

A class of 2D non-Abelian gauged linear sigma models

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Motivation

- 4D type II supersymmetric string theories:

$$\mathbb{M}^{1,3} \times \underbrace{\mathcal{N} = (2,2) \text{ SCFT with } c=9}_{\text{internal worldsheet theory}}$$

- **Aim:** Realize internal SCFT as IR RG-flow fixed point of 2D (non-Abelian) gauge theories
- **Result:** Non-trivial type II quantum moduli space arising from a strong-weak coupling duality in 2D gauge theories

Gauged linear sigma model: Essentials

- $\mathcal{N} = (2, 2)$ two-dimensional gauge theory, given by Witten, 93; ...

① Gauge group: $G = U(1)^\ell \times$ semi-simple Lie group

Fayet-Iliopoulos term for each $U(1)$ \Rightarrow Kähler moduli t , (D-terms)

② Chiral superfields $\{\Phi_1, \dots, \Phi_N\}$ and their G -representation

③ Vector R -charges: $q_V(\Phi_i)$

- Superpotential $W(\Phi_i)$ \Rightarrow Complex structure moduli z , (F-terms)

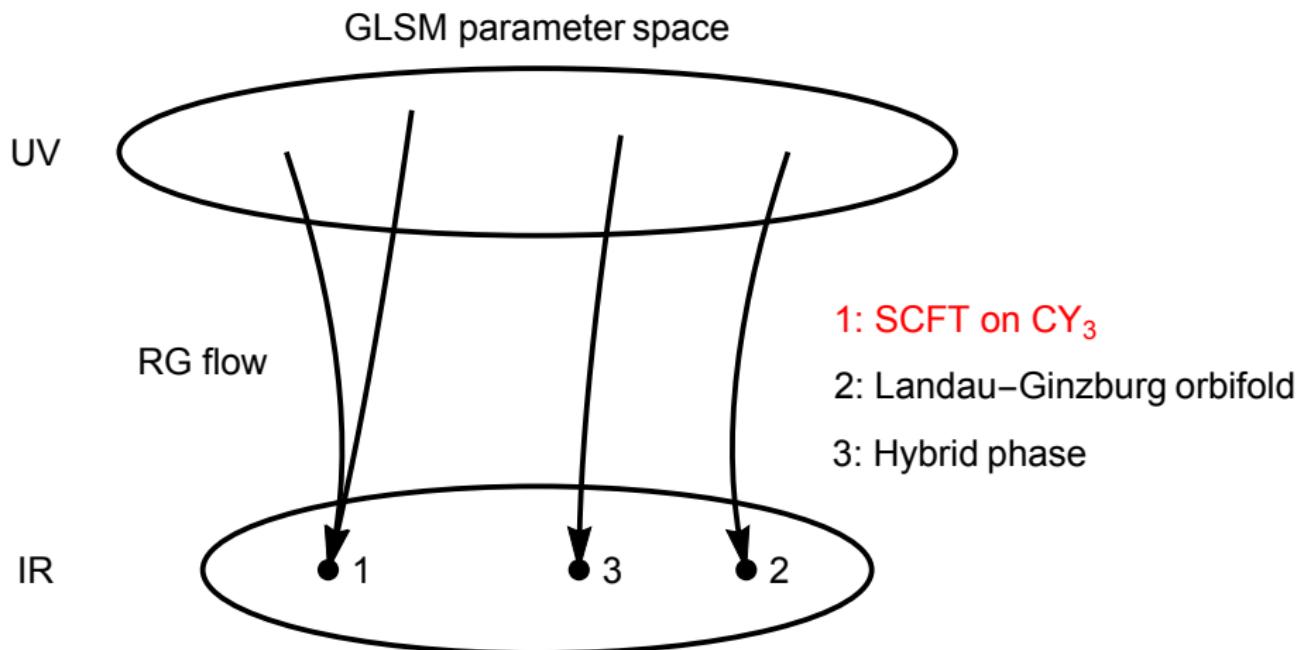
- Moduli dependent semi-classical scalar potential

