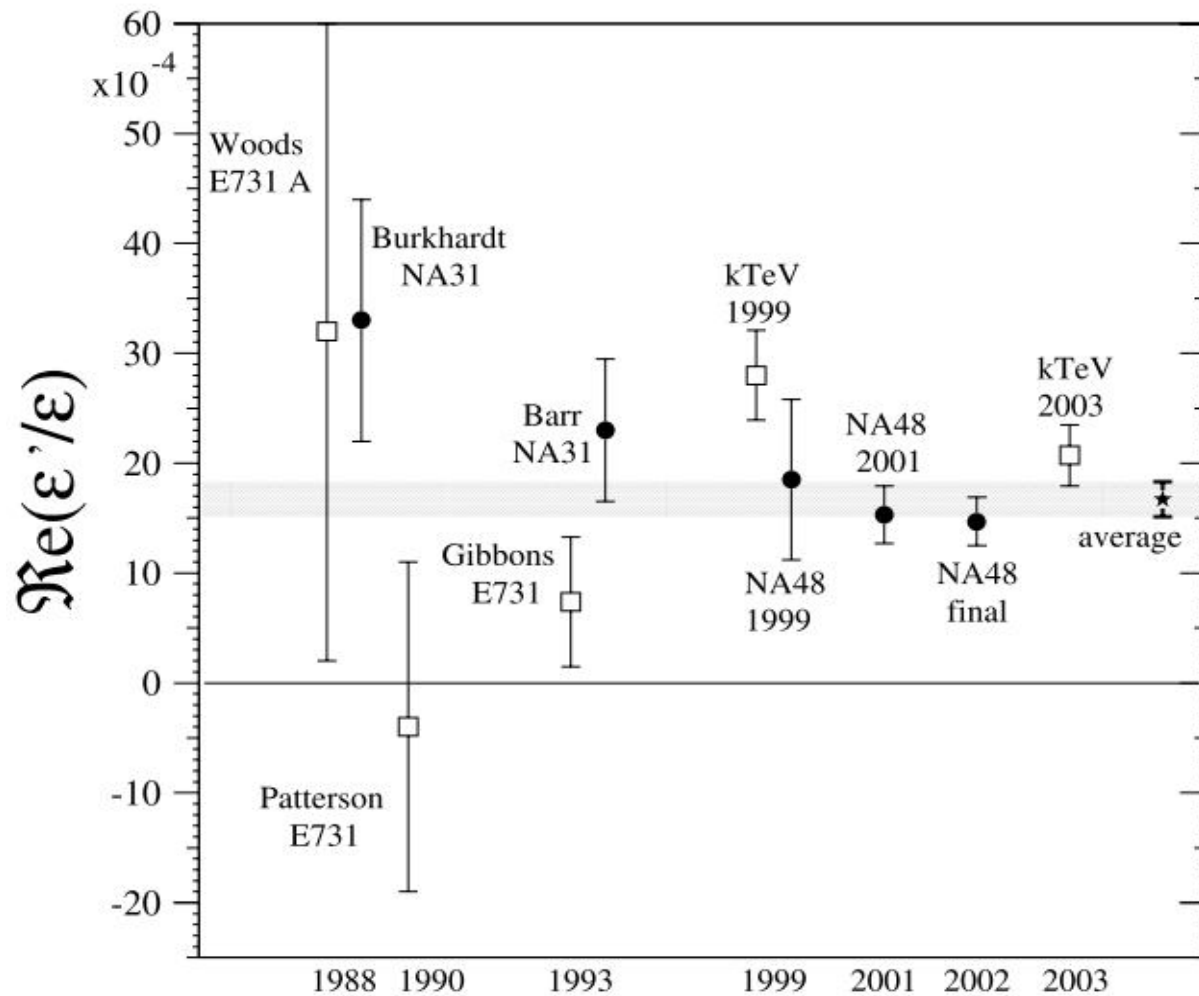# ε′/ε from the lattice and some of its implications

Amarjit Soni, HET-BNL

([adlersoni@gmail.com](mailto:adlersoni@gmail.com))

**EPS 2015; 07/24/15**

Vienna

**Based on RBC-UKQCD arXiv:1505.07683 And manuscript in prep with Lehner And Lunghi**

Konrad kleinknecht "Uncovering CPV"

$16.6(2.3) \times 10^{-4}$

PDG 2014

# outline

- **Long, long time coming: Obstacles aglore!**

- **Reminder of essential basics**

- **Method of choice: Direct K=> $\pi\pi$ a la Lellouch-Luscher**
- **1$^{st}$ results**
- **Few implications**
- **Outlook**

# The RBC&UKQCDcollaborations

_BNL and RBRC_

Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin, Taichi
Kawanai
**Christopher Kelly**
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

_CERN_

Marina Marinkovic

_ColumbiaUniversity_

Ziyuan Bai
Norman Christ
Xu Feng

Luchang Jin
Bob Mawhinney
Greg McGlynn
David Murphy
**Daiqian Zhang**

_UniversityofConnecticut_

Tom Blum

_EdinburghUniversity_

Peter Boyle
Luigi Del Debbio
Julien Frison
Richard Kenway Ava
Khamseh Brian
Pendleton Oliver
Witzel Azusa
Yamaguchi

_PlymouthUniversity_

Nicolas Garron

_UniversityofSouthampton_

Jonathan Flynn
Tadeusz Janowski
Andreas Juettner
Andrew Lawson Edwin
Lizarazo Antonin
Portelli Chris Sachrajda
Francesco Sanfilippo
Matthew Spraggs Tobias
Tsang

_YorkUniversity(Toronto)_

Renwick Hudspith

# MOTHER of all (lattice) calculations to date: A Personal Perspective

- ~1/3 of a century

- 9 PhD thesis: Terry Draper (UCLA), George Hockney(UCLA), Cristian Calin (Columbia=CU), Jack Laiho(Princeton), Sam Li(CU), Matthew Lightman(CU), Elaine Goode(Southampton), Qi Liu(CU), Daiqian Zhang(CU)

- Post-docs & such: Tom Blum (U Conn), Matthew Wingate (Cambridge), Chris Dawson(google), Chris Kelly (RIKEN-BNL-RC)

| | | |
|---|---|---|
| I. **Wilson Fermions with Bernard ~'82**<br>**See also Martinelli et al [WF]**<br>**Giusti et al [WF]**<br>**Sharpe et al [Stag F]** | **Lattice χ S is a pre-requisite for this physics**<br>**Off-shoot B-physics important observables identified & studied=> evolved into UT** | |
| II (a) DWF with Blum ~ '95<br>II(b) DWF with RBC[with Blum, Christ and Mawhinney became "flagship" project of RBC] ~'97. | LOχPT; Quenched approx.[QA]<br>Same<br>QA is disastrous for this physics [Golterman-Pallante] pathologies; NPR of full $\Delta S=1$ accomplished for the 1st time used since then. | CRAY @ NERSC<br><br>QCDSP ~ 1 TF |
| III. DWF with full QCD RBC, ~ '02 | Used LOχPT + full QCD<br>Large chiral corrections | QCDSP ~ 1TF |
| IV. DWF with full QCD RBC + UKQCD, ~ '06 | Direct K=>$\pi\pi$, [Lellouch-Luscher method] @ threshold | QCDOC ~ 10 TF |
| V. DWF with full QCD, RBC + UKQCD ~ '11 | Direct K=>$\pi\pi$, [Lellouch-Luscher method] ; physical kinematics | BG/Q ~ 100TF@BNL; RBRC;ANL; Edinburgh |
| Vi. Same ~now | Same | Seeking new hardware<br><br>~1.5PF;NERSC;ANL;BNL |

