Phenomenological analysis using Difftop: impact of differential top quark-pair production on the gluon distribution.

M. Guzzi, K. Lipka*, S. Moch

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Probing QCD with top-quark pair production at the LHC

Top-quark pair production in $pp$ collisions probes gluon distribution, top mass and $\alpha_S$

with available NNLO theory for inclusive $t\bar{t}$ cross sections:

✓ correlations of $g(x)$, $m_t$, $\alpha_S$ studied

[CMS Collaboration, PLB 728 (2013) 496]
[S. Alekhin, J. Bluemlein, S. Moch PRD 89 (2014) 054028]

✓ impact on $g(x)$ is quantified

[Rojo et al., JHEP 1307 (2013) 167]

top-quark pair production reaches x-range, poorly known due to lack of experimental data
Probing gluon with $t\bar{t}$ differential cross sections

High sensitivity of $\sigma_{tt}(p_T^t)$ to the gluon distribution is expected at high $x$

NLO calculation too imprecise...

Additional constraints from the shape of top-quark transverse momenta?

Need a higher-order calculation suitable for a QCD analysis

Need effective description of the kinematic distributions capturing the main features of the full calculation
Difftop: differential $t\bar{t}$ cross sections at aNNLO

Differential cross sections of heavy-quark pair production in proton- (anti)proton collisions calculated in pQCD at approximate NNLO ($\alpha_s^4$) using methods of threshold resummation beyond the leading logarithmic accuracy.

Predictions for the single-particle inclusive kinematics (top-quark $p_t$, $y_t$)


Open-source code: http://difftop.hepforge.org/

Version 1.0.0: choose process, order, energy, PDFs, $m_t$, $\alpha_s$, scales (NB: so far $\mu_r=\mu_f$)

Designed for fast QCD analyses: allows for various phenomenological studies
Phenomenology uncertainty on $t\bar{t}$ kinematics

full uncertainty for the predicted $\sigma_{t\bar{t}}(p_T, y^t)$ LHC 7 TeV for different PDFs

PDF alters the normalization and shape of the predicted kinematics
Phenomenology uncertainty on $t\bar{t}$ kinematics

breakdown of different contributions to the theory uncertainty

Dominant contribution to the theory uncertainty is due to PDF uncertainty
Comparison with LHC measurements: 7 TeV

Difftop predictions agree well with data, agreement on different level for different PDFs

CMS [EPJ C73 (2013) 2339]
ATLAS [PRD 90 (2014) 7, 072004]
Comparison with LHC measurements: 8 TeV

CMS Collaboration
[arXiv:1505.04480]
Comparison with LHC measurements: 8 TeV

CMS Collaboration
[arXiv:1505.04480]

+ break-down of theory uncertainty:

dominant contributions: top-quark mass variation and PDF
QCD analysis: general strategy

Factorisation: \[ \sigma = \sum_k \sigma_k \times f_k \]

- \( \sigma_k \) - Wilson coefficient
- \( k \) - number of active flavours in the proton

PDF for flavor \( k \): \( f_k = f_k(x, Q^2) \)
- \( Q^2 \) - dependence predicted by QCD
- \( x \) - dependence determined from data

PDF determination

- parameterize PDFs at a starting scale \( Q^2_0 \):
  \[ f(x) = Ax^B(1-x)^C(1+Dx+Ex^2) \]
  - A: normalization, B: small-\( x \) behavior, C: \( x \to 1 \) shape
- evolve these PDFs to \( Q^2 > Q^2_0 \)
- construct cross sections from PDFs and coefficients:
  - predictions for every data point in \( (x, Q^2) \) – plane
- \( \chi^2 \)- fit to the experimental data
Experiments sensitive to PDFs

Measurements probing proton structure

- HERA DIS: quarks, gluon @ low, medium x
- HERA heavy-quarks: gluon, $m_c$, $m_b$
- Neutrino scattering: s-quark @ high x
- LHC W,Z: light quarks at low and high x
- LHC W+c: s-quark medium x
- LHC jets: gluon at medium x
- LHC single top u, d and b quarks
- LHC top-pairs: gluon at high x
open-source code to test impact of the measurements on e.g. PDFs during data analysis

**experimental input**
- experiments: HERA, Tevatron, LHC, fixed target
- processes: NC, CC DIS, jets, diffraction, heavy quarks (c,b,t)
- Drell-Yan, W production

**theoretical calculations/tools**
- Heavy quark schemes: MSTW, CTEQ, ABM
- Jets, W, Z production: fastNLO, Applgrid
- Top-quark production: NNLO (Hathor, Difftop)
- QCD Evolution: DGLAP (QCDNUM)
- Alternative tools: NNPDF reweighting
- Other models: Dipole model
  - Different error treatment models
  - Tools for data combination (HERAaverager)