$$U_{t,z}(\phi_i) = |\text{D-terms}|^2 + |\text{F-terms}|^2$$

- Semi-classical IR target space $\mathcal{X}_{t,z}$ = space of inequivalent vacua

$$\mathcal{X}_{t,z} = \frac{U_{t,z}^{-1}(0)}{G}$$

Gauged linear sigma model: Illustration



Skew Symplectic Sigma Models: $SSSM_{k,m,n}$

- Class of skew symplectic sigma models $SSSM_{k,m,n}$:

- ① Gauge group: $G = \frac{U(1) \times USp(2k)}{\mathbb{Z}_2} \Rightarrow$ 1 Kähler parameter t
- ② Matter spectrum:

| chiral multiplets | G representation | multiplicity | q_V |
|---------------------------------|--------------------|-----------------|----------|
| $P_{[ij]}, 1 \leq i < j \leq n$ | $\mathbf{1}_{-2}$ | $(\frac{n}{2})$ | $2 - 2q$ |
| Q | $2\mathbf{k}_{-3}$ | 1 | $2 - 3q$ |
| $\phi_a, a = 1, \dots, m$ | $\mathbf{1}_2$ | m | $2q$ |
| $X_i, i = 1, \dots, n$ | $2\mathbf{k}_1$ | n | q |

- ③ Superpotential: $W(P, Q, \phi, X)$

Conformal models with CY_3 target space

$$SSSM_{1,12,6} \quad \text{and} \quad SSSM_{2,9,6}$$

$SSSM_{k,m,n}$ phase $r \gg 0$: Weak coupling

- Solving D-terms = F-terms = 0 gives

① Not all ϕ_a vanish simultaneously: $\phi \in \mathbb{P}^{m-1}$

② For skew-symmetric matrix $A(\phi)$, $X = (X_1, \dots, X_n)$

$$A(\phi) = -X^T \epsilon X \quad \text{with} \quad \epsilon = \begin{pmatrix} 0 & \mathbf{1}_{k \times k} \\ -\mathbf{1}_{k \times k} & 0 \end{pmatrix}$$

$$\Leftrightarrow \operatorname{rk} A(\phi) \leq 2k \quad \text{since} \quad X \quad \text{is } 2k \times n \text{ matrix}$$

③ For n -dimensional vector $B(\phi)$: $A(\phi) \cdot B(\phi) = 0$

Semi-classical target space variety $r \gg 0$

$$\mathcal{X}_{k,m,n} = \{\phi \in \mathbb{P}^{m-1} \mid \operatorname{rk} A(\phi) \leq 2k \text{ and } A(\phi) \cdot B(\phi) = 0\}$$

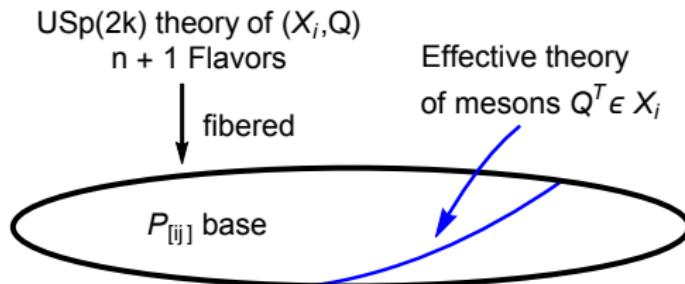
c.f. Galkin (talk), 13

- $\operatorname{USp}(2k)$ broken \Rightarrow phase weakly coupled

$SSSM_{k,m,n}$ phase $r \ll 0$: Strong coupling

- Gauge group broken to $USp(2k)$
⇒ non-Abelian **strong coupling** dynamics important