$\Delta$ S=1 H$_W$

WL to NLO
Buchalla, Buros, Lautenbacher
RMP /96; Ciuchini etl
95

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[ z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu).$$

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}.$$

**Tree**

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

**QCDP**

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_R,$$

$$Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_L,$$

EWP

$s \underline{\qquad} u$

$\to 0$

$m_q \to 0$

$\longrightarrow const$
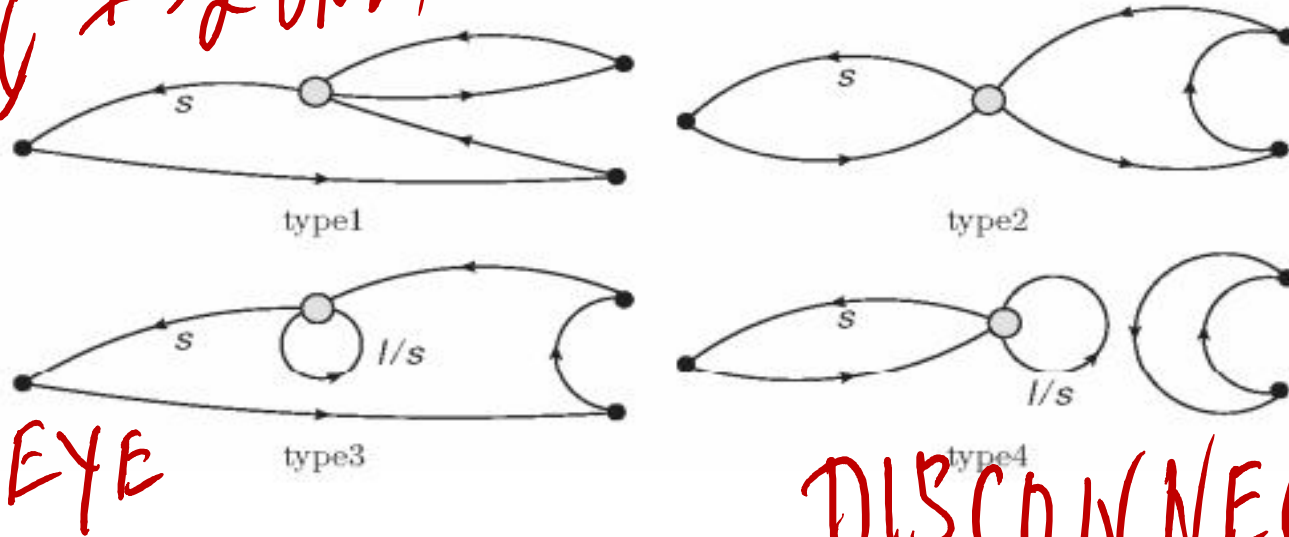
$m \to 0$

QCDP

EWP

I=0
All

I=2 ONLY

EYE

DISCONNECTED

FIG. 1. Examples of the four types of diagram contributing to the $\Delta I = 1/2$, $K \to \pi\pi$ decay. Lines labeled $\ell$ or $s$ represent light or strange quarks. Unlabeled lines are light quarks.

type1  type2  type3  type4

# Ensemble

- $32^3 \times 64$ Mobius DWF ensemble with IDSDR gauge action at $\beta$=1.75. Coarse lattice spacing ($a^{-1}$=1.378(7) GeV) but large, (4.6 fm)$^3$ box.

- Using Mobius params (b+c)=32/12 and $L_s$ =12 obtain same explicit $\chi$SB as the $L_s$=32 Shamir DWF + IDSDR ens. used for $\Delta I$=3/2 but at reduced cost.

- Utilized USQCD 512-node BG/Q machine at BNL, the DOE "Mira" BG/Q machines at ANL and the STFC BG/Q "DiRAC" machines at Edinburgh, UK.

- Performed 216 independent measurements (4 MDTU sep.).

- Cost is ~1 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).

- G-parity BCs in 3 spatial directions results in close matching of kaon and $\pi\pi$ energies:

$PHYSICAL\ MASSES$
$of\ Kinematics\ !$

$m_K$=490.6(2.4) MeV $\leftarrow$

$E_{\pi\pi}$(I=0) = 498(11) MeV

$E_{\pi\pi}$(I=2) = 573.0(2.9) MeV

$E_\pi$=274.6(1.4) MeV $\quad$ ($m_\pi$ = 143.1(2.0) MeV) $\leftarrow$

85% 〉15%

<1% ←

$Q_2 = \dfrac{\bar{W} \bar{u} d}{s \quad u}$

large
↑ cancellate
→ dominant

| i | Re($A_0$)(GeV) | Im($A_0$)(GeV) |
|---|---|---|
| 1 | $1.02(0.20)(0.07) \times 10^{-7}$ | 0 |
| 2 | $3.63(0.91)(0.28) \times 10^{-7}$ | 0 |
| 3 | $-1.19(1.58)(1.12) \times 10^{-10}$ | $1.54(2.04)(1.45) \times 10^{-12}$ |
| 4 | $-1.86(0.63)(0.33) \times 10^{-9}$ | $1.82(0.62)(0.32) \times 10^{-11}$ |
| 5 | $-8.72(2.17)(1.80) \times 10^{-10}$ | $1.57(0.39)(0.32) \times 10^{-12}$ |
| 6 | $3.33(0.85)(0.22) \times 10^{-9}$ | $-3.57(0.91)(0.24) \times 10^{-11}$ |
| 7 | $2.40(0.41)(0.00) \times 10^{-11}$ | $8.55(1.45)(0.00) \times 10^{-14}$ |
| 8 | $-1.33(0.04)(0.00) \times 10^{-10}$ | $-1.71(0.05)(0.00) \times 10^{-12}$ |
| 9 | $-7.12(1.90)(0.46) \times 10^{-12}$ | $-2.43(0.65)(0.16) \times 10^{-12}$ |
| 10 | $7.57(2.72)(0.71) \times 10^{-12}$ | $-4.74(1.70)(0.44) \times 10^{-13}$ |
| Tot | $4.66(0.96)(0.27) \times 10^{-7}$ | $-1.90(1.19)(0.32) \times 10^{-11}$ |

TABLE I. Contributions to $A_0$ from the ten continuum, $\overline{\text{MS}}$ operators $Q_i(\mu)$, for $\mu = 1.53$ GeV. Two statistical errors are shown: one from the lattice matrix element (left) and one from the lattice to $\overline{\text{MS}}$ conversion (right).

| Description | Error | Description | Error |
|---|---|---|---|
| Finite lattice spacing | 8% | Finite volume | 7% |
| Wilson coefficients | 12% | Excited states | $\leq 5\%$ |
| Parametric errors | 5% | Operator renormalization | 15% |
| Unphysical kinematics | $\leq 3\%$ | Lellouch-Lüscher factor | 11% |
| Total (added in quadrature) | | | 26% |

