# Re-examining $\sin 2\beta$ and $\Delta m_d$ from evolution of $B_d^0$ mesons with decoherence

S Uma Sankar

Department of Physics
Indian Institute of Technology Bombay
Mumbai, India

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# Plan of talk

1. Motivation
2. Determination of $\sin 2\beta$
3. Determination of $\Delta m_d$
4. An example of estimation of decoherence parameter
5. Conclusions

Based on the paper by
Ashutosh Kumar Alok, Subhashish Banerjee and S. Uma Sankar,
arXiv:1504.02893

# Motivation

- The time evolution of neutral mesons are used to measure a number of important parameters in flavor physics.
- In the time evolution of neutral meson systems, a perfect quantum coherence is usually assumed.
- However, any real system interacts with its environment and this interaction can lead to a loss of quantum coherence.
- Hence with the inclusion of decoherence effects, the measured values of some of the parameters can get masked.

We study the effect of decoherence on the important observables in the $B_d^0$ meson system, such as the CP violating parameter $\sin 2\beta$ and the $B_d^0 - \bar{B}_d^0$ mixing parameter $\Delta m_d$.

# Decoherence

Decoherence is an unavoidable phenomenon as any physical system is inherently open due to its inescapable interactions with a pervasive environment.

## Possible environment

- Environmental effects may arise at a fundamental level, such as the fluctuations in a quantum gravity space-time background.
- They may also arise due to the detector environment itself.

- The effect of environment on the neutral meson systems can be taken into account by using the ideas of open quantum systems.
- We use an effective desription which is phenomenological in nature. It is independent of the details of the actual dynamics between the system and environment.

Our phenomenological approach provides a universal framework for the study of quantum decoherence effects.

# Open time evolution of B mesons

- We are interested in the decays of $B^0$ and $\bar{B}^0$ mesons as well as $B^0 \leftrightarrow \bar{B}^0$ oscillations.
- To describe the time evolution of all these transitions, we need a basis of three states: $\left|B^0\right\rangle$, $\left|\bar{B}^0\right\rangle$ and $|0\rangle$, where $|0\rangle$ reprents a state with no $B$ meson and is required for describing the decays.
- We use the density matrix formalism to represent the time evolution of the $B^0$ system: $\rho_{B^0(\bar{B}^0)}(0)$ is the initial density matrix for the state which starts out as $B^0(\bar{B}^0)$.
- The time evolution of these matrices is governed by the Kraus operators $K_i(t)$ as $\rho(t) = \sum_i K_i(t)\rho(0)K_i^\dagger(t)$.

The Kraus operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment.

# Open time evolution of B mesons

## Time dependent density matrices

$$\frac{\rho_{B^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} a_{ch} + e^{-\lambda t} a_c & -a_{sh} - i e^{-\lambda t} a_s & 0 \\ -a_{sh} + i e^{-\lambda t} a_s & a_{ch} - e^{-\lambda t} a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix}$$

$$\frac{\rho_{\bar{B}^0}(t)}{\frac{1}{2}e^{-\Gamma t}} = \begin{pmatrix} a_{ch} - e^{-\lambda t} a_c & -a_{sh} + i e^{-\lambda t} a_s & 0 \\ -a_{sh} - i e^{-\lambda t} a_s & a_{ch} + e^{-\lambda t} a_c & 0 \\ 0 & 0 & 2(e^{\Gamma t} - a_{ch}) \end{pmatrix}$$

- $a_{ch} = \cosh\left(\frac{\Delta\Gamma\, t}{2}\right)$, $a_{sh} = \sinh\left(\frac{\Delta\Gamma\, t}{2}\right)$, $a_c = \cos(\Delta m\, t)$, $a_s = \sin(\Delta m\, t)$.
- $\Gamma = (\Gamma_L + \Gamma_H)/2$, $\Delta\Gamma = \Gamma_L - \Gamma_H$: $\Gamma_L$ and $\Gamma_H$ are the respective decay widths of the decay eigenstates $B^0_L$ and $B^0_H$.
- $\lambda$ is the decoherence parameter, due to the interaction between one-particle system and its environment.