c.f. Hori, Knapp, 13



- $USp(2k)$ theory dual to a weakly coupled theory of mesons

Hori, Tong, 06; Hori, 11

Semi-classical target space variety $r \ll 0$

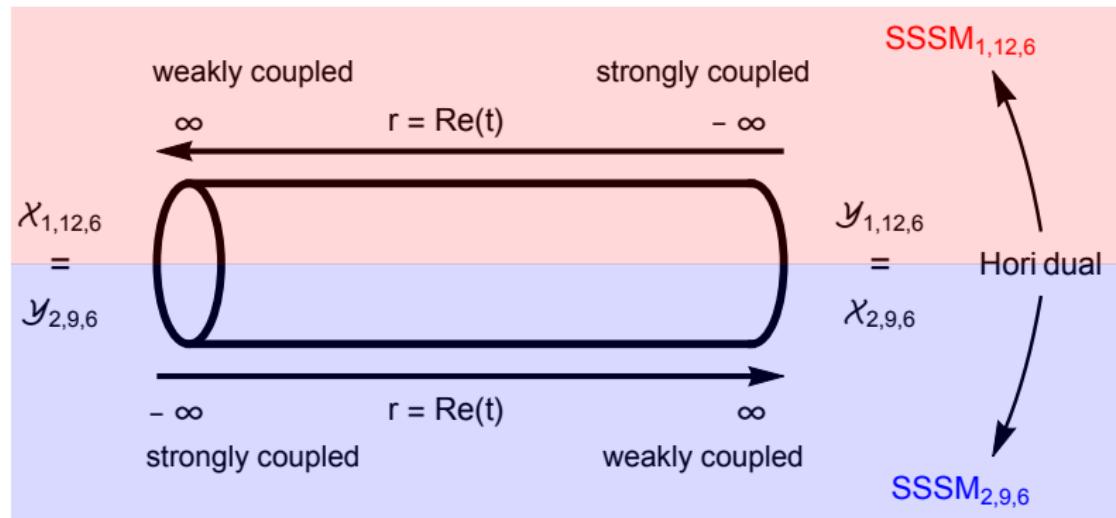
$$\mathcal{Y}_{k,m,n} \simeq \mathcal{X}_{\tilde{k}, \tilde{m}, n} \quad \text{with} \quad \tilde{k} = \frac{n}{2} - k, \quad \tilde{m} = \frac{n(n+1)}{2} - m, \quad n \text{ even}$$

- In particular:

$$\mathcal{Y}_{1,12,6} \simeq \mathcal{X}_{2,9,6}$$

$$\mathcal{Y}_{2,9,6} \simeq \mathcal{X}_{1,12,6}$$

Phase structure and duality



- Further evidence: $Z_{S^2}(SSSM_{1,12,6}, t) = Z_{S^2}(SSSM_{2,9,6}, -t)$

- Geometric data

Miura, 13; Galkin (talk), 13; van Straten's database

$$\chi_{1,12,6} : \quad \chi = -102, \quad \deg = 33, \quad N_d = 252, 1854 \dots$$

$$\gamma_{1,12,6} : \quad \chi = -102, \quad \deg = 21, \quad N_d = 387, 4671 \dots$$

Summary

- We have analysed class of non-Abelian models GLSMs, $SSSM_{k,m,n}$
- $SSSM_{1,12,6}$ and $SSSM_{2,9,6}$: NLSM interpretation on CY₃ target space
- Strong and weak coupling phases

Weakly coupled: $\mathcal{X}_{1,12,6}$

Strongly coupled: $\mathcal{Y}_{1,12,6}$

Weakly coupled: $\mathcal{X}_{2,9,6}$

Strongly coupled: $\mathcal{Y}_{2,9,6}$

- Duality of $SSSM_{1,12,6}$ and $SSSM_{2,9,6}$, with interchange of strong and weak coupling phases

$$\mathcal{X}_{1,12,6} \simeq \mathcal{Y}_{2,9,6} \quad \text{and} \quad \mathcal{X}_{2,9,6} \simeq \mathcal{Y}_{1,12,6}$$

- Future work: Relation between Hori duality and phases of non-Abelian GLSMs?