TABLE II. Representative, fractional systematic errors for the individual operator contributions to $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$.

$$\mathrm{Re}(A_0) = 4.66(1.00)(1.21) \times 10^{-7} \, \mathrm{GeV}$$

$$\mathrm{Im}(A_0) = -1.90(1.23)(1.04) \times 10^{-11} \, \mathrm{GeV}$$

**3.32 × 10⁻⁷ GeV expt** → $3.32 \times 10^{-7}$ GeV expt

**1.48 × 10⁻⁸ Expt** → $1.48 \times 10^{-8}$ Expt

$$\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}} 10^{-8} \, \mathrm{GeV},$$

$$\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13} \, \mathrm{GeV}.$$

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8} \, \mathrm{GeV};$$

$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13} \, \mathrm{GeV}.$$

2012 PRD
$a^{-1} = 1.364$ GeV
$32^3 \times 64 \times 32$

$a^{-1} = 1.728$ GeV $\, 48^3 \times 96 \times 24$
$= 2.358$ GeV $\, 64^3 \times 128 \times 12$

→ 2015 PRD
Continuum

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{EWP}} = -(6.6 \pm 1.0) \times 10^{-4}$$

For A2 error is now completely dominated by perturbation theory calculation of Wilson coeffs.

13

# Results for ε'

*EWP*

*QCDP*

- Using Re(A$_0$) and Re(A$_2$) from experiment       and our lattice values for Im(A$_0$) and Im(A$_2$) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 1.38(5.15)(4.43) \times 10^{-4}, \quad \text{(this work)}$$
$$16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

Bearing in mind the largish errors in this first calculation, we interpret that our result agrees with experiment at ~2σ level

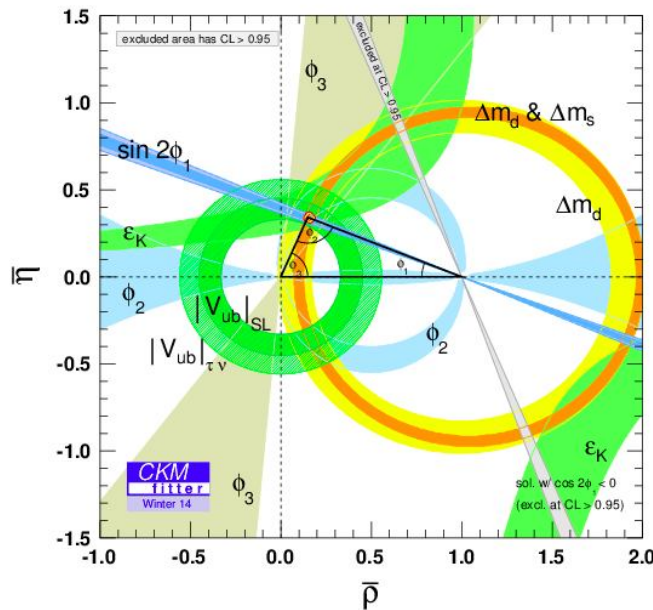# Proof of the pudding: underlying method is systematically improvable

- BK in full QCD with DWF '07 error O(7%)
- ~2012 many discretizations , WA error O(1-2%)
- KI3 O(1/2%), A2 O(10%) , fB's O(few %) , BB's O(few%)........


- 0 doubt that A0, A2 for ε' will not go that way for quite
  sometime to come.........to ~10% total

After that EM & isospin effects will have to be ascertained quantitatively.
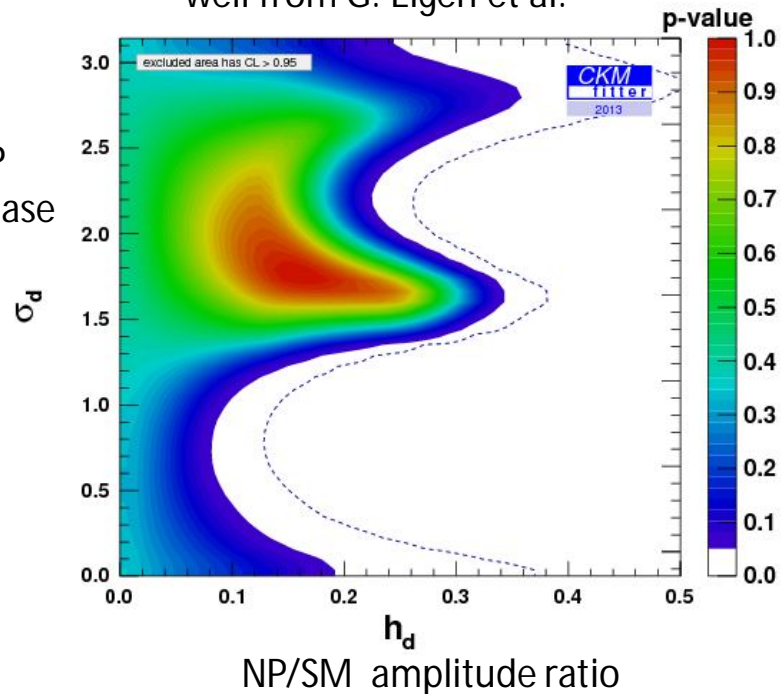
# Results from Global Fits to Data (CKMFitter Group)

**Great progress on $\varphi_3$ or $\gamma$ (first from B factories and now in the last two years from LHCb (several new results at ICHEP2014). These measure the phase of $V_{ub}$**

ICHEP2014: Similar results from UTFIT (D. Derkach) as well from G. Eigen et al.

NP Phase

NP/SM amplitude ratio

Looks good
(except for an issue with $|V_{ub}|$)

But a 10-20% NP amplitude in $B_d$ mixing is perfectly compatible with all current data.

# *A lesson from history (I)*

-----------------------------------------------------------------------------------------------------

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single $K_L \rightarrow \pi^+ \pi^-$ event among 600 decays into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

     -Lev Okun, "The Vacuum as Seen from Moscow"

-----------------------------------------------------------------------------------------------------

1964: BF= 2 x 10$^{-3}$

A failure of imagination ? Lack of patience ?

CHRISTENSON, CRONIN, FITCH + TURLAY BNL '64

=> **Precision! Precision! Precision! Need of the day**.
=> **Also, since we are  searching for small effects, using different probes may be valuable**

=

- In B's, in conjunction with experiments, Lattice WME helped in attaining a milestone in our understanding of CP

- Analogously can lattice sharpen tests now via K's?

- Since $m_K$ is ~10 times lighter, the non-perturbative effects are much more difficult and quantitatively a lot bigger, can the lattice meet this long-standing challenge and render K-tests become precise?

# Promising developments on the lattice in K-decays.........RBC-UKQCD
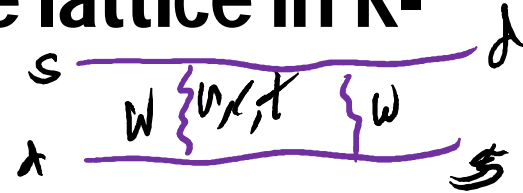
- In the process of taming $\varepsilon'$ also

- Long-distance (non-local) effects; most interesting & important in

 $\Delta mK$ because of extreme sensitivity to chiral structure of Heff see Beall, Bander + AS, PRL '82 ....$\delta O(40\%)$ Brod & Gorban

   See N.Christ et al PRD'13; PRL'14...  Look forward to

$\Delta mK$ from lattice as a useful observable for constraining NP.