**PDF or uPDF or DPDF**

**αS (Mz), mc, mb, mt, fs,...**

**Theory predictions**

**Benchmarking**

**Comparison of schemes**

**Difftop interfaced to HERAFitter via fastNLOtoolkit**

**HERAFitter QCD analysis framework**

**EPJ C75 (2015) 7, 304**

https://www.herafitter.org/HERAFitter
CMS muon charge asymmetry at $\sqrt{s}=7$ TeV (L=4.7 fb$^{-1}$) *Phys. Rev. D* 90 (2014) 032004

+ Top-quark pair production cross-sections:

- **total**

- **differential**

NB: top-pair production at TEVATRON qq-dominated, need better constraint on light quarks
→ use CMS W asymmetry data
Use HERAFitter, DGLAP evolution at NNLO, parametrization at $Q_0$:

\[
\begin{align*}
x u_v(x) &= A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} (1 + D_{u_v} x + E_{u_v} x^2) \\
x d_v(x) &= A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}} \\
x \bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} \\
x \bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}} \\
x g(x) &= A_g x^{B_g} (1 - x)^{C_g} + A'_g x^{B'_g} (1 - x)^{C'_g}
\end{align*}
\]

\[
x \bar{U} = x \bar{u} \\
x \bar{D} = x \bar{d} + x \bar{s} \\
B_{\bar{U}} = B_{\bar{D}} \\
A_{\bar{U}} = A_{\bar{D}} (1 - f_s) \\
f_s = \bar{s} / (\bar{d} + \bar{s}) \equiv 0.31 \pm 0.08
\]

$Au_v, Ad_v, Ag$ are determined by QCD sum rules

**Experimental (Hessian) uncertainties:**

* originate from uncertainties of the data considered, criterion $\Delta \chi^2 = 1$ is applied

**Details of the theory calculations and model input**

- Calculation for $W$ production via MCFM, interfaced via APPLGRID, K-factors by using FEWZ
- Calculation for top-quark pair production via fastNLO x Difftop
- Starting scale of PDF evolution, $Q^2_0 = 1.9$ GeV$^2$
- Heavy quark treatment: general mass variable flavor number (GMVFN) scheme by Thorne-Roberts (TR), $m_c = 1.4$ GeV, $m_b = 4.75$ GeV.
Impact on the gluon distribution

HERA-only vs HERA+W asymmetry:
- very similar effects on u and d-distributions as in CMS NLO fit [PRD 90 (2014) 032004]
- slight improvement in g(x) only due to Sum Rules

HERA-only + HERA+W asymmetry + TOP data:
- Moderate improvement of the uncertainty on the gluon distribution for x > 0.1
- Significant change of the shape of the gluon distribution

NB: only experimental uncertainty shown
Differential cross sections of top-pair production can be used in PDF fits @ NNLO

Data are yet not very precise, expect more effect on g(x) using Run II measurements

The correlations between the total and differential cross sections are NOT used since those are NOT yet available

Needs from experimental measurements for PDF fits for RUN II data:

✓ absolute differential cross sections with list of sources of uncorrelated and correlated statistical and systematic uncertainties

or

✓ normalized differential cross sections with list of sources of uncorrelated and correlated statistical and systematic uncertainties
+ provided correlations with the total cross section
Back-up
Single-particle inclusive (1PI) kinematics

In our calculation, heavy-quark hadroproduction near the threshold is approximated by considering the partonic subprocesses

\[ q(k_1) + \bar{q}(k_2) \rightarrow t(p_1) + X[\bar{t}](p_2'), \]
\[ g(k_1) + g(k_2) \rightarrow t(p_1) + X[\bar{t}](p_2') \quad p_2' = \bar{p}_2 + k, \quad (1) \]

where is \( k \) any additional radiation, and \( s_4 = p_2' - m^2 \rightarrow 0 \) momentum at the threshold.

This kinematic is used to determine the \( p^t_\perp \) and rapidity \( y^t \) distribution of the final-state top-quark.

Hard scattering functions are expanded in terms of

\[
\left[ \ln^l \left( \frac{s_4}{m^2_t} \right) \right]_+ = \lim_{\Delta \to 0} \left\{ \ln^l \left( \frac{s_4}{m^2_t} \right) \theta(s_4 - \Delta) + \frac{1}{l + 1} \ln^{l+1} \left( \frac{\Delta}{m^2_t} \right) \delta(s_4) \right\},
\]

where corrections are denoted as leading-logarithmic (LL) if \( l = 2i + 1 \) at \( \mathcal{O}(\alpha_s^{i+3}) \) with \( i = 0, 1, \ldots \), as next-to-leading logarithm (NLL) if \( l = 2i \), as next-to-next-to-leading logarithm (NNLL) if \( l = 2i - 1 \), and so on.
The hard scattering expansion