# CP asymmetry in $B_d^0 \to J/\psi K_S$

- We study the effect of decoherence on the important observables in the $B_d^0$ meson system, such as the CP violating parameter $\sin 2\beta$ and the $B_d^0 - \bar{B}_d^0$ mixing parameter $\Delta m_d$.
- To keep expressions simple, CP violation in mixing is neglected.

**Hermitian operator describing decay $B^0 \to f$ and $\bar{B}^0 \to f$**

$$\mathcal{O}_f = \begin{pmatrix} |A_f|^2 & {A_f}^* \bar{A}_f & 0 \\ A_f \bar{A}_f^* & |\bar{A}_f|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- $A_f \equiv A(B^0 \to f)$ and $\bar{A}_f \equiv A(\bar{B}^0 \to f)$.
- The probability, $P_f(B^0/\bar{B}^0; t)$, of an initial $B^0/\bar{B}^0$ decaying into the state $f$ at time $t$ is given by $\mathrm{Tr}\left[\mathcal{O}_f \, \rho_{B^0(\bar{B}^0)(t)}\right]$.

# CP asymmetry of $B_d^0 \to J/\psi K_S$ decay

$$\mathcal{A}_{J/\psi K_S}(t) = \frac{P_{J/\psi K_S}(\bar{B}_d^0; t) - P_{J/\psi K_S}(B_d^0; t)}{P_{J/\psi K_S}(\bar{B}_d^0; t) + P_{J/\psi K_S}(B_d^0; t)}$$

$$= \frac{\left(|\lambda_f|^2 - 1\right) \cos\left(\Delta m_d t\right) + 2\mathrm{Im}(\lambda_f) \sin\left(\Delta m_d t\right)}{(1 + |\lambda_f|^2) \cosh\left(\frac{\Delta\Gamma_d t}{2}\right) - 2\mathrm{Re}(\lambda_f) \sinh\left(\frac{\Delta\Gamma_d t}{2}\right)} e^{-\lambda t}$$

- $\lambda_f = A(\bar{B}_d^0 \to J/\psi K_S))/A(B_d^0 \to J/\psi K_S)$.
- The standard expression for $\mathcal{A}_{J/\psi K_S}(t)$ is obtained by putting $\lambda = 0$.

With the approximations $\Delta\Gamma_d \approx 0$, $|\lambda_f| \approx 1$ and $\mathrm{Im}(\lambda_f) \approx \sin 2\beta$, we get

$$\mathcal{A}_{J/\psi K_S}(t) \approx \sin 2\beta \, e^{-\lambda t} \sin\left(\Delta m_d \, t\right)$$

The coefficient of $\sin\left(\Delta m_d \, t\right)$ is $\sin 2\beta \, e^{-\lambda t}$ and not $\sin 2\beta$!

The measurement of $\sin 2\beta$ is masked by the presence of decoherence.

# Determination of $\Delta m_d$

In order to determine $\sin 2\beta$, we need to know $\Delta m_d$.

Is the measurement of $\Delta m_d$ also affected by the presence of decoherence?

**LHCb, CDF** and **D0** experiments determine $\Delta m_d$ by measuring rates that a state that is pure $B_d^0$ at time $t = 0$, decays as either as $B_d^0$ or $\bar{B}_d^0$ as function of proper decay time.

In the presence of decoherence, the survival (oscillation) probability of initial $B_d^0$ meson to decay as $B_d^0(\bar{B}_d^0)$ at a proper decay time $t$ is:

**$B_d^0$ survival (oscillation) probability**

$$P_{\pm}(t,\lambda) = \frac{e^{-\Gamma t}}{2}\left[\cosh(\Delta\Gamma_d t/2) \pm e^{-\lambda t}\cos(\Delta m_d t)\right]$$

The positive (negative) sign implies $B_d^0$ meson decaying with the same (opposite) flavor as its production.