- $\varepsilon K$ LD .....$\delta O(7\%)$.....N.Christ talk @LAT'15 & many more

- $K+ => \pi \nu \nu$......$\delta O(few\%)$.......Xu Feng talk @ lat'15

- $K => \pi e e$..........A. Lawson talk @ Lat'15; [A.Portelli]; C. Sachrajda @LAT'14

- => Pathways to K-UT
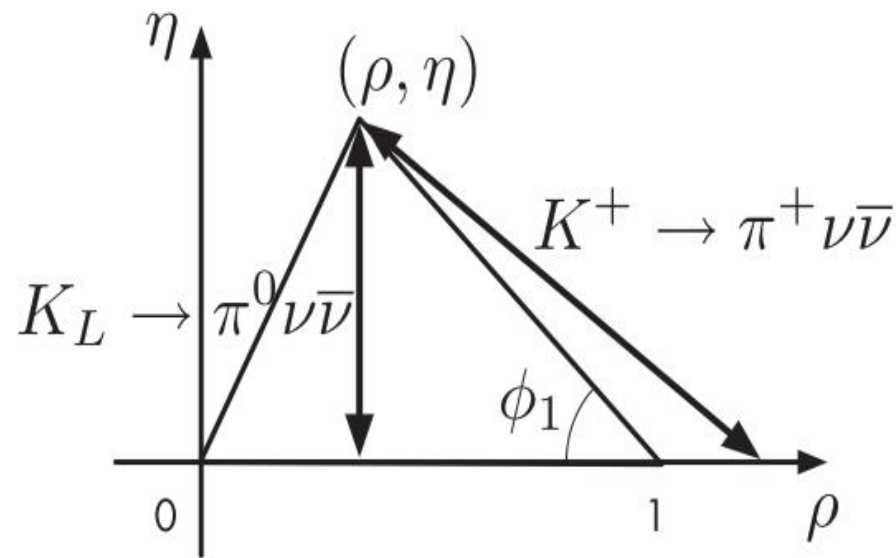
# A dream for some

Blucher, Winstein and Yamanaka '09

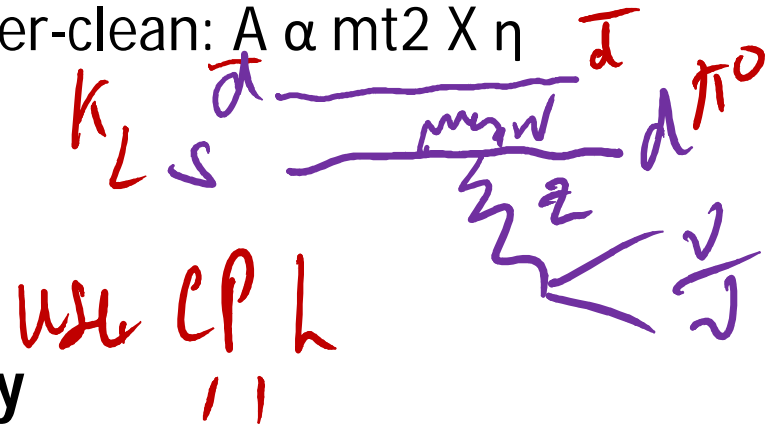

Fig. 14. Unitarity triangle.

A Faster way in the offing?

# More on K-decays=>rare K's

- KL => π0 ν ν  ..."Gold-plated", i.e Theory super-clean: $A \propto mt2 \times \eta$

*(handwritten, red)* LITTENBERG PRD '89

*(handwritten, red)* Nothing → Nothing

*(handwritten, red)* Taku Yamanaka @ CKM2014

*(handwritten, red)* use CP L

- **Observe: The above expt is exceedingly**

challenging (esp for precision) and expensive.

- **Assertion: Once the (exptal) community realizes we mean business by reducing errors on Im A0 to around ~ 20% they will get the message loud and clear: It is much more cost effective to invest in better lattice calculation(s) of eps' ......**

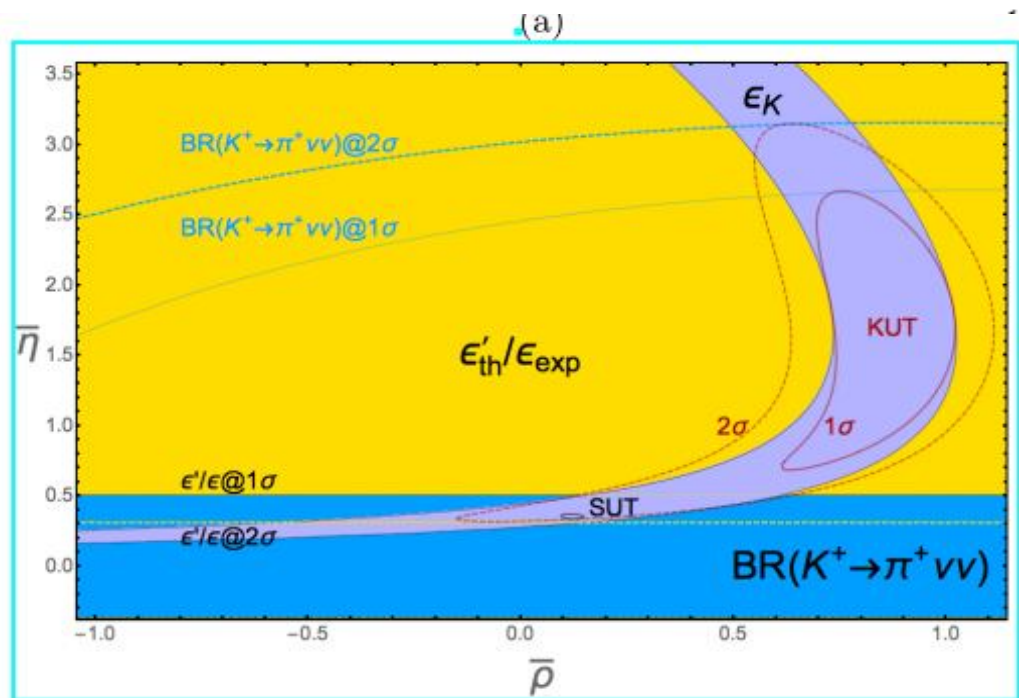# Lattice ε'/ε & SUT ≅ The UT.
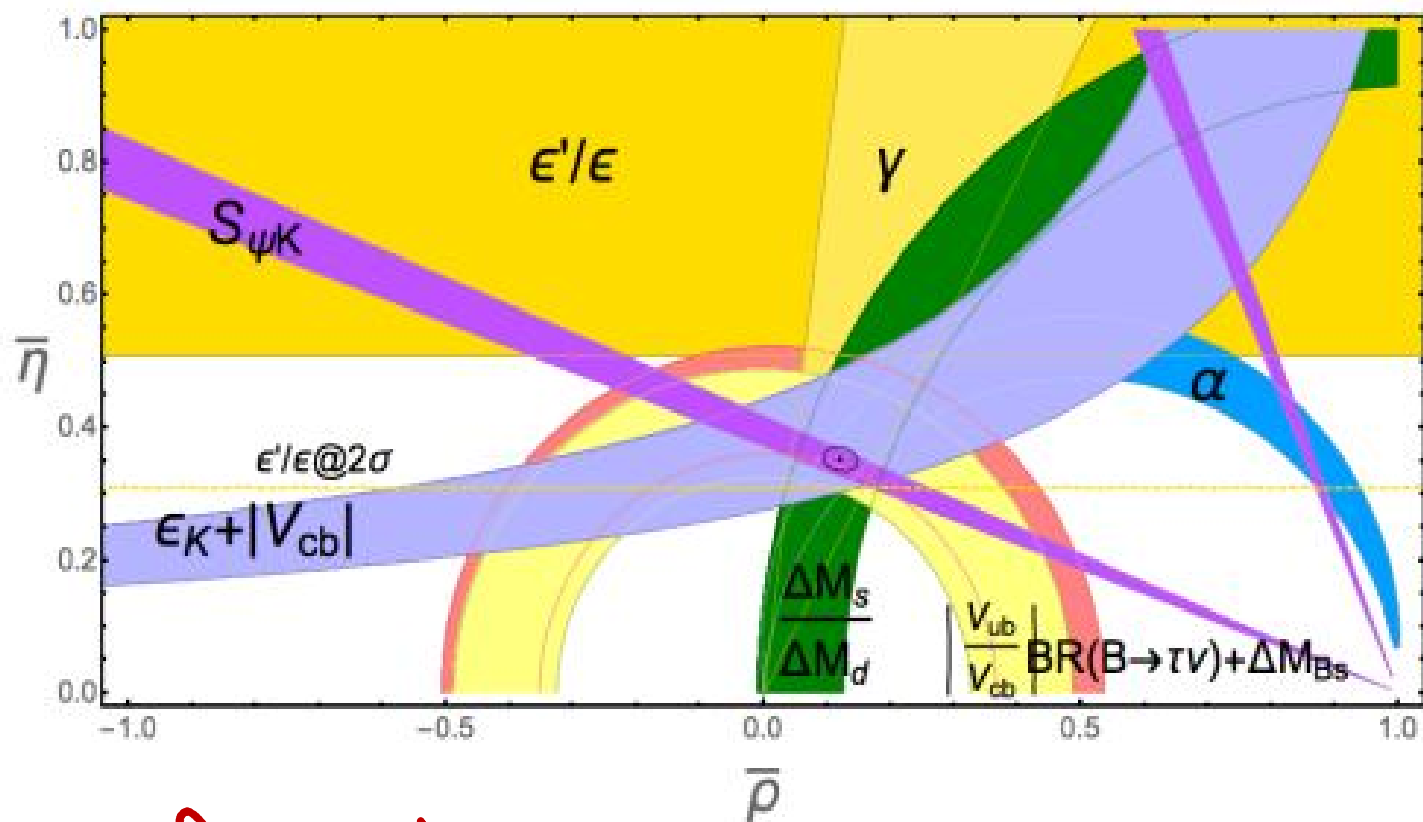
LLS in prep

