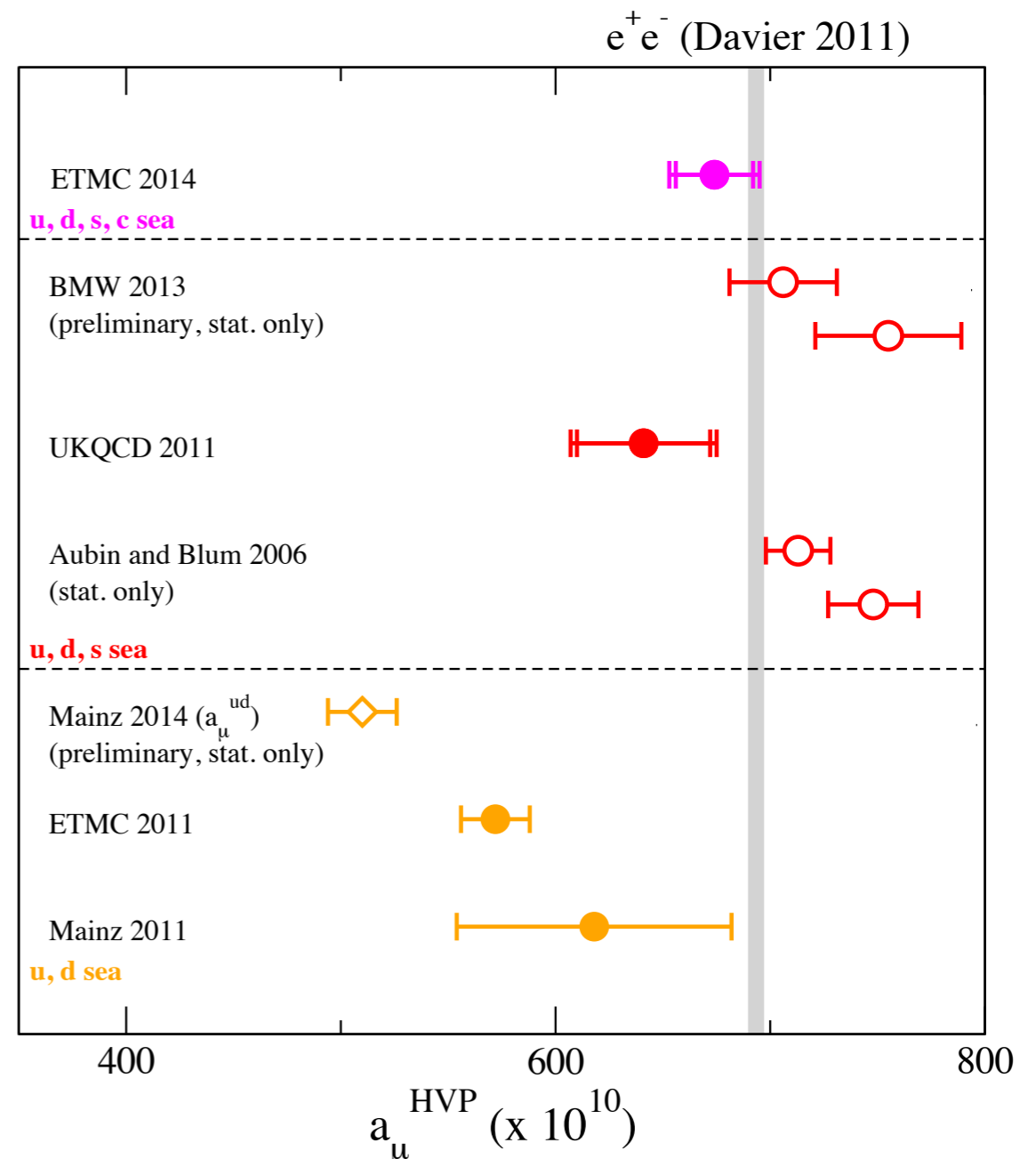
$$i\Pi_{\mu\nu}(q^2) = \text{wavy line } q_\mu \text{ --- } \text{QCD} \text{ --- } \text{wavy line } q_\nu$$



# Recent lattice progress on $a_\mu^{\text{HVP}}$

- ◆ **Statistical-error-reduction techniques:** all-mode averaging [Blum *et al.*, PRD 88 (2013) 094503], stochastic sources [Marinkovic *et al.*, 1502.05308]
- ◆ **Methods for controlling  $q^2$  extrapolation:**
  - ❖ Padé approximants to remove hidden model systematics in  $q^2$  extrapolation [Aubin *et al.*, PRD86 (2012) 054509]
  - ❖ Twisted boundary conditions to access smaller momentum values below  $(2\pi/L)$  [Della Morte *et al.*, JHEP 1203 (2012) 055; Aubin *et al.*, PRD88 (2013) 7, 074505]
  - ❖ **Moments method to sidestep  $q^2 \rightarrow 0$  extrapolation** by expressing  $a_\mu^{\text{HVP}}$  in terms of derivatives of  $\Pi(q^2)$  at  $q^2=0$  [Chakraborty *et al.* (HPQCD), PRD89 (2014) 11, 114501]

Active research area with several independent ongoing efforts!

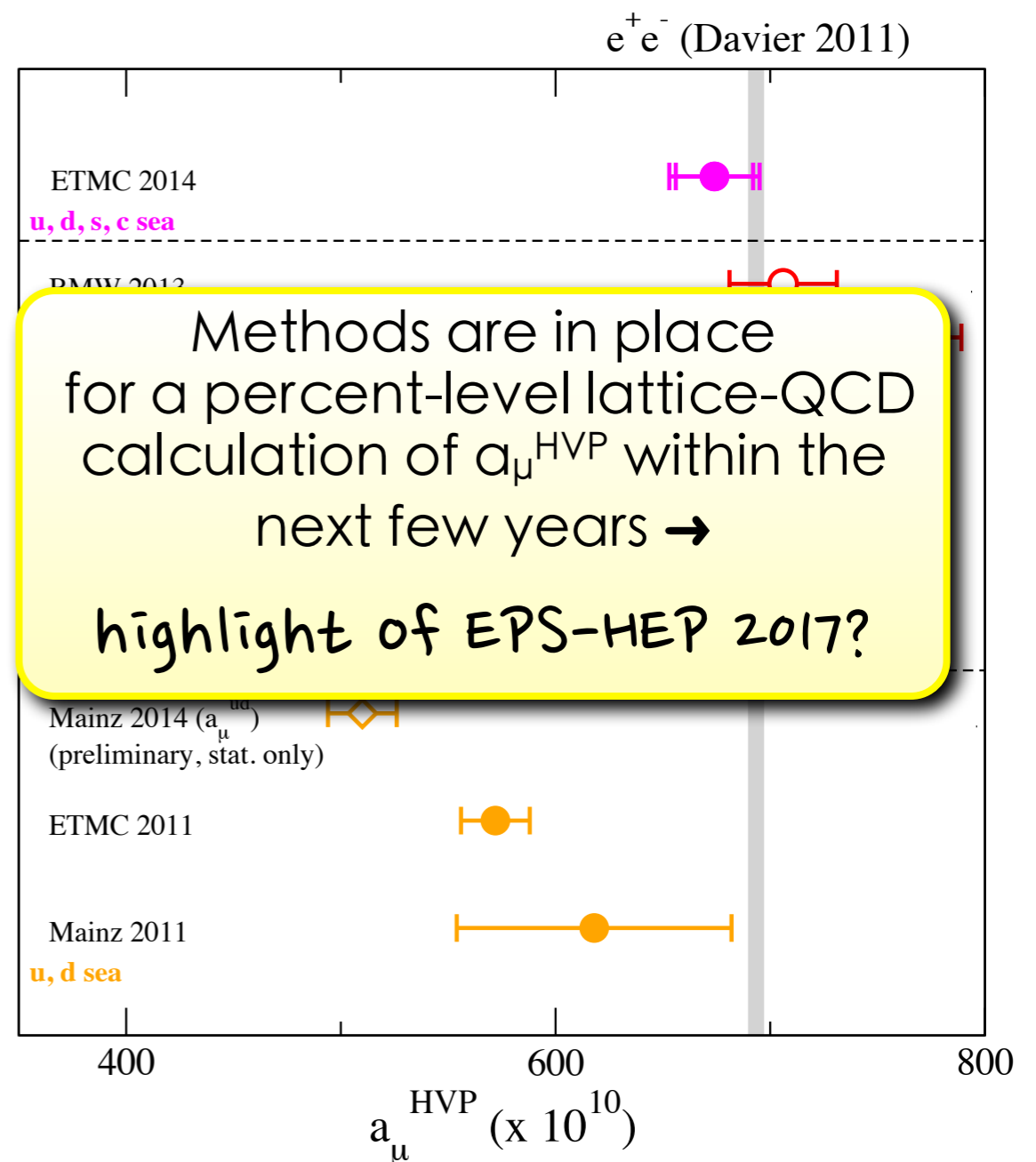




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The background of the slide is a deep blue space filled with numerous small white stars. In the center, there are two large, semi-transparent spheres. The sphere on the left is smaller and contains three smaller spheres in red, blue, and yellow. The sphere on the right is larger and contains a red sphere at the top and a yellow sphere at the bottom. The word "Conclusions" is written in a large, yellow, sans-serif font across the middle of the image, overlapping the spheres.

# Conclusions

“[An] area of striking progress has been lattice gauge theory. ... It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years.”

– **Snowmass Executive Summary**

# Summary & outlook

- ◆ **Lattice-QCD calculations needed throughout current & future experimental high-energy physics program**
  - ❖ For precision measurements of rare kaon and B decays, muon  $g-2$ , neutrino oscillation parameters, Higgs properties, ...
  - ❖ For searches for  $\mu \rightarrow e$  conversion, dark matter, proton decay, nucleon EDMs, ...
- ◆ **Ambitious worldwide lattice-QCD program in place to deliver the needed theoretical support** by
  - ❖ **Increasing precision of present calculations** of parameters of the QCD Lagrangian & simplest quark flavor-changing matrix elements
  - ❖ **Providing first reliable QCD calculations of new, more challenging quantities** such as muon  $g-2$ , long-distance amplitudes, decays to QCD resonances or multi-hadron final states, ...
- ◆ **Theoretical innovation in methods & algorithms drive progress in lattice QCD, but sufficient computational resources are absolutely essential!**

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  - ❖ For searches for  $\mu \rightarrow e$  conversion, dark matter, proton decay, nucleon EDMs, ...
- ◆ **Ambitious world-class theoretical studies needed**
  - ❖ **Increasingly precise calculations of the simplest quantities needed to understand the QCD Lagrangian & its phenomenology**
    - Future experiments, aided by lattice-QCD calculations, will continue to tighten the noose on the Standard Model and probe the nature of whatever new particles and forces lie beyond
  - ❖ **Providing first reliable QCD calculations of new, more challenging quantities** such as muon  $g-2$ , long-distance amplitudes, decays to QCD resonances or multi-hadron final states, ...
- ◆ **Theoretical innovation in methods & algorithms drive progress in lattice QCD, but sufficient computational resources are absolutely essential!**

# Further reading

See Lattice 2015 website for more hot-off-the-press results!

## Lattice 2014 review talks:

- ❖ [“Weak interactions of pions and kaons”](#)
- ❖ [“Testing the Standard Model under the weight of heavy flavors”](#)

## 2013 FLAG report:

- ❖ [“Review of lattice results concerning low-energy particle physics”](#)

## Snowmass working-group reports:

- ❖ [“Charged Leptons”](#)
- ❖ [“Higgs Working Group Report ...”](#)
- ❖ [“Lattice field theory ... Scientific goals and computing needs”](#)
- ❖ [“Report of ... QCD Working Group”](#)
- ❖ [“Report of ... Quark Flavor Physics Working Group”](#)



**LATTICE 2015**  
The 33rd International Symposium on Lattice Field Theory  
Kobe International Conference Center, Kobe, Japan  
Tuesday, July 14 - Saturday, July 18, 2015  
<http://www.aics.riken.jp/sympo/lattice2015>

**Topics**

- ◆ Algorithms and Machines
- ◆ Applications Beyond QCD
- ◆ Chiral Symmetry
- ◆ Hadron Spectroscopy and Interactions
- ◆ Hadron Structure
- ◆ Nonzero Temperature and Density
- ◆ Physics Beyond the Standard Model
- ◆ Standard Model Parameters and Renormalization
- ◆ Theoretical Developments
- ◆ Vacuum Structure and Confinement
- ◆ Weak Decays and Matrix Elements

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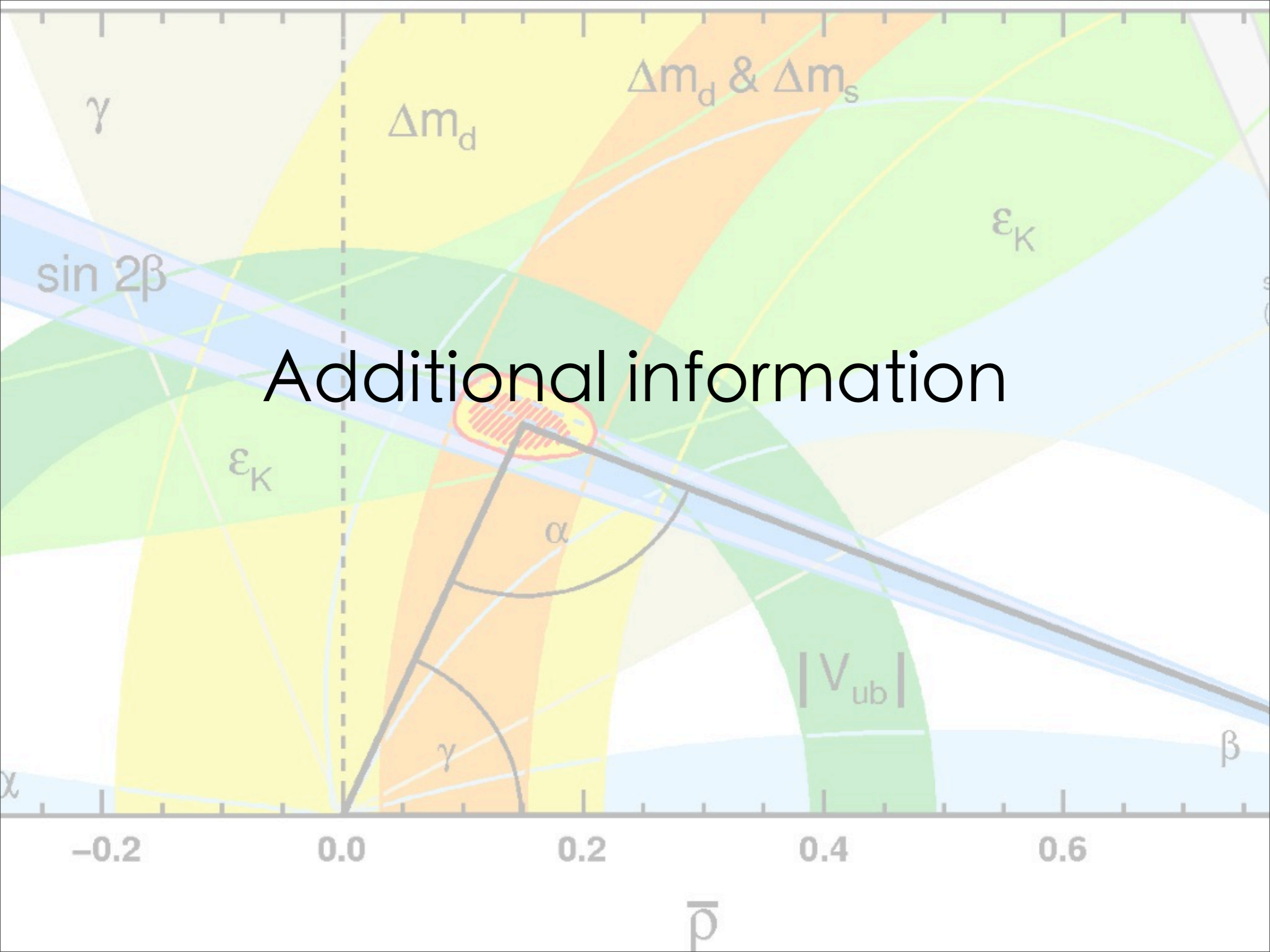
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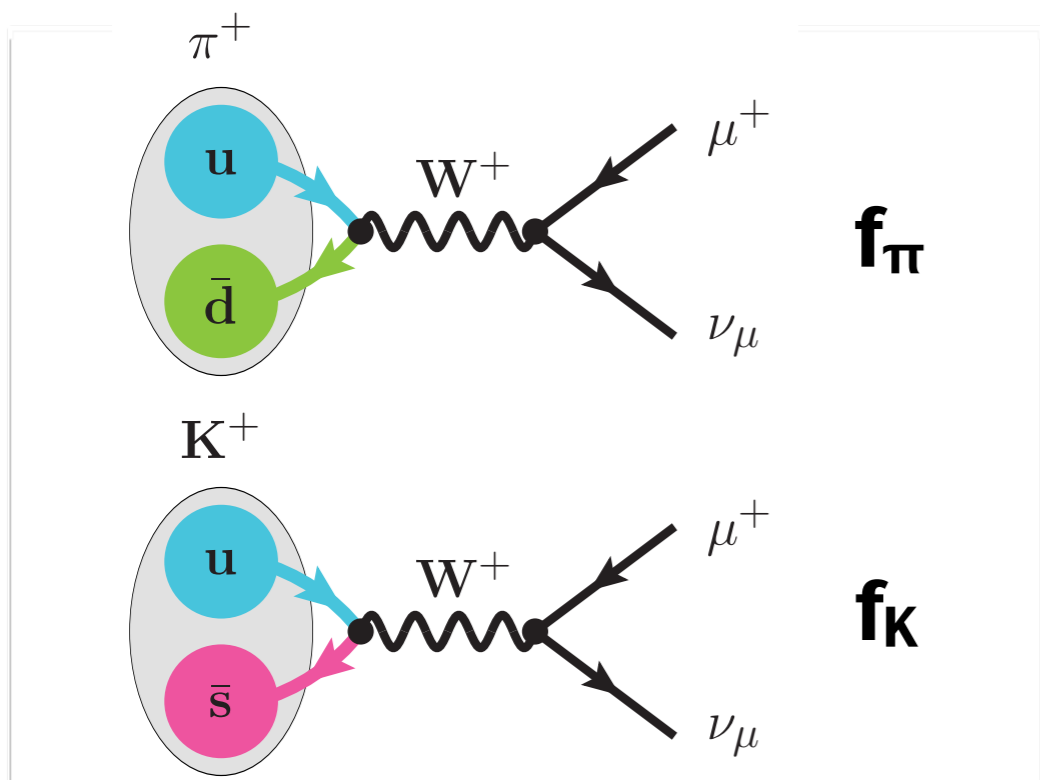
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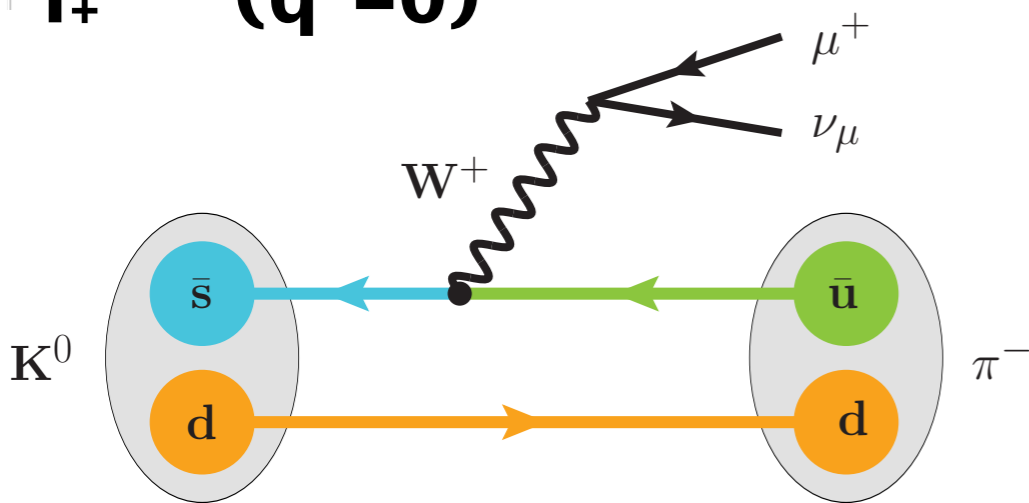
# Kaon overview

See Lattice '15 light-flavor review by [Jüttner](#) & ICHEP '14 meson-mixing review by [Carrasco](#)