# Determination of $\Delta m_d$

$\Delta m_d$ is determined from the following time dependent asymmetry:

$$A_{\rm mix}(t,\lambda) = \frac{P_+(t,\lambda) - P_-(t,\lambda)}{P_+(t,\lambda) + P_-(t,\lambda)} = e^{-\lambda t}\frac{\cos(\Delta m_d t)}{\cosh(\Delta\Gamma_d t/2)}.$$

At **BaBar** and **Belle**, $\Delta m_d$ is determined by measuring the $B_d^0 \bar{B}_d^0$ decay probability $P_+(\Delta t)$ and the $B_d^0 B_d^0/\bar{B}_d^0 \bar{B}_d^0$ decay probability $P_-(\Delta t)$ for entangled neutral $B_d$ meson pair produced by the decay of the $\Upsilon(4S)$ resonance.

The expressions for $P_\pm(\Delta t)$ are the same as those of $P_\pm(t)$, except that the proper time $t$ is replaced by proper decay-time difference $\Delta t$ between the decays of the two neutral $B_d$ mesons.

# Determination of $\Delta m_d$

Determination of $\Delta m_d$ at LHCb, CDF, D0, Belle and BaBar experiments is masked by the presence of $\lambda$.

It can be shown that the time independent observables $r_d$ (measured by ARGUS and CLEO) and $\chi_d$ (measured by the LEP experiments), used to determine $\Delta m_d$, are also affected by the presence of decoherence.

The true value of $\Delta m_d$, along with $\Delta\Gamma_d$, can be determined by a three parameter ($\Delta m_d$, $\Delta\Gamma_d$, $\lambda$) fit to the time dependent mixing asymmetry $A_{\rm mix}(t, \lambda)$. This in turn will enable a determination of true value of $\sin 2\beta$.

# Estimation of $\lambda$: An example

We make an attempt to determine $\lambda$ $\Delta m_d$ and $\Delta\Gamma_d$ by performing a $\chi^2$ fit to $A_{\rm mix}(\Delta t, \lambda)$, using the efficiency corrected distributions given by Belle Collaboration:

TABLE I. Time-dependent asymmetry in $\Delta t$ bins, corrected for experimental effects, with statistical and systematic uncertainties. Contributions from event selection, background subtraction, wrong tag correction, and deconvolution are also shown.

| | | | | Systematic errors | | | | |
|---|---|---|---|---|---|---|---|---|
| $\Delta t$ bin | Window [ps] | $A$ and total error | Statistical error | Total | Event sel. | Bkgd sub. | Wrong tags | Deconvolution |
| 1 | 0.0–0.5 | $1.013 \pm 0.028$ | 0.020 | 0.019 | 0.005 | 0.006 | 0.010 | 0.014 |
| 2 | 0.5–1.0 | $0.916 \pm 0.022$ | 0.015 | 0.016 | 0.006 | 0.007 | 0.010 | 0.009 |
| 3 | 1.0–2.0 | $0.699 \pm 0.038$ | 0.029 | 0.024 | 0.013 | 0.005 | 0.009 | 0.017 |
| 4 | 2.0–3.0 | $0.339 \pm 0.056$ | 0.047 | 0.031 | 0.008 | 0.005 | 0.007 | 0.029 |
| 5 | 3.0–4.0 | $-0.136 \pm 0.075$ | 0.060 | 0.045 | 0.009 | 0.009 | 0.007 | 0.042 |
| 6 | 4.0–5.0 | $-0.634 \pm 0.084$ | 0.062 | 0.057 | 0.021 | 0.014 | 0.013 | 0.049 |
| 7 | 5.0–6.0 | $-0.961 \pm 0.077$ | 0.060 | 0.048 | 0.0120 | 0.017 | 0.012 | 0.038 |
| 8 | 6.0–7.0 | $-0.974 \pm 0.080$ | 0.060 | 0.053 | 0.034 | 0.025 | 0.020 | 0.025 |
| 9 | 7.0–9.0 | $-0.675 \pm 0.109$ | 0.092 | 0.058 | 0.041 | 0.027 | 0.022 | 0.022 |
| 10 | 9.0–13.0 | $0.089 \pm 0.193$ | 0.161 | 0.107 | 0.067 | 0.063 | 0.038 | 0.039 |
| 11 | 13.0–20.0 | $0.243 \pm 0.435$ | 0.240 | 0.363 | 0.145 | 0.226 | 0.080 | 0.231 |