# Sketch of an emerging K-UT

$$\mathrm{BR}(K^+ \to \pi^+ \nu \bar{\nu}) = \begin{cases} (8.64 \pm 0.60) \times 10^{-11} & \text{SM} \\ (17.3^{+11.5}_{-10.5}) \times 10^{-11} & \text{E949} \end{cases}$$

BNL



(a)

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_K = \begin{cases} (16.7 \pm 1.6) \times 10^{-4} \\ (1.36 \pm 5.21_{\text{stat}} \pm 4.49_{\text{syst}}) \times 10^{-4} \end{cases}$$
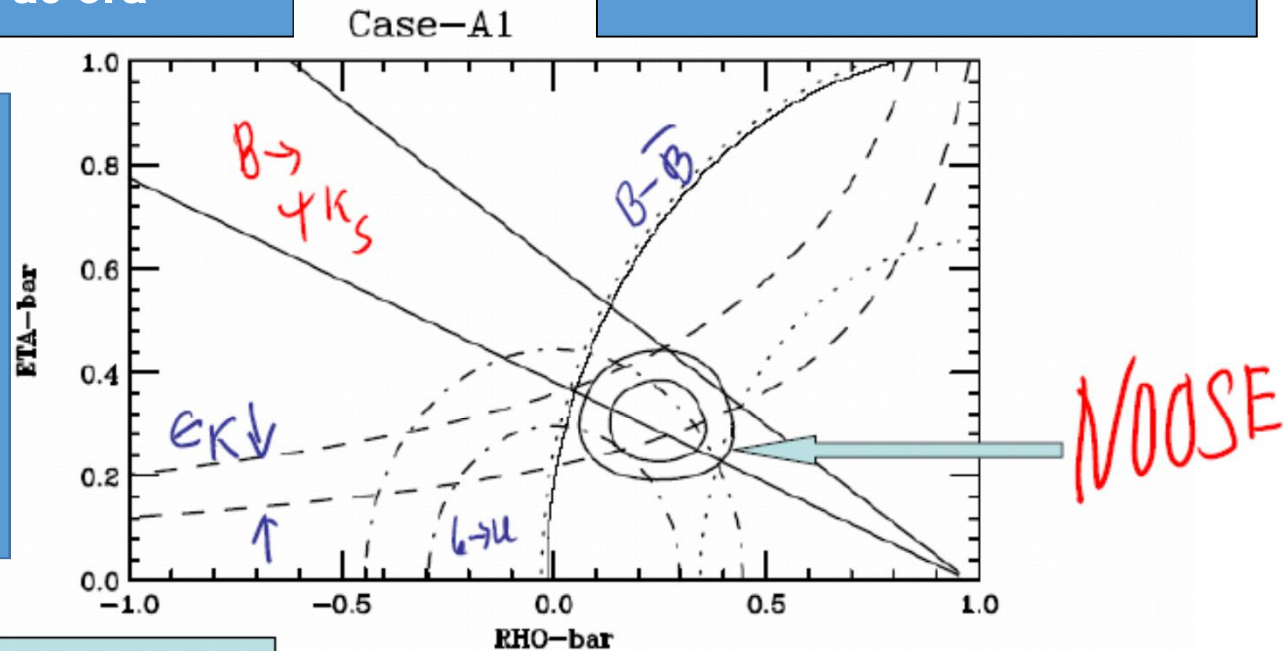
PDG-2015