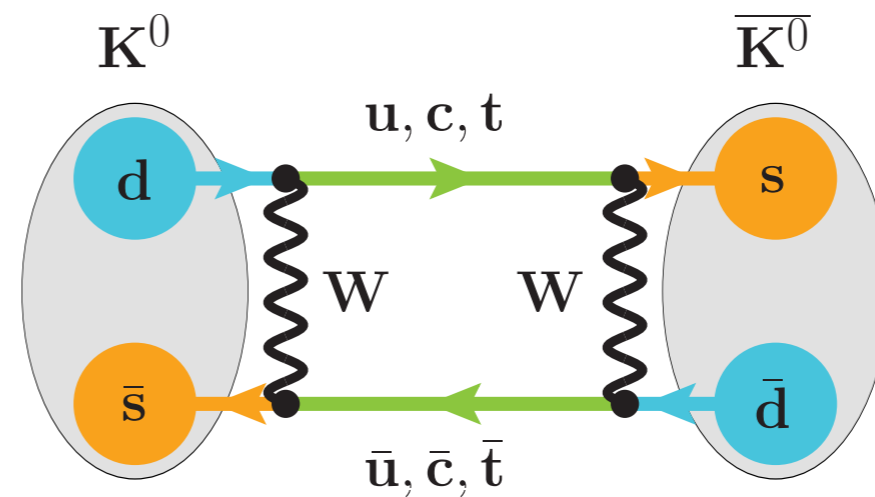


- ◆ For all simple quantities
  - ❖ **Physical light-quark masses**
  - ❖ **Nonperturbative or no renormalization**
  - ❖ Confirmation from independent results
- ◆ (Sub-)percent precision → **EM & isospin-breaking becoming relevant**

$\mathbf{f}_+^{K \rightarrow \pi}(q^2=0)$

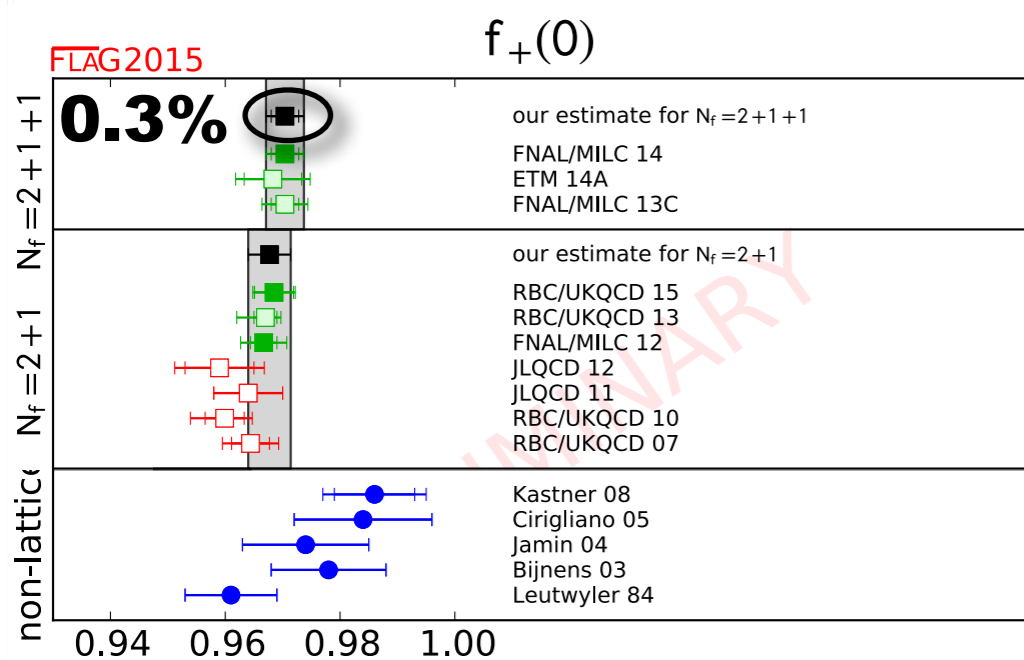
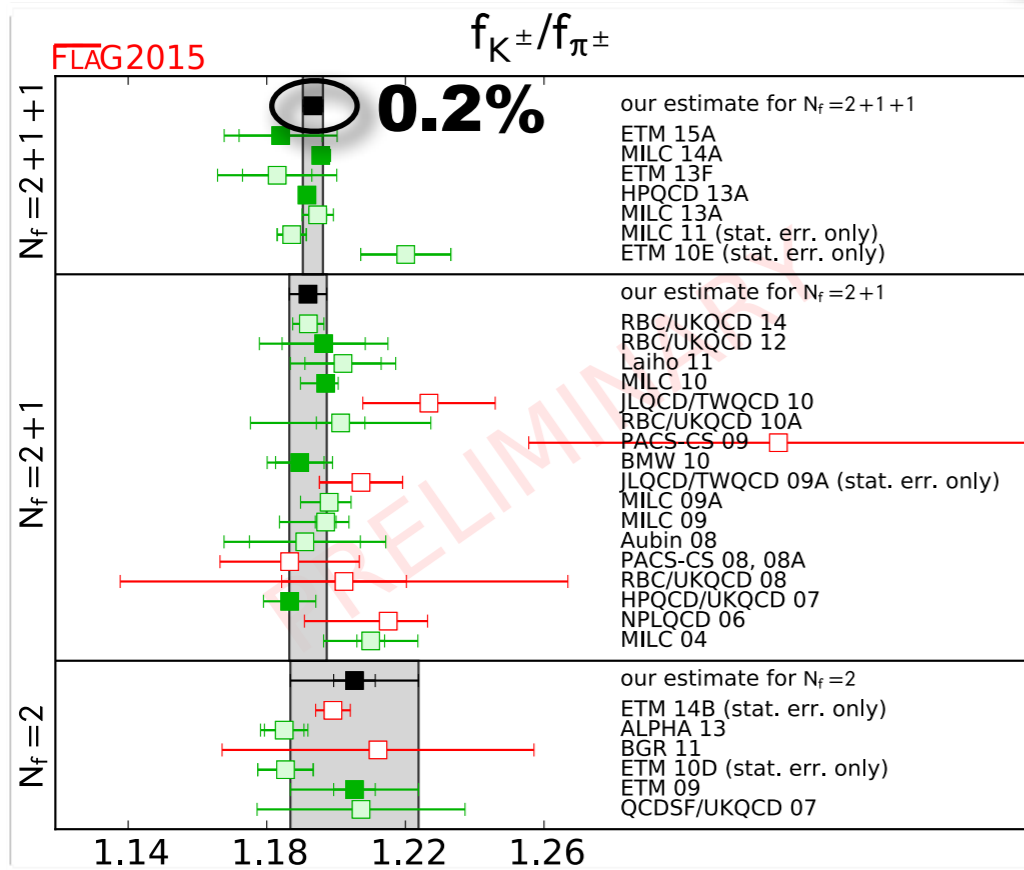


$\mathbf{B}_K$

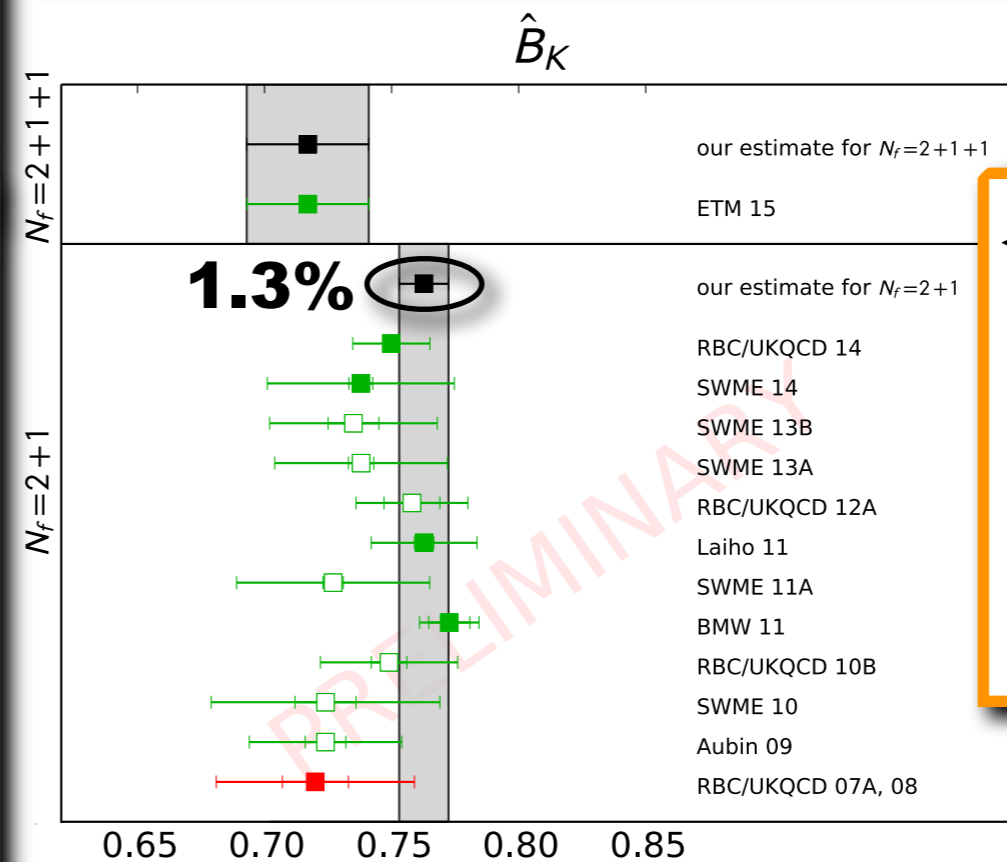


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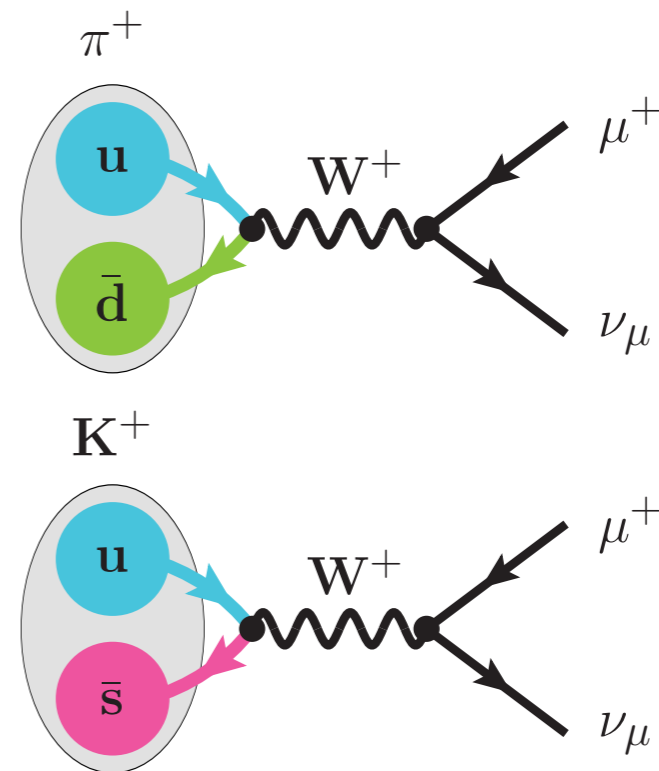
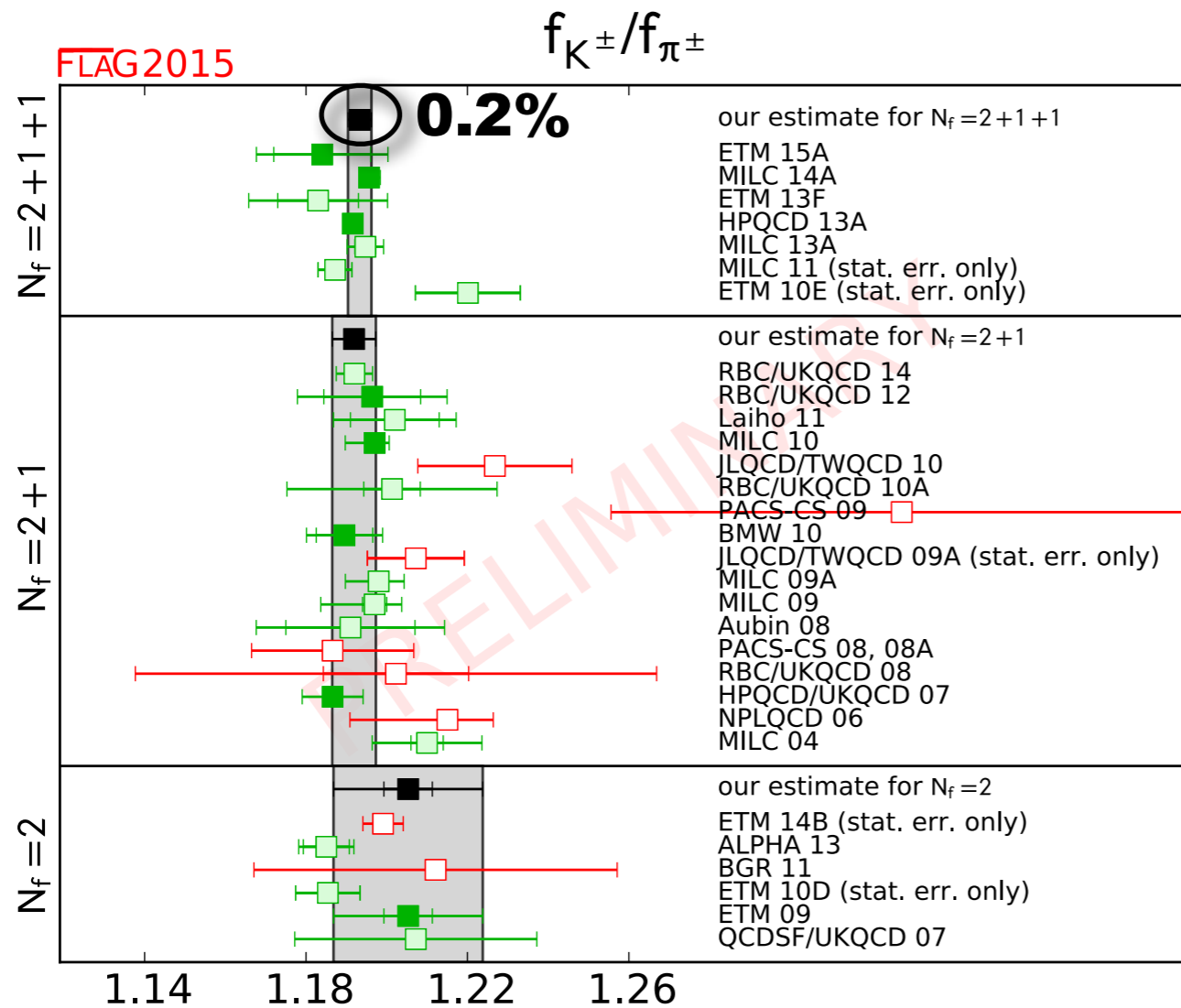
◆ Kaon-mixing matrix elements of all possible  $\Delta S=2$  BSM operators also available



# Pion & kaon decay constants

◆ **Decay-constant ratio  $f_K/f_\pi$  can be computed to sub-% precision with lattice QCD:**

- ❖ Statistical fluctuations correlated between numerator & denominator
- ❖  $f_K=f_\pi$  in SU(3) limit  $m_s=m_{ud}$ , so some systematics suppressed by  $(m_s-m_{ud})/\Lambda_{\text{QCD}}$



Combination with experimental decay rates enables  $|V_{ud}|/|V_{us}|$  **determination** [Marciano]

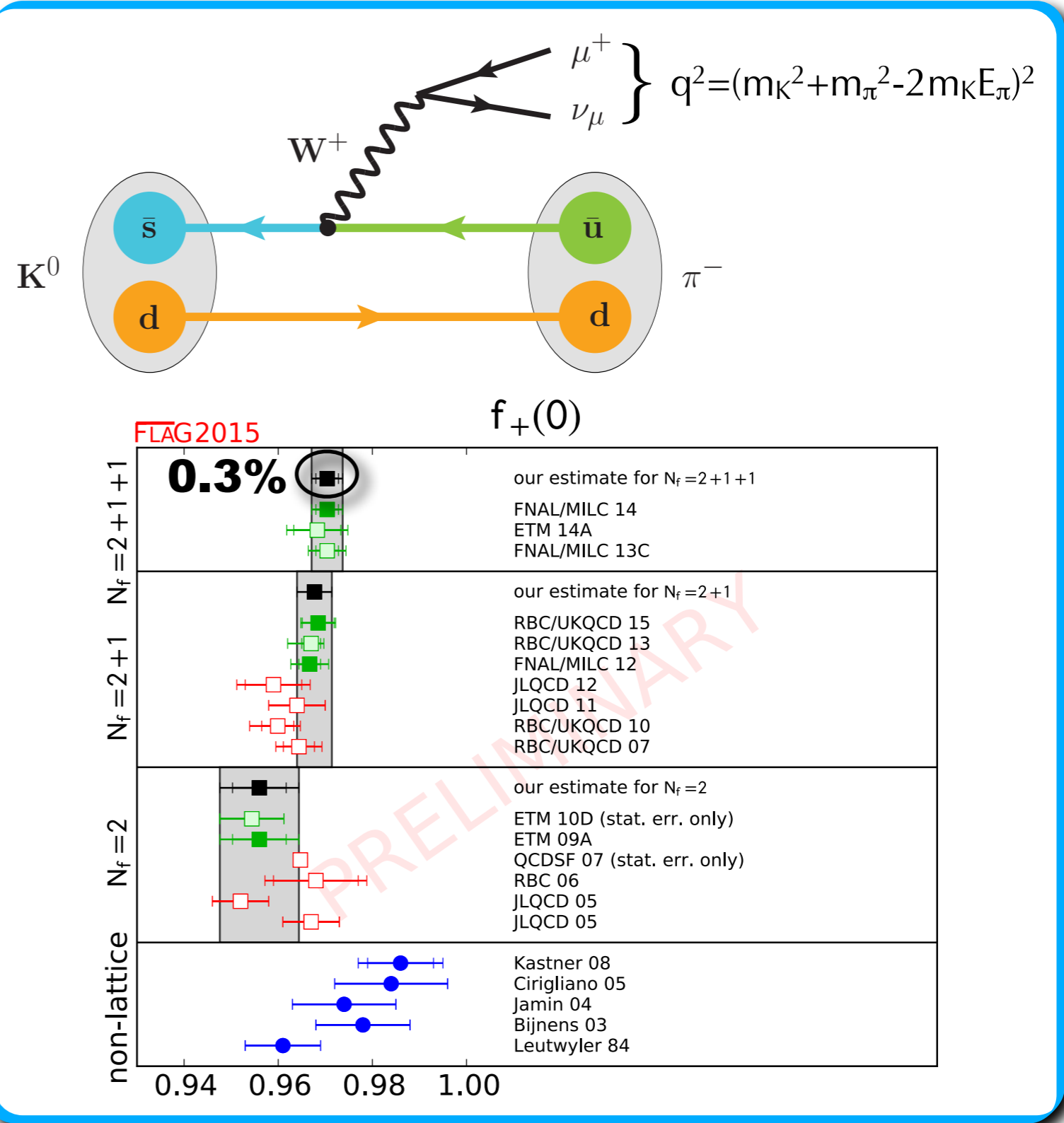
# $K \rightarrow \pi \ell \nu$ semileptonic form factor

◆ **Zero-recoil form factor  $f_+(q^2=0)$  highly constrained by  $SU(3)_f$  and chiral symmetries:**

- ❖  $f_+(0) = 1$  in  $SU(3)$  limit  $m_s = m_{ud}$
- ❖ Leading-order correction to unity is known function of  $\{m_\pi, m_K, f_\pi\}$  [**Leutwyler & Roos**]
- ❖  $f_2 = -0.023$  numerically small because second-order in  $(m_K^2 - m_\pi^2)$  [**Ademollo-Gatto**]