The factorized differential cross section is written as

\[
S^2 \frac{d^2 \sigma(S, T_1, U_1)}{dT_1 dU_1} = \sum_{i,j=q,\bar{q},g} \int_{x_1^-}^1 \frac{dx_1}{x_1} \int_{x_2^-}^1 \frac{dx_2}{x_2} f_{i/H_1}(x_1, \mu_F^2) f_{j/H_2}(x_2, \mu_F^2) \\
\times \omega_{ij}(s, t_1, u_1, m_t^2, \mu_F^2, \alpha_s(\mu^2_R)) + \mathcal{O}(\Lambda^2/m_t^2),
\]

\[
\omega_{ij}(s_4, s, t_1, u_1) = \omega_{ij}^{(0)} + \frac{\alpha_s}{\pi} \omega_{ij}^{(1)} + (\frac{\alpha_s}{\pi})^2 \omega_{ij}^{(2)} + \cdots
\]

where \(\omega_{ij}^{(2)}\) at parton level in 1PI is

\[
\omega_{ij}^{(2)} = s^2 \frac{\hat{\sigma}_{ij}^{(2)}}{du_1 dt_1} \bigg|_{1PI} = F_{ij}^{\text{Born}} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ D_{ij}^{(3)} \left[ \frac{\ln^3(s_4/m_t^2)}{s_4} \right] + \right.
\]

\[
+ D_{ij}^{(2)} \left[ \frac{\ln^2(s_4/m_t^2)}{s_4} \right] + D_{ij}^{(1)} \left[ \frac{\ln(s_4/m_t^2)}{s_4} \right] + D_{ij}^{(0)} \left[ \frac{1}{s_4} \right] + R_{ij}^{(2)} \delta(s_4) \right\}. 
\]
Few more details...

- **Hard and soft functions:** $H_{ij} = H_{ij}^{(0)} + \left(\alpha_s/\pi\right)H_{ij}^{(1)} + \cdots$ and
  $S_{ij} = S_{ij}^{(0)} + \left(\alpha_s/\pi\right)S_{ij}^{(1)} + \cdots$,
  $H_{ij}^{(2)}$ and $S_{ij}^{(2)}$ are set to zero.

- **Soft anomalous dimension matrices:**
  $\Gamma_S = \left(\alpha_s/\pi\right)\Gamma_S^{(1)} + \left(\alpha_s/\pi\right)^2\Gamma_S^{(2)} + \cdots$

  In our calculation, $\Gamma_S^{(2)}$ at two-loop for the massive case is included. Becher (2009), Kidonakis (2009).

- **Anomalous dimensions of the quantum fields** $i = q, g$:
  $\gamma_i = \left(\alpha_s/\pi\right)\gamma_i^{(1)} + \left(\alpha_s/\pi\right)^2\gamma_i^{(2)} + \cdots$

- **The Coulomb interactions**, due to gluon exchange between the final-state heavy quarks, are included at 1-loop level.
Matching

The matching conditions are determined by comparing the expansion in the Mellin moment space to the exact results for the partonic cross section.

Matching terms at NLO

$$Tr\{H^{(1)} S^{(0)} + H^{(0)} S^{(1)}\}$$

(2)


Matching terms at NNLO

$$Tr\{H^{(1)} S^{(1)}\}, \ Tr\{H^{(0)} S^{(2)}\}, \ Tr\{H^{(2)} S^{(0)}\}$$

(3)

are set to zero.
Systematic uncertainties due to missing terms

The uncertainties due to missing contributions in $D_{ij}^{(0)}$ and $R_2$ are part of the systematic uncertainty associated to approximate calculations of this kind which are based on threshold expansions.

Left: The coefficient $C_0^{(2)}$ (scale ind. contribution in $D_{ij}^{(0)}$) is varied within its 5% while $R_2$ is kept fixed. Right: here the coefficient $R_2$ is varied by adding and subtracting $2R_2$ while $C_0^{(2)}$ is kept fixed.
QCD Threshold expansions: “pros and cons”

Approximate calculations based on threshold expansions are not perfect, but can be (easily) highly improved once the full NNLO calculation will be available.

😊 provide a local effective description of the $p_T$ and $y$ distributions that captures the main features of the full calculation.

😊 relatively easy interface to FastNLO or ApplGrid.

😊 provide a fast tool for taking into account correlations ($\alpha_s$, $m_t$, gluon); easy to implement different heavy-quark mass definitions. Dowling, Moch (2014)

😢 Very sensitive to the missing contribution in $D^{(0)}$ and $R_2$.

😢 Scale uncertainty is also affected (at approx NNLO is underestimated at the moment. We’ll improve on this)

😊 At the moment the description is valid near the threshold.