RBC+UK QCD '15

LLS '15

**In the "beginning"   "Dawn " of the asymmetric B-Fac era**

**Atwood &AS, hepph/0103197**

B-CP Feb'01 Ise, Japan

**1st Hint of confirmation of CKM CP description**

Case—A1



Most bands due
To theory errors

**New physics will be a perturbation, important to use clean theory and lots of statistics.**

*Legendary American Philosopher* (handwritten)

| Yogi Berra |
|---|

Catcher / Manager/ Outfielder
**Born:** May 12, 1925
St. Louis, Missouri

| Batted: Left | Threw: Right |
|---|---|

**MLB debut**
September 22, 1946 for the New York Yankees

**Last MLB appearance**
May 9, 1965 for the New York Mets

# The Future

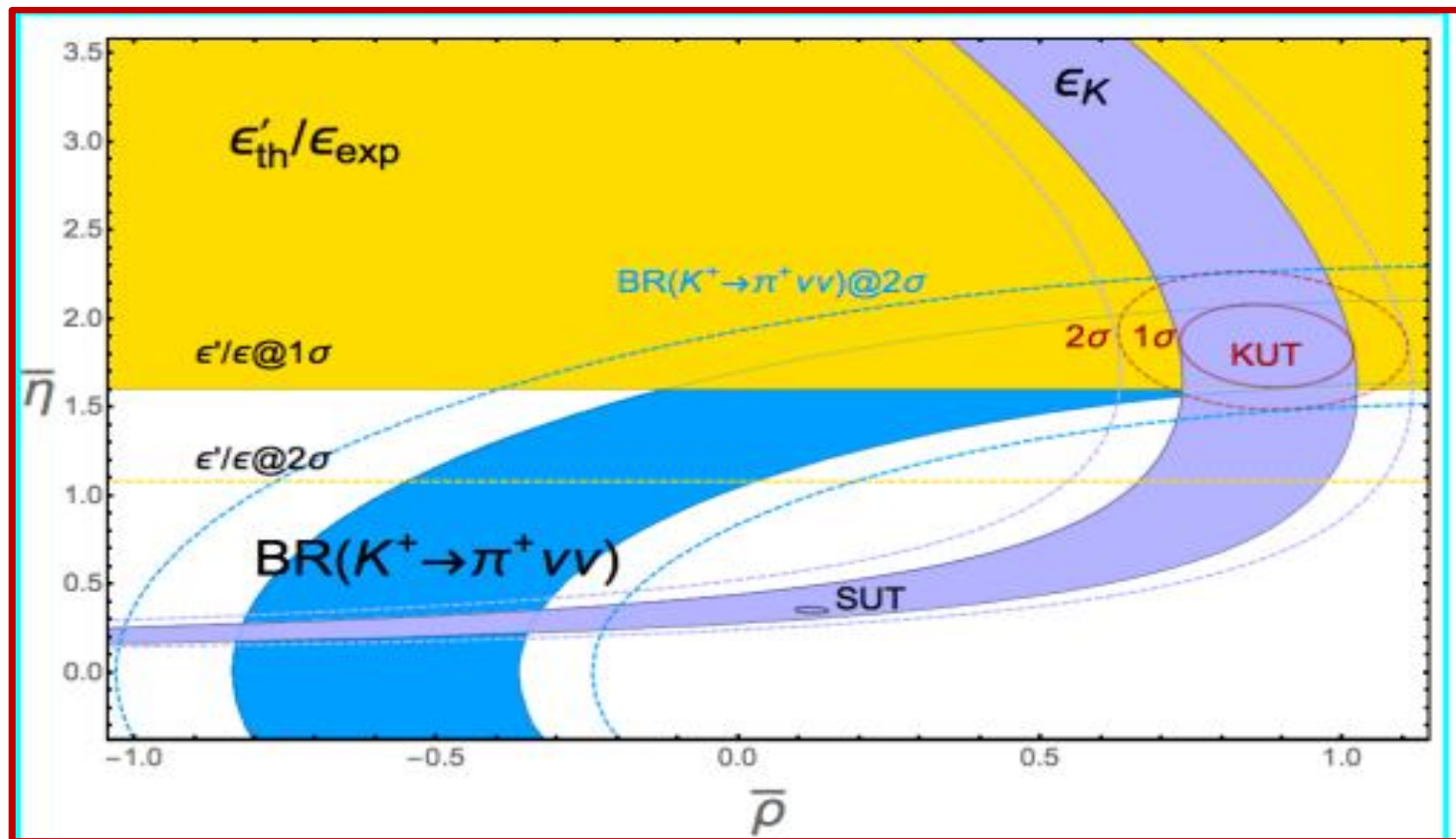- Yogi Berra: "Its difficult to make predictions, especially about the future" *

- New York Yankees (1964, 1984-1985)
- New York Mets (1972-1975)

**Career highlights and awards**
- 15× All-Star selection (1948, 1949, 1950, 1951, 1952, 1953, 1954, 1955, 1956, 1957, 1958, 1959, 1960, 1961, 1962)
- 13× World Series champion (1947, 1949, 1950, 1951, 1952, 1953, 1956, 1958, 1961, 1962, 1969, 1977, 1978)
- 3× AL MVP (1951, 1954, 1955)
- New York Yankees #8 retired
- Major League Baseball All-Century Team

Member of the National
Baseball Hall of Fame

| Induction | 1972 |
|---|---|
| Vote | 85.61% (second ballot) |

*Stolen from Sheldon Stone* (handwritten)

# POSSIBLE KUT CIRCA 2020

ILLUSTRATION



NO
unique
$\bar{\rho}, \bar{\eta}$ !
use
NA62
$K^+$
10 /

# A new observable on the horizon

$\epsilon'_{th}/\epsilon_K$

CP Conserving observable



(a)                                        (b)

π⁰ e e
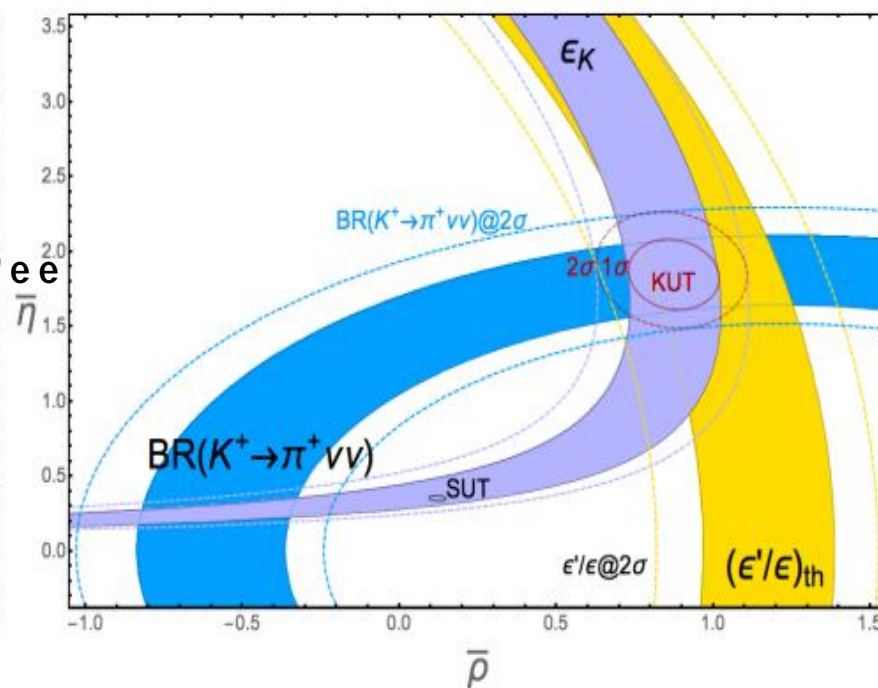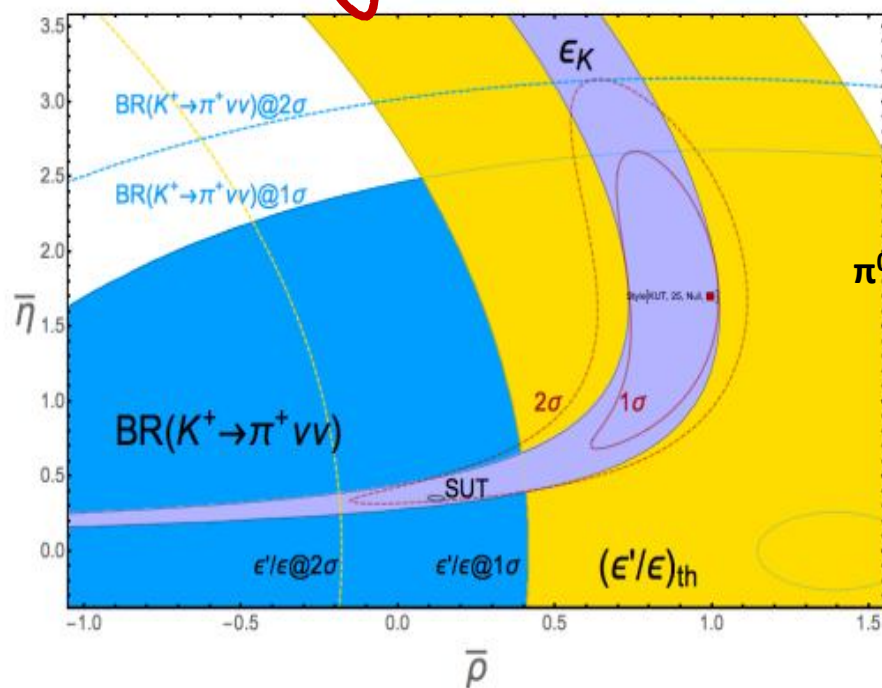
See Lehner, Lunghi AS In prep