# Estimation of $\lambda$: An example

First, the fit is done by assuming no decoherence, i.e., $\lambda = 0$.
In this case, we find $\Delta m_d = (0.489 \pm 0.010)\ \mathrm{ps}^{-1}$ and $\Delta\Gamma_d = (0.087 \pm 0.054)\ \mathrm{ps}^{-1}$ with a $\chi^2/d.o.f. = 8.42/9$.

We then redo the fit including decoherence.
This gives $\lambda = (-0.012 \pm 0.019)\ \mathrm{ps}^{-1}$, $\Delta m_d = 0.490 \pm 0.010\ \mathrm{ps}^{-1}$ and $\Delta\Gamma_d = (0.144 \pm 0.088)\ \mathrm{ps}^{-1}$ with a $\chi^2/d.o.f. = 8.02/8$.

- Decoherence parameter $\lambda$ is very loosely bounded.
- The upper limit on $\lambda$ is $0.018\ \mathrm{ps}^{-1}$ at 90% C.L.
- $\Delta m_d$ is numerically unaffected.

Given the wealth of data coming from LHCb and expected from the KEK Super B factory, a clear picture is expected to emerge.

# Comment on Approximations made

- The decoherence is expected to emerge from a scale much finer than that of the flavor physics.
- Hence for an accurate determination, one should include all the known effects, such as CP violation in mixing, which are usually neglected in the extraction of $\sin 2\beta$.

The theoretical precision for the extraction of CP violating phase $\sin 2\beta$ from the CP asymmetry of $B_d^0 \to J/\psi K_S$ decay is limited by contributions from doubly Cabibbo-suppressed penguin topologies which cannot be calculated in a reliable way within QCD.
However, $B_s^0 \to J/\psi K_S$ is related to $B_d^0 \to J/\psi K_S$ through the $U$-spin symmetry of strong interactions and it offers a tool to control the penguin effects.

# Decoherence in $B_s$ systems

- The present analysis can easily be extended to the $B_s^0$ system as well.
- The expression for the time dependent CP asymmetry in the mode $B_s^0 \to J/\psi\phi$ will be a function of four parameters: $\lambda$, $\sin 2\beta_s$, $\Delta m_s$ and $\Delta\Gamma_s$.
- The time dependent mixing asymmetry will determine $\lambda$, $\Delta m_s$ and $\Delta\Gamma_s$.
- These two time-dependent asymmetries should be re-analysed using a four parameter fit for a clean determination of $\sin 2\beta_s$, $\Delta m_s$, $\Delta\Gamma_s$ and $\lambda$.

Like $\sin 2\beta_d$, the extraction of $\sin 2\beta_s$ from time dependent CP asymmetry in the mode $B_s^0 \to J/\psi\phi$ is restricted due to penguin pollution. The penguin contribution to $B_s^0 \to J/\psi\phi$ can be estimated using decays $B_d^0 \to J/\psi\rho$ and $B_s^0 \to J/\psi\bar{K}^*$.

# Conclusions

- In this work, we have studied the effect of decoherence on two important observables $\sin 2\beta$ and $\Delta m_d$ in a neutral meson system.
- We find that the asymmetries which determine these quantities are also functions of the decoherence parameter $\lambda$.
- Hence it is imperative to measure $\lambda$ for a clean determination of these quantities.
- We suggest a re-analysis of the data on the above asymmetries for an accurate measurement of all the three quantities $\lambda$, $\sin 2\beta$ and $\Delta m_d$.
- The present analysis can easily be extended to the $B_s^0$ system as well.


Thank You