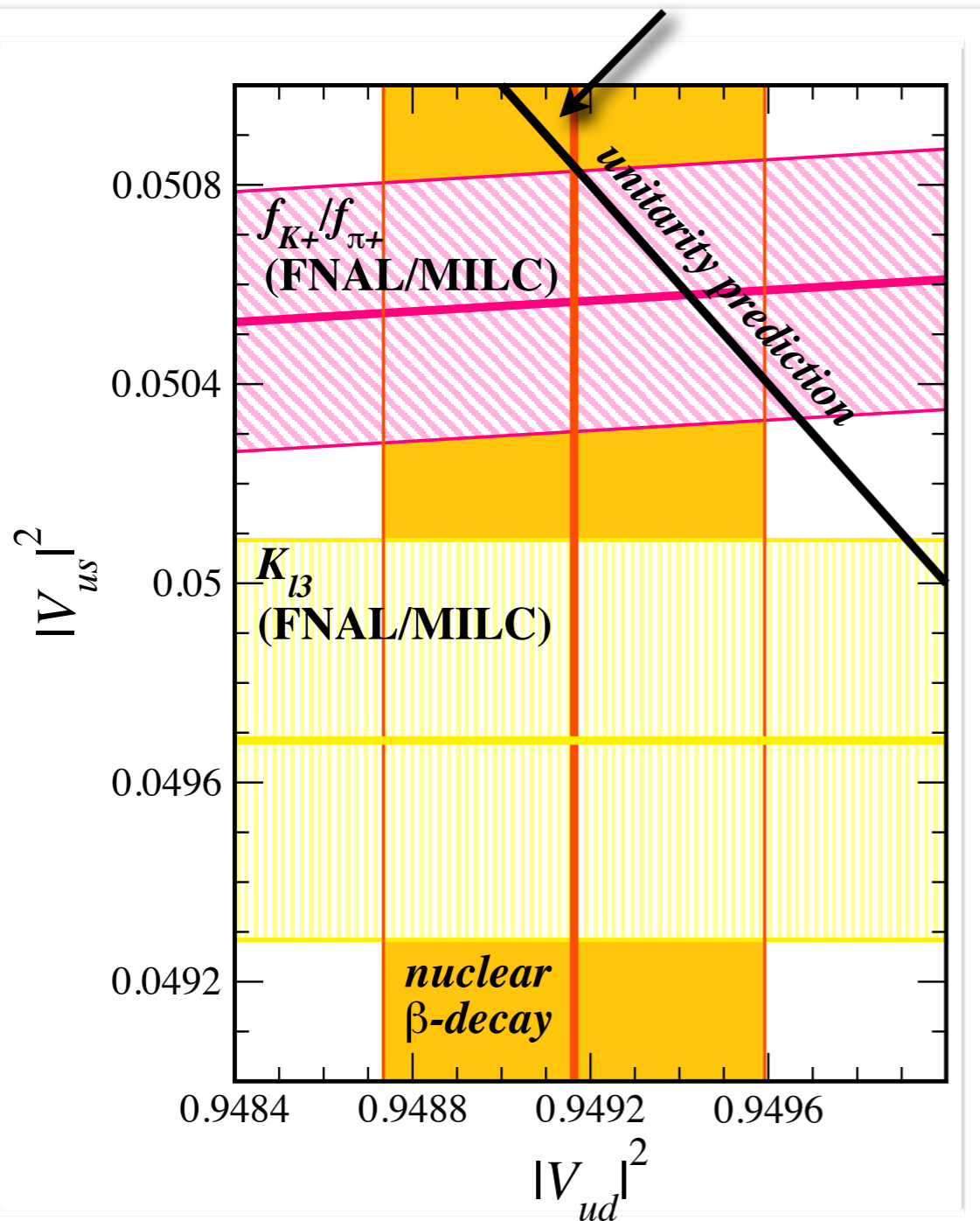
◆ **Lattice-QCD calculation does not require renormalization**

Combination with experimental rate enables  **$|V_{us}|$  determination**



# Implications for 1<sup>st</sup> row of CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$



- Using preliminary 2015  $K \rightarrow \pi l \nu$  form factor from **FLAG**, latest experimental  $f_+^{K\pi}(0) \times |V_{us}|$  from **Moulson [1411.5252]** and  $|V_{ud}|$  from **Hardy & Towner [1411.5987]**:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0021(29)_{V_{us}}(41)_{V_{ud}}$$

- Error on  $|V_{us}|^2$  now smaller than on  $|V_{ud}|^2$**   
 → Motivates revisiting error on  $|V_{ud}|$  from nuclear  $\beta$  decays
- 2.1  $\sigma$  tension between  $K \rightarrow \pi l \nu$ ,  $f_K/f_\pi$ , and unitarity prediction**

# Neutral kaon mixing parameter

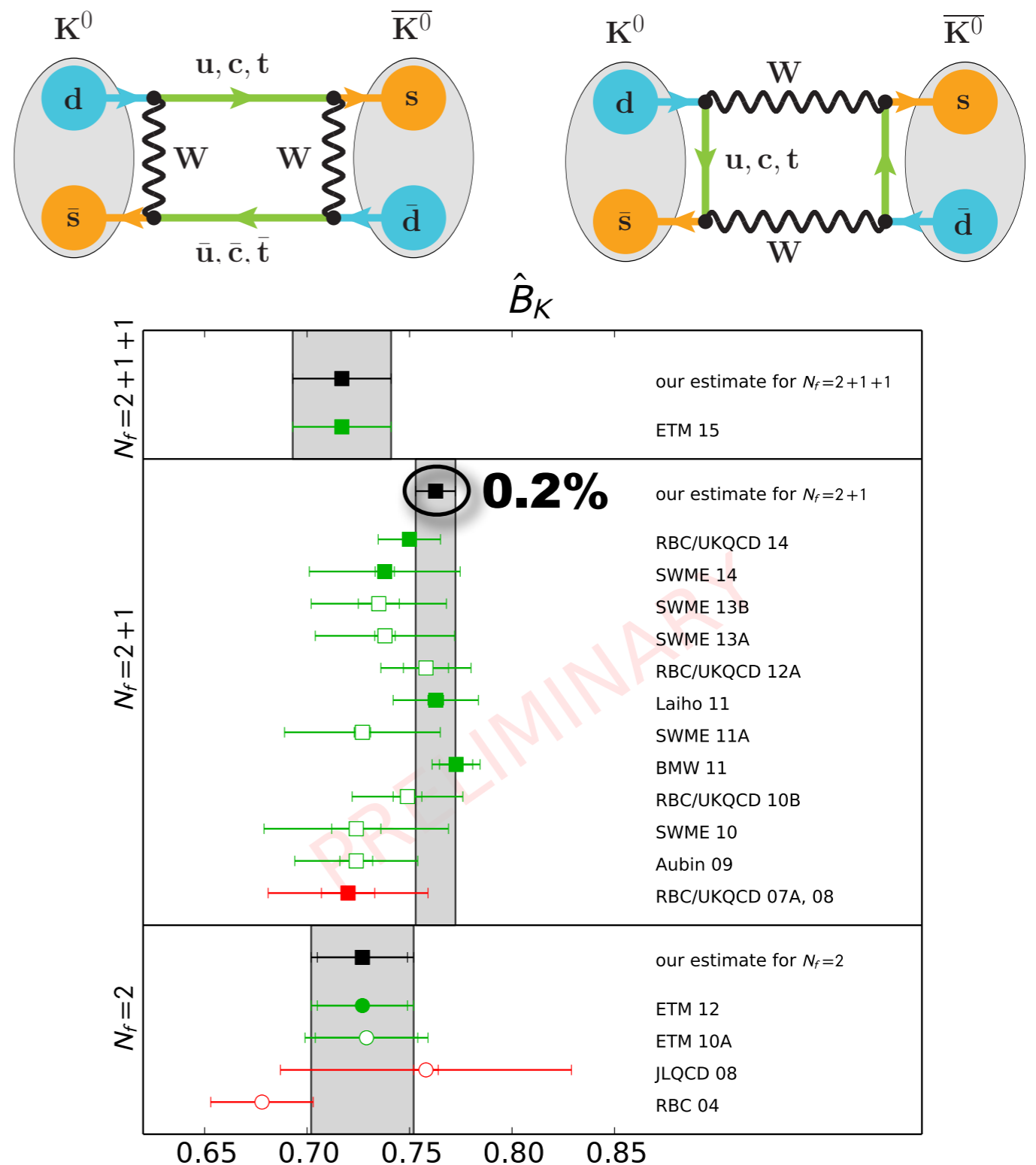
◆ Percent-level lattice-QCD calculation of  $B_K$  enabled by:

- ❖ Chiral fermion actions
- ❖ Nonperturbative renormalization

◆ Kaon mixing constrains scale of new physics with generic  $O(1)$  flavor couplings to  $\gtrsim 10,000$  TeV [Isidori, Nir, Perez (2010)]

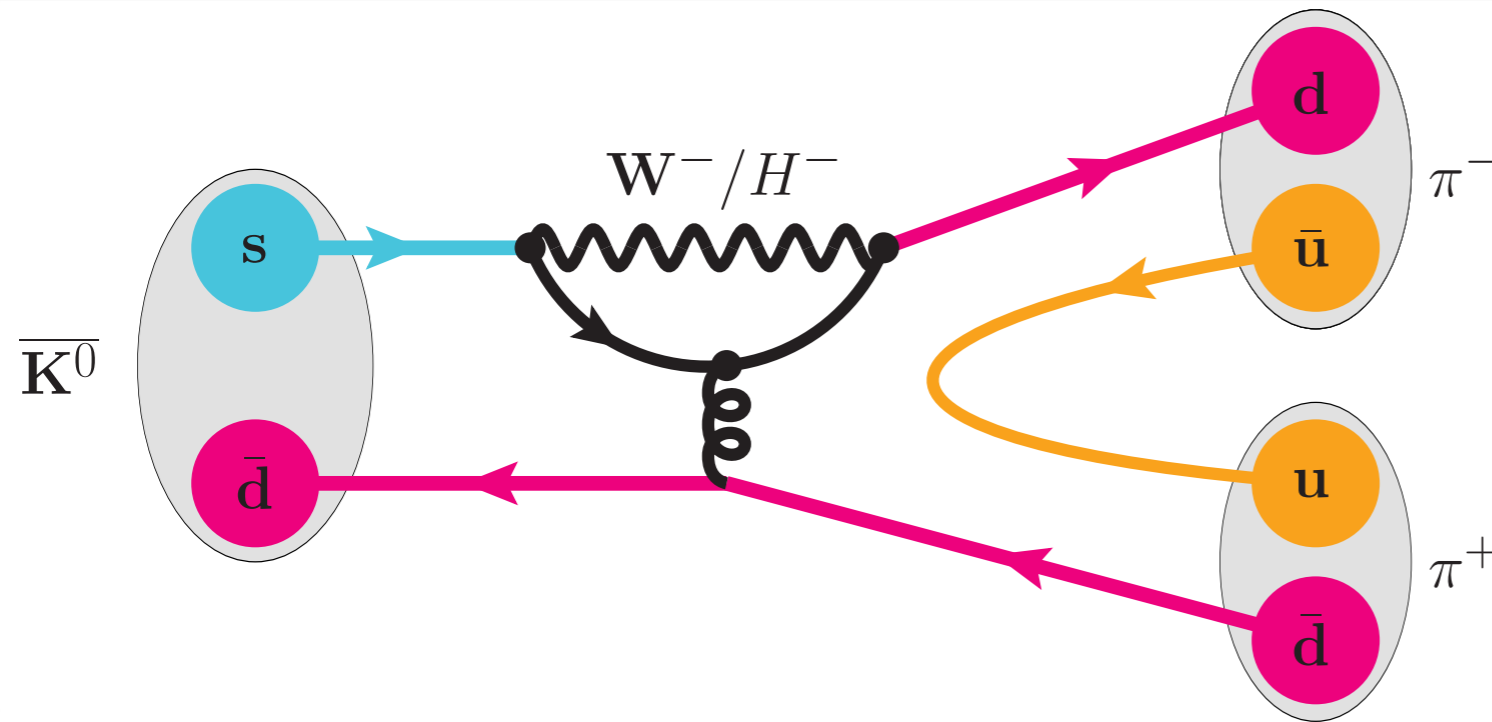
- ❖ Lattice-QCD results for matrix elements of all possible  $\Delta S=2$  BSM operators available for model building

Combination with measurement of indirect CP violation in kaon system ( $\epsilon_K$ ) **constrains CKM phase**



# $K \rightarrow \pi\pi$ decays

See [Kelly Lattice 2015 Review](#)



- Sensitive to new particles, interactions, sources of CP violation
- “ $\Delta I=1/2$  rule”: empirically observe enhancement  $\text{Re}A_0/\text{Re}A_2 \approx 22.5$

- ◆ Describe  $\Delta S=1$  FCNC transitions with **effective Hamiltonian**

$$\mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \left( V_{us}^* V_{ud} z_i(\mu) - V_{ts}^* V_{td} y_i(\mu) \right) Q_i(\mu)$$

- ❖ Short-distance effects factorized in Wilson coefficients  $\rightarrow$  continuum perturbation theory
- ❖ Long-distance effects factorized in **matrix elements**  $\langle \pi\pi | Q_i | K \rangle \rightarrow$  **lattice QCD**
- ◆ New physics above EW scale modifies Wilson coefficients, but hadronic matrix elements remain the same

# New! $K \rightarrow \pi\pi$ matrix elements

- \* **Lattice complication:** additional Lüscher formalism needed to relate amplitudes calculated in Euclidean box to physical observables in Minkowski space  
[Briceño review, PoS LATTICE2014 (2015) 008]
- ◆ **First complete three-flavor  $K \rightarrow \pi\pi$  amplitudes with controlled errors using domain-wall (chiral) fermions**
- ◆ Also first Wilson-fermion results from Ishizuka et al. [arXiv:1505.05289] with heavy, zero-momentum pions

*Emerging explanation of  $\Delta I=1/2$  rule:*  
Significant cancellation between dominant contributions to  $\text{Re}(A_2)$  which does not occur for  $\text{Re}(A_0)$

## $\Delta I=3/2$ amplitude ( $A_2$ )

[RBC/UKQCD, PRDD91 (2015) 7, 074502]

- ◆ Physical-mass pions, continuum limit, and approximately physical kinematics
- ◆  $\rightarrow$   **$\sim 10\%$  errors on  $\text{Re}(A_2)$ ,  $\text{Im}(A_2)$**
- ◆ Dominant uncertainty from perturbative truncation error in continuum Wilson coefficients

## $\Delta I=1/2$ amplitude ( $A_0$ )

[RBC/UKQCD, arXiv:1505.07863]

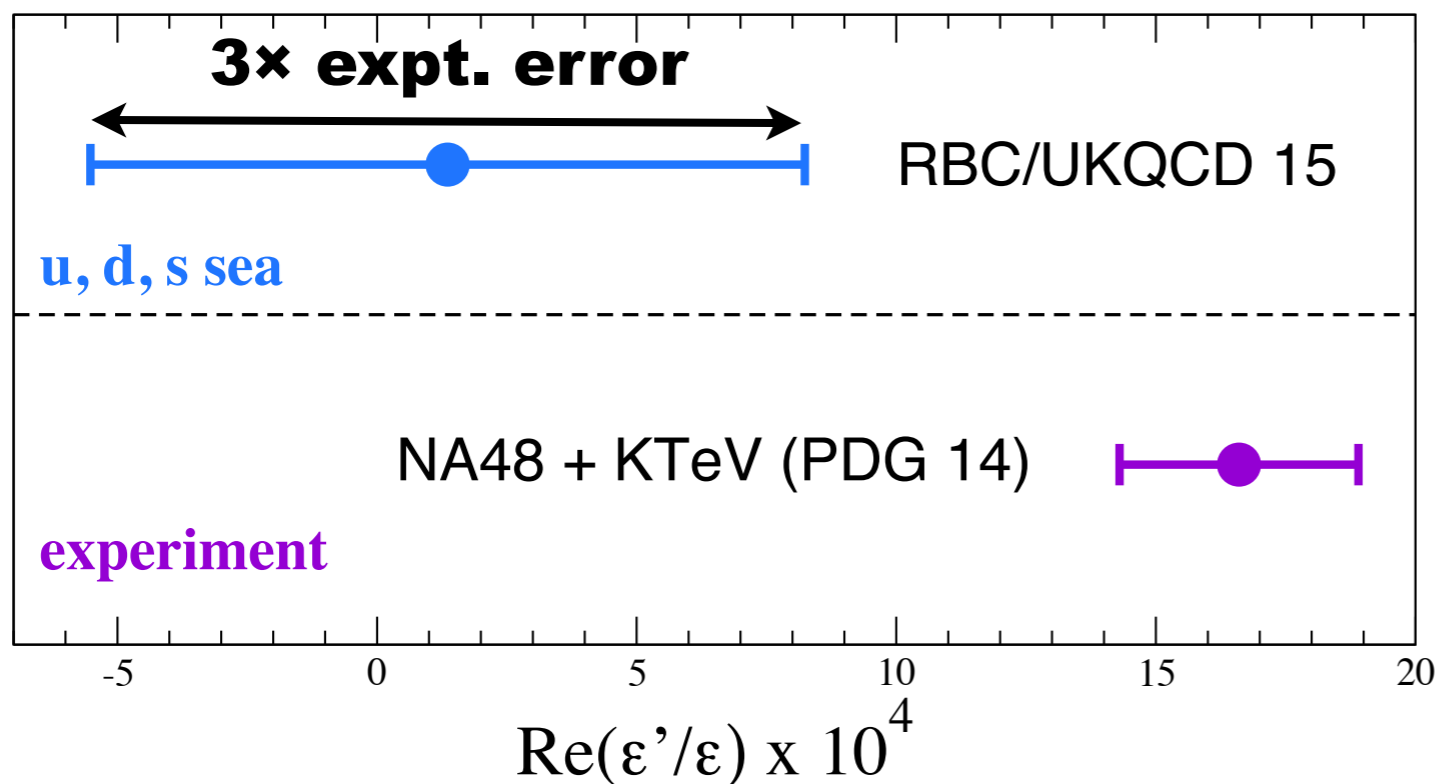
- ◆ 170 MeV pions with physical kinematics
- ◆ Small spatial volume  $m_\pi L \approx 3.2$  and single lattice spacing  $a \sim 0.14$  fm
- ◆  $\rightarrow$   **$\sim 35\%$  error on  $\text{Re}(A_0)$**
- ◆ Will be reduced with higher statistics, larger volumes, continuum limit, ...

# New! $\text{Re}(\epsilon'/\epsilon)$ in the Standard Model

[RBC/UKQCD, arXiv:1505.07863]

- ◆ Measures direct CP violation in  $K \rightarrow \pi\pi$  decays

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) \approx \frac{1}{6} \left[ \frac{\Gamma(K_L \rightarrow \pi^+\pi^-)/\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_L \rightarrow \pi^0\pi^0)/\Gamma(K_S \rightarrow \pi^0\pi^0)} - 1 \right]$$