# Summary + outlook

- Significant progress in K=> ππ with physical masses and kinematics

- Presented 1$^{st}$ computation of ε'/ε with controlled errors:

- $1.38(5.15)(4.43) \times 10^{-4}$    ; $16.6(2.3) \times 10^{-4}$ expt

- Trying hard to reduce syst and stat errors

- Fall '15 detailed paper, hopefully with some improvements

- New (faster) hardware later this year or '16=> should have significantly reduced errors in 1-3 years

- Expect errors < ~ 10% in ~5 years; thence EM & isospin needs tackling

- Experimentalists ought to think of improved measurements of ε', error now ~15%

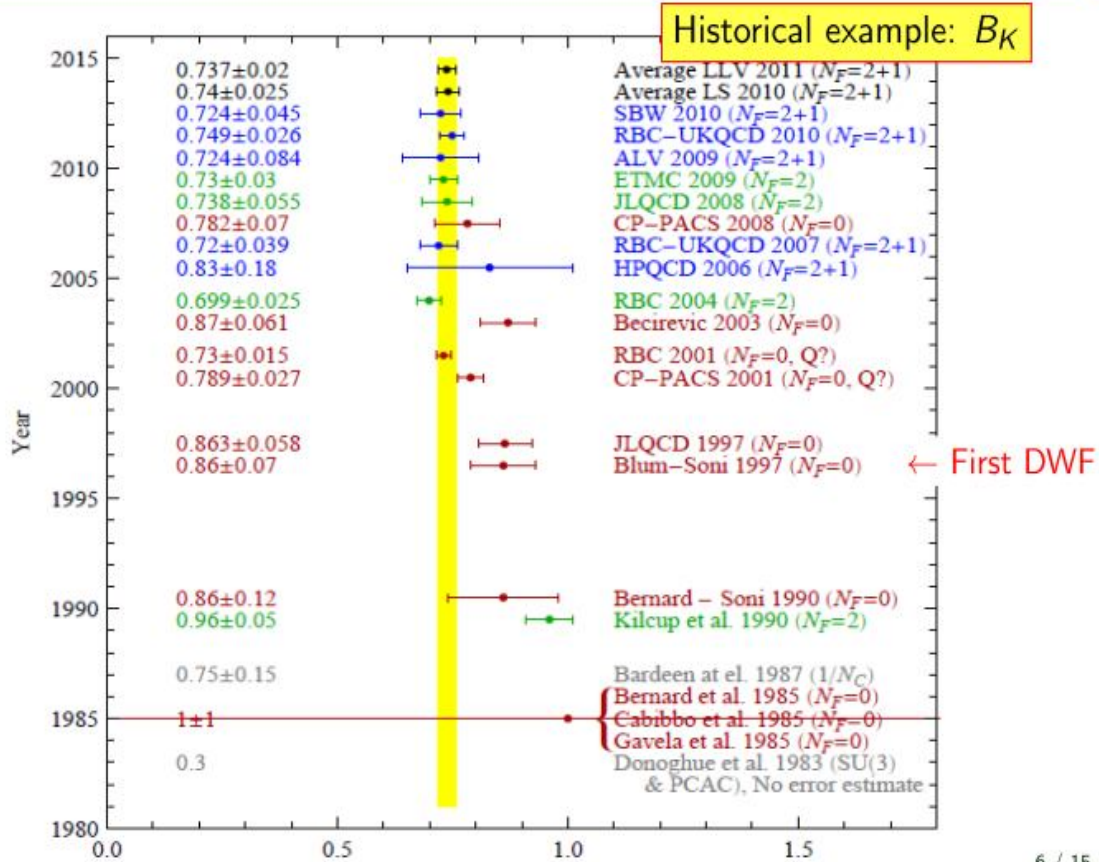- Perhaps easier than precise measurement of KL=>πνν

# xtras
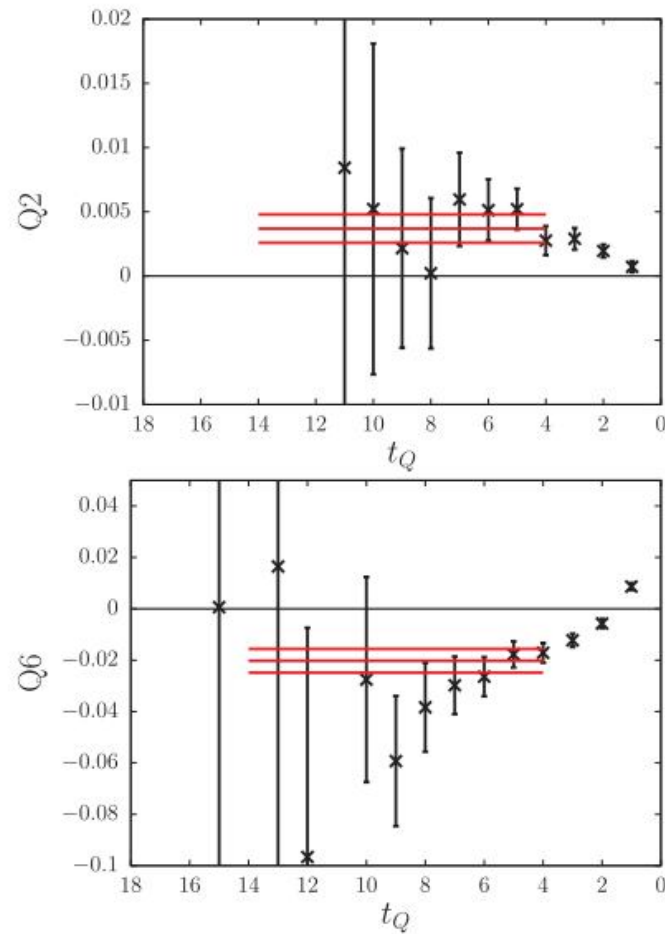
EXAMPLES

For now, signal is rather weak; a lot more statistics is needed

Power of the lattice: Only method to systematically reduce the NP error!

