Non-zero θ_{13} and δ_{CP} in a Neutrino Mass Model with A_4 Symmetry

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Plan of talk

1. A_4 Symmetric Neutrino mass model
2. $Z_2 \times Z_2$ perturbations in charged lepton sector
3. $Z_2 \times Z_2$ perturbations in neutrino sector
4. Conclusions

Based on the paper by
Abhish Dev, P. Ramadevi and S. Uma Sankar, arXiv:1504.04034

A_4 Symmetric Neutrino mass model

The neutrino mass model we consider was proposed by
X.-G. He, Y.-Y. Keum and R. R. Volkas (hep-ph/0601001)
JHEP **0604** 039 (2006).
The fields and their group charges are given in the table below.

	$SU(2)$	$U(1)$	A_4	
D_{iL}	$\underline{\frac{1}{2}}$	Y=-1	$\underline{3}$	left-handed doublets
l_{iR}	$\underline{0}$	Y=-2	$\underline{1} \oplus \underline{1}' \oplus \underline{1}''$	right-handed charged lepton singlets
ν_{iR}	$\underline{0}$	Y= 0	$\underline{3}$	right-handed neutrino singlets
ϕ_i	$\underline{\frac{1}{2}}$	Y= 1	$\underline{3}$	complex scalar SU(2) doublet
ϕ_0	$\underline{\frac{1}{2}}$	Y= 1	$\underline{1}$	complex scalar SU(2) doublet
χ_i	$\underline{0}$	Y= 0	$\underline{3}$	real scalar SU(2) singlet

Table : Assignments of lepton and scalar fields to various irreps of $SU(2)$, $U(1)$, and A_4.

A_4 Symmetric Neutrino mass model

One can write down the most general $SU(2)_L \times U(1)_Y \times A_4$ invariant Lagrangian in terms of these fields.

An additional $U(1)_X$ symmetry is also imposed to forbid some unwanted neutrino mass terms which spoil the tribimaximal form of the leptonic mixing matrix.

Spontaneous symmetry breaking leads VEVs for various scalars: v_i for ϕ_i, w_i for χ_i and v_0 for ϕ_0.

This leads to mass terms of the form

$$-\bar{l}_L M_l^0 l_R - \bar{\nu}_L M_D \nu_R + \frac{1}{2}\nu_R^T C^{-1} M_R \nu_R + h.c.,$$

where

$$M_l^0 = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 v_2 \omega^2 & h_3 v_2 \omega \\ h_1 v_3 & h_2 v_3 \omega & h_3 v_3 \omega^2 \end{pmatrix}, \ M_R = \begin{pmatrix} M & h_\chi w_3 & h_\chi w_2 \\ h_\chi w_3 & M & h_\chi w_1 \\ h_\chi w_2 & h_\chi w_1 & M \end{pmatrix},$$

and $M_D = h_0 v_0 I$.

A_4 Symmetric Neutrino mass model

Tribimaximal mixing requires a special vacuum alignment

$$v_1 = v_2 = v_3 = v, \;\; w_1 = w_3 = 0, \text{and } h_\chi w_2 = M^{'}.$$

With these VEVs, the charged lepton mass matrix can be put in diagonal form by the transformation $U_\omega M_l^0 I$ and the Majorana mass matrix of the neutrinos is transformed to diagonal form by $U_\nu M_R U_\nu^\dagger$.
The matrices U_ω and U_ν are given by

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

The PMNS matrix $U = U_\omega U_\nu$ is in the tribimaximal form upto phases on both the sides.

Non-zero U_{e3} by perturbing charged lepton sector

- U_{e3} element of the tribimaximal form is zero because the 11 and 13 elements of U_ω are equal. By disturbing this equality we can get non-zero U_{e3}.
- A multiplicative factor i^{th} row of the charged lepton mass matrix leads to reciprocal factor in the i^{th} column of U_ω.
- We introduce perturbations in the first and third rows of the charged lepton mass matrix so that corresponding changes occur in the first and third column of U_ω and its 11 and 13 elements will no longer be equal and U_{e3} will be non-zero.
- This can be done in a simple way by introducing $Z_2 \times Z_2$ perturbations in the charged lepton sector.

$Z_2 \times Z_2$ perturbation in charged lepton sector

$Z_2 \times Z_2$ invariant perturbation, for the charged lepton mass matrix, is

$$\underset{(\underline{\mathbf{3}})}{h_1 \bar{\mathrm{D}}_{\mathrm{L}}} M_1 \underset{(\underline{\mathbf{3}})}{\phi} \underset{(\underline{\hat{\mathbf{1}}})}{l_{1\mathrm{R}}} + \underset{(\underline{\mathbf{3}})}{h_2 \bar{\mathrm{D}}_{\mathrm{L}}} M_2 \underset{(\underline{\mathbf{3}})}{\phi} \underset{(\underline{\hat{\mathbf{1}}})}{l_{2\mathrm{R}}} + \underset{(\underline{\mathbf{3}})}{h_3 \bar{\mathrm{D}}_{\mathrm{L}}} M_3 \underset{(\underline{\mathbf{3}})}{\phi} \underset{(\underline{\hat{\mathbf{1}}})}{l_{3\mathrm{R}}},$$