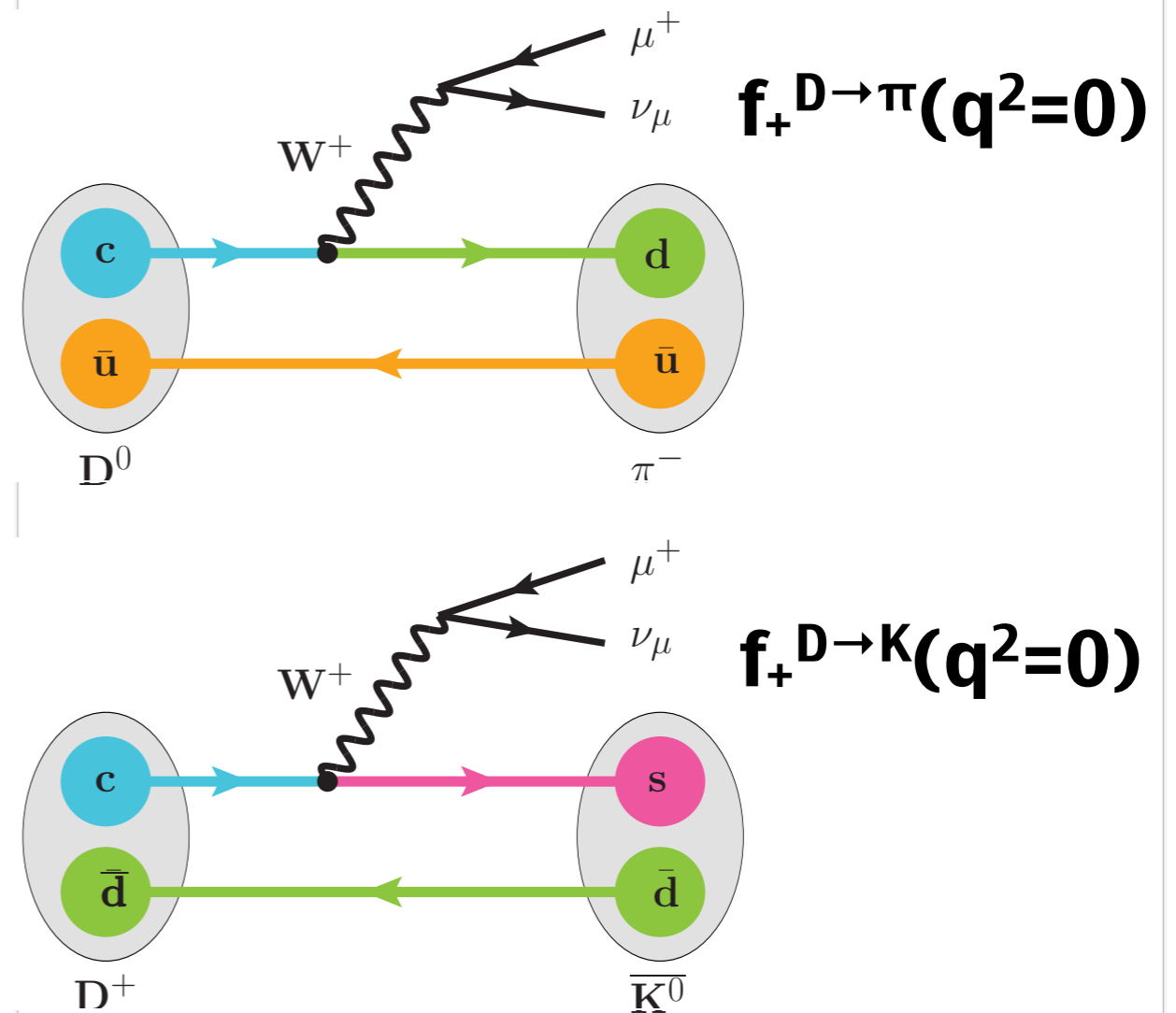
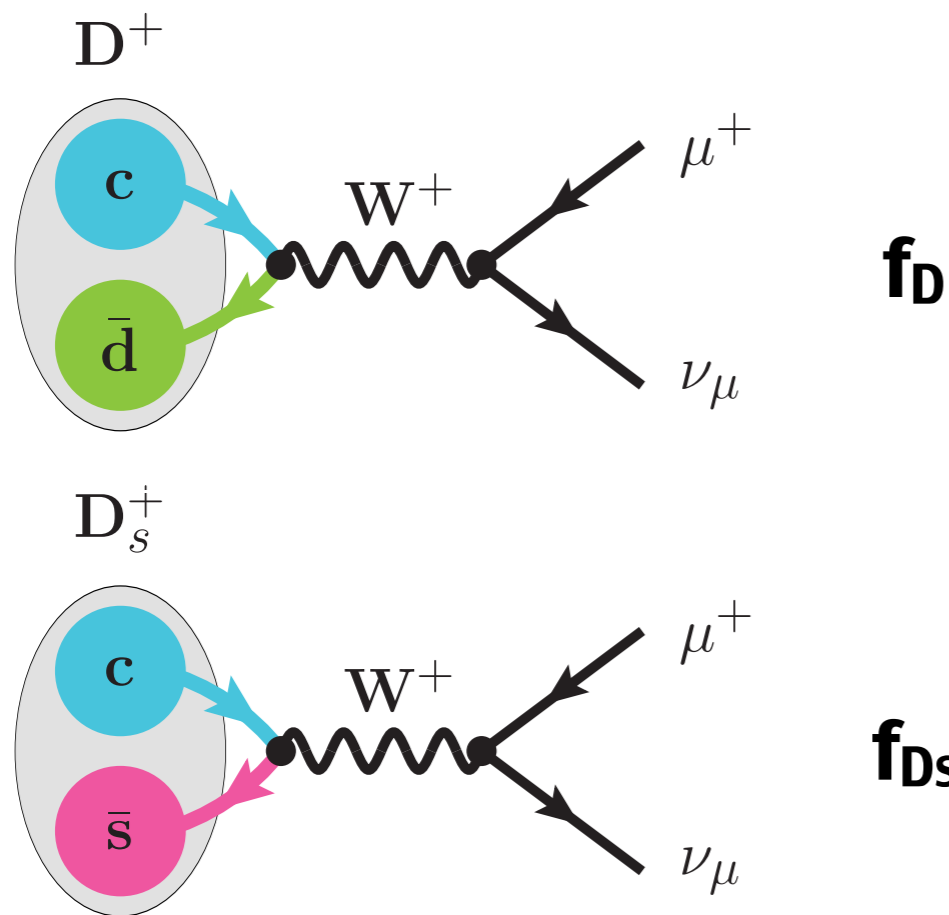
- ◆ Lattice  $\text{Im}(A_0, A_2)$  + experimental  $\text{Re}(A_0, A_2)$   
→ first  $\text{Re}(\epsilon'/\epsilon)$  in Standard Model with controlled errors

Lattice-QCD calculation of  $\text{Re}(\epsilon'/\epsilon)$  with ~10% uncertainty achievable in the foreseeable future!

# D-meson summary

See [Pena Lattice '15](#) & [Bouchard Lattice '14](#) heavy-flavor reviews

- ◆ Small errors due to
  - ❖ **Physical light-quark masses**
  - ❖ **Improved charm-quark actions**  
+ fine lattice spacings
  - ❖ **No renormalization** (PCAC)





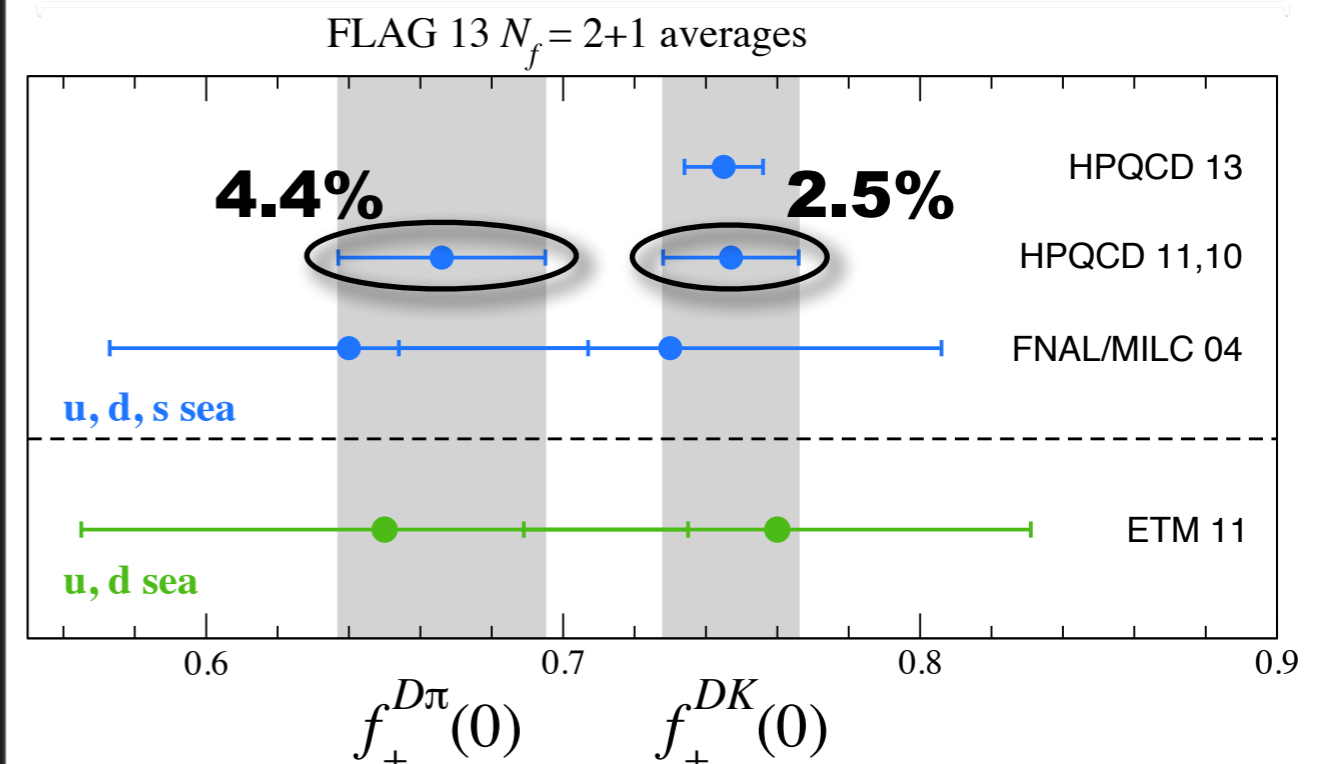
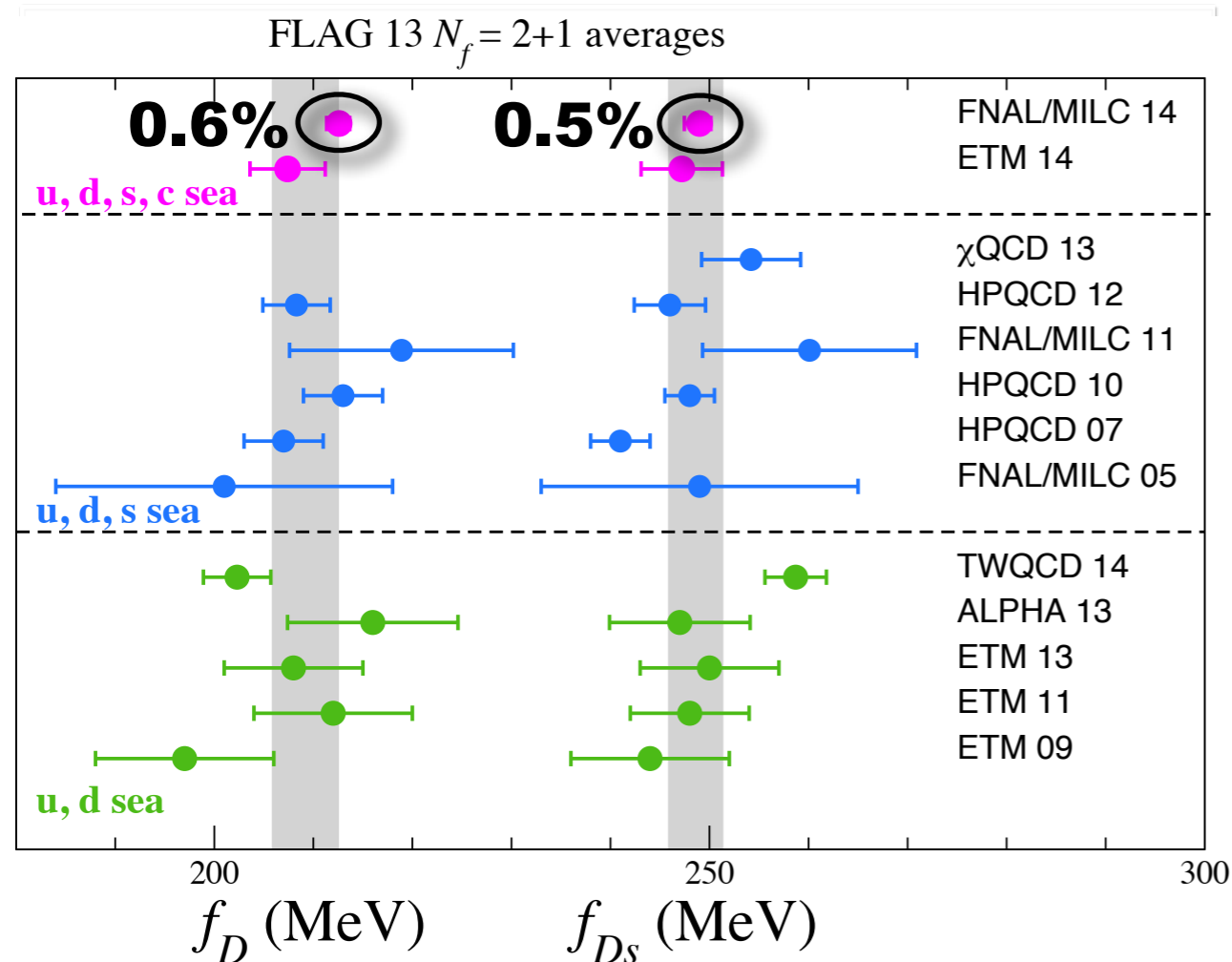
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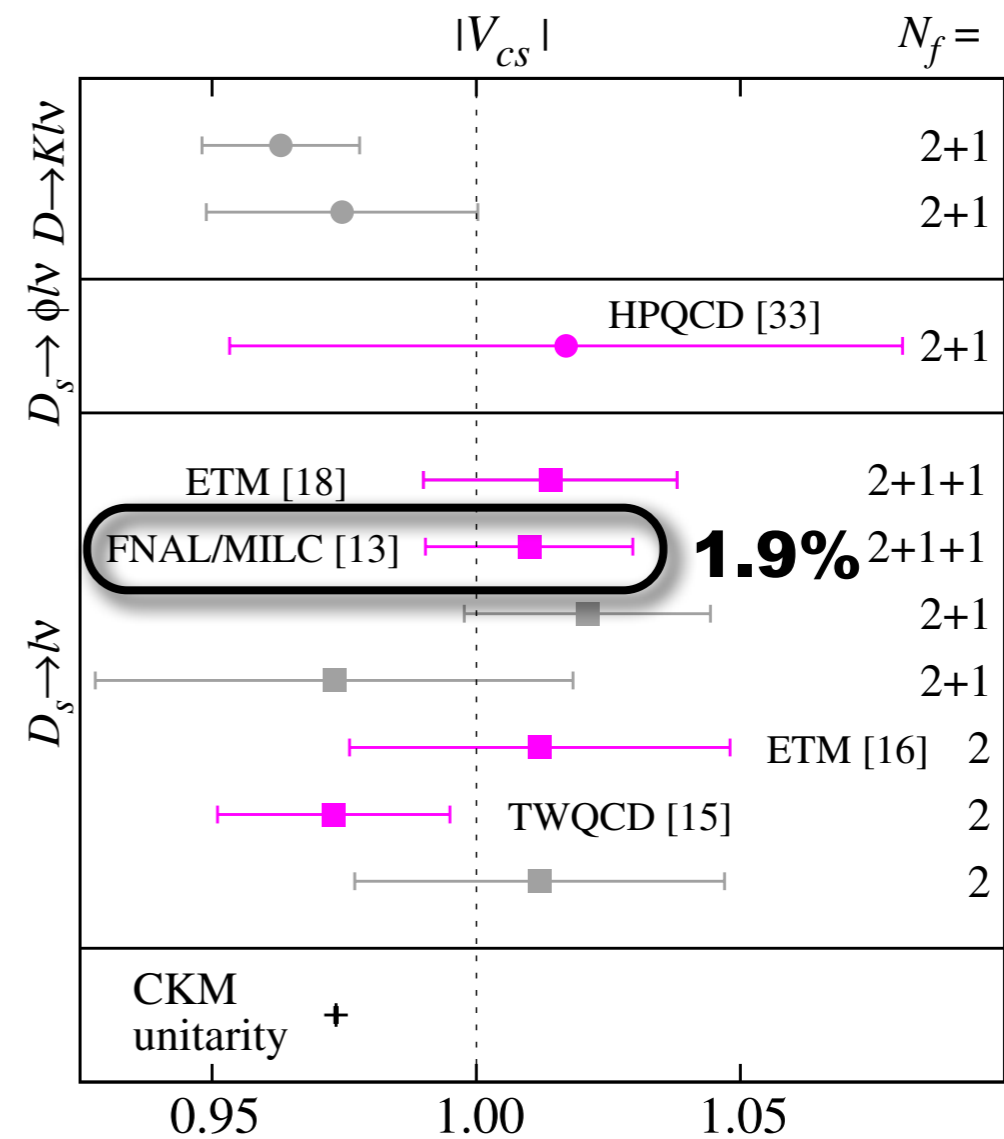
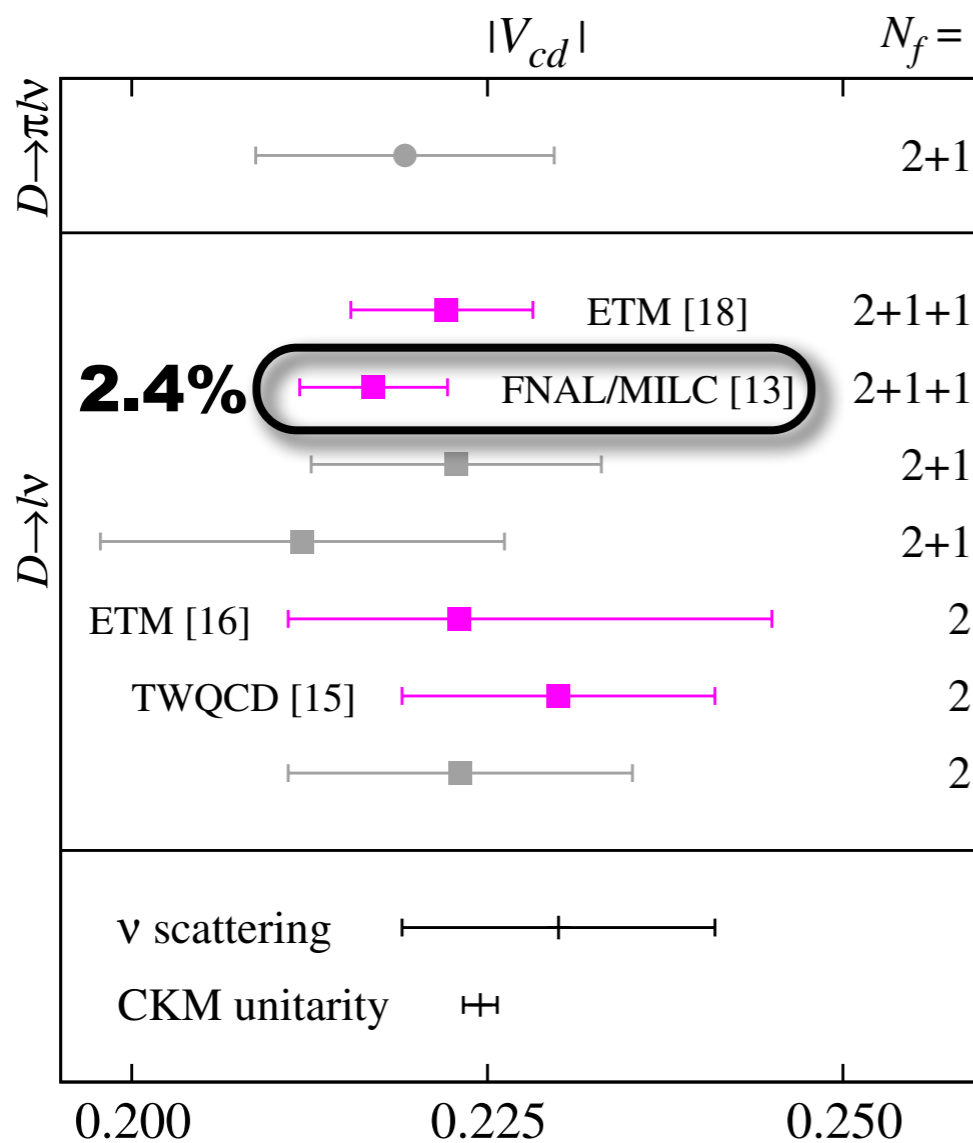
## ◆ *Recent highlights:*

- ◆ First 3-flavor  $D_s \rightarrow \varphi l \nu$  form factors [[HPQCD, PRD90 \(2014\) 7, 074506](#)]
- ◆ First 4-flavor D-mixing matrix elements (all five  $\Delta C=2$  SM & BSM operators, only short-distance contributions) [[ETM, 1505.06639](#)]



# Implications for 2<sup>nd</sup> row of CKM matrix

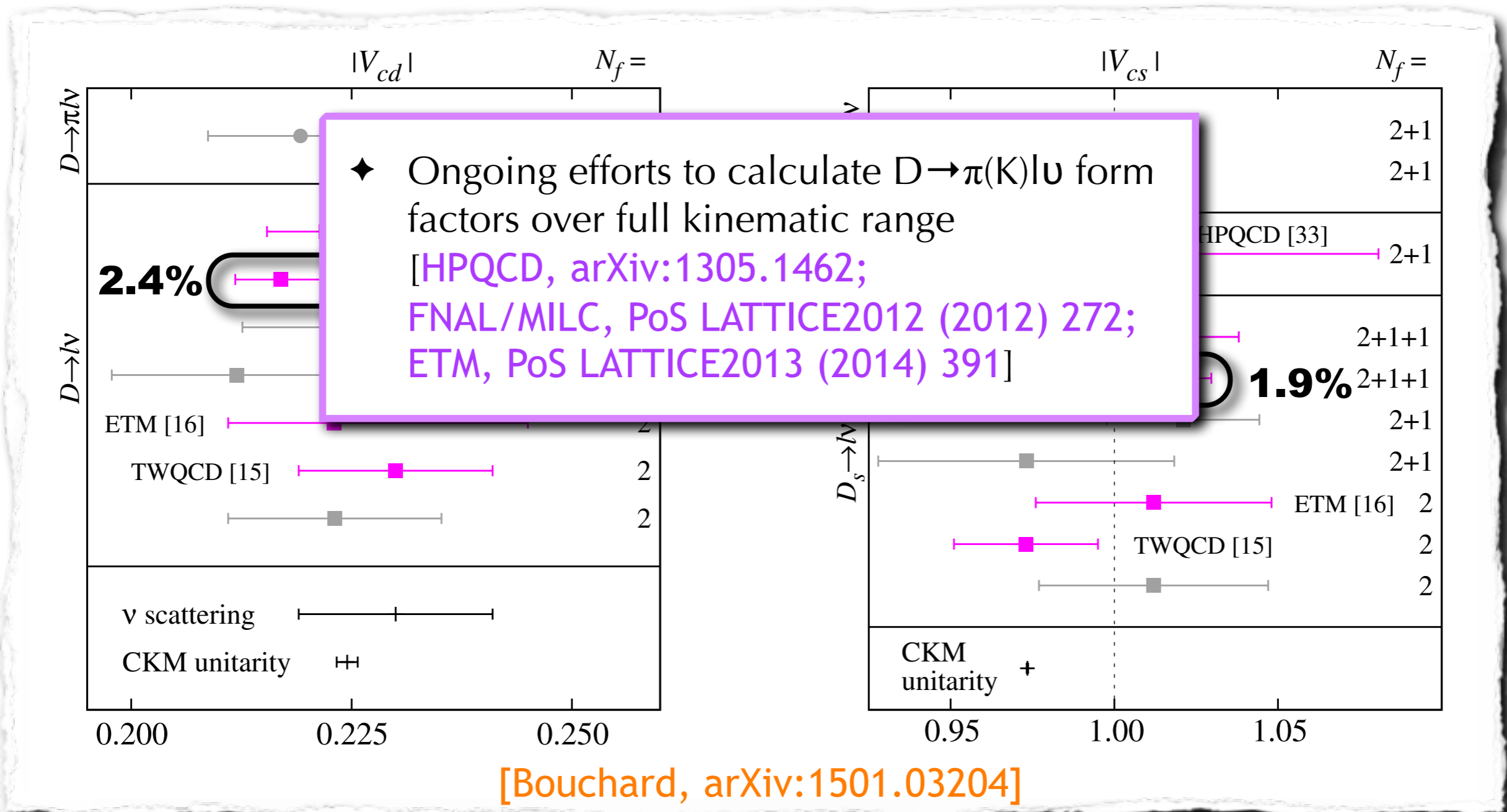
- ◆  $|V_{cd}|$  &  $|V_{cs}|$  from leptonic  $D_{(s)}$  decays limited by experimental errors
- ◆ Slight tension between  $|V_{cs}|$  from leptonic & semileptonic decay