Historical example: $B_K$

← First DWF

AB-initio Calculation

$$B_K \equiv \frac{\langle K \mid (\bar{s}\gamma_\mu d)^2 \mid \bar{K} \rangle}{8/3 \, f_K^2 \, m_K^2}$$

Status before lattice 2014

FLAG [Aoki et al., '13-14]

*Garron LAT14*

$\hat{B}_K$

FLAG2013

our estimate for $N_f = 2+1$
SWME 13
RBC/UKQCD 12
Laiho 11
SWME 11A
BMW 11
RBC/UKQCD 10B
SWME 10
Aubin 09
RBC/UKQCD 07A, 08

$N_f = 2+1$

our estimate for $N_f = 2$
ETM 10A
JLQCD 08
RBC 04

$N_f = 2$

0.65  0.70  0.75  0.80  0.85
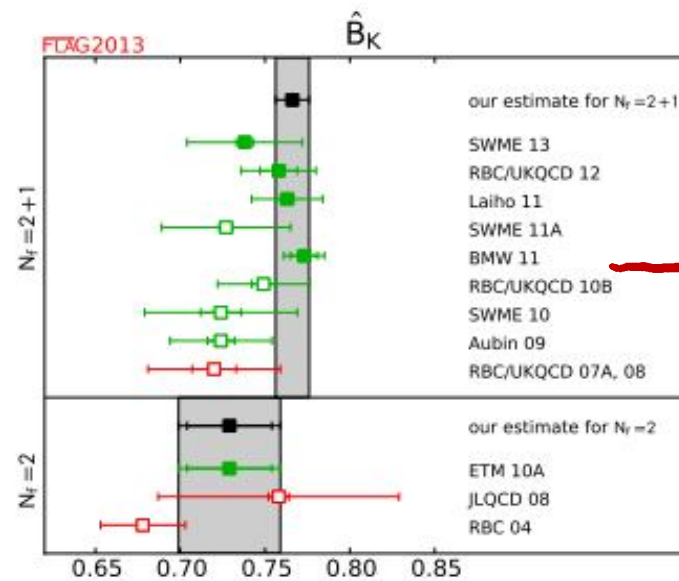
*FLAG 2013*

$$N_f = 2+1: \qquad \hat{B}_K = 0.7661(99),$$

*~ 1.3%.*

Lattice weak decays, IIT Guwahati   A. Soni

33

# 10 ops are not linearly independent

$$Q_4 = -Q_1 + Q_2 + Q_3,$$

$$Q_9 = \frac{3}{2} Q_1 - \frac{1}{2} Q_3,$$

$$Q_{10} = \frac{1}{2} Q_1 + Q_2 - \frac{1}{2} Q_3.$$

RBC    QA; ChPT    PRD ~02

TABLE XLIX. Our final values for physical quantities using one-loop full QCD extrapolations to the physical kaon mass (choice 2) and a value of $\mu = 2.13$ GeV for the matching between the lattice and continuum. The errors for our calculation are statistical only. ←

| Quantity | Experiment | This calculation (statistical errors only) |
|---|---|---|
| Re $A_0$(GeV) | $3.33 \times 10^{-7}$ | $(2.96 \pm 0.17) \times 10^{-7}$ |
| Re $A_2$(GeV) | $1.50 \times 10^{-8}$ | $(1.172 \pm 0.053) \times 10^{-8}$ |
| $\omega^{-1}$ | 22.2 | $(25.3 \pm 1.8)$ |
| Re$(\epsilon'/\epsilon)$ | $(15.3 \pm 2.6) \times 10^{-4}$(NA 48) | $(-4.0 \pm 2.3) \times 10^{-4}$ |
| | $(20.7 \pm 2.8) \times 10^{-4}$(KTEV) | |

C ALSO CRAAES

RBC = RMRC + ANL + C.W

# Extremely serious quench patholgy

- Most important for Q6 as it LR=> (S+P)(S-P);  AND it makes the most important contribution to ε′

Source of problem is that H_eff for ΔS=1 has operators such as Q6 with Quark content

$(\bar{S}d)$ $(\bar{u}u)$ → quark loop from weak interaction

Quench approx

$Q_6$ gets unphysical contribution for $Q_8$

$(8,1)$                                                                                    $(8,8)$

*Full QCD) But ChPTa la BDSPW*

## Conclusion

(Sam)Shu Li, PhD thesis,
Columbia '08

| Quantity | This analysis | Quenched | Experiment |
|---|---|---|---|
| $\text{Re}A_0$ (GeV) | $4.5(11)(53) \times 10^{-7}$ | $2.96(17) \times 10^{-7}$ | $3.33 \times 10^{-7}$ |
| $\text{Re}A_2$ (GeV) | $8.57(99)(300) \times 10^{-9}$ | $1.172(53) \times 10^{-8}$ | $1.50 \times 10^{-8}$ |
| $\text{Im}A_0$ (GeV) | $-6.5(18)(77) \times 10^{-11}$ | $-2.35(40) \times 10^{-11}$ | |
| $\text{Im}A_2$ (GeV) | $-7.9(16)(39) \times 10^{-13}$ | $-1.264(72) \times 10^{-12}$ | |
| $1/\omega$ | $50(13)(62)$ | $25.3(1.8)$ | $22.2$ |
| $\text{Re}(\epsilon'/\epsilon)$ | $7.6(68)(256) \times 10^{-4}$ | $-4.0(2.3) \times 10^{-4}$ | $1.65 \times 10^{-3}$ |

- ChPT approach to $K \rightarrow \pi\pi$ faces <u>severe difficulties</u>.

- RBC/UKQCD studying physical $\pi\pi$ final states.

- DWF on coarse lattices and large volumes: $4 \rightarrow 5$ fm?

- Vranas auxiliary determinant (Renfrew talk on Wed.)

*LARGE SYSTEMATIC errors DUE ChPT*

Lattice

N. Christ @LAT08

# Mass depends of ReA2, A0

| | $a^{-1}$ [GeV] | $m_\pi$ [MeV] | $m_K$ [MeV] | ReA$_2$ [$10^{-8}$ GeV] | ReA$_0$ [$10^{-8}$ GeV] | $\frac{\text{ReA}_0}{\text{ReA}_2}$ | notes |
|---|---|---|---|---|---|---|---|
| $16^3$ Iwasaki | 1.73(3) | 422(7) | 878(15) | 4.911(31) | 45(10) | 9.1(2.1) | threshold calculation |
| $24^3$ Iwasaki | 1.73(3) | 329(6) | 662(11) | 2.668(14) | 32.1(4.6) | 12.0(1.7) | threshold calculation |
| IDSDR | 1.36(1) | 142.9(1.1) | 511.3(3.9) | 1.38(5)(26) | - | - | physical kinematics |
| Experiment | – | 135 - 140 | 494 - 498 | 1.479(4) | 33.2(2) | 22.45(6) | |

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

**Due to the cancellation, 3/2 amplitude decreases significantly as the
pion mass is lowered towards its physical value**