where the matrices M_1, M_2 and M_3 are diagonal.
To keep the discussion simple, we choose a particular form of $M_i = \mathrm{diag}(\bar{z}, 0, \omega^{i-1} z)$, where all the matrix elements are parametrized by a single complex number z.
This leads to the following perturbation in the charged lepton mass matrix:

$$\Delta M_l = \begin{pmatrix} h_1 v \bar{z} & h_2 v \bar{z} & h_3 v \bar{z} \\ 0 & 0 & 0 \\ h_1 v z & h_2 v z \omega & h_3 v z \omega^2 \end{pmatrix}.$$

Such a ΔM_l can arise from higher order effects of the theory.

$Z_2 \times Z_2$ perturbation in for charged lepton sector

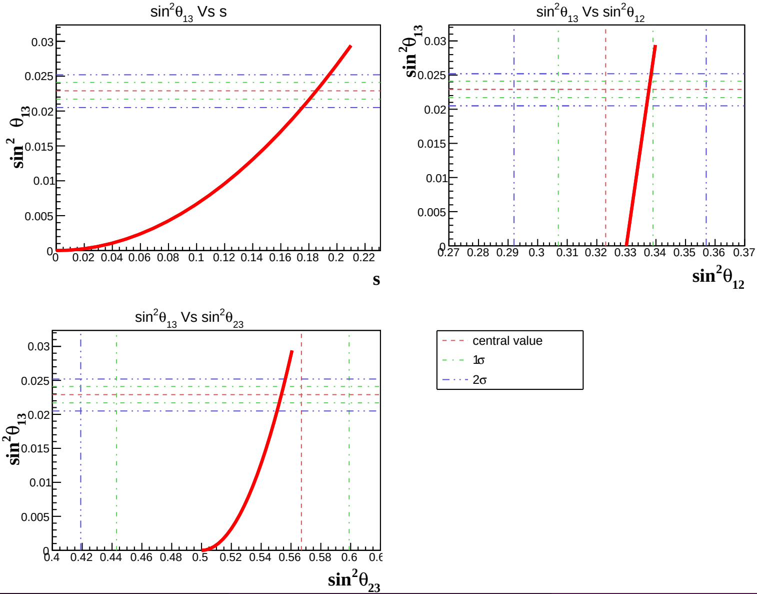
Requring the diagonalizing matrix of $M_l^0 + \Delta M_l$ to be unitary leads to the constraint $z = -1 + \sqrt{1 - s^2} + is$, where s is a small real number. Parametrizing $s = \sin\alpha$, we get the modified PMNS matrix to be

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

We can obtain the following expressions for the mixing angles

$$\begin{aligned} \sin^2\theta_{13} &= \frac{2}{3}\sin^2\alpha &&= \frac{2s^2}{3} \\ \sin^2\theta_{12} &= \frac{1}{2+\cos 2\alpha} &&= \frac{1}{3} + \frac{2s^2}{9} + O(s^3) \\ \sin^2\theta_{23} &= \frac{2+\cos 2\alpha + \sqrt{3}\sin 2\alpha}{2(2+\cos\, 2\alpha)} &&= \frac{1}{2} + \frac{s}{\sqrt{3}} + O(s^3). \end{aligned} \tag{1}$$

Variation of Mixing angles with s



$Z_2 \times Z_2$ perturbation in Neutrino Sector

- Another reason for the vanishing of U_{e3} is because the 13 and 33 elements of U_ν are $\mp 1/\sqrt{2}$ respectively. This is due to the degeneracy of the 11 and 33 elements of the Majorana mass matrix M_R.
- If these diagonal elements of M_R are not degenerate, then the diagonalizing matrix is parametrized by an arbitrary mixing angle and U_{e3} will be non-zero.
- These perturbations, being diagonal, are again $Z_2 \times Z_2$ invariant.
- Since the diagonal entrees of the Majorana mass matrix are soft terms in the Lagrangian, these perturbations can also be introduced through the same route.