[Bouchard, arXiv:1501.03204]

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# New! Neutral B-mixing matrix elements

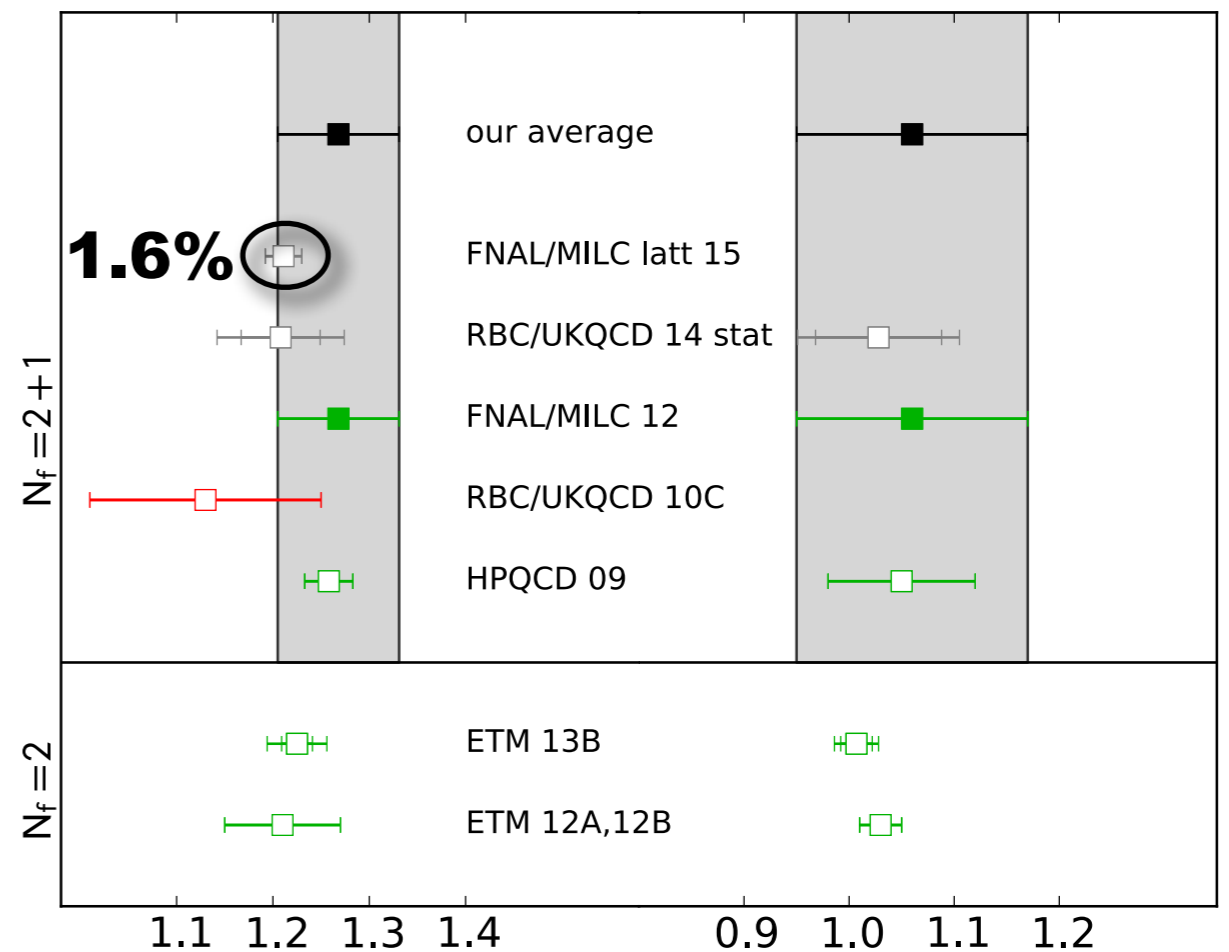
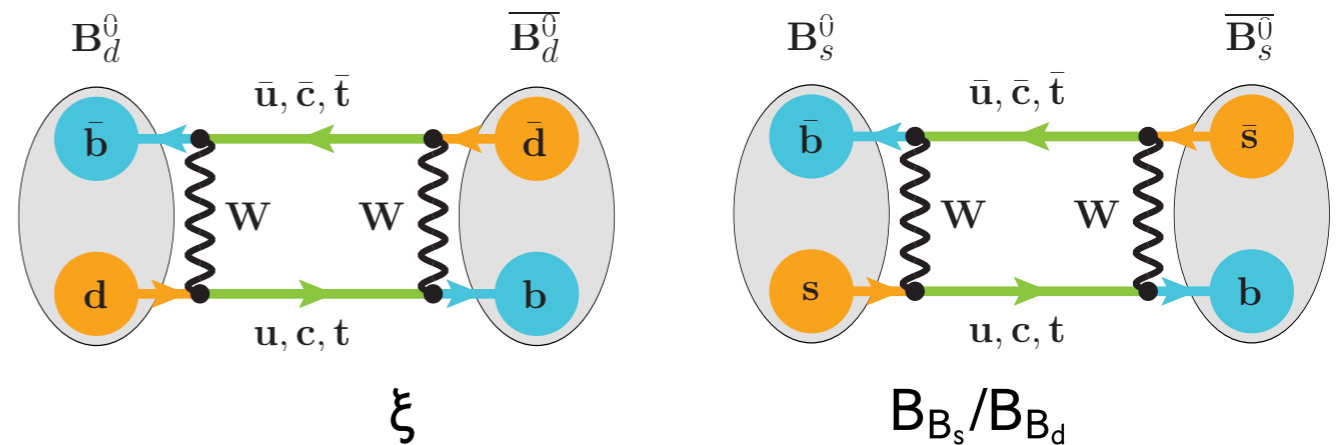
[Bouchard, Freeland, et. al [FNAL/MILC], presented at Lattice 2015]

Combination with measurement of  $B_d/B_s$  oscillation frequencies constrains apex of CKM unitarity triangle

- ◆ **Matrix-element ratio**  
 $\xi \equiv f_{B_d} \sqrt{B_{B_d}} / f_{B_s} \sqrt{B_{B_s}}$  can be computed precisely because:
  - ❖ Statistical fluctuations correlated
  - ❖  $B_d$  &  $B_s$  matrix elements equal in SU(3) limit, so some systematics suppressed by  $(m_s - m_{ud})/\Lambda_{\text{QCD}}$
- ◆ Preliminary FNAL/MILC calculation with increased statistics, lighter quark masses, & finer lattice spacings

$$\xi = 1.211(19) \rightarrow$$

$$|V_{td}|/|V_{ts}| = 0.2069(6)_{\text{exp}}(32)_{\text{LQCD}}$$



[figure from Pena, Lattice 2015]

# New! Neutral B-mixing matrix elements

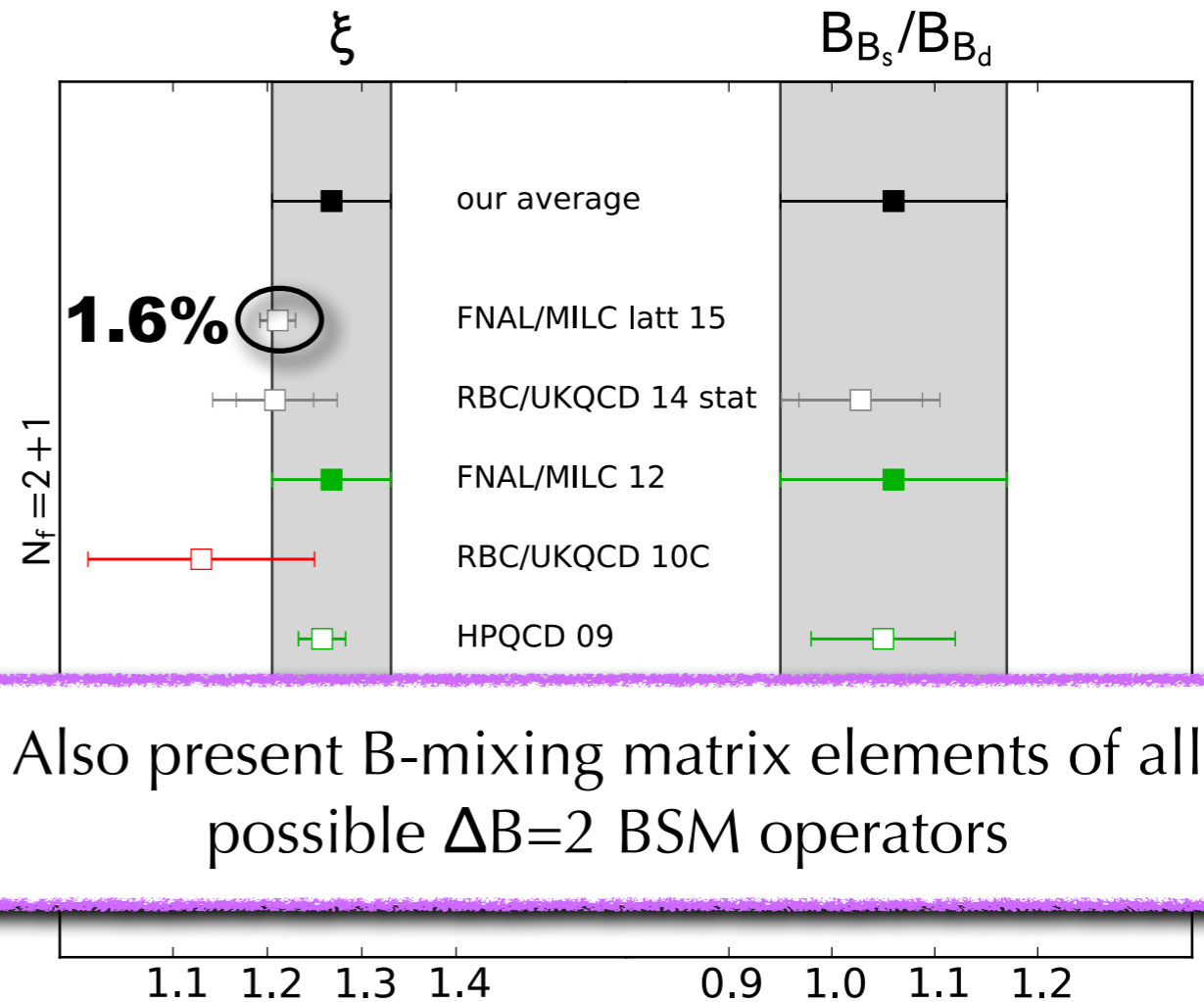
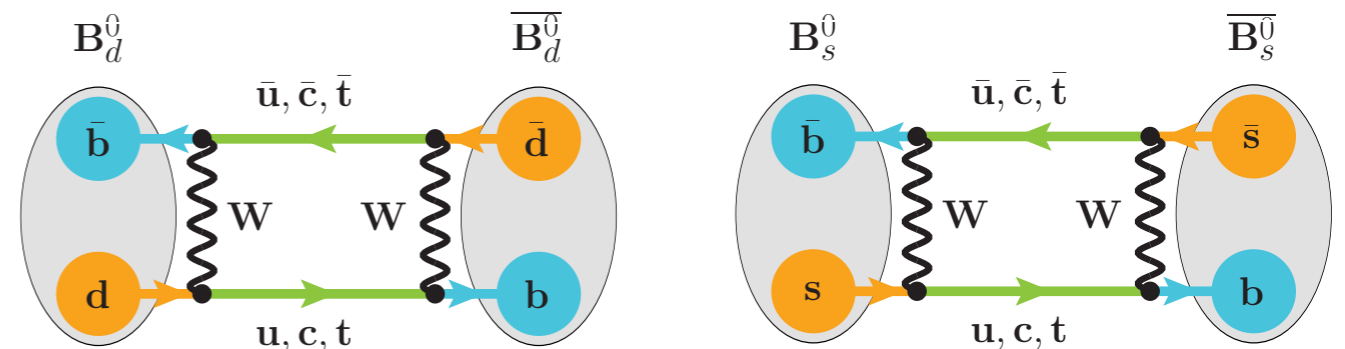
[Bouchard, Freeland, et. al [FNAL/MILC], presented at Lattice 2015]

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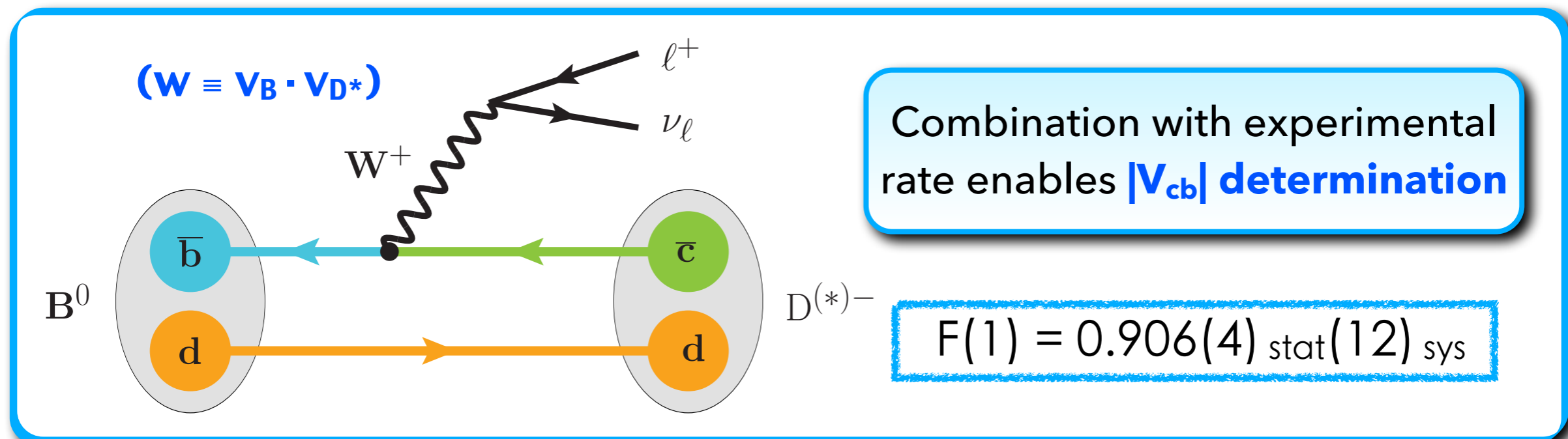
Also present B-mixing matrix elements of all possible  $\Delta B=2$  BSM operators

[figure from Pena, Lattice 2015]

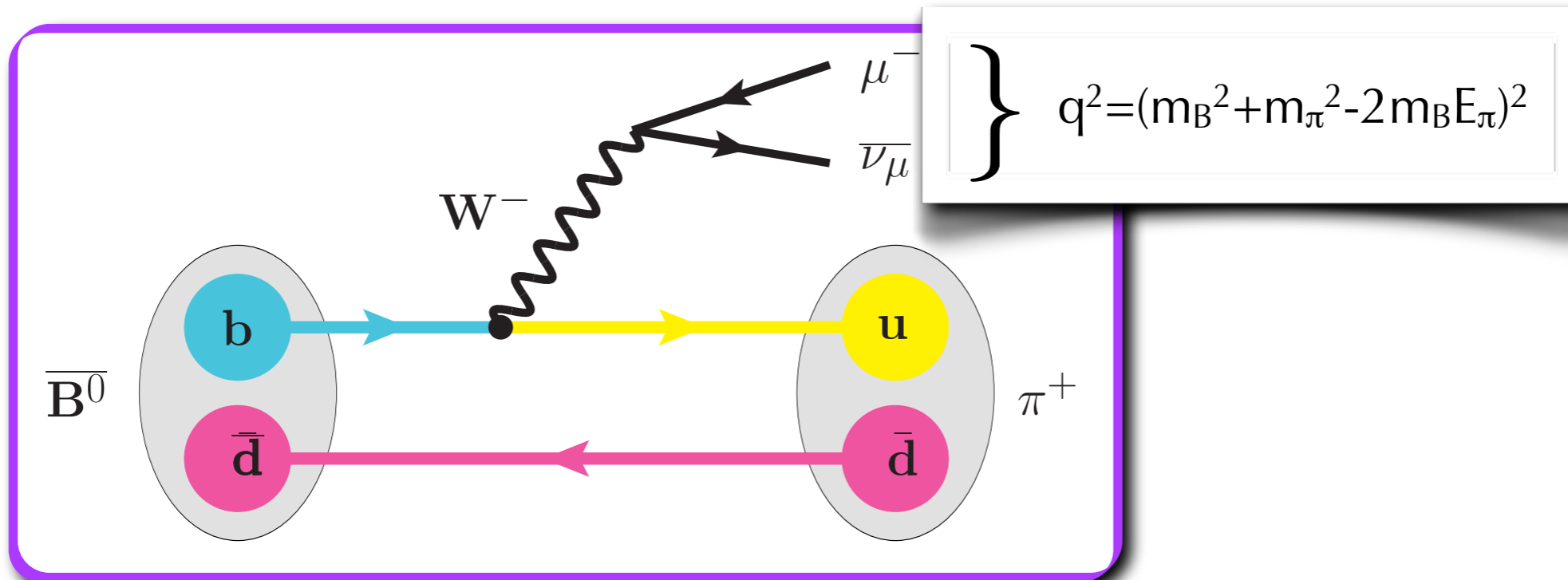
# New! $B \rightarrow D^* \ell \nu$ form factor @ zero recoil

[Bailey et al. [FNAL/MILC], PRD89 (2014) 11, 114504]

- ◆ Only need one normalization point from lattice QCD → **choose zero recoil ( $w=1$ ) where it can be computed most precisely**
- ❖  $F(1) \rightarrow 1$  in the static limit ( $m_b = m_c \rightarrow \infty$ ) [Isgur & Wise], and **Luke's theorem** ensures that the leading heavy-quark corrections to  $F(1)$  are of  $\mathcal{O}(1/m_b^2, 1/m_c^2)$
- ❖ Can compute form factor using double ratio of lattice three-point correlation functions in which statistical and systematic errors largely cancel
- ◆ New FNAL/MILC calculation with increased statistics, lighter quark masses, & finer lattice spacings → **1.4% precision on  $F(1)$**



# $|V_{ub}|$ from $B \rightarrow \pi \ell \nu$ decay



- ◆  $B \rightarrow \pi \ell \nu$  semileptonic decay enables determination of  $|V_{ub}|$  via:

$$\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} [(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2]^{3/2} |V_{ub}|^2 |f_+(q^2)|^2$$

- ◆ Few percent determination of exclusive  $|V_{ub}|$  challenging because:

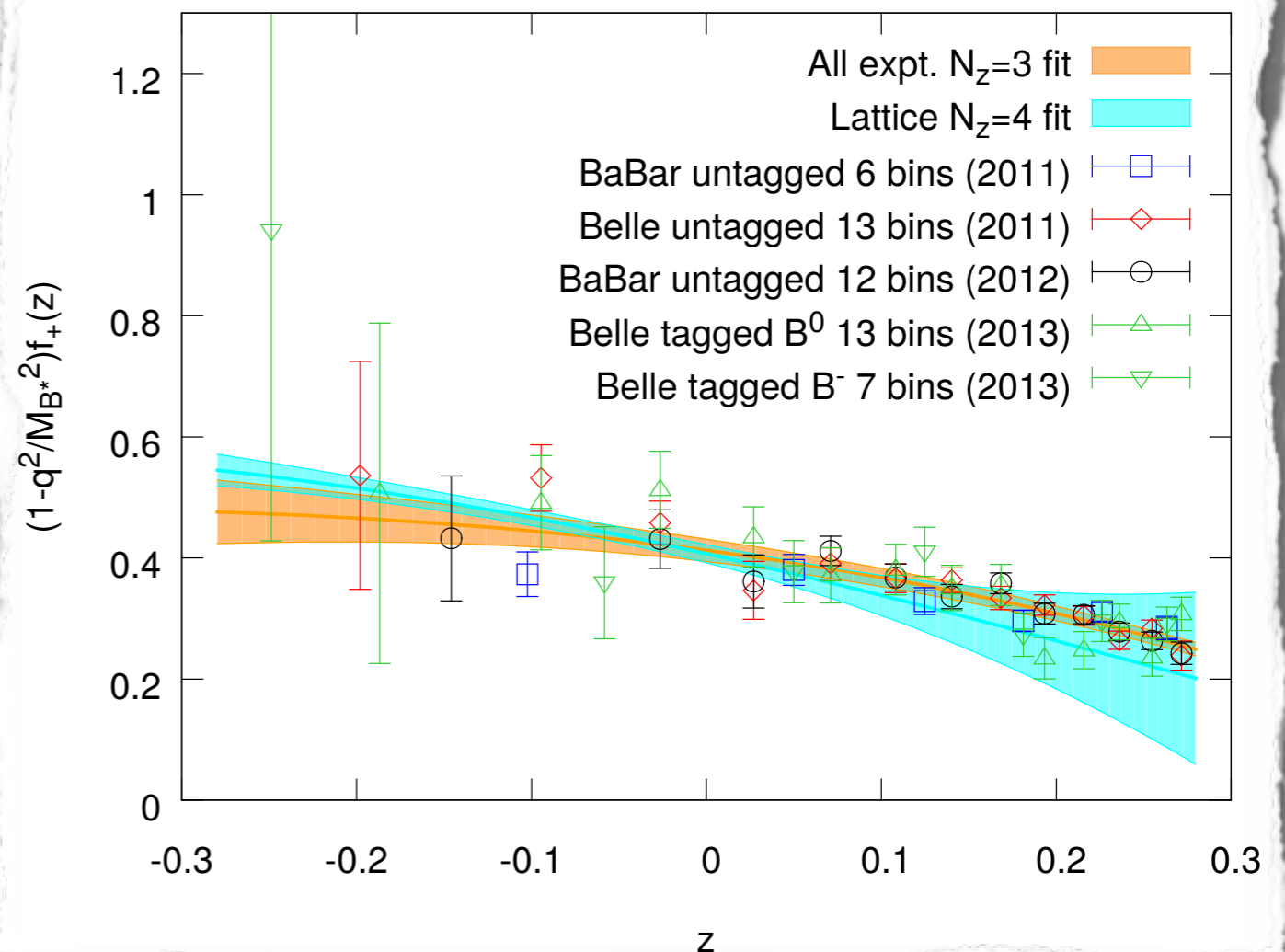
- ❖ Lattice statistical & discretization errors grow with increasing pion momentum  $\rightarrow$  determination of **hadronic form factor  $f_+(q^2)$**  best at large momentum-transfer ( $q^2$ )
- ❖ Experimental branching fraction most precise at low  $q^2$

# Lattice + experiment fit for $|V_{ub}|$

- ◆ Fit lattice-QCD and experimental data together to model-independent parameterization ("z-expansion") based on analyticity, unitarity, and crossing symmetry [Boyd, Grinstein, Lebed, PRL74, 4603 (1995); Bourrely, Caprini, Lellouch, PRD79 (2009) 013008]
- ◆ Relative normalization ( $|V_{ub}|$ ) free parameter determined in fit
- ◆ Combined fit to Belle, BaBar, and 2015 FNAL/MILC lattice data yields  $|V_{ub}|$  with a 4.3% uncertainty:

$$|V_{ub}| = 3.72(16) \times 10^{-3}$$

[FNAL/MILC], arXiv:1503.07839]



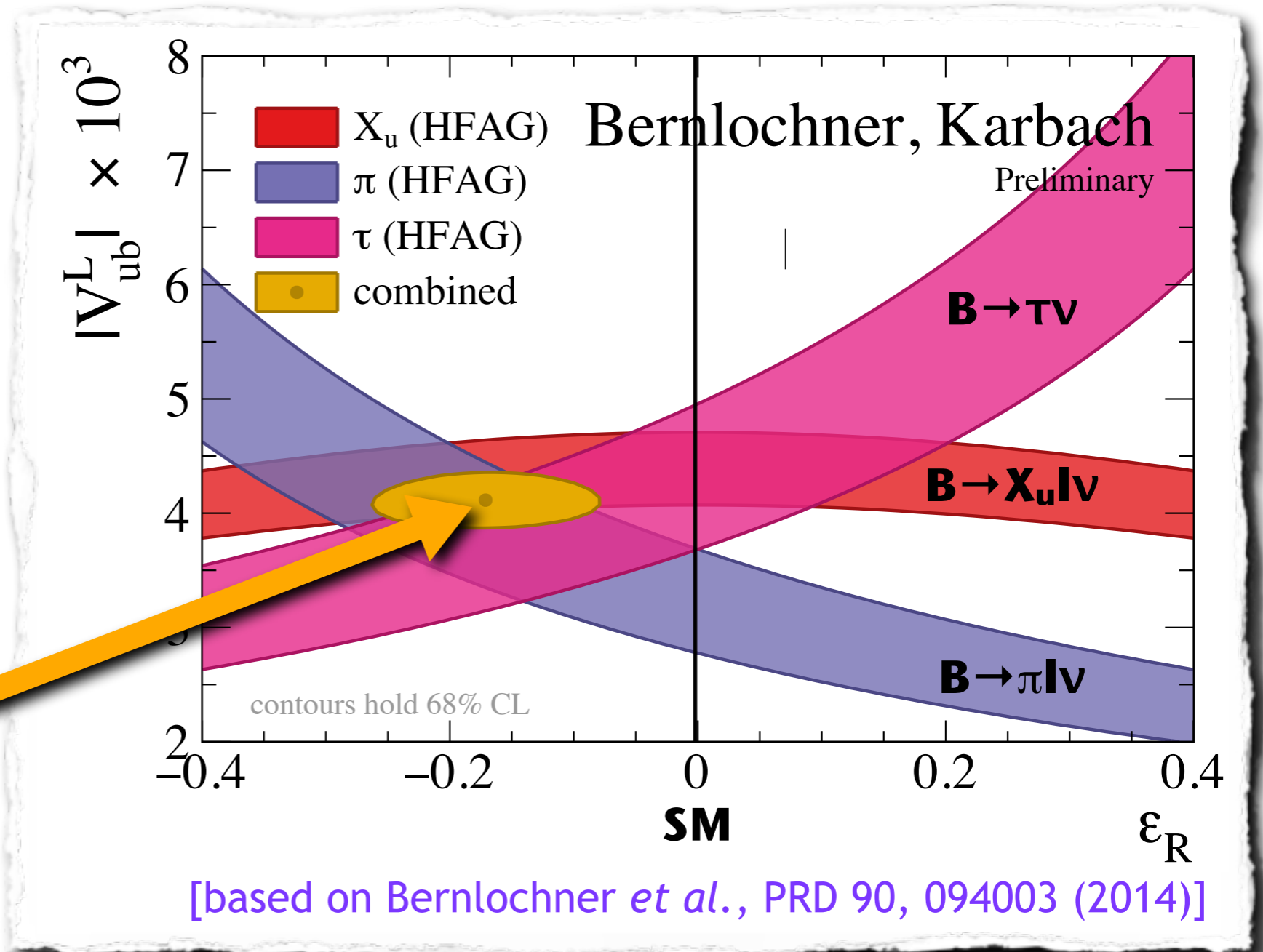
Use of all experimental & theoretical information minimizes error in  $|V_{ub}|$



# A sign of new physics?

- ◆ Can ease  $|V_{ub}|$  tension by allowing small right-handed contribution to Standard-Model weak current [Crivellin, PRD81 (2010) 031301]

~15% admixture of RH current

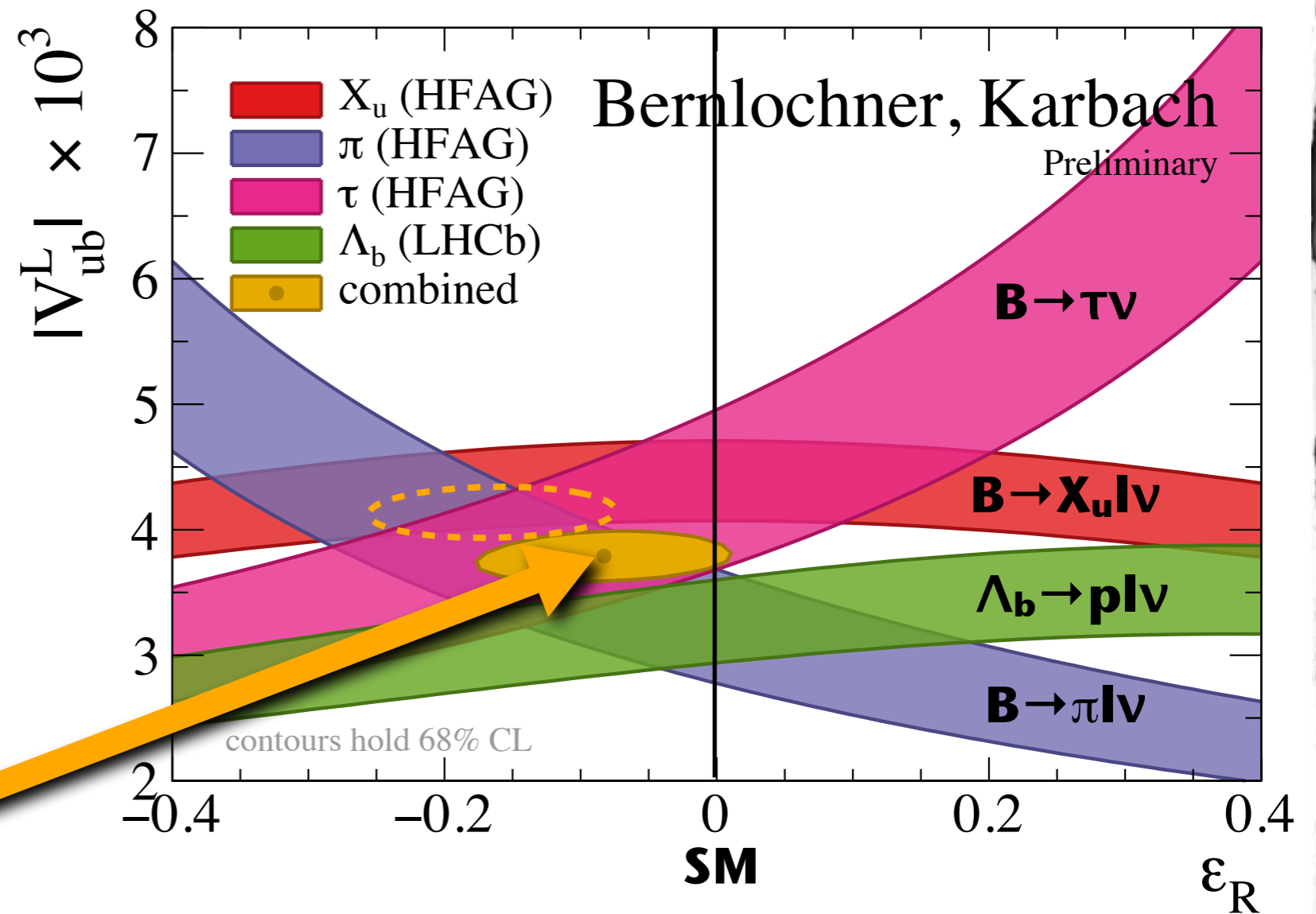


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◆ RH currents disfavored by  $\Lambda_b$  decays (taking  $|V_{cb}|$  from  $B \rightarrow D^* l \nu$  + HFAG to obtain  $|V_{ub}|$ )

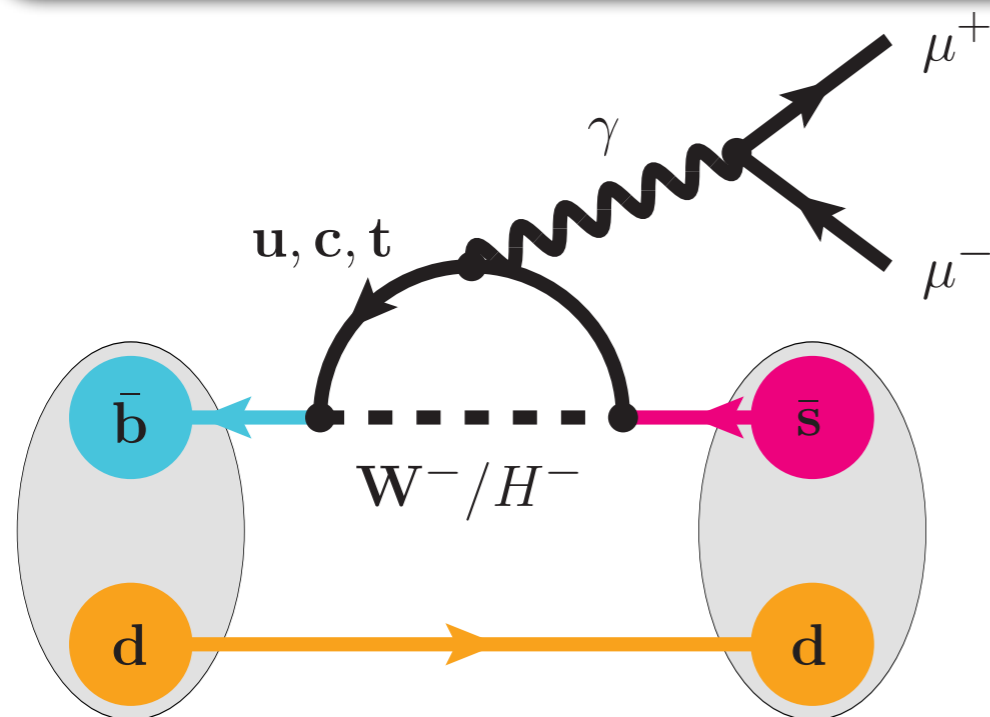
$p=0.03$



[based on Bernlochner *et al.*, PRD 90, 094003 (2014)]

# New! $B \rightarrow K^{(*)} e^+ e^-$ form factors

$b \rightarrow s$  flavor-changing neutral-current processes **sensitive to new particles & interactions**



- ◆ **First three-flavor lattice form factors**
- ◆ Enable predictions of observables (decay rate, forward-backward asymmetry, ...) in Standard Model & all BSM theories

## $B \rightarrow K l^+ l^-$

[HPQCD, PRD88 (2013) 054509]

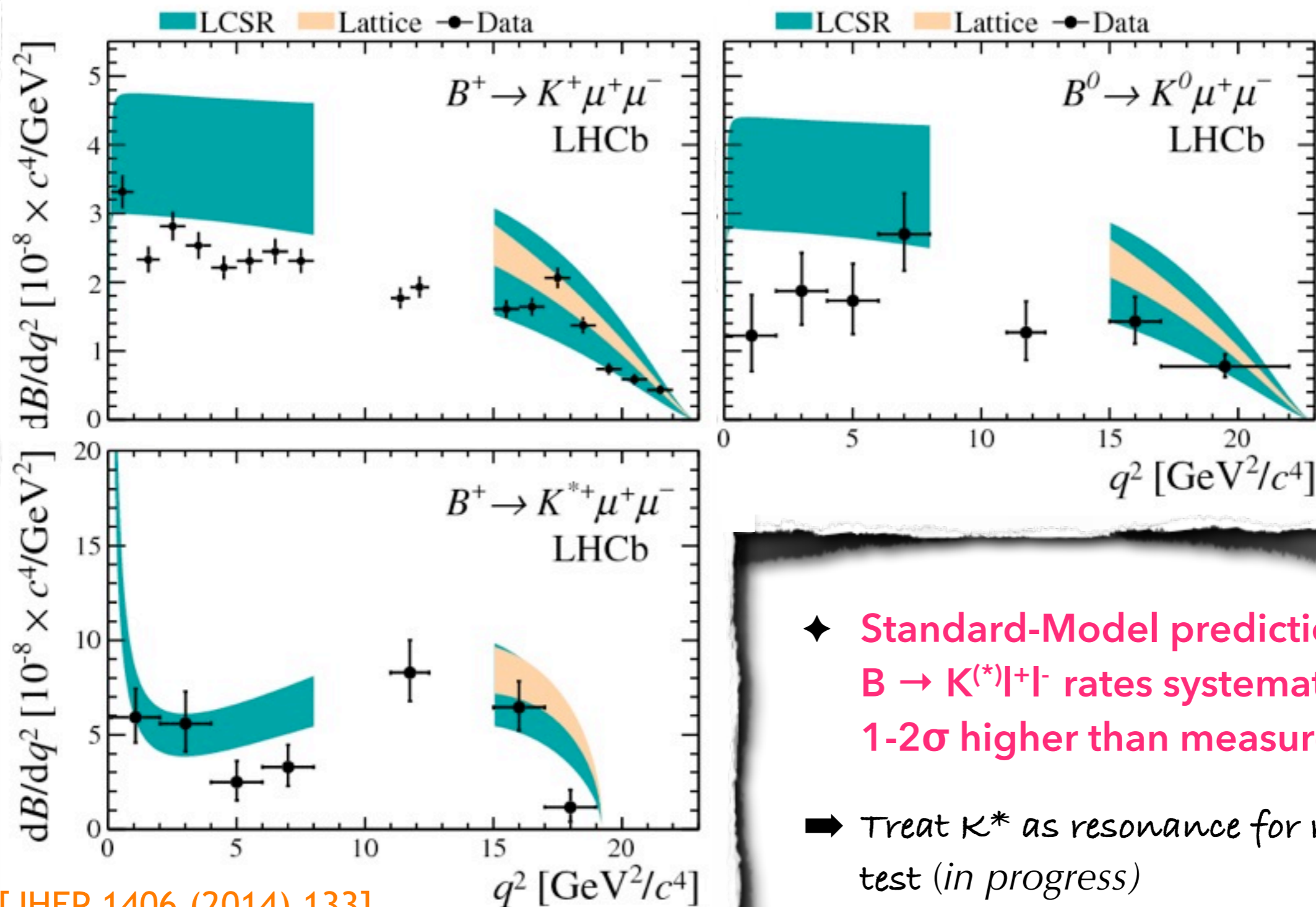
- ◆ Vector, scalar, and tensor form factors
- ◆ Use model-independent  $z$ -expansion to extend results over full  $q^2$  range

## $B_{(s)} \rightarrow K^*$ and $B_s \rightarrow \varphi$

[Horgan *et al.*, PRDD89 (2014) 9, 094501]

- ◆ Conference update [arXiv:1501.00367] includes correlations between form factors through kinematic constraints
- ◆ Use modified  $z$ -expansion to extend results over full  $q^2$  range
- ◆ **\*Caveat:**  $K^*$  and  $\varphi$  treated as stable hadrons in simulations

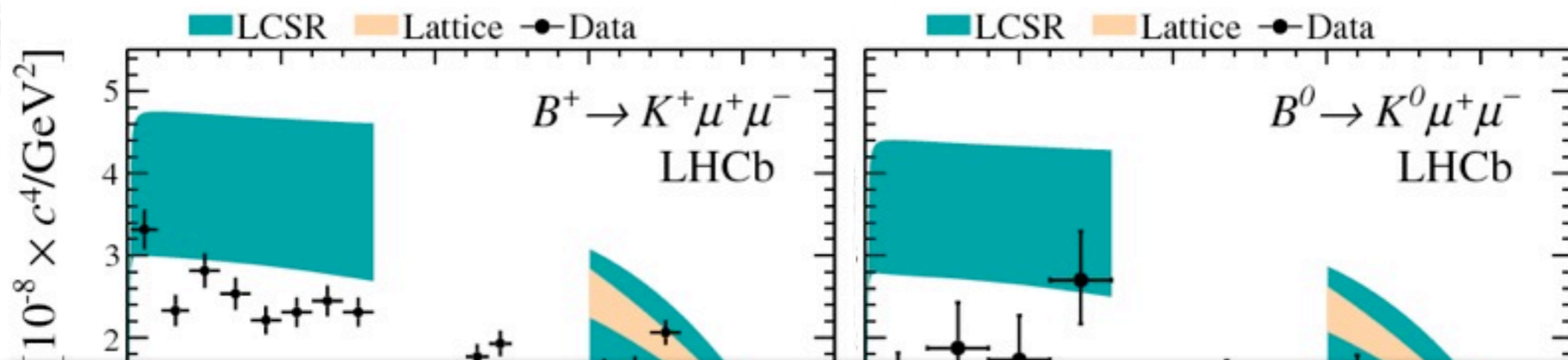
# Implications for new-physics searches



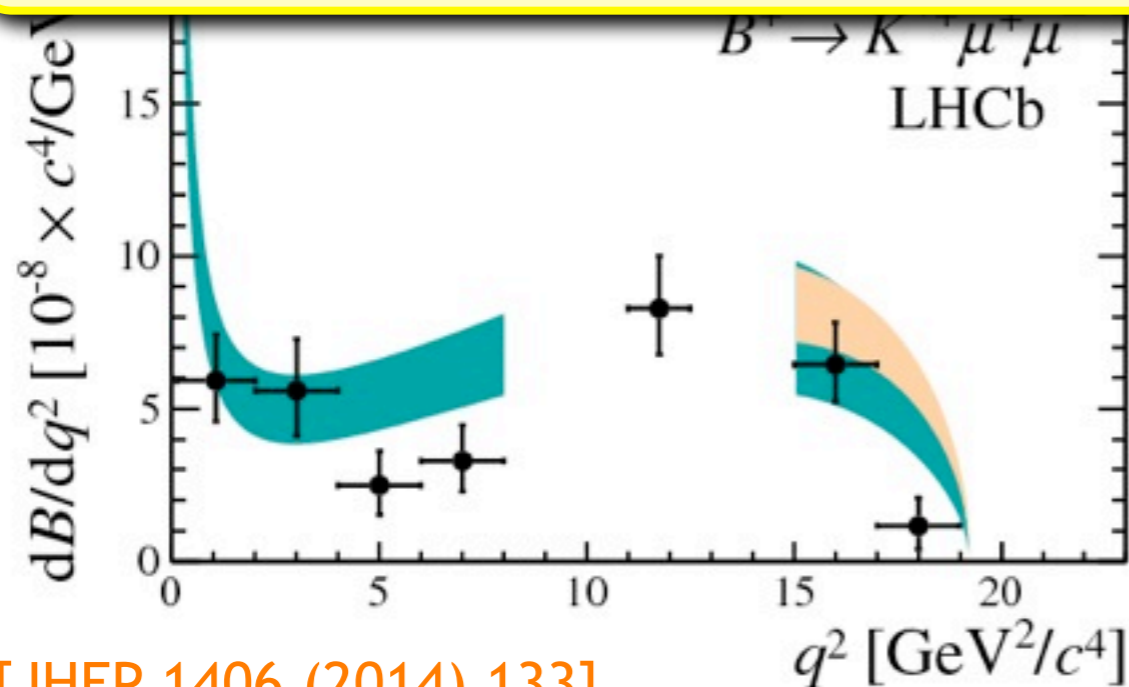
[JHEP 1406 (2014) 133]

- ◆ Standard-Model predictions for  $B \rightarrow K^{(*)} \mu^+ \mu^-$  rates systematically 1-2 $\sigma$  higher than measurements
- ➡ Treat  $K^*$  as resonance for robust test (in progress)

# Implications for new-physics searches



If deviations in  $b \rightarrow s$  transitions grow in significance, correlations between  $B \rightarrow K$ ,  $B \rightarrow K^*$ , and  $B_s \rightarrow \phi$  can provide information on masses & couplings of underlying new physics [see e.g. [Altmannshofer & Straub, arXiv:1503.06199](#)]



[JHEP 1406 (2014) 133]

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# $B_s \rightarrow \mu^+ \mu^-$ decay



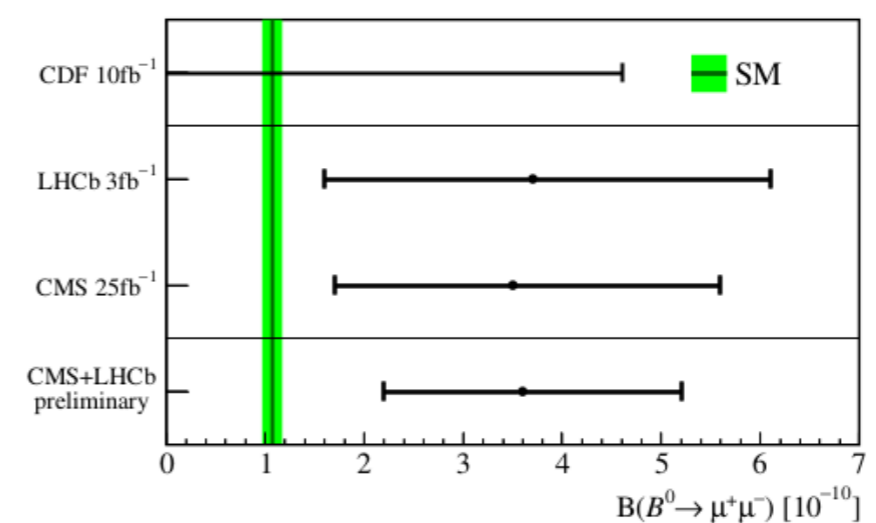
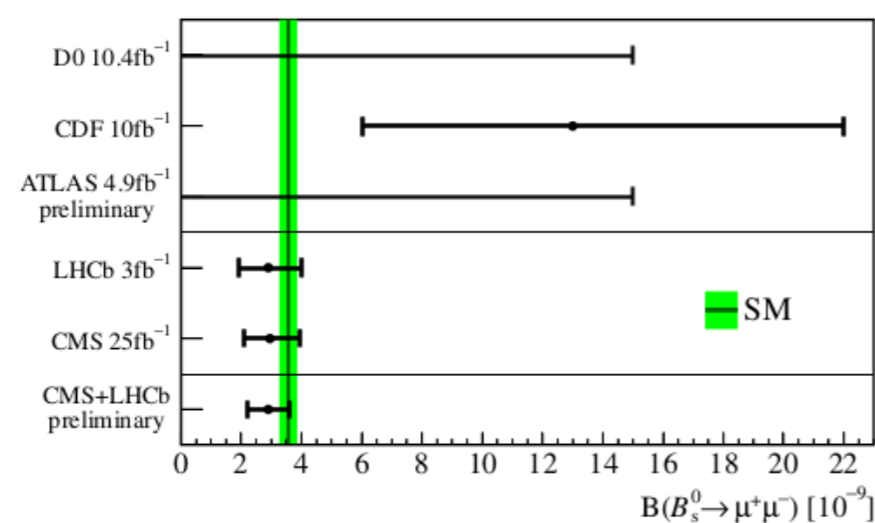
## Combined LHCb + CMS Result

new @ EPS2013

Observation:  
$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$



$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) = 3.6_{-1.4}^{+1.6} \times 10^{-10}$$



LHCb-CONF-2013-012, CMS-PAS-BPH-13-007

[S. Hansmann-Menzemer, EPS 2013]

# $B_s \rightarrow \mu^+ \mu^-$ decay

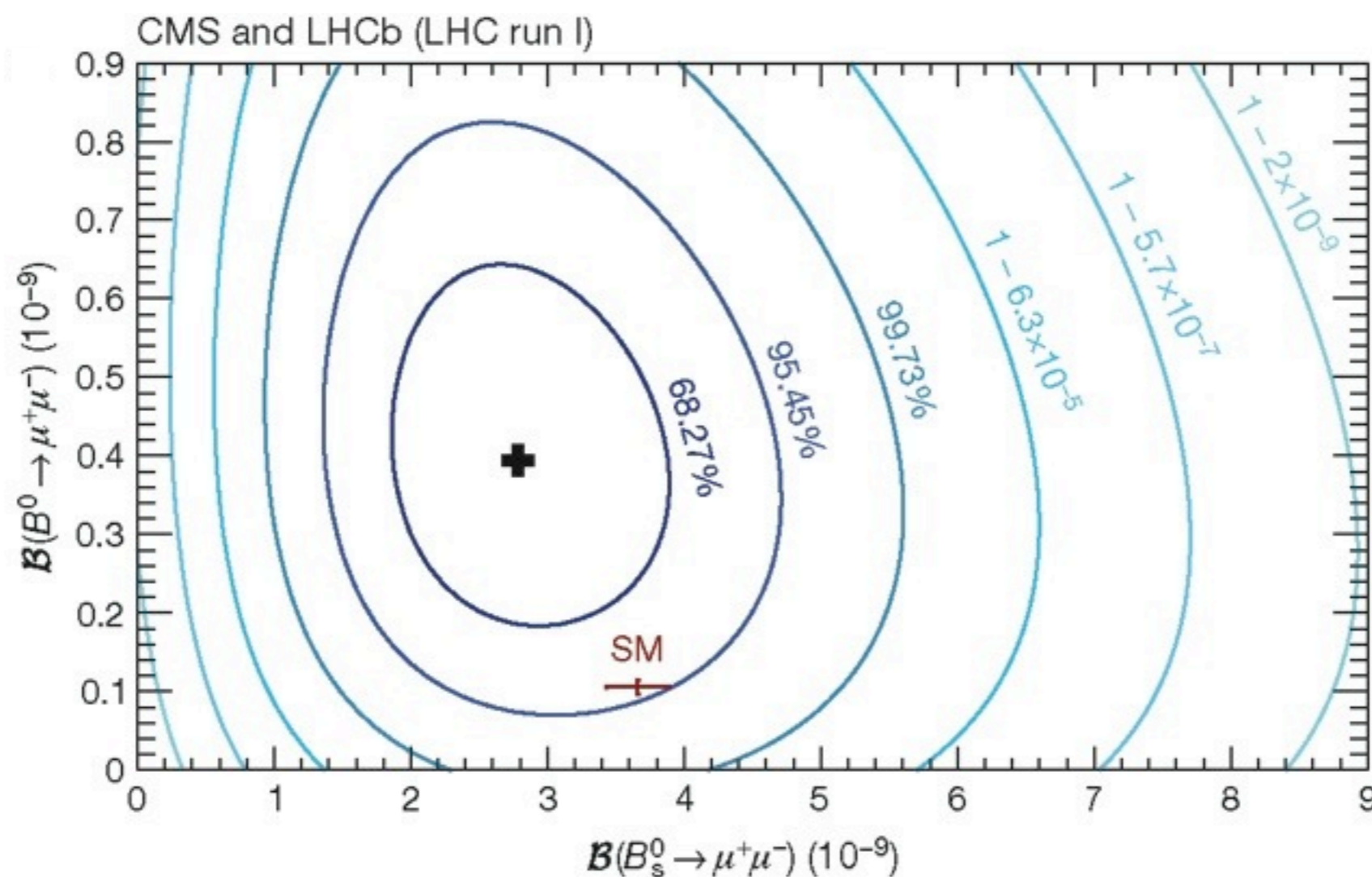


LETTER

nature

doi:10.1038/nature14474

Observation of the rare  $B_s^0 \rightarrow \mu^+ \mu^-$  decay from the combined analysis of CMS and LHCb data



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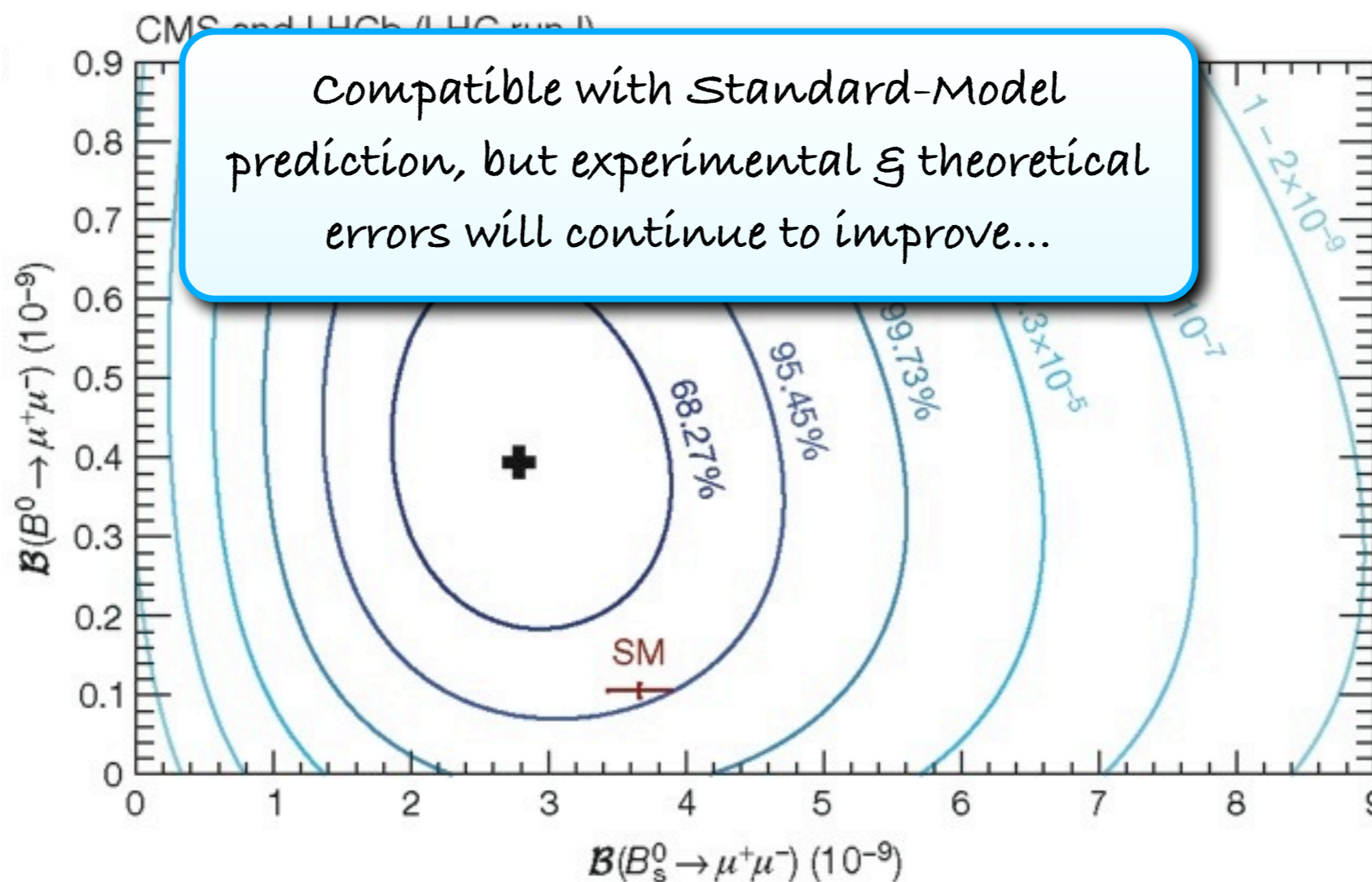


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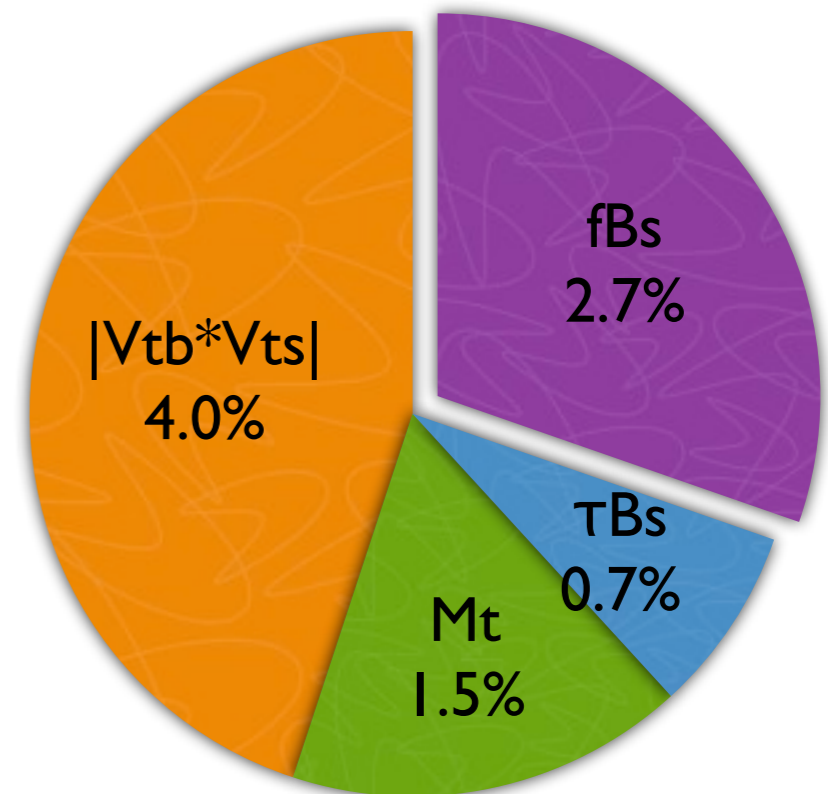
# Standard-Model prediction for $B_s \rightarrow \mu^+ \mu^-$

- Standard-Model rate proportional to **pseudoscalar decay constant  $f_{B_s}$**

$$\Gamma(B_s \rightarrow \mu^+ \mu^-) = \frac{G_F^2}{\pi} \underbrace{Y \left( \frac{\alpha}{4\pi \sin^2 \Theta_W} \right)^2}_{\text{perturbative QCD \& EW corrections}} m_{B_s} f_{B_s}^2 |V_{tb}^* V_{ts}|^2 m_\mu^2 \sqrt{1 - 4 \frac{m_\mu^2}{m_B^2}}$$

- Current **uncertainty in Standard-Model prediction ~5% using 2011 lattice-QCD calculation of  $f_{B_s}$**  from HPQCD  
[McNiele *et al.*, PRD85 (2012) 031503 ]
- Error in  $f_{B_s}$  will continue to improve with analysis of data at even finer lattice spacings  $\rightarrow$  limited by error on  $|V_{tb}^* V_{ts}|$  for foreseeable future

## BR( $B_s \rightarrow \mu^+ \mu^-$ ) USING $f_{B_s}$

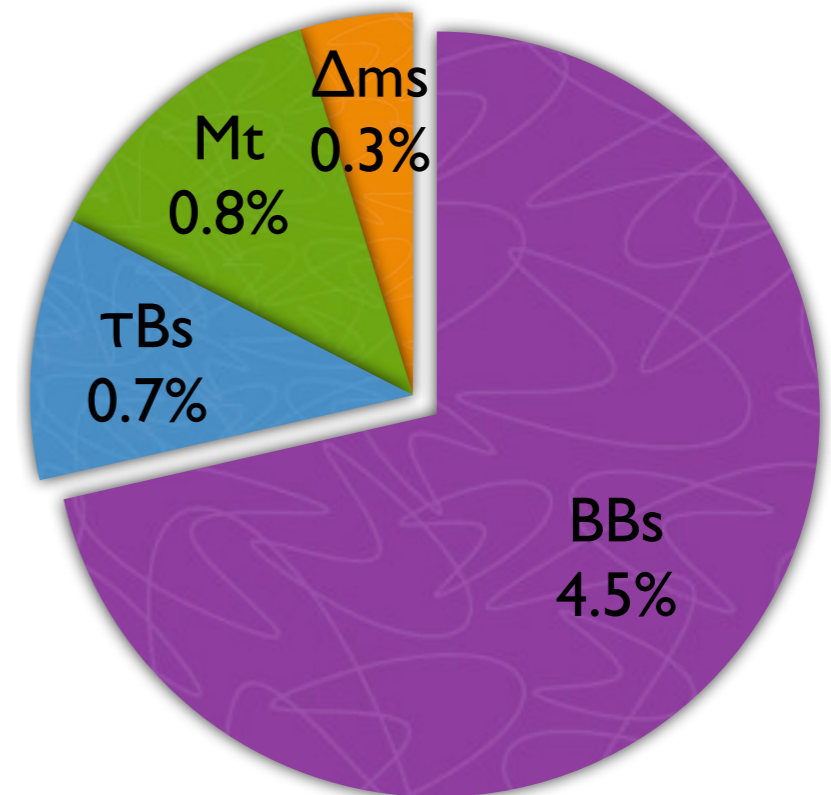


[Buras *et al.*, JHEP 1307 (2013) 77]

# Standard-Model prediction for $B_s \rightarrow \mu^+ \mu^-$

- ◆ Alternatively, can express Standard-Model rate in terms of oscillation frequency  $\Delta m_s$  and **hadronic matrix element  $B_{B_s}$**
- ◆ Obtain similar precision for **uncertainty in Standard-Model prediction using 2009 lattice-QCD calculation of  $B_{B_s}$**  from HPQCD [[Gamiz et al., PRD80 \(2009\) 014503](#)]
- ◆ Uncertainty in Standard-Model theory prediction via this approach will shrink with anticipated improvements in  $B_s$ -mixing matrix elements