$Z_2 \times Z_2$ perturbation in Neutrino Sector

The perturbed Majorana mass matrix becomes

$$\begin{pmatrix} M + aM & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M - aM \end{pmatrix},$$

where a is a small real number characterizing the perturbation.
This matrix can be diagonalized by a rotation matrix of angle x where $\tan 2x = M'/aM$. We define a dimensionless parameter $\zeta = aM/M'$.
The modified PMNS matrix, under the combined perturbations in the charged lepton and neutrino sectors, is

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \cos x & 0 & -\sin x \\ 0 & 1 & 0 \\ \sin x & 0 & \cos x \end{pmatrix}$$

$Z_2 \times Z_2$ perturbation in both sectors

From the PMNS matrix, we obtain the expressions for the mixing angles upto second order in s and ζ.

$$\sin^2\theta_{13} = \frac{1}{3}(1-\cos 2\alpha \sin 2x) = \frac{\zeta^2}{6} + \frac{2}{3}s^2 - \frac{\zeta^2 s^2}{3},$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\alpha \sin 2x} = \frac{1}{3} + \frac{\zeta^2}{18} + \frac{2}{9}s^2 - \frac{\zeta^2 s^2}{27},$$

$$\sin^2\theta_{23} = \frac{2+\cos 2\alpha \sin 2x + \sqrt{3}\sin 2x \sin 2\alpha}{4+2\cos 2\alpha \sin 2x} = \frac{1}{2} + \frac{s}{\sqrt{3}} - \frac{\zeta^2 s}{3\sqrt{3}}.$$

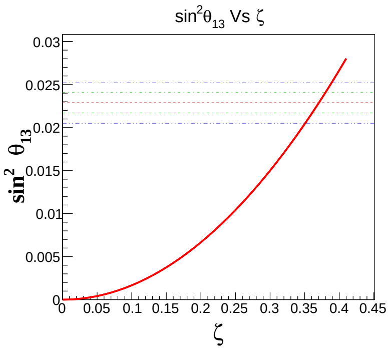
We can also calculate the Jarlskog invariant and obtain an expression for $\sin\delta_{CP}$ to the second order in s and ζ

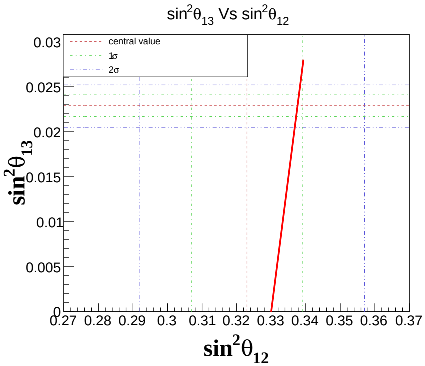
$$\sin\ \delta_{CP} = -\frac{\zeta}{\sqrt{4s^2+\zeta^2-\frac{16s^2\zeta^2}{3}}}.$$

$Z_2 \times Z_2$ perturbation in both sectors

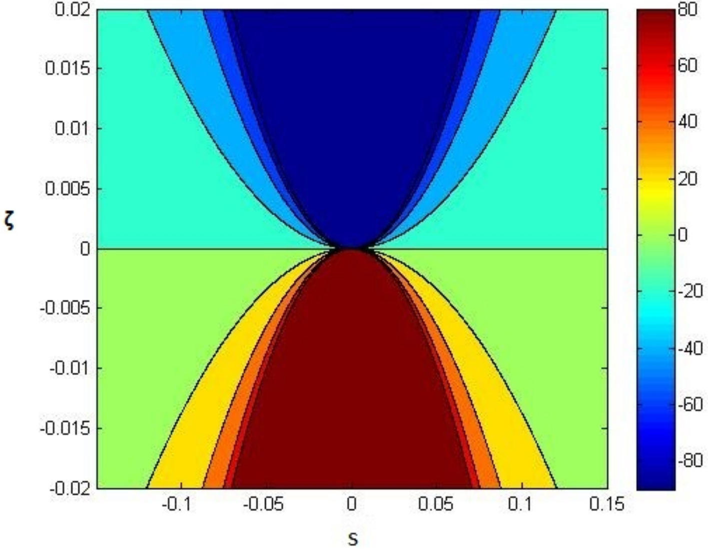
- $\sin\delta_{CP}$ vanishes when ζ is zero, *i.e.* when there is no perturbation in the neutrino sector.
- $\sin\delta_{CP} = \pm\pi/2$ when $s = 0$, *i.e.* when the perturbation is only in the neutrino sector, we have **maximal** CP violation.
- For perturbation purely in neutrino sector, we need $\zeta \approx 0.36$ to obtain $\sin^2\theta_{13}$ consistent with experiment.
- Eventhough ζ seems a bit large, the perturbation parameter in the Majorana mass matrix $a = \zeta M'/M$ will be quite small because $M' \ll M$.
- Arbitrary values of $\sin\delta_{CP}$ can be obtained by choosing appropriate values for s and ζ.

Variation of Mixing angles with ζ





δ_{CP} dependence on s and ζ



Conclusions

- We started with a neutrino mass model with A_4 symmetry, which predicts a tribimaximal form for the PMNS matrix.
- We considered the effect of introducing $Z_2 \times Z_2$ perturbations in the charged lepton sector and in the neutrino sector.
- With perturbation only in the charged lepton sector, we obtained realistic values for θ_{13} and for the other mixing angles but no CP violation.
- With perturbation only in the neutrino sector, we obtain maximal CP violation along with realistic values for θ_{13} and the other mixing angles.
- Any desired value of δ_{CP} can be obtained by adjusting the perturbations in the charged lepton and the neutrino sectors.


THANK
you !