## BR( $B_s \rightarrow \mu^+ \mu^-$ ) USING $B_{B_s}$

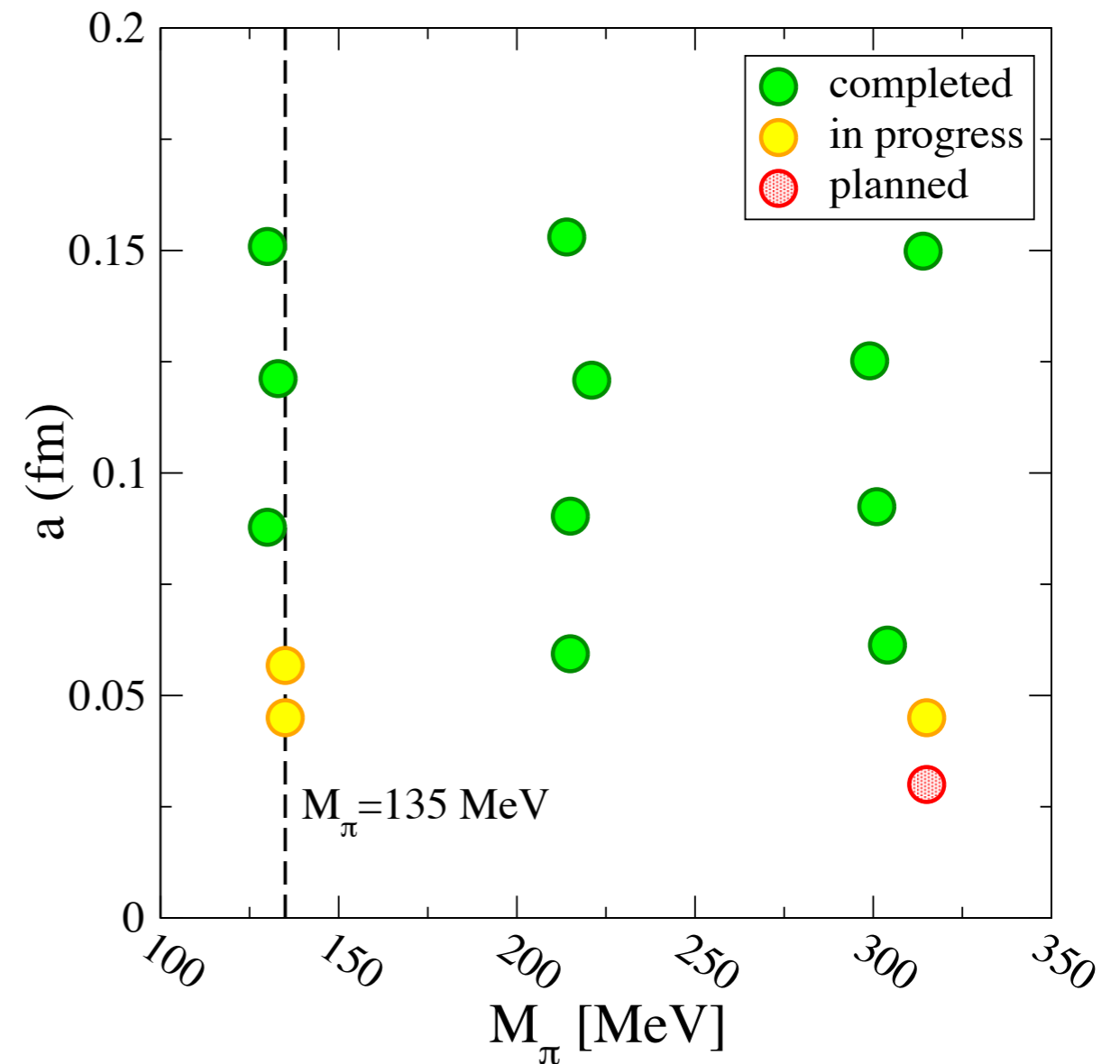


[[Buras et al., JHEP 1307 \(2013\) 77](#)]

# Controlling lattice-QCD systematics

- ◆ Statistical errors dictated by number of gauge-field configurations (sample size)
- ◆ **To control systematic errors, generate sets of gauge-field ensembles with different parameters, e.g.:**

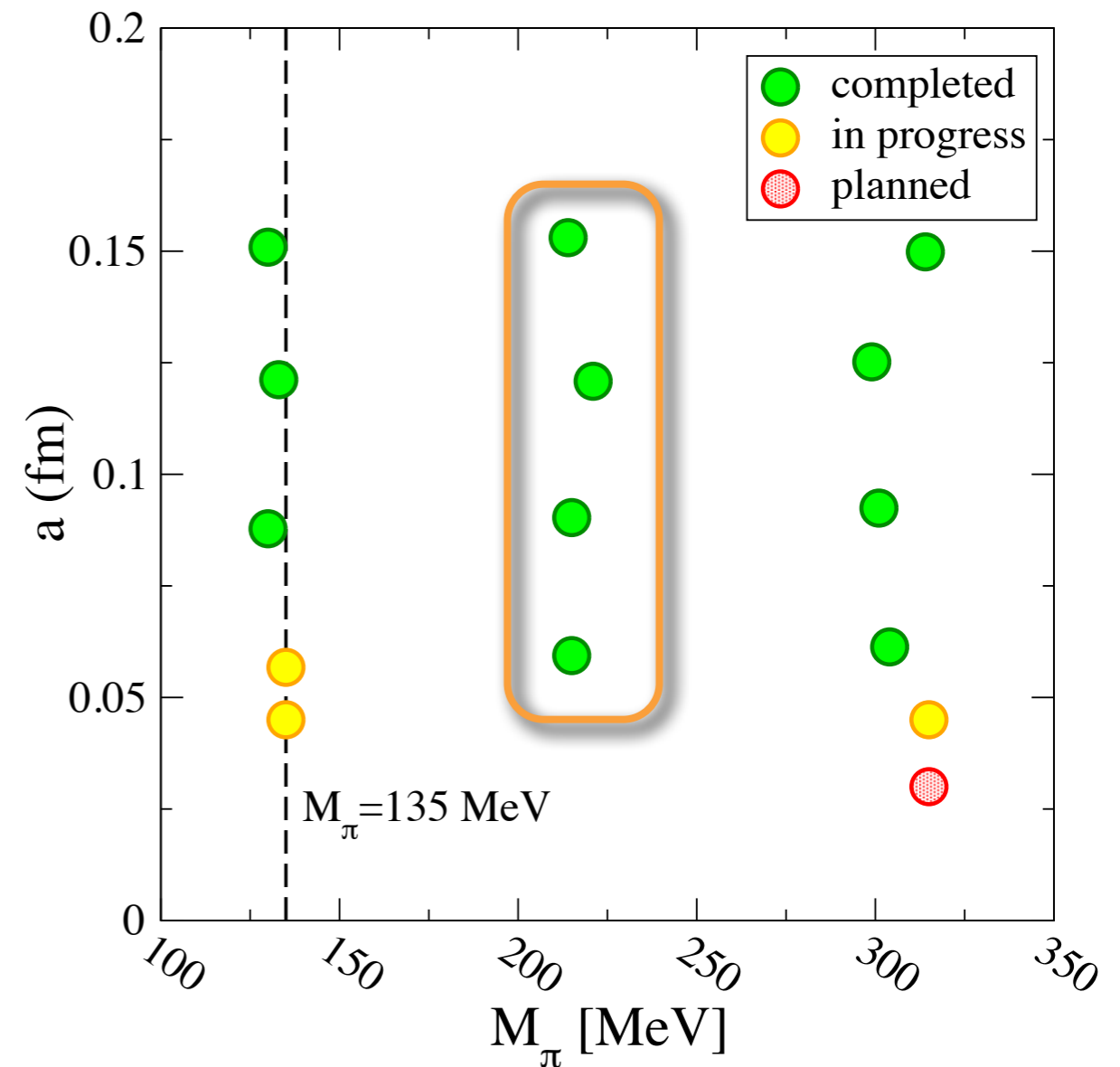
**MILC Collaboration's QCD ensembles**



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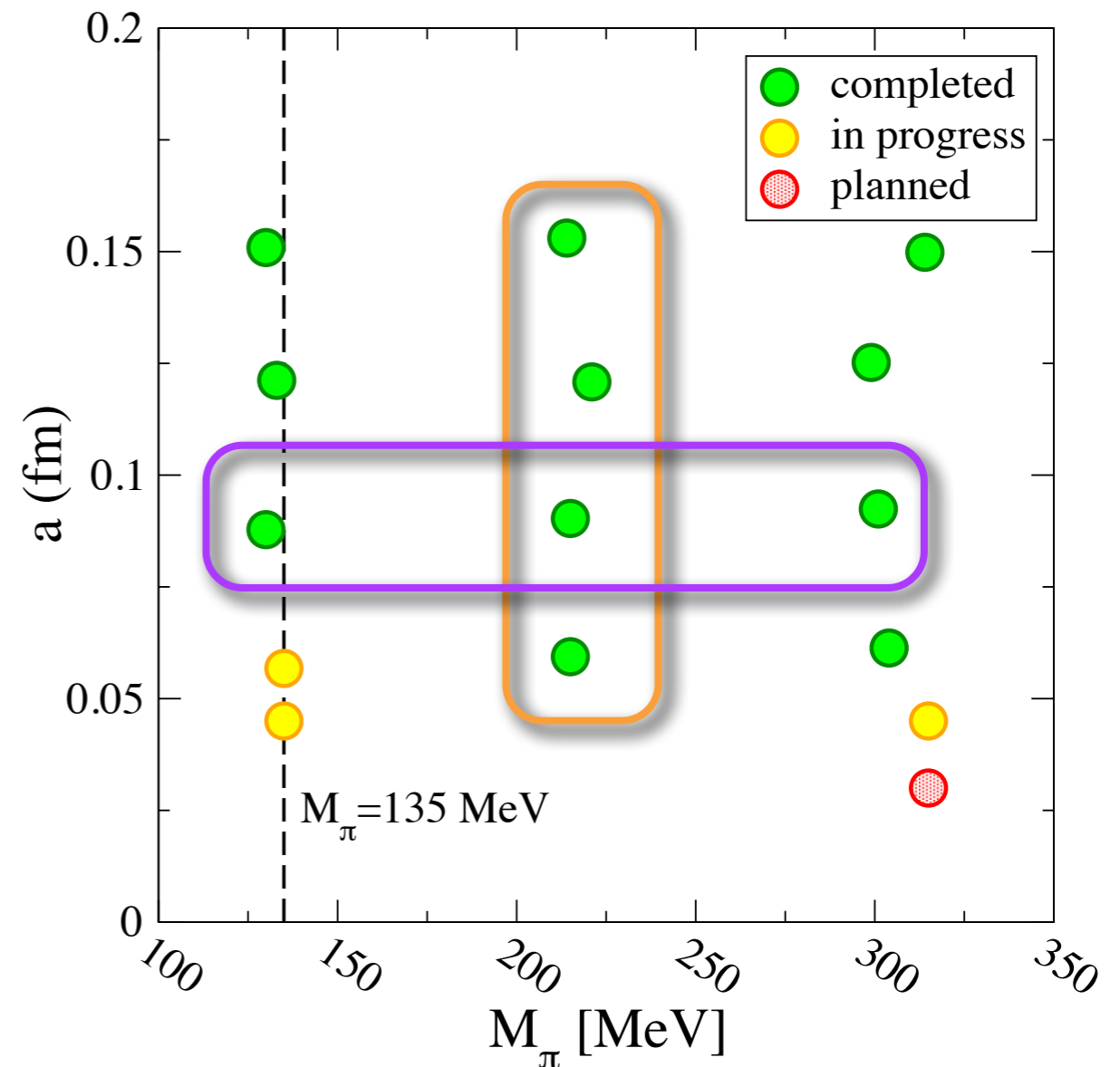
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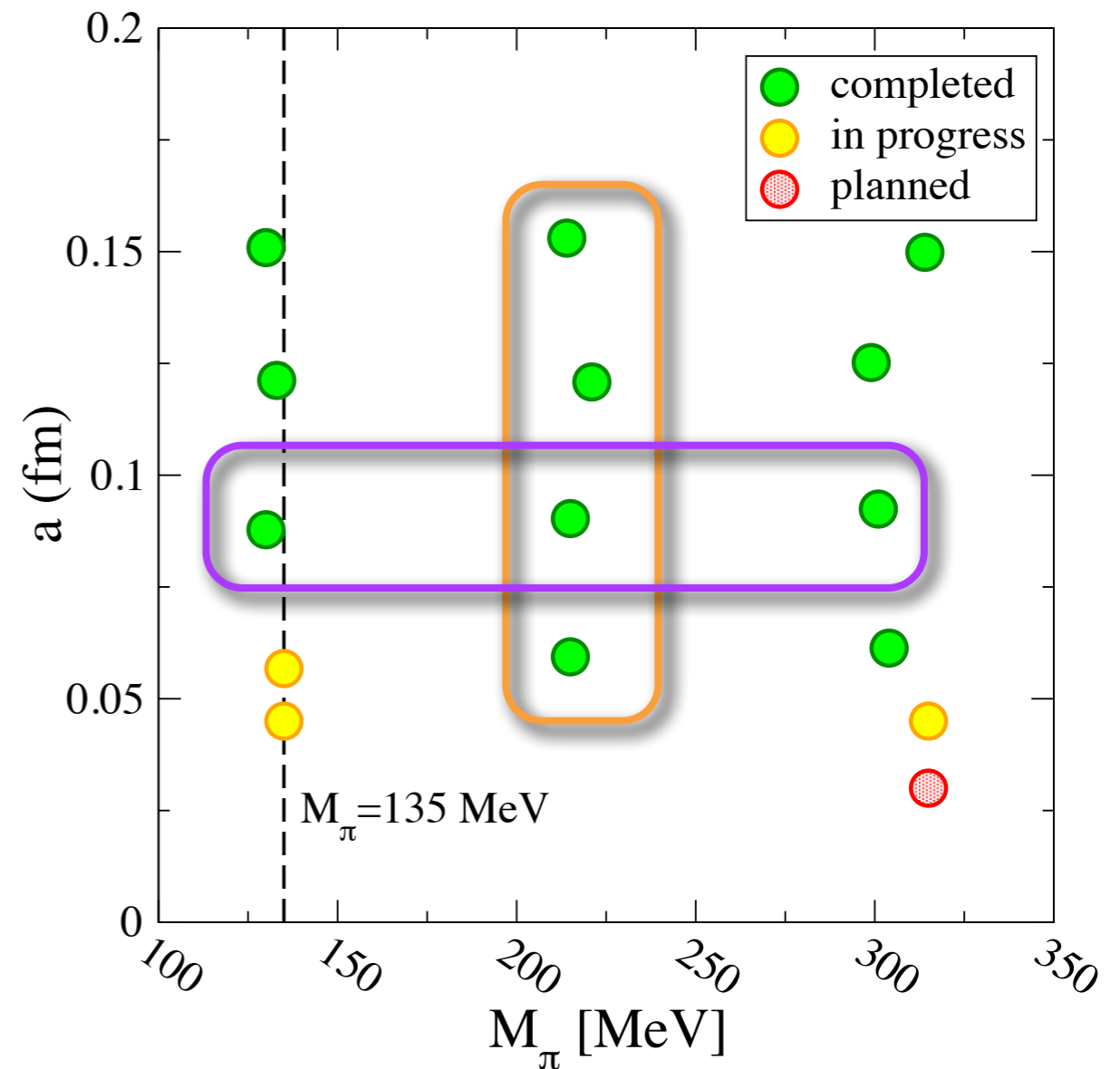
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MILC Collaboration's QCD ensembles

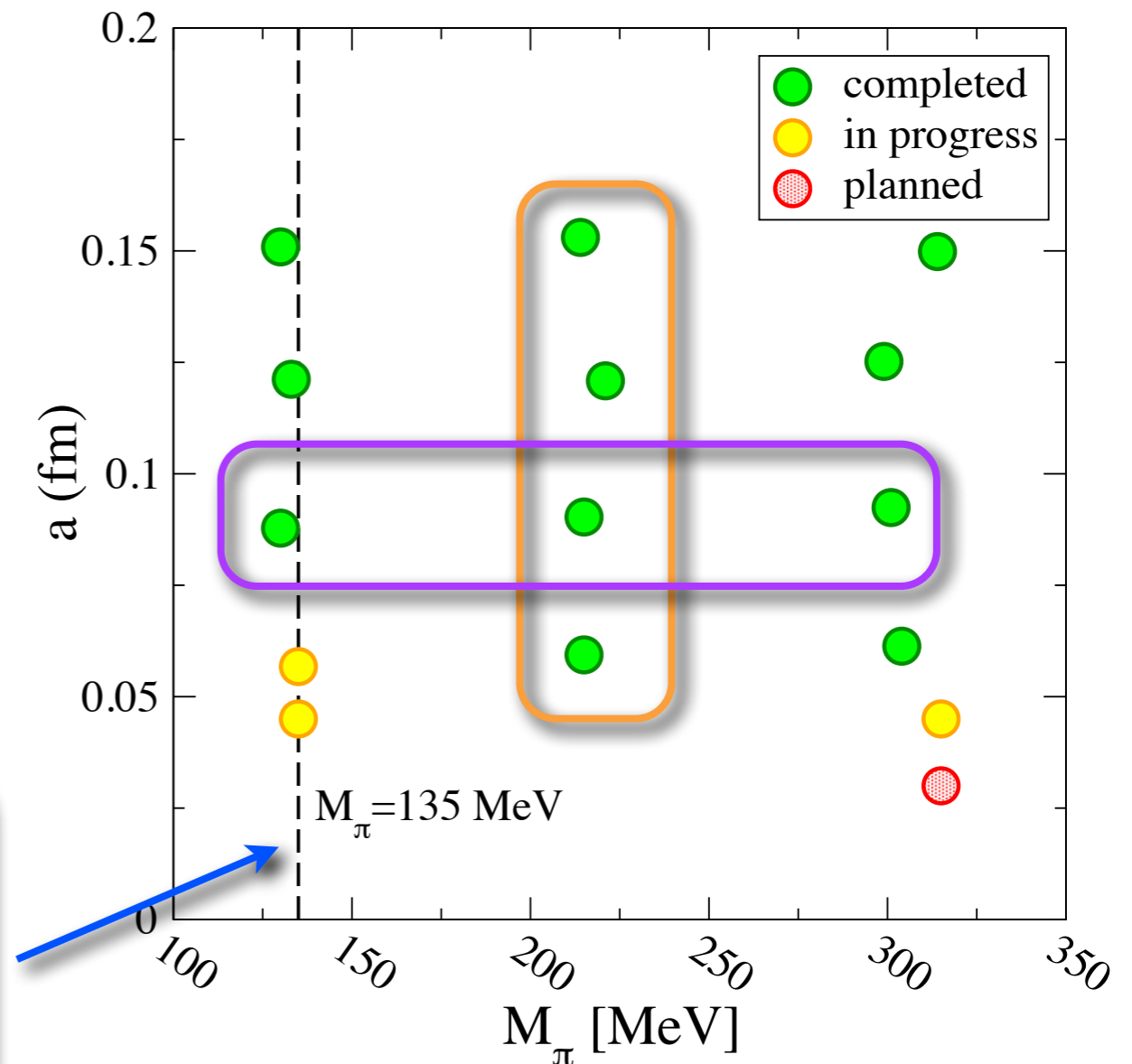


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Latest generation supercomputers enabling simulations with physical u,d-quark masses, & number of results obtained at physical point is growing rapidly!

MILC Collaboration's QCD ensembles



# New-physics complementarity

	LHT	RSc	4G	2HDM	RHMFV	
$D^0 - \bar{D}^0$ (CPV)	★★★★	★★★★	★★	★★		
$\epsilon_K$	★★	★★★★	★★	★★	★★	
$S_{\psi\phi}$	★★★★	★★★★	★★★★	★★★★	★★★★	
$S_{\phi K_S}$	QUARK FLAVOR		★★			
$A_{CP}(B \rightarrow X_s \gamma)$			★			
$A_{7,8}(K^* \mu^+ \mu^-)$	LEPTON FLAVOR		★★			
$B_s \rightarrow \mu^+ \mu^-$			★	★	★★★★	★★★★
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	★★★★	★★★★	★★★★		★★	
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	★★★★	★★★★	★★★★		★★	
$\mu \rightarrow e \gamma$	EDMs		★★	★★★★		
$\tau \rightarrow \mu \gamma$			★★	★★★★		
$\mu + N \rightarrow e + N$			★★	★★★★		
$d_n$	EDMs		★★	★★★★		
$d_e$			★★	★★★★		
$(g-2)_\mu$	★	★★	★			

★★★★★ = sizeable NP effects  
 ★★ = moderate to small NP effects  
 ★ = no visible NP effects

- ◆ Different processes & observables sensitive to different new-physics scenarios
- ❖ Pattern of measurements can distinguish between models & constrain model parameters

We do not know where the new physics lies → *cast a wide net!*

[Buras, Acta Phys.Polon.B41:2487-2